**MERCHANT SHIPPING SECRETARIAT**
**GOVERNMENT OF SRI LANKA**
**CERTIFICATE OF COMPETENCY EXAMINATION**

GRADE : OFFICER IN CHARGE OF A NAVIGATIONAL WATCH ON SHIPS OF 500
GT OR MORE (UNLIMITED)
SUBJECT : MATHEMATICS
DATE : 30th April 2024      Time : 0900 to 1200 hrs

Time allowed **THREE hours**      Total marks : 120
**ANSWER ANY SIX (06) QUESTIONS**      Pass marks : 50%

Formulae and all intermediate steps taken in reaching your answer should be clearly shown. You may draw sketches wherever required. Electronic devices capable of storing and retrieving are **not** allowed.

1. a) Find numerical value of the followings: (06 marks)

     i. $\dfrac{x^2 + x^{5/2}}{x^{-1/2}}$      ii. $\log_{1/2} 16$

   b) Find the value of $3\log_{10} 2 - \log_{10} 12 + \log_{10} 3 + \left(\dfrac{1}{2}\right)\log_{10} 25$ (06 marks)

   c) solve $\log_2(3^{2x-2} + 7) = 2 + \log_2(3^{x-1} + 1)$ (08 marks)

2. In the binomial expansion of $\left(x - \dfrac{1}{x}\right)^6$, Find

   a) the coefficient of $x^4$ (08 marks)
   b) the term independent of x. (08 marks)
   c) Show that there is no $x^3$ term in above binomial expansion. (04 marks)

3. a) Without using tables or calculator, evaluate (06 marks)
     i. $\sin 15^0 \cos 15^0$      ii. $\cos^2 22.5^0 - \sin^2 22.5^0$
   b) Prove that (08 marks)

     i. $\dfrac{\cos x - \sin x}{\cos x + \sin x} = \dfrac{1 - \sin 2x}{\cos 2x}$      ii. $\sin 3x = 3\sin x - 4\sin^3 x$

   c) Given that $0 \le x \le 2\pi$, solve the equation $\sqrt{3}\sin x + \cos x = 1$ (06 marks)

4. a) Solve the quadratic equation $2x^2 - 3x - 3 = 0$. (06 marks)
   b) Determine the nature of roots of quadratic equation $nx^2 - mx - n = 0$. (08 marks)
   c) Determine the range of value of **k** for which the quadratic equation
     $kx^2 + 6(k-2)x + 3(k+2) = 0$ has real distinct roots. (06 marks)

5. The function is given by $y = x^2 + 2x - 8$

    a) Find the coordinate of turning points.     (06 marks)

    b) Find the coordinate of x and y intercepts.     (08 marks)

    c) Sketch the graph of $y = x^2 + 2x - 8$.     (06 marks)

6. a) Fit a straight line to the following data regarding x as independent variable. (06 marks)

| x | 1 | 2 | 3 | 4 | 6 | 8 |
|---|---|---|---|---|---|---|
| y | 2.4 | 3 | 3.6 | 4 | 5 | 6 |

    b) The diagram represents a swimming pool.
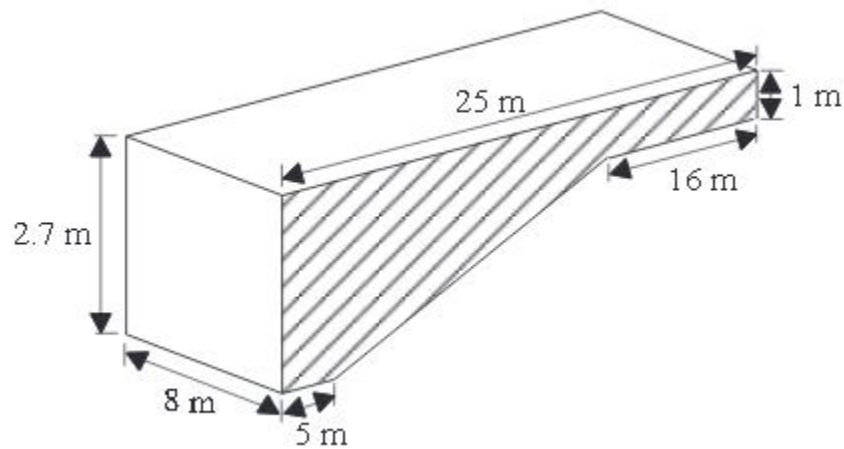


    Calculate the area of the shaded cross section.     (08 marks)

    c) i.     If the swimming pool is completely filled with water, calculate the volume of water in the pool.     (03 marks)

    ii.     If 64 m$^3$ leaks out of the pool, calculate the distance by which the water level falls.     (03 marks)

7.  a)  Find the centre and radius of the circle $x^2 + y^2 - 2x + 4y = 8$ (08 marks)

    b)  A rough sketch of cross sectional view of an oil tank is illustrated in the figure below and the height of the tank has been measured by the intervals of **50 cm** distance.
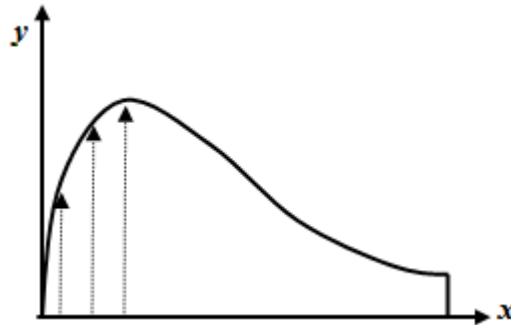


| x(m) | 0 | 0.5 | 1.0 | 1.5 | 2.0 | 2.5 | 3.0 | 3.5 | 4.0 | 4.5 | 5.0 | 5.5 | 6.0 |
|------|---|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| y(m) | 0 | 2.2 | 4.3 | 5.1 | 4.9 | 4.6 | 4.1 | 3.8 | 3.2 | 2.0 | 1.70 | 1.68 | 1.68 |

Determine the area of the layer (Hint : Use the Simpson's 1/3 Rule) (12 marks)

8.  a)  Divide $2x^3 - 5x^2 + 7x + 3$ by $x - 2$ (06 marks)

    b)  Factorize $x^3 - 7x^2 + 7x + 15$ using factor theorem. (06 marks)

    c)  Solve the inequality $x^2 - 3x \leq 10$. (08 marks)

End.