MECHANICAL ENGINEERING

A Series of Textbooks and Reference Books

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GEAR DRIVE SYSTEMS

DESIGN AND APPLICATION

Peter Lynwander
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PREFACE

Gear drives are critical components of mechanical systems used in such diverse industries as turbo-machinery, process, refinery, steel, construction, mining, and marine. In all these fields there is a continuing trend toward higher reliability and improved technology in mechanical components. Higher reliability is desired to reduce downtime. In many applications, the cost of one day's lost production due to a gearbox malfunction far exceeds the initial cost of the unit; therefore, in critical installations there is a strong emphasis on conservative design and quality manufacture. In addition to achieving high reliability, mechanical systems must be increasingly efficient to conserve energy. Gear manufacturers are constantly refining their analytical, design, and manufacturing techniques to take advantage of new technologies and provide reliable, efficient gearboxes at minimum cost.

The purpose of this book is to present practical gearbox design and application information to individuals responsible for the specification and operation of mechanical systems incorporating gear drives. Sufficient theoretical information is included to enable the engineer interested in gear analysis and design to understand how gear units are rated and detail gear tooth geometry is defined. The major emphasis is on parallel shaft and planetary units using spur and helical gearing.

In addition to basic data on gear design and manufacture, such subjects as installation, operation, maintenance, troubleshooting, failure analysis, and economics are covered. Material on lubrication systems, bearings, couplings, and seals is presented in order to cover all aspects of gear system operation.

Several new trends in the gear industry, due in part to the emphasis on energy conservation, are discussed.

1. As mechanical equipment such as pumps, motors, compressors, turbines, etc. are designed for higher efficiencies, rotating speeds are increased and, therefore, higher speed transmissions are required. High speed gearing characteristics are featured throughout the book.
2. Also, as a result of energy consciousness, there is a tendency to package smaller mechanical systems; therefore, there is a trend developing in the United States toward the use of planetary gear units which are far more compact than parallel shaft designs. Included in the book is a section on planetary gear design and application.
3. In order to achieve the highest load carrying capability in a minimum envelope, case hardened and ground gear tooth designs are finding wide application. This technology is covered in the book.
4. The book attempts to take a systems approach to gearbox application. It has become apparent that gear units, when incorporated into a system of rotating machinery, are susceptible to a variety of problems. All characteristics of the drive system from the driver to the driven equipment, including the lubrication system and accessories, can influence gearbox operation and must be considered in the specification, installation, operation, and maintenance of the unit.

Throughout the book, standards and practices developed by the American Gear Manufacturers Association are referred to. Successful selection, rating, and operation of gearboxes can be accomplished by the use of AGMA publications and the gear designer and user should be familiar with the Standards system. The AGMA is located at 1901 North Fort Myer Drive, Arlington, Virginia 22209.

I would like to thank Mr. Alvin Meyer and Mr. Alan Swirnow for their assistance and comments. I am also indebted to American Lohmann Corporation, Hillside, New Jersey for its support.

Peter Lynwander

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GEAR DRIVE SYSTEMS

DESIGN AND APPLICATION

1

TYPES OF GEAR DRIVES: ARRANGEMENTS, TOOTH FORMS

The function of a gearbox is to transmit rotational motion from a driving prime mover to a driven machine. The driving and driven equipment may operate at different speeds, requiring a speed-increasing or speed-decreasing unit. The gearbox therefore allows both machines to operate at their most efficient speeds. Gearboxes are also used to change the sense of rotation or bridge an angle between driving and driven machinery.

The gearbox configuration chosen for a given application is most strongly influenced by three parameters:

Physical arrangement of the machinery
Ratio required between input and output speeds
Torque loading (combination of horsepower and speed)

Other factors that must be considered when specifying a gear drive are:

Efficiency
Space and weight limitations
Physical environment

PHYSICAL ARRANGEMENT

The location of the driving and driven equipment in the mechanical system defines the input and output shaft geometrical relationship. Shaft arrangements can be parallel offset, concentric, right angle, or skewed as shown in Figure 1.1. The material presented in this book focuses on parallel offset and concentric designs.

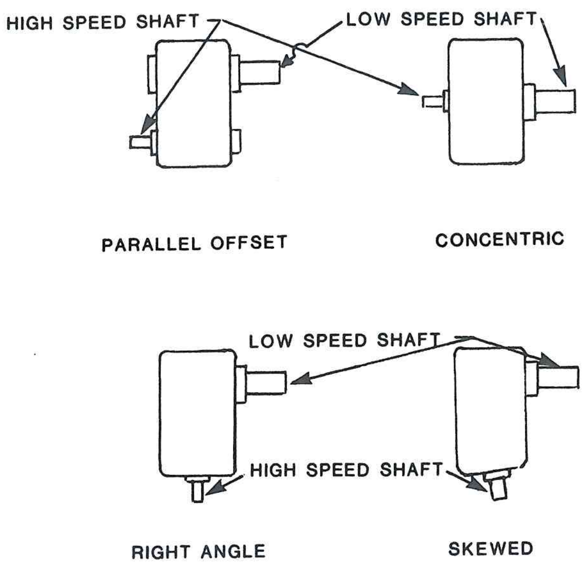

Figure 1.1 Gearbox shaft arrangements.

In the majority of parallel offset units in use, the input and output shafts are horizontally offset; however, vertical offsets are used and any orientation of input to output shaft is possible. Figure 1.2 illustrates a typical horizontally offset parallel shaft gearbox and Figure 1.3 presents a cutaway view of such a unit. In this case there is one input shaft and one output shaft located on opposite sides of the unit. There are many different options available as far as the input and output shaft extensions are concerned. Figure 1.4 shows the various possibilities and presents a system for defining the extensions desired on a gearbox. There may be two inputs driving a single output, such as dual turbines powering a large generator, or two outputs with a single input, such as an electric motor driving a two-stage compressor. Often shaft extensions are used to drive accessories such as pumps or starters.

The minimum amount of offset required is determined by gear tooth stress considerations. The offset of a gearbox incorporating a single mesh, as shown in Figure 1.3, is the sum of the pitch radii of the pinion and gear, otherwise known as the center distance. The pitch radii must be sufficiently large to transmit the system load. An offset greater than the minimum may be required to provide enough space for the machinery incorporated in the system. Figure 1.5 illustrates an accessory drive where the input and output shafts are offset through two meshes to separate the machinery located at these shafts.

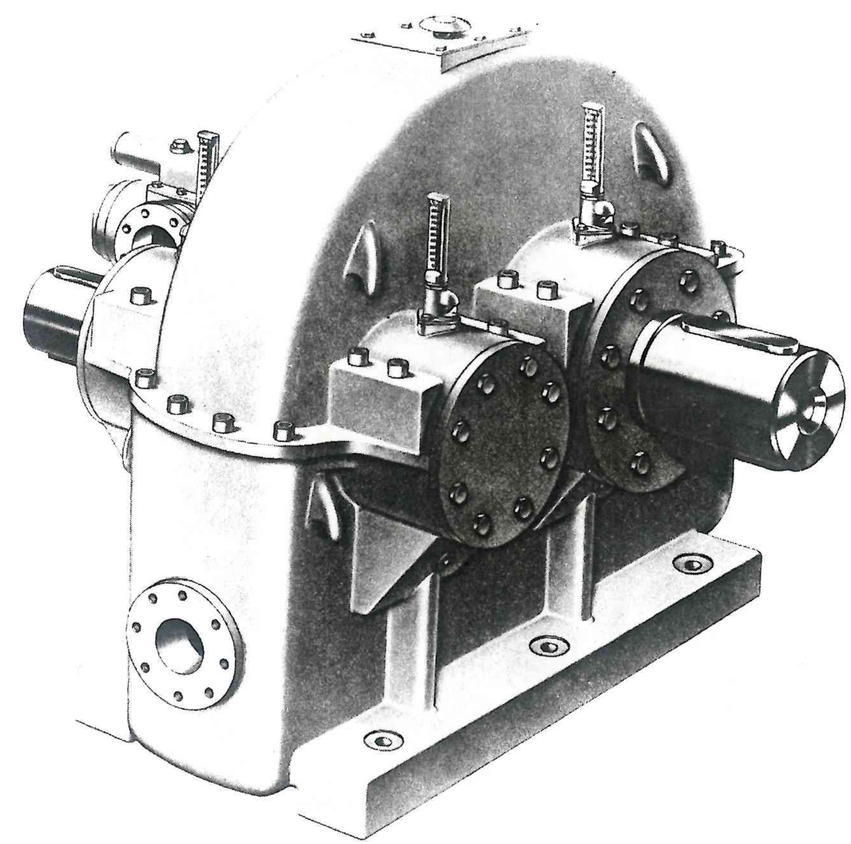

Figure 1.2 Parallel offset gearbox. (Courtesy of American Lohmann Corporation, Hillside, N.J.)

Figure 1.6 illustrates a gearbox with concentric input and output shafts. The driving and driven machinery will therefore be in line. Planetary gearing (described in the next section) has concentric shafts and is used to achieve high ratios in minimum space. It is also possible to package parallel shaft gearing such that the input and output shafts are in line when such a configuration is desired. Figure 1.7 presents external and internal views of a right-angle gear drive.

The gearboxes in Figures 1.3, 1.6, and 1.7 are foot mounted; that is, they are meant to be bolted to a horizontal base through a flange at the bottom of

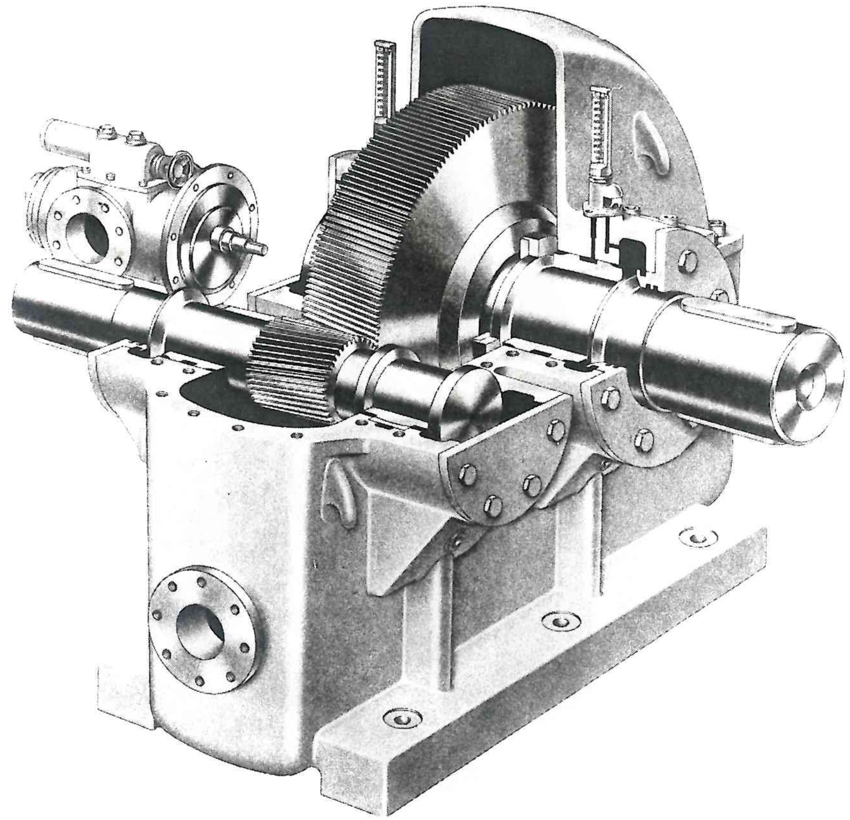

Figure 1.3 Parallel offset gearbox sectional view. (Courtesy of American Lohmann Corporation, Hillside, N.J.)

the gear casing. Although this is the most common design, gearboxes can be mounted in many other configurations and operate in attitudes other than horizontal. Figure 1.8 illustrates a flange-mounted unit. Such a gearbox can be operated horizontally or be vertically mounted on a horizontal base. Vertically operating gearbox designs must have special lubrication provisions to provide lubricant to the upper components in the unit and seal the lower end from oil leakage. Figure 1.9 shows yet another mounting configuration. This unit is shaft mounted with a support arm that is fixed to ground to react the gearbox housing torque.

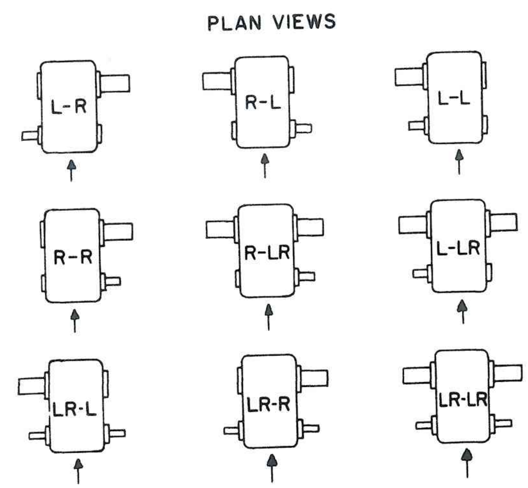

Figure 1.4 Definition of shaft extensions. Parallel shaft-helical and herringbone gear reducers; single, double, and triple reduction. Code; L = left; R = right; arrows indicate line of sight to determine direction of shaft extensions; letters preceding the hyphen refer to number and direction of highspeed shaft extensions; letters following the hyphen refer to number and direction of low speed shaft extensions. (From Ref. 1.)

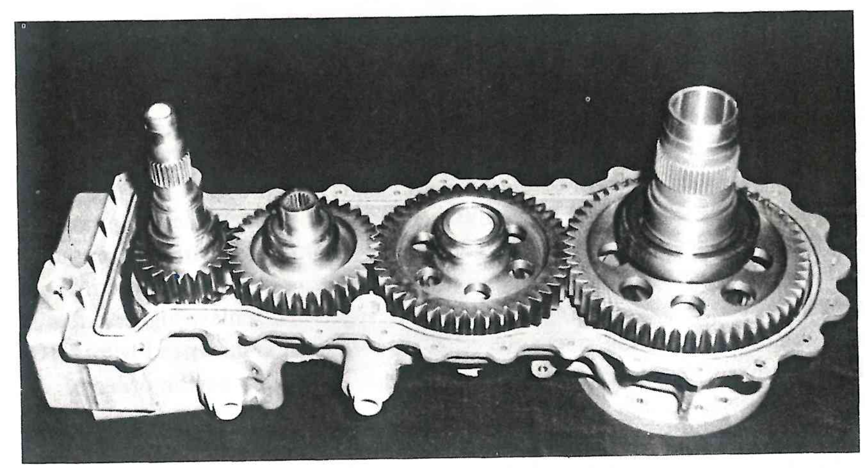

Figure 1.5 Parallel offset accessory drive gearbox. (Courtesy of American Lohmann Corporation, Hillside, N.J.)

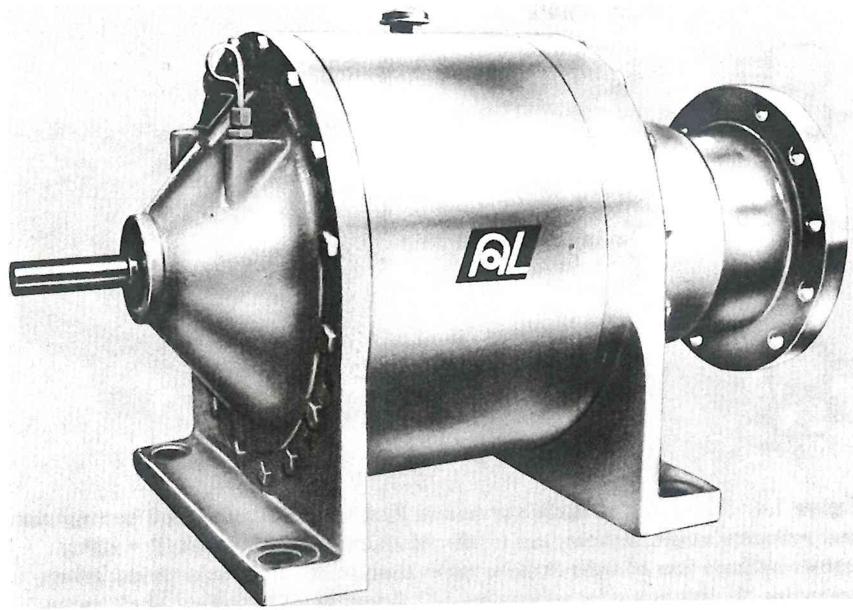

Figure 1.6 Gearbox with concentric input and output shafts. (Courtesy of American Lohmann Corporation, Hillside, N.J.)

GEAR RATIO

There is no limit to the reduction or speed increasing ratio that can be achieved using gearing; however, for high ratios the arrangement of the components can be quite complex. In a simple gear mesh a maximum ratio in the order of 8:1 to 10:1 can be achieved. The amount of speed reduction or increase is simply the ratio of the pitch diameter of the larger gear to the smaller gear. The number of teeth in a gear pair is related to the pitch diameters, so the speed ratio can also be calculated by dividing the larger number of teeth by the smaller. The smaller gear is often called the pinion. To attain a ratio of 10:1, therefore, the gear must be 10 times larger than the pinion and there usually are stress or geometrical limitations on the pinion when this ratio is exceeded. To achieve higher ratios with parallel shaft gearing, stages of meshes are combined as shown in Figure 1.10. This unit has three stages of reduction and achieves ratios on the order of 100:1.

An efficient method of achieving high reduction ratios in minimum space is the use of planetary gearing. This design, completely described in Chapter 9, is illustrated in Figure 1.11. The high-speed sun gear meshes with a number of

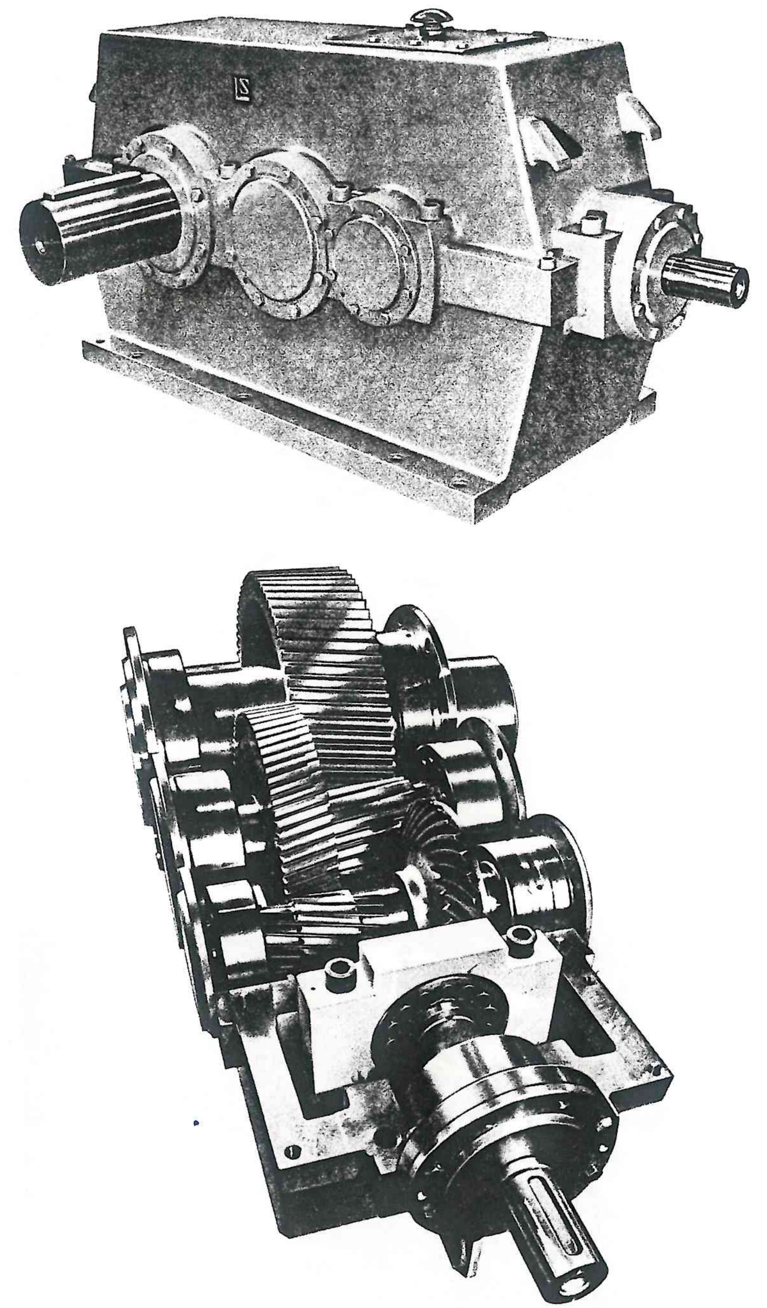

Figure 1.7 Right-angle gear drive. (Courtesy of American Lohmann Corporation, Hillside, N.J.)

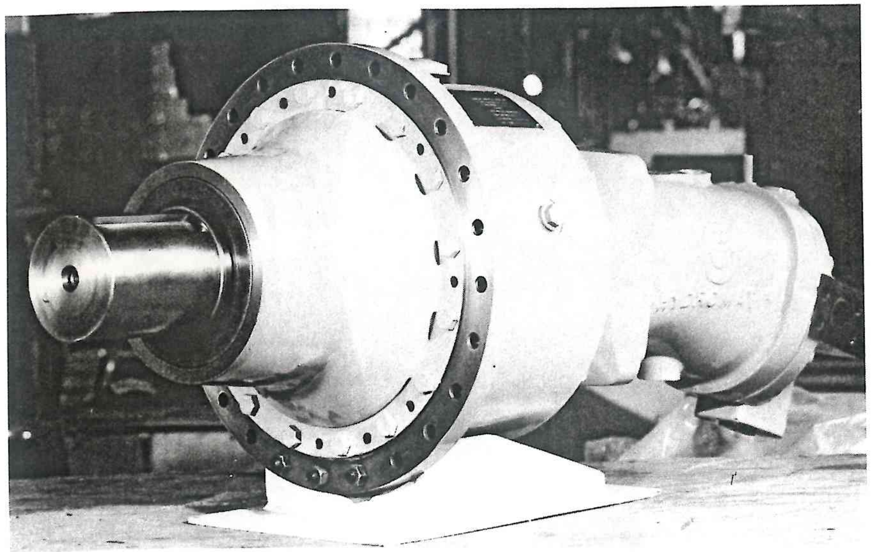

Figure 1.8 Flange-mounted gear unit. (Courtesy of American Lohmann Corporation, Hillside, N.J.)

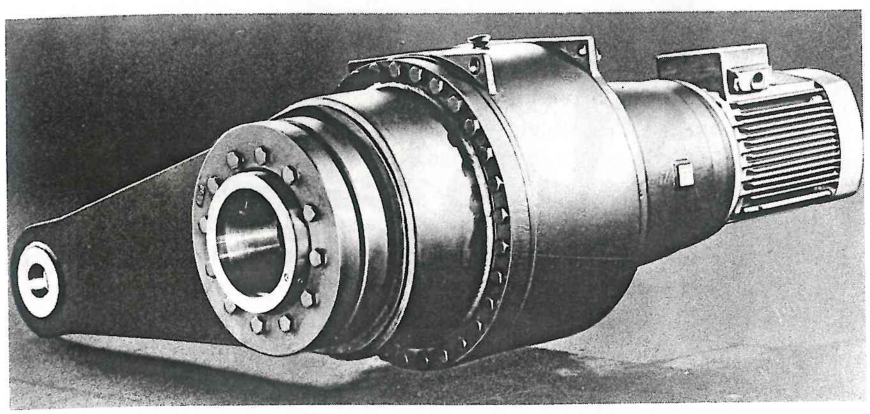

Figure 1.9 Shaft-mounted gearbox. (Courtesy of American Lohmann Corporation, Hillside, N.J.)

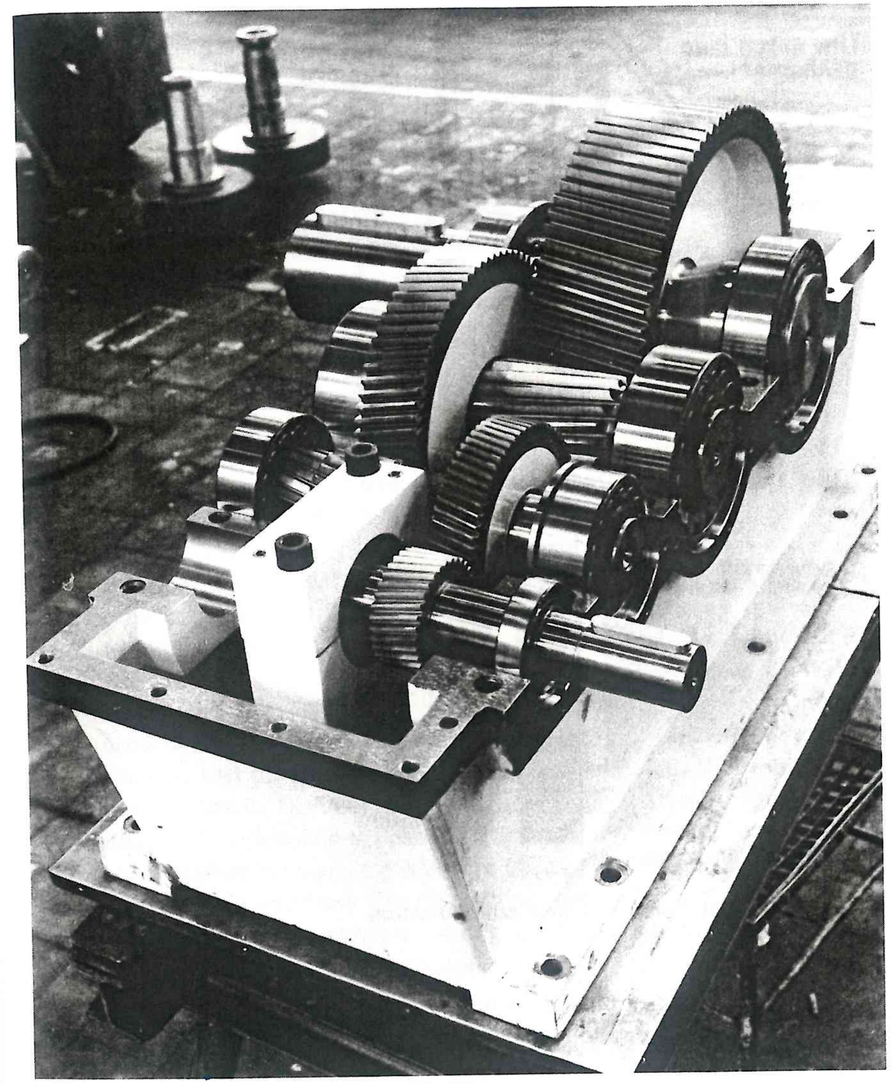

Figure 1.10 Multi-stage parallel shaft gearbox. (Courtesy of American Lohmann Corporation, Hillside, N.J.)

planets, usually three, which in turn mesh with a ring gear. The ring gear has internal teeth. Either the ring gear or the planet carrier rotate at the low speed of the gear set. Occasionally, all three members are connected to rotating equipment. When the low-speed shaft is either the ring gear or the planet carrier, the ratios in a planetary gearset are:

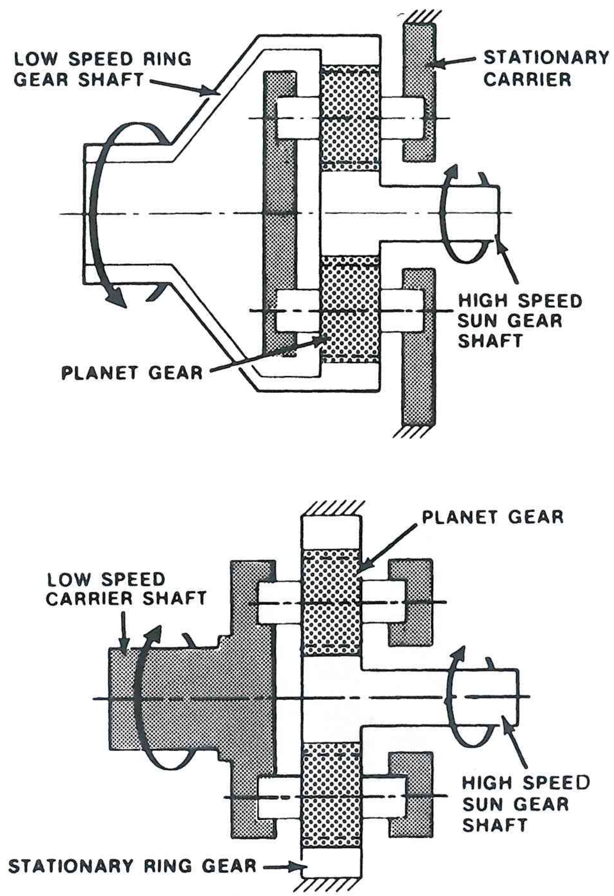

Figure 1.11 Basic planetary gear configurations.

$$\text{Ratio} = \frac{\text{ring gear pitch diameter}}{\text{sun gear pitch diameter}} \qquad \text{for rotating ring gear}$$

$$\text{Ratio} = 1 + \frac{\text{ring gear pitch diameter}}{\text{sun gear pitch diameter}} \qquad \text{for rotating planet carrier}$$

Because of the multiple load path of planetary gearing the horsepower transmitted is divided between several planet meshes and the gear size can be reduced significantly compared to parallel shaft designs. Planetary stages can be linked together to achieve high ratios, as shown in Figure 1.12. This is a three-stage planetary gear with a ratio of 630:1. The first-stage planet carrier drives the second-stage sun gear and the second-stage carrier drives the third-stage sun gear.

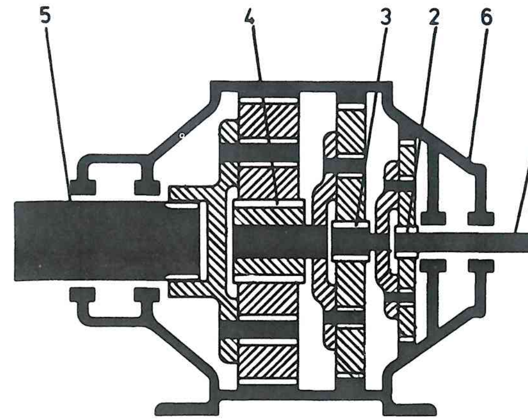

Figure 1.12 Multistage planetary gearbox. High-speed shaft, 1; first planetary stage, 2; second stage, 3; third stage, 4; low-speed shaft, 5; gear housing, 6.

In theory, any parallel shaft or planetary gearbox can be used either as a speed reducer or increaser. There may be details within a gearbox, however, that require modification if such a changeover is made. The same holds true if it is desired to reverse the direction of rotation for which the gearbox was initially designed. For instance, one side of the teeth may have been favored in the finishing process when the initial design was manufactured. If the gearbox is used in such a manner that the initially unloaded face is now loaded, poor tooth performance may result. The direction of rotation of the input shaft with respect to the output shaft depends on the gear design chosen. For a simple parallel shaft gear mesh the sense of rotation will change through the mesh. A planetary arrangement with a stationary ring gear will not change the sense of rotation between input and output, while a rotating ring gear will turn in the opposite sense compared to the sun gear's rotation.

TORQUE LOADING

The size of gearbox required for a given application is dependent primarily on how large the gear pitch diameters and face widths are. These dimensions are determined on the basis of tooth stresses which are imposed by the transmitted tooth load. The tooth load is simply the torque on a given gear divided by the gear pitch radius:

$$\text{Tooth load (lb)} = \frac{\text{torque (in.-lb)}}{\text{pitch radius (in.)}}$$

Torque is calculated from the horsepower transmitted and the speed of the rotating component in question:

$$\text{Input torque (in.-lb)} = \frac{63{,}025\ (\text{hp})}{\text{input rpm}}$$

$$\text{Output torque (in.-lb)} = \frac{63{,}025\ (\text{hp})}{\text{output rpm}}$$

When designing a gearset one cannot consider torque alone. The operating speed of the gears has a significant effect on the design definition. As an illustration of this point, consider a high-speed unit transmitting 2000 hp at 20,000 rpm input. The input torque would be the same as a low-speed unit operating at 2000 rpm input with a transmitted horsepower of 200. On a simple stress basis, if the ratio of both gearboxes were the same, the same gearbox could be used for both applications; however, the high-speed design must differ from the low-speed design in the following respects:

At high speeds, component geometry discrepancies such as tooth spacing error, shafting unbalance, and so on, generate significant dynamic loading, and these dynamic effects must be taken into account in the design process. Also, the components experience high numbers of load cycles and are more prone than low-speed units to fatigue failures. For all of these reasons, high-speed components must be of high accuracy to minimize dynamic problems.

Heat generation within the unit is proportional to speed; therefore, high-speed units usually require pressure jet lubrication systems and external cooling systems. Low-speed units often operate with integral splash lubrication, the heat being dissipated through the gear casing.

The bearing design is strongly dependent on shaft speeds. Low-speed units generally incorporate antifriction bearings, while high-speed industrial gearboxes typically use journal bearing designs.

There is no clear demarcation between low-speed and high-speed gearing. Units with several gear meshes may have some of each. An arbitrary definition sometimes used is that units with pinion speeds exceeding 3600 rpm or pitch line velocities exceeding 5000 fpm are considered high speed [1].

Pitch line velocity is a measure of the peripheral speed of a gear:

$$\text{Pitch line velocity (fpm)} = \frac{\pi\ (\text{pitch diameter, in.})\ (\text{rpm})}{12}$$

The pitch line velocity of a gear is a better index of speed than is rotational velocity, since a large gear operating at a relatively low rpm may experience the same velocity effects as a small gear operating at high rpm. Standard high-speed gear units operate at pitch line velocities up to approximately 20,000 fpm.

Applications exceeding this speed must be considered special and exceptional care must be taken in their design and manufacture. Pitch line velocities of 40,000 fpm have been attained in practice.

Parallel offset or concentric shaft gearboxes incorporate gears with spur, single helical, or double helical tooth forms. The face of a spur gear is parallel to the axis of rotation, whereas a helical gear tooth face is at an angle, as shown in Figure 1.13. The figure illustrates that helical gears have an overlap in the axial direction, which results in the following advantages:

Helical gears have more face width in contact than do spur gears of the same size; therefore, they have greater load-carrying capability.

With conventional spur gearing the load is transmitted by either one or two teeth at any instant; thus the elastic flexibility is continuously changing as load is transferred from single-tooth to double-tooth contact and back. With helical gearing the load is shared between sufficient teeth to allow a smoother transference and a more constant elastic flexibility; therefore, helical gearing generates less noise and vibration than spur gearing.

The disadvantage of helical gearing in relation to spur gearing is that axial thrust is generated in a helical gear, which necessitates the incorporation of a thrust bearing on each helical gear shaft.

To take advantage of the helical gearing benefits described above, yet not generate axial thrust loads, double helical gearing is used (Figure 1.14). The two halves generate opposite thrust loads, which cancel out. When the two helices are cut adjacent to one another with no gap between, the gearing is termed herringbone. Because helical gear thrust is proportional to the tangent of the helix angle, single helical gears tend to have lower helix angles than do double helical designs, where the thrust loads cancel. Typical single helical helix angles are 6 to 15°. Double helical gearsets have helix angles of up to 35°.

Another advantage of double helical gears is that the ratio of face width to pitch diameter in each half can be held to reasonable limits. When the face widths become longer than the pitch diameters in spur or single helical gearing it is difficult to achieve complete tooth contact since thermal distortion, load deflections, and manufacturing errors tend to load the gear teeth unevenly. A double helical gear with a face width/pitch diameter ratio of 1 will have twice the face width of a spur or single helical gear with the same L/D ratio and therefore greater load-carrying capability.

Double helical gearing has two disadvantages. Because the two halves of each gear cannot be perfectly matched, one member of the gearset must be free to float axially. This gear will be continually shifting to achieve axial force equilibrium since the thrust loads of each half will rarely cancel exactly. This shifting can lead to detrimental axial vibrations if tooth geometry errors are excessive. Another potential problem with double helical gearing is that external

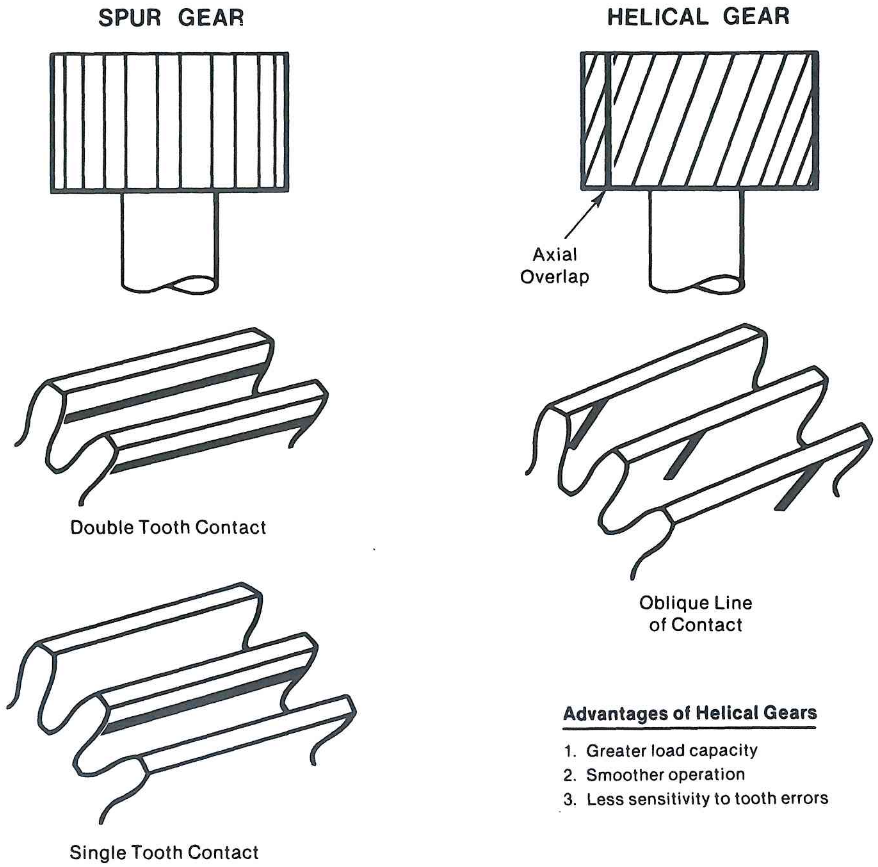

Figure 1.13 Comparison of spur and helical gear teeth.

thrust loads will tend to overload one helix. For instance, if a double helical gear is attached to a gear tooth type of coupling and the coupling locks up axially due to tooth friction, axial loads transmitted through the coupling will be reacted by the teeth of one-half of the gearset. With single helical gears an external axial load will either add to or subtract from the gear tooth load and be reacted by the thrust bearing.

Gear metallurgy, although not mentioned heretofore, is one of the most significant factors in determining gearbox size, since the strength of a gear tooth is proportional to the hardness of the steel. Most gears are in the hardness ranges of approximately Rc 30 to 38 or Rc 55 to 64. The region from Rc 30 to 38 is usually termed "through-hardened," while the range Rc 55 to 64 is almost always "surface-hardened," where the tooth has a hard surface case and a softer inner core. Through-hardened gears are cut by such processes as hobbing,

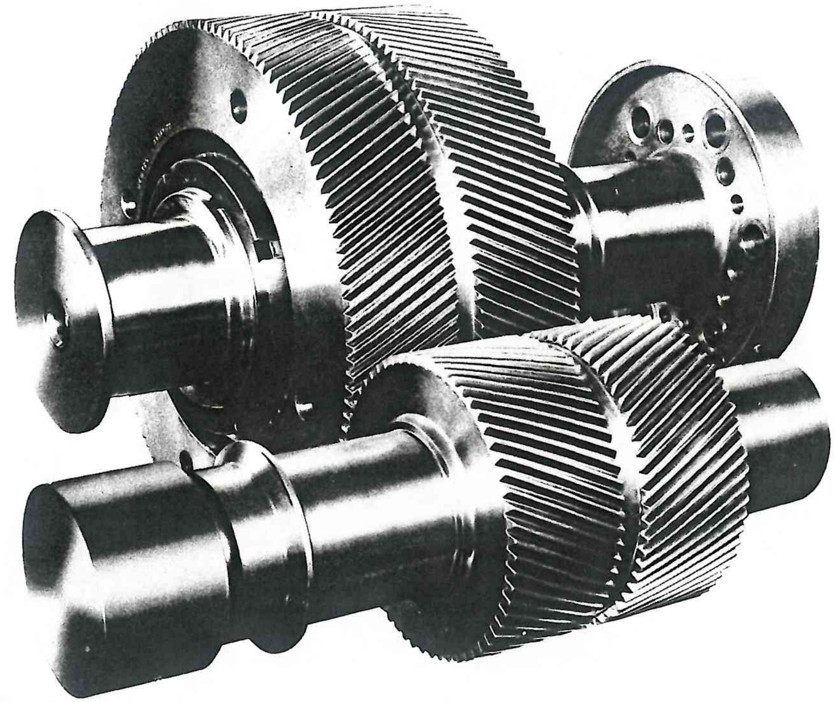

Figure 1.14 Double helical gearing.

shaping, and shaving. Surface-hardened gears are cut and then hardened. They may be used in this state, but the more accurate surface-hardened gears are ground after heat treatment. Spur, single helical, and double helical gearing may be produced by any of the methods noted above. Generally, double helical gearing is through-hardened and cut. It is possible to harden and grind double helical gearing; however, to grind a one-piece double helical gear a large central gap is required between the two helices to allow runout of the grinding wheel. Gears can be ground in halves and then assembled, but this presents serious alignment and attachment problems.

To achieve minimum envelope and maximum reliability, the latest technology utilizes single helical, hardened, and precision ground gearing. With single helical gears the thrust load axially locates the gear shaft against the thrust bearing. Bearing design has progressed to the point where thrust loads are routinely handled either by hydrodynamic tapered land or tilting pad configurations or an antifriction thrust bearings. Because case-hardened gears have maximum

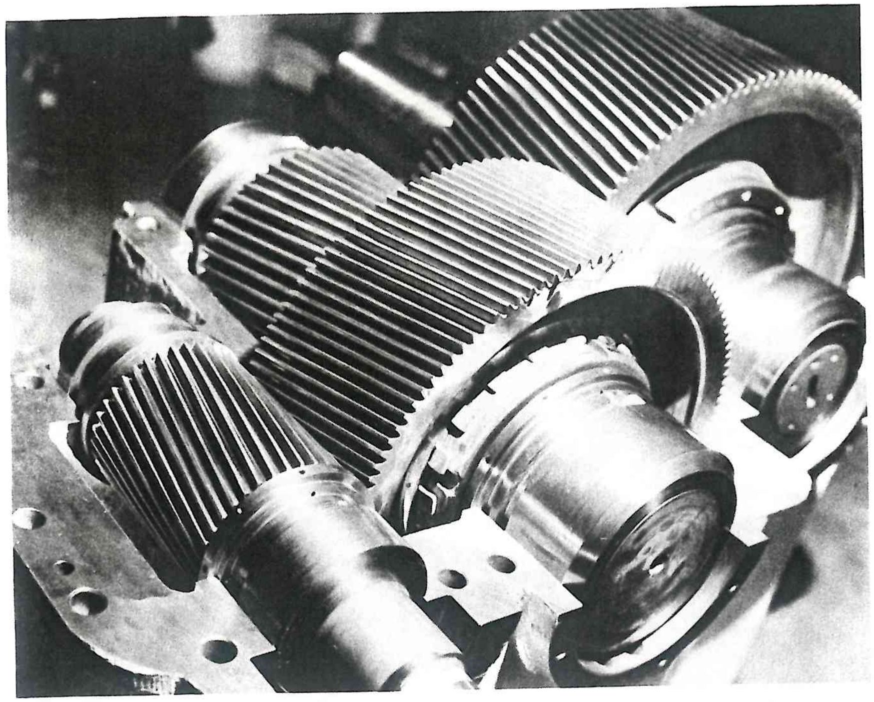

Figure 1.15 High-speed single helical hardened and ground gearset. (Courtesy of American Lohmann Corporation, Hillside, N.J.)

load-carrying capacity, gear size can be minimized; therefore, the ratio of face width to diameter of a single helical gear can be held to reasonable limits. Pitch line velocities are minimized, reducing dynamic effects. Also, the bearing span with single helical gears is short, resulting in lower elastic deflection. Figure 1.15 illustrates a generator drive gearbox with two stages of single helical gearing. This unit transmits 4500 hp at an input speed of 14,500 rpm. The high-speed mesh pitch line velocity is 18,000 fpm. Gearbox weight is 3500 lb.

A single helical hardened and ground gearset can reduce by up to one-half the envelope and weight of a through-hardened double helical gearbox with equivalent capacity. The inherent precision of the grinding process results in accurate tooth geometry, leading to minimum noise and vibration.

EFFICIENCY

Gearbox efficiency is a much discussed subject, but accurate values are very difficult to determine. Analytical estimates must be confirmed by testing to gain

a degree of confidence in the procedure. With good design and manufacturing practice, efficiencies of 99% per mesh and better are possible. Often, lubrication system development is required to attain the highest efficiency potential.

Power losses in a gearbox are divided between friction losses at the gear and bearing contacts and windage losses as the rotating components churn the oil and air. In high-speed units the churning losses may exceed the friction losses; therefore, the type and amount of lubricant, and its introduction and evacuation, are critical in terms of efficiency. Journal bearings require significantly more oil flow than do antifriction bearings and generate higher power losses.

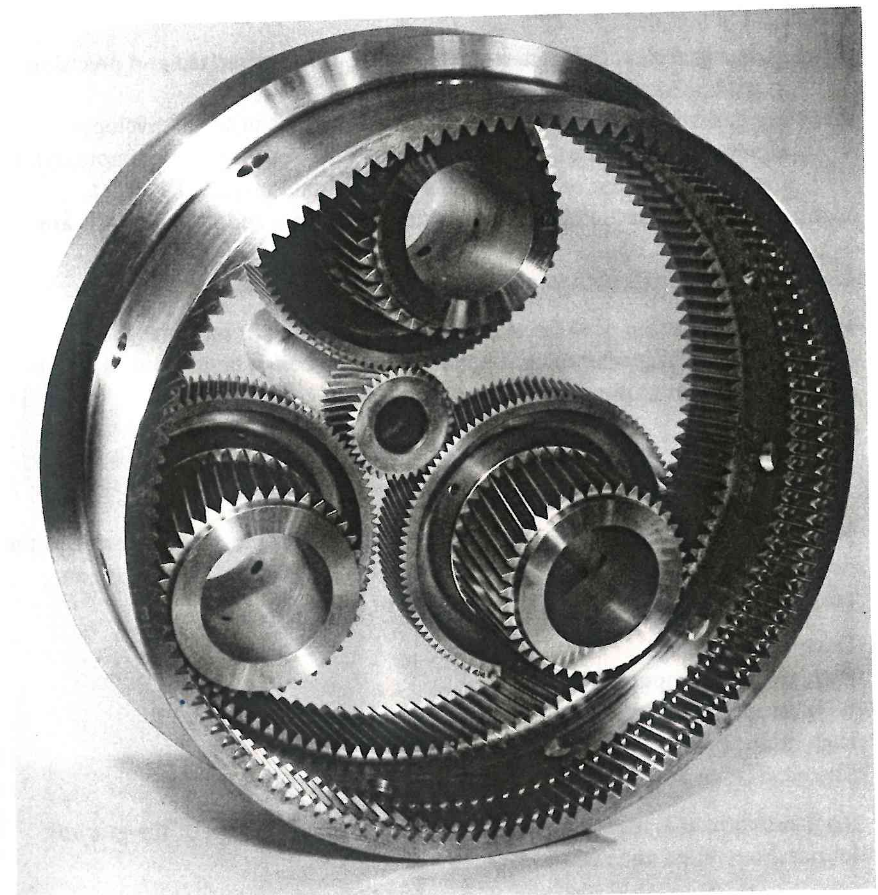

Figure 1.16 Compound planetary gearset. (Courtesy of American Lohmann Corporation, Hillside, N.J.)

A reasonable estimate of efficiency for industrial gear boxes is 1 to 2% power loss per mesh. A three-stage unit, therefore, might be expected to have an efficiency in the range 94 to 97%. The efficiency is quoted at the design load and speed conditions. At full speeds and lower loads the efficiency will drop off because the churning losses will remain constant.

SPACE AND WEIGHT LIMITATIONS

There are industrial applications where gearbox space and weight is limited. For instance, generator-drive gearboxes on offshore oil platforms or units used on mobile equipment must have minimum envelope. To achieve small gear units, several techniques can be used:

To minimize gear size, the highest-quality steel, case carburized and precision ground, is incorporated in the unit.

Planetary configurations are used to achieve high ratios in small envelopes. Figure 1.16 illustrates a compound planetary gearset which demonstrates a very efficient use of space, in the approximate range 9:1 to 12:1.

Lightweight design techniques such as thin-wall casings and hollow shafts are employed.

Lightweight materials such as aluminum housings are used.

Maximum application of these techniques can be found in the aerospace industry. An aircraft gearbox might handle the same design conditions as a conventional unit but at one-fiftieth the weight [2].

PHYSICAL ENVIRONMENT

When specifying a gear drive, the physical environment must be addressed in the design stage. Listed below are detrimental environments which can have an adverse effect on lubricant, bearings, gears, or seals:

Dusty atmosphere
High ambient temperature
Wide temperature variation
High humidity
Chemical-laden atmosphere

Such environments require special consideration in the design of the gearbox lubrication system and seals.

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1. AGMA Standard 420.04, Practice for Enclosed Speed Reducers or Increasers Using Spur, Helical, Herringbone and Spiral Bevel Gears, American Gear Manufacturers Association, Arlington, Va., December 1975.
2. Dudley, D. W., *Gear Handbook*, McGraw-Hill, New York, 1962, pp. 3–5.

2
GEAR TOOTH DESIGN

The purpose of gearing is to transmit power and/or motion from one shaft to another at a constant angular velocity. The tooth form almost universally used is the involute, which has properties that make it particularly desirable for these functions. It will be shown that in order to attain constant angular velocity, the meshing tooth forms must have specific geometrical characteristics which are easily obtained with an involute system.

In order to understand gear tooth drives it is useful to observe the dynamics of simpler power transmission systems, such as friction disks or belt drives (Figure 2.1), both of which are capable of transmitting power at a constant velocity ratio. The velocity ratio is inversely proportional to the ratio of the diameters:

$$\frac{W_A}{W_B} = \frac{D_B}{D_A}$$

where

W = angular velocity, rad/sec
D = diameter, in.

If $D_A = \frac{1}{2}D_B$ it can be seen that friction disk A has to make two revolutions for each revolution of disk B if the circumferences of the disks are rolling on one another without slipping. Another way of looking at it is that at the point of contact both disks have the same tangential velocity V_T in inches per second, and

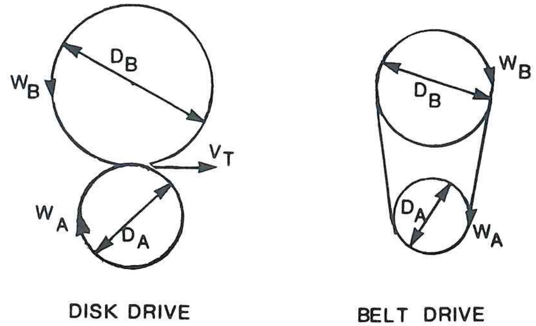


Figure 2.1 Friction disk and belt drives.

$$V_T = \frac{W_A D_A}{2} = \frac{W_B D_B}{2}$$

Therefore,

$$\frac{W_A}{W_B} = \frac{D_B}{D_A}$$

Similarly, the ratio of angular velocity of sheaves A and B in the belt drive are proportional to the ratio of diameter B to diameter A.

Disks and belt drives are power and speed limited and sometimes slip. A more positive method of transmitting power is through gear teeth, which can be illustrated as two cam profiles acting on one another (Figure 2.2). The force of the driving cam on the driven at any instant of time acts normal to the point of tangency of the curved surfaces. This normal line, shown as AA in Figure 2.2, is known as the line of action. Line AA intersects a line drawn between the two centers of rotation at point X. R_A and R_B are then the instantaneous pitch radii of the two cams. The angular velocity ratio of the cams at a given instant is inversely proportional to the ratio of the instantaneous pitch radii. For the angular velocity ratio to remain constant, the respective pitch radii must be the same at all points of contact. If this condition is met, the two profiles are said to be conjugate. Two cam profiles chosen at random will rarely be conjugate; however, given one profile a conjugate mating profile can be developed mathematically. The problem is that these two conjugate profiles may not be practical from an operating or manufacturing point of view.

This leads to one major reason why the involute curve is widely used for gear teeth. Two mating involutes will always be conjugate and the tooth forms relatively easy to manufacture with standardized tooling.

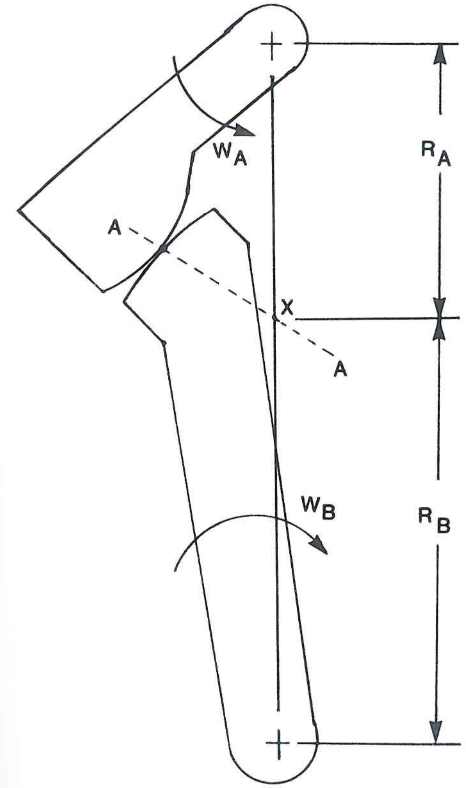


Figure 2.2 Cam drive.

THE INVOLUTE CURVE

Figure 2.3 illustrates the involute curve, which may be visualized as the locus of points generated by the end of a string which is held in tension as it is unwound from a drum. The drum is known as the base circle and once the base circle diameter is known, the involute curve is completely defined. Mathematically, the involute is expressed as a vectorial angle θ in radians:

$$\theta = \tan\phi - \phi = \text{Inv}\,\phi$$

where ϕ is the pressure angle at any diameter, in radians. In order to plot the involute curve for a base circle of radius R_B, simply assume values for the pressure angle ϕ. All the terms shown in Figure 2.3 can then be calculated.

R is the radius to any point on the involute and is related to R_B by the cosine of the pressure angle:

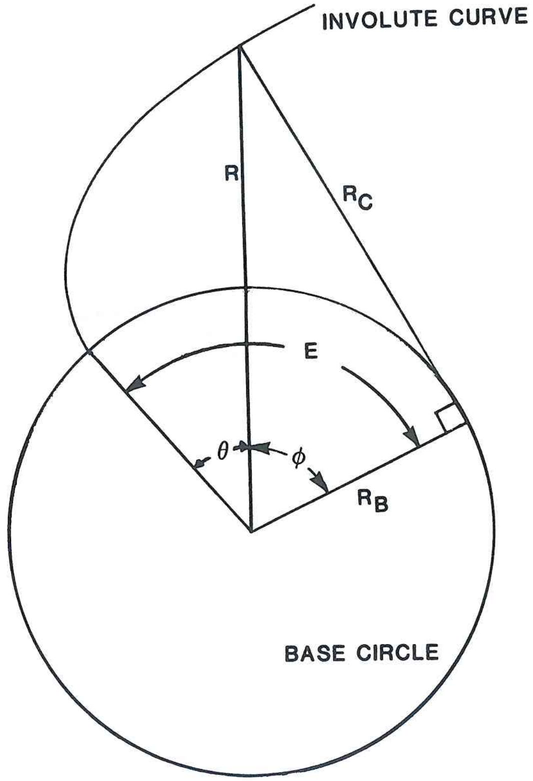


Figure 2.3 The involute curve.

$$\cos \phi = \frac{R_B}{R}$$

R_C is the radius of curvature of any point on the involute at radius R. Inspection of Figure 2.3 reveals that R_C is also the length of string unrolled from the base circle as the base circle rolls through an angle E; therefore,

$$E \cdot R_B = R_C$$

$$R_C = \sqrt{R^2 - R_B^2}$$

$$\theta = E - \phi = \frac{R_C}{R_B} - \tan^{-1} \frac{R_C}{R_B}$$

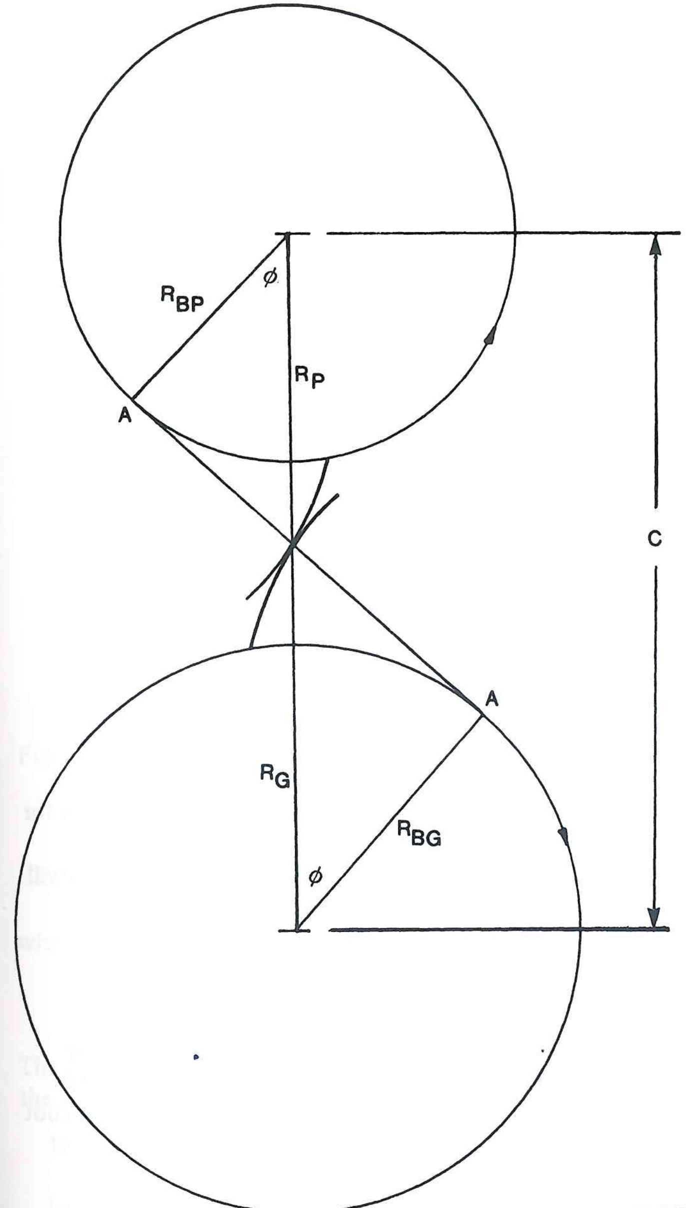


Figure 2.4 Gear teeth in mesh.

or

$$\theta = \tan\phi - \phi$$

Thus, knowing the base circle radius R_B and assuming values for ϕ, R, and θ, the polar coordinates of the involute can be plotted. The involute in terms of the pressure angle is useful in many gear tooth calculations, and a table of involutes is presented in the Appendix.

Let us now mesh two involute curves together at a center distance C, as shown in Figure 2.4. The angle ϕ is now the operating pressure angle of the gear mesh and R_P and R_G are the operating pitch radii. The subscripts P and G stand for pinion and gear, with the pinion always being the smaller of the two meshing gears. AA, the common tangent between the two base circles, is the line of action and the two involutes are shown meshing at the pitch point. It is important to understand that if the center distance C is increased or decreased, the involutes will contact at different points and have different operating pressure angles and pitch diameters. The velocity ratio, however, will not change since it is dependent only on the ratio of the two base diameters. The relationship of the base radius, pitch radius, and pressure angle is

$$\cos\phi = \frac{R_{BP}}{R_P} = \frac{R_{BG}}{R_G}$$

The center distance C in terms of the other parameters is

$$C = R_P + R_G = \frac{R_{BP} + R_{BG}}{\cos\phi}$$

The insensitivity of the involute to center distance variation is another reason for using this curve for gear teeth. Also, it can be seen that a whole system of involute gearing can be established where within certain limits any two gears will mesh with one another and transmit uniform rotary motion.

GEAR TOOTH DEFINITIONS

Figure 2.5 depicts a practical gear tooth. As shown, a portion of the involute curve bounded by the outside diameter and root diameter is used as the tooth profile. In a properly designed gear mesh the involute curve merges with the root fillet at a point below the final contact of the mating gear. This intersection of involute and root fillet, called the form diameter, is discussed later in the chapter. From Figure 2.5 we can define an important parameter, the diametral pitch, which is a measure of tooth size. The diametral pitch is the number of teeth per inch of pitch diameter:

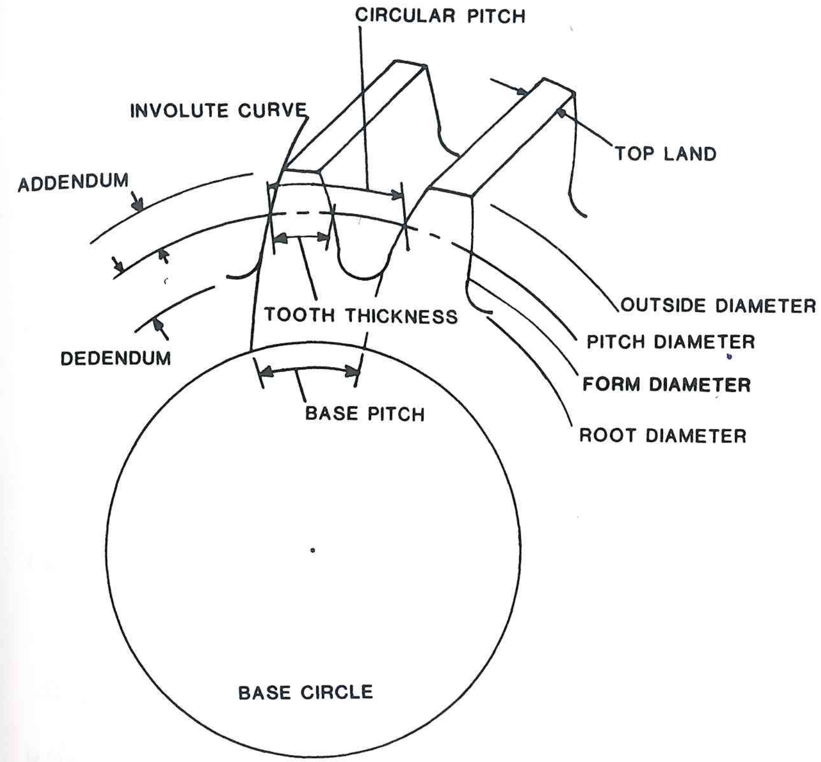


Figure 2.5 Gear tooth nomenclature.

$$DP = \frac{N}{PD}$$

where

N = number of teeth
PD = pitch diameter, in.

The circumference of the pitch diameter divided by the number of teeth is called the circular pitch:

$$CP = \pi \cdot \frac{PD}{N}$$

The circumference of the base diameter divided by the number of teeth is called the base pitch P_B:

$$P_B = \pi \frac{BD}{N}$$

The relationship between circular pitch and diametral pitch is

$$CP \cdot DP = \pi$$

The area between the pitch diameter and the outside diameter is called the addendum and on a standard gear tooth is 1.0/DP. The area between the outside diameter and the root diameter is the whole depth, which is the addendum plus dedendum. The whole depth of a standard gear tooth is generally 2.25 to 2.4 divided by the diametral pitch.

GEAR TOOTH GENERATION

Let us look at the generation of a gear tooth. Figure 2.6 shows a straight-sided cutting tool, such as that used in the hobbing process, generating an involute tooth. By "generating" it is meant that the tool is cutting a conjugate form. Such a straight-sided tooth is sometimes referred to as a rack. As the tool traverses and the work rotates, an involute is generated on the gear tooth flank and a trochoid in the root fillet, as shown in Figure 2.7.

Figure 2.8 is a closer look at a hob tooth. This is a hob of pressure angle ϕ and diametral pitch $\pi/(TH + TP)$. It is capable of cutting a whole family of gears with the same pressure angle and diametral pitch. Such tools are standard, as, for instance, a 20° pressure angle and 8 pitch (diametral pitch) hob, and are easily obtainable. Gears cut by this hob will be capable of meshing with one another. The distance (B + R) on the hob tooth is equal to the dedendum of the

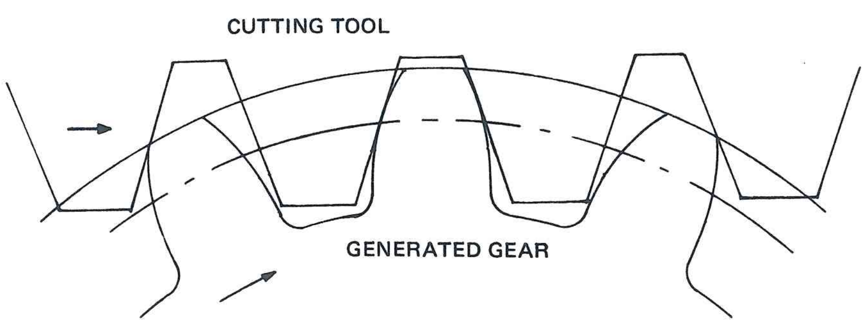


Figure 2.6 Generating a gear tooth.

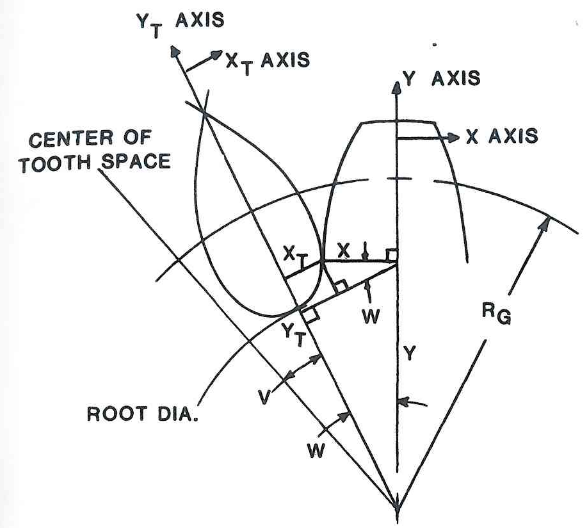


Figure 2.7 Root fillet trochoid and involute.

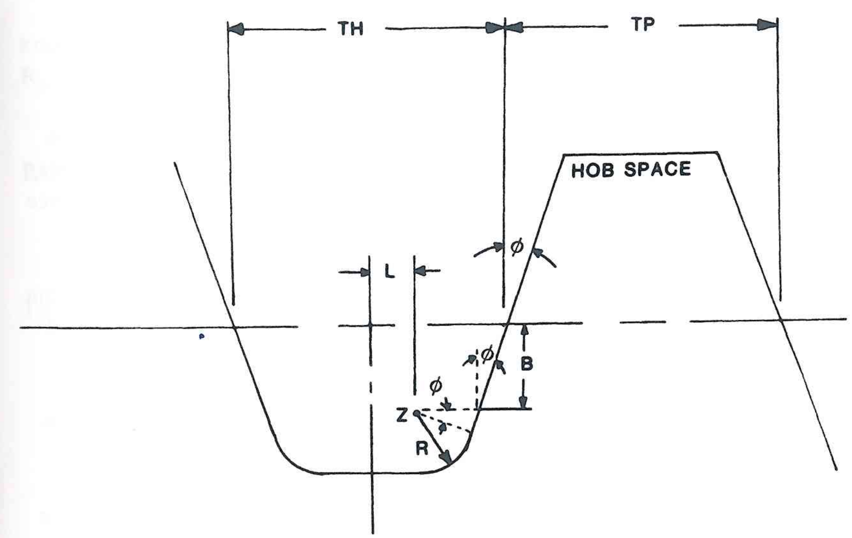


Figure 2.8 Hob geometry.

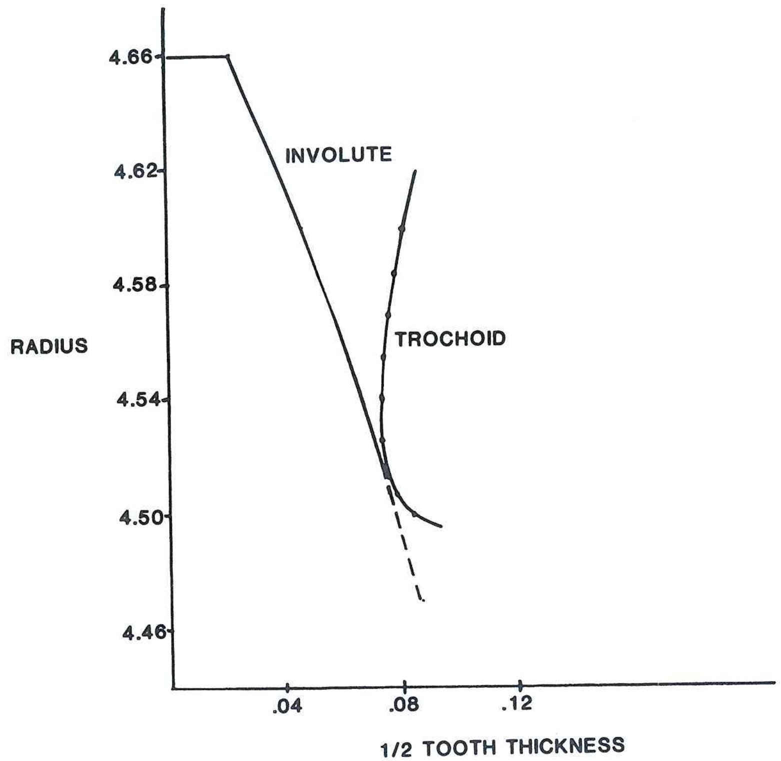


Figure 2.9 Plot of involute and trochoid.

generated gear tooth. R is the hob tip radius with its center at point Z. TP, the hob tooth space, is equal to the tooth thickness of the generated gear at the gear pitch diameter. TH is the hob tooth thickness. When the hob traverses a distance (TP + TH), the gear rotates through an angle (TP + TH)/R_G, where R_G is the gear pitch radius. (TP + TH) is the circular pitch of the gear.

The involute and trochoid can be plotted on a Cartesian coordinate system emanating at the center of the gear with the Y axis going through the center of the gear tooth, as shown in Figure 2.7. Figure 2.9 is a plot of the profile of a hobbed 20° pressure angle tooth on a Cartesian coordinate system. In the following paragraphs equations are developed to generate this plot. These equations are easily programmed and the coordinates can be plotted automatically.

First, the involute coordinates will be obtained. Previously, it was shown that if the base circle radius is known, the involute angle θ and the radius to the curve can be found for any assumed pressure angle. To find the coordinates with respect to the center of the tooth, the tooth thickness at any radius must be

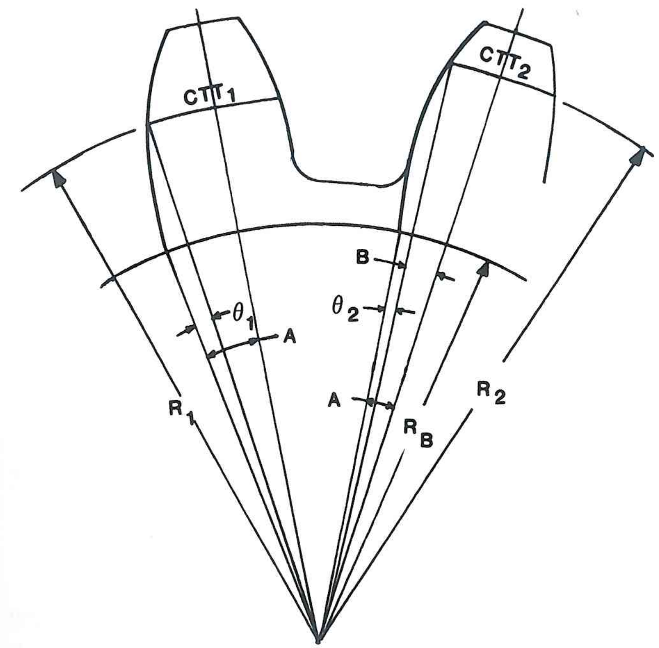


Figure 2.10 Tooth thickness calculation.

known. Let us start at the pitch diameter with a pressure angle ϕ_1, a pitch radius R_1, and a circular tooth thickness CTT_1. The involute angle is

$$\theta_1 = \tan\phi_1 - \phi_1$$

Referring to Figure 2.10, it can be seen that the angle A is

$$A = \theta_1 + \frac{\frac{1}{2}CTT_1}{R_1}$$

To find the circular tooth thickness at any other radius R_2, we use

$$B = A - \theta_2 = \theta_1 + \frac{\frac{1}{2}CTT_1}{R_1} - \theta_2$$

$$\phi_2 = \cos^{-1}\frac{R_B}{R_2}$$

$$\theta_2 = \tan\phi_2 - \phi_2$$

and

$$CTT_2 = 2R_2\left(\frac{\frac{1}{2}CTT_1}{R_1} + \theta_1 - \theta_2\right)$$

To find the X and Y coordinates of the involute at any radius R_2, we use

$X = R_2 \sin B$

$Y = R_2 \cos B$

Following are the steps that would be used in a computer routine to calculate the involute profile:

R_1 = pitch radius

ϕ_1 = pressure angle at pitch radius

CTT_1 = circular tooth thickness at pitch radius

R_B = base radius

$\theta_1 = \tan\phi_1 - \phi_1$

$\phi_2 = 0.0$

1 $\theta_2 = \tan\phi_2 - \phi_2$

$R_2 = \dfrac{R_B}{\cos\phi_2}$

$CTT_2 = 2R_2\left(\dfrac{\frac{1}{2}CTT_1}{R_1} + \theta_1 - \theta_2\right)$

$B = \dfrac{\frac{1}{2}CTT_2}{R_2}$

$X = R_2 \sin B$

$Y = R_2 \cos B$

Write (X, Y)

If (CTT_2 · LE · 0.005) go to 2

$\phi_2 = \phi_2 + \dfrac{\pi}{180}$

Go to 1

2 Continue

The routine starts at the base circle, where the pressure angle is 0.0, and calculates coordinates at pressure angle intervals of $\pi/180$ rad until it reaches a point near the tip of the tooth where the thickness is less than 0.005 in.

In the hobbing process depicted in Figure 2.6, the point on the corner of the tool generates the shape of the root fillet. This shape, which is bounded by the root diameter and the involute, is called a trochoid. In Figure 2.7 the complete curve is shown as a loop on the X_T and Y_T axes. Only that portion of the curve that lies between the involute and root diameter is of interest. To calculate the trochoid coordinates, the following procedure is used:

1. Calculate the trochoid generated by the center of the hob tip radius, point Z on Figure 2.8.
2. Find the normal to this trochoid at any point in order to add the radius R. This step would not be necessary if the hob had a sharp corner since the corner point would cut a trochoid, but practical hobs have rounded tips.
3. The trochoid coordinates calculated will be with respect to the center of the trochoid (Figure 2.7, X_T and Y_T axes). To find the coordinates on the desired X-Y system through the center of the tooth they must be shifted through the angle W (Figure 2.7).

When the hob traverses a distance (TH + TP) (Figure 2.8), the gear rotates through an angle $(TH + TP)/R_G$; therefore, the angle (W + V) between the center of the gear tooth and the center of the tooth space is

$$W + V = \frac{\frac{1}{2}(TH + TP)}{R_G}$$

where R_G is the gear pitch radius in inches. Angle V in Figure 2.7 can be calculated as follows:

$$V = \frac{L}{R_G}$$

where

L = distance between the center of the hob tooth and point Z on Figure 2.8, in.

$$L = \frac{TH}{2} - B\tan\phi - \frac{R}{\cos\phi}$$

and

$$W = \frac{\frac{1}{2}(TH + TP) - L}{R_G}$$

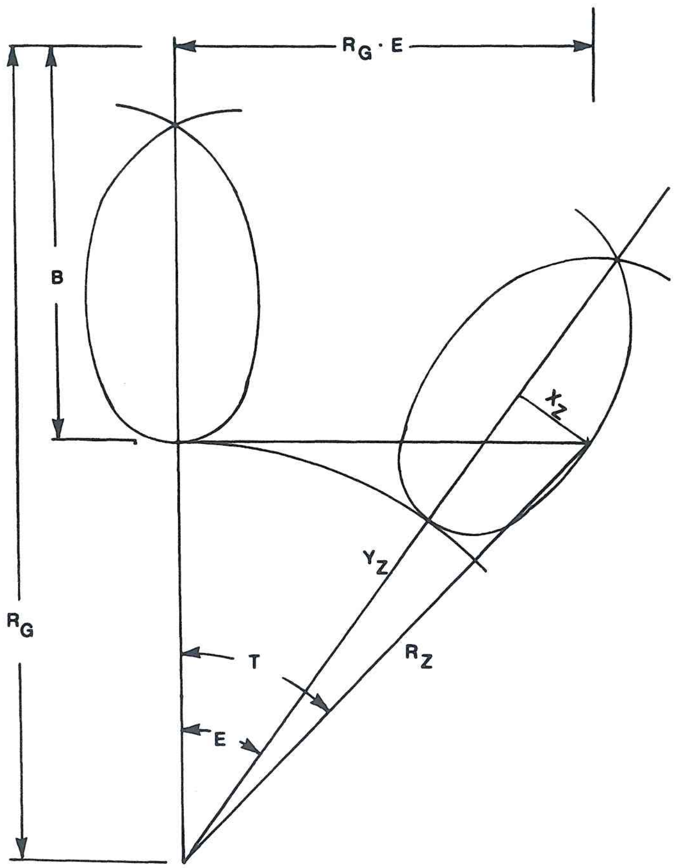


Figure 2.11 Trochoid generated by point Z.

Figure 2.11 shows the trochoid generated by point Z at its starting point and after the hob has moved a distance $R_G \cdot E$ and the gear has rotated through an angle E. The coordinates are:

$$X_Z = R_Z \sin (T - E)$$
$$= R_Z(\sin T \cos E - \cos T \sin E)$$
$$Y_Z = R_Z \cos (T - E)$$
$$= R_Z(\cos T \cos E + \sin T \sin E)$$

where

$$\cos T = \frac{R_G - B}{R_Z}$$

$$\sin T = \frac{R_G \cdot E}{R_Z}$$

Therefore,

$$X_Z = (R_G \cdot E) \cos E - (R_G - B) \sin E$$
$$Y_Z = (R_G - B) \cos E + (R_G \cdot E) \sin E \qquad (2.1)$$

Figure 2.12 shows how to calculate the actual trochoid coordinates, adding the hob tip radius R to the trochoid generated by point Z:

$$X_T = X_Z + R \cos A$$
$$Y_T = Y_Z - R \sin A \qquad (2.2)$$

A is the angle formed by a line normal to the trochoid generated by point Z and the Y_T axis and can be found as follows:

$$\tan A = \frac{dX_Z}{dY_Z}$$

To find dX_Z/dY_Z:

$$\frac{dX_Z}{dE} = -(R_G \cdot E) \sin E + R_G \cos E - R_G \cos E + B \cos E$$

$$\frac{dY_Z}{dE} = -R_G \sin E + B \sin E + (R_G \cdot E) \cos E + R_G \sin E$$

and dX_Z/dY_Z is

$$\frac{dX_Z}{dY_Z} = \frac{-(R_G \cdot E) \sin E + B \cos E}{B \sin E + (R_G \cdot E) \cos E}$$

Finally, to obtain the trochoid coordinates with respect to the system through the gear tooth center, refer to Figure 2.7:

$$\sin W = \frac{X_T + X \cos W}{Y}$$

$$\cos W = \frac{Y_T - X \sin W}{Y}$$

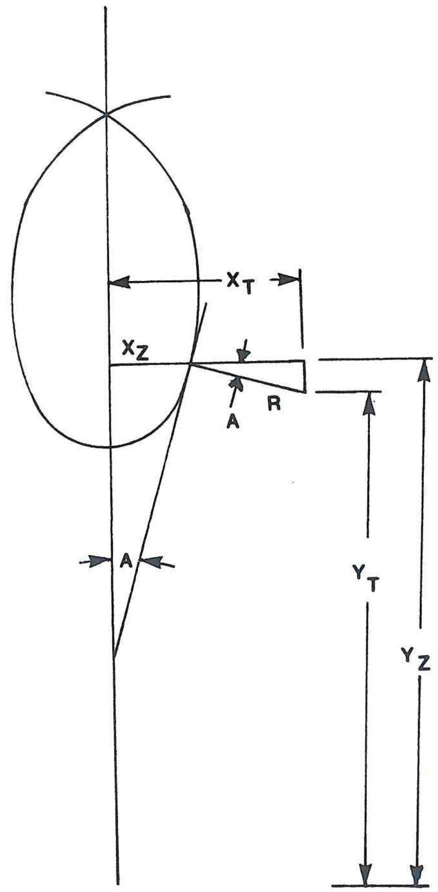


Figure 2.12 Trochoid coordinates.

$$\frac{X_T + X \cos W}{\sin W} = \frac{Y_T - X \sin W}{\cos W}$$

$$X_T \cos W + X \cos^2 W = Y_T \sin W - X \sin^2 W \qquad (2.3)$$

$$X = Y_T \sin W - X_T \cos W$$

$$Y = Y_T \cos W + X_T \sin W$$

Let us review the procedure for plotting the trochoid coordinates:

1. Choose angles E at random and calculate the trochoid coordinates of point Z using Eqs. (2.1).

2. Calculate the angle A at each point and define the actual trochoid coordinates using Eqs. (2.2).
3. Convert to the Cartesian coordinate system through the tooth center using Eqs. (2.3).

Following is a computer routine to carry out the process described above, starting with angle E equal to 0 rad (where angle A is 90° or $\pi/2$ rad) and ending with E equal to $25\pi/180$ rad.

$$\text{Counter} = 25 \cdot \frac{\pi}{180}$$

$$L = \tfrac{1}{2}TH - B \tan \phi - \frac{R}{\cos \phi}$$

$$W = \frac{(TH + TP)/2 - L}{R_G}$$

$$E = 0.0$$

1 $$\frac{dX_Z}{dE} = -(R_G \cdot E) \sin E + B \cos E$$

$$\frac{dY_Z}{dE} = B \sin E + (R_G \cdot E) \cos E$$

$$X_Z = (R_G \cdot E) \cos E - (R_G - B) \sin E$$

$$Y_Z = (R_G - B) \cos E + (R_G \cdot E) \sin E$$

If (E · LE · 0.0) go to 2

$$A = \tan^{-1} \frac{dX_Z}{dY_Z}$$

Go to 3

2 $$A = \frac{\pi}{2}$$

3 $$X_T = X_Z + R \cos A$$

$$Y_T = Y_Z - R \sin A$$

$$X = Y_T \sin W - X_T \cos W$$

$$Y = Y_T \cos W + X_T \sin W$$

If (E · GE · counter), go to 4

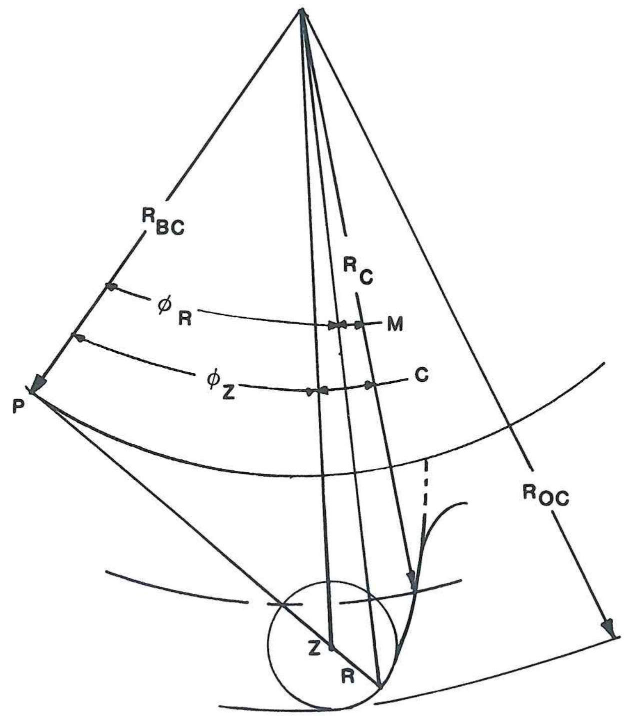


Figure 2.13 Shaper cutter tooth.

Write X, Y

$$E = E + \frac{0.25\,\pi}{180}$$

Go to 1

4 Continue

The previous discussion was concerned with a straight-sided cutter such as a hob. Another type of cutting tool is in the form of a gear tooth and is called a shaper cutter. The shaper cutter meshes with the work and generates a mating gear. Figure 2.13 shows a shaper cutter tooth with the profile in the form of an involute and with a rounded tip edge of radius R.

R_C = cutter pitch radius, in.

R_{BC} = cutter base circle radius, in.

R = round edge radius, in.

R_{OC} = cutter outside radius, in.

Z = center of round edge

As the cutter rolls through angle C the gear it is cutting will roll through an angle $C \cdot R_C/R_G$, where R_G is the gear pitch radius. Referring back to Figure 2.7, the angle W between the center of the trochoid and the center of the gear tooth is

$$W = \frac{\frac{1}{2}CTT_G}{R_G} + \left(\frac{C \cdot R_C}{R_G}\right)$$

where CTT_G is the circular tooth thickness of the gear at the pitch diameter, in inches. From Figure 2.13 angle C is found as follows:

$$C = \phi_R + M - \phi_Z$$

$$M = \text{Inv}\,\phi_R - \text{Inv}\,\phi$$

$$\phi_Z = \cos^{-1}\frac{R_{BC}}{R_{OC} - R}$$

$$PZ = R_{BC}\tan\phi_Z$$

$$\phi_R = \tan^{-1}\frac{PZ + R}{R_{BC}}$$

where ϕ is the pressure angle at the pitch diameter.

Figure 2.14 shows how the coordinates of the trochoid of point Z, the center of the shaper cutter round edge radius, are arrived at:

$$X_Z = R_Z \sin(T - E_G)$$

$$Y_Z = R_Z \cos(T - E_G)$$

$$\sin T = (R_{OC} - R)\frac{\sin E_C}{R_Z}$$

$$\cos T = \frac{R_Z^2 + (R_C + R_G)^2 - (R_{OC} - R)^2}{2(R_Z)(R_C + R_G)}$$

$$R_Z^2 = (R_{OC} - R)^2 + (R_C + R_G)^2 - 2(R_{OC} - R)(R_C + R_G)\cos E_C$$

Therefore,

$$X_Z = (R_{OC} - R)\sin(E_C + E_G) - (R_C + R_G)\sin E_G$$

$$Y_Z = (R_C + R_G)\cos E_G - (R_{OC} - R)\cos(E_C + E_G)$$

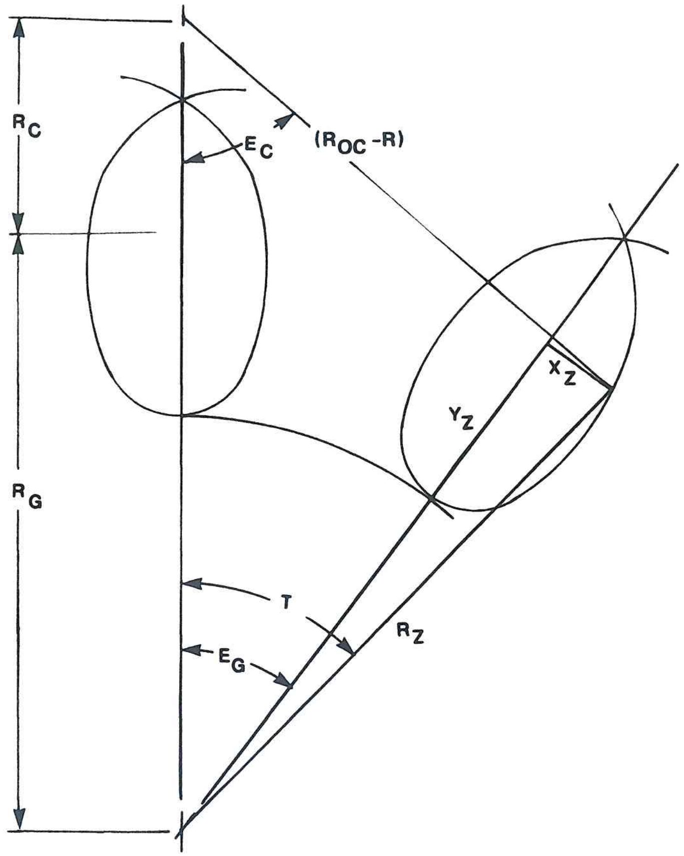


Figure 2.14 Shaper cutter trochoid coordinates.

To find the trochoid coordinates with respect to the center of the gear tooth, the same procedure is followed as was used for the hob-type cutter:

1. From Figure 2.12 find the X_T and Y_T coordinates differentiating the foregoing equations to calculate tan A using Eqs. (2.2).
2. From Figure 2.7, shift to the X, Y coordinate system by using Eqs. (2.3).

GEAR TEETH IN ACTION

Figure 2.15 shows a gear mesh with the driving pinion tooth on the left just coming into mesh at point T and the two teeth on the right meshing at point S.

Notice that contact starts at point T where the outside diameter of the gear crosses the line of action and ends where the outside diameter of the pinion crosses the line of action, point R.

Z is the length of the line of action. In other words, a tooth will be in contact from point T to point R. P_B is the base pitch, the distance from one involute to the next along a radius of curvature. It was shown earlier that

$$P_B = \pi \frac{BD}{N}$$

where

BD = base diameter, in.
N = number of teeth

Point T, where contact initiates, is called the lowest point of contact on the pinion tooth and also the highest point of contact on the gear tooth. Similarly, point R is the highest point of contact on the pinion tooth and the lowest point of contact on the gear tooth. Point S is the highest point of single tooth contact on the pinion and the lowest point of single tooth contact on the gear. In other words, if one imagines the gears in Figure 2.15 to begin rotating, just prior to

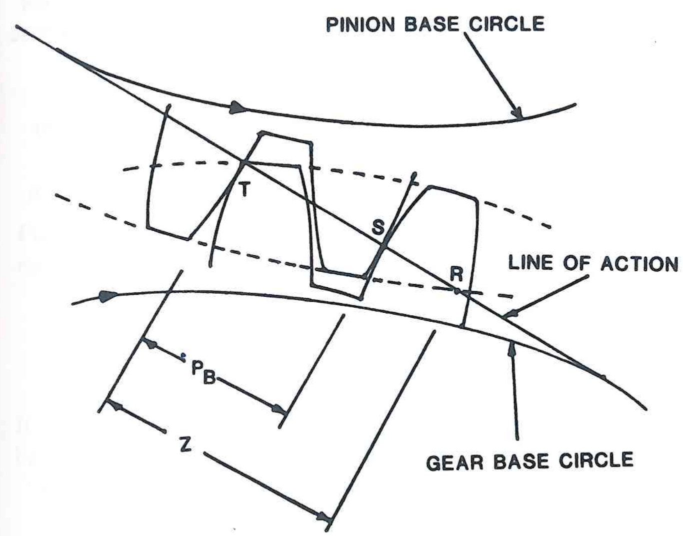


Figure 2.15 Gear tooth action.

meshing at point T a single pair of teeth was carrying the load. As the gears continue turning and the pinion tooth on the right moves from point S to R, two pairs of teeth are carrying the load and after point R a single pair again carries the load until the next two teeth mesh at point T.

Thus it can be seen that for some period of time one tooth mesh carries the load and for another period of time two tooth meshes share the load. A measure of the percentage of time two meshes share the load is the profile contact ratio M_P. For instance, a profile contact ratio of 1.0 would mean that one tooth is in contact 100% of the time. A contact ratio of 1.6 means that two pairs of teeth are in contact 60% of the time and one pair carries the load 40% of the time. Contact ratios for conventional gearing are generally in the range 1.4 to 1.6.

Let us now derive the profile contact ratio in terms of parameters easily obtainable:

$$M_P = \frac{E_{TR}}{360°/N}$$

where

E_{TR} = degrees of roll to traverse the length of the line of action from point T to R
N = number of teeth

This equation may not be obvious, but it can be understood if it is remembered that from one tooth to another the base circle must roll 360°/N. Thus if the base circle rolls 360°/N while going from point T to R, the profile contact ratio is 1.0. If the base circle rolls more than 360°/N going from T to R, the profile contact ratio is greater than 1, indicating that for some percentage of time two pairs of teeth are in contact.

E_{TR}, the total degrees of roll, is equal to the degrees of roll to the pinion tooth tip minus the degrees of roll to the pinion lowest point of contact:

$$E_{TR} = E_{ODP} - E_{TIFP}$$

where

E_{ODP} = degrees of roll to pinion outside diameter
E_{TIFP} = degrees of roll to true involute form diameter on the pinion (lowest point of contact) (point T on Figure 2.15)

Figure 2.16 shows how the degrees of roll to the pinion outside diameter is calculated:

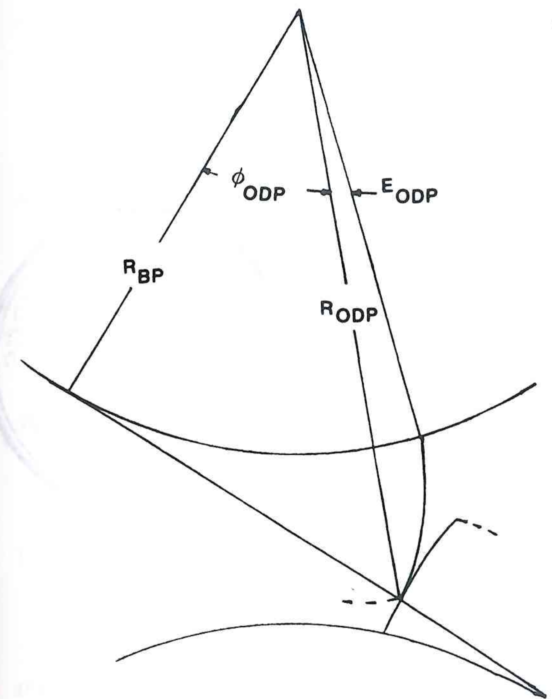


Figure 2.16 Degrees of roll to pinion outside diameter.

$$E_{ODP} = \tan \phi_{ODP}\left(\frac{180}{\pi}\right) = \frac{\sqrt{R_{ODP}^2 - R_{BP}^2}}{R_{BP}}\left(\frac{180}{\pi}\right)$$

Figure 2.17 shows how the degrees of roll to the pinion form diameter is calculated:

$$E_{TIFP} = \tan \phi_{TIFP}\left(\frac{180}{\pi}\right) = \frac{\sqrt{R_{TIFP}^2 - R_{BP}^2}}{R_{BP}}\left(\frac{180}{\pi}\right)$$

It is more convenient to express E_{TIFP} in terms of the gear outside radius and base radius:

$$XX = C \sin \phi_{PD}$$

$$\sqrt{R_{TIFP}^2 - R_{BP}^2} = C \sin \phi_{PD} - \sqrt{R_{ODG}^2 - R_{BG}^2}$$

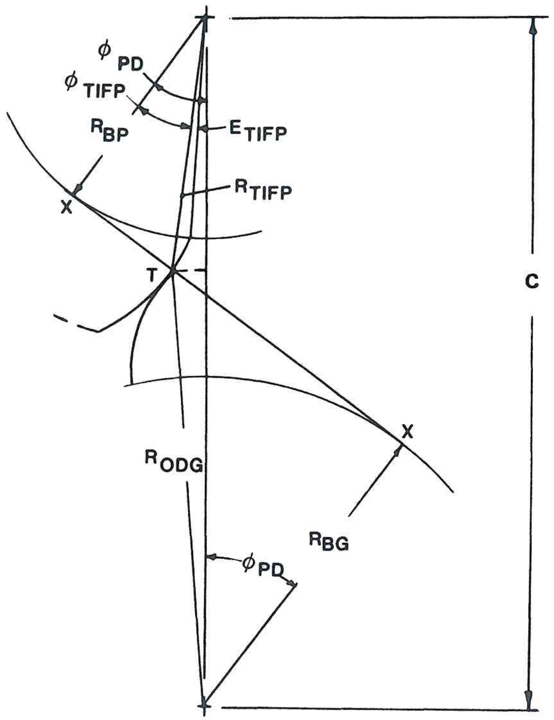


Figure 2.17 Degrees of roll to pinion true involute form diameter.

Therefore,

$$E_{TIFP} = \frac{C \sin \phi_{PD} - \sqrt{R^2_{ODG} - R^2_{BG}}}{R_{BP}} \left(\frac{180}{\pi}\right)$$

and

$$M_P = \frac{\sqrt{R^2_{ODP} - R^2_{BP}} - C \sin \phi_{PD} + \sqrt{R^2_{ODG} - R^2_{BG}}}{R_{BP}} \; \frac{180N}{\pi \cdot 360}$$

where

N = number of pinion teeth
R_{BP} = $\frac{1}{2} PD_P \cos \phi_{PD}$
$\frac{N}{PD_P \cdot \pi} = \frac{1}{CP}$

Therefore,

$$M_P = \frac{\sqrt{R^2_{ODP} - R^2_{BP}} - C \sin \phi_{PD} + \sqrt{R^2_{ODG} - R^2_{BG}}}{CP \cos \phi_{PD}}$$

Another way of expressing the profile contact ratio is

$$CP \cos \phi_{PD} = P_B$$

because

$$\cos \phi_{PD} = \frac{BD}{PD}$$

and

$$CP = \pi \frac{PD}{N}$$

Therefore,

$$CP \cos \phi_{PD} = \pi \frac{BD}{N} = P_B$$

and

$$\sqrt{R^2_{ODP} - R^2_{BP}} - C \sin \phi_{PD} + \sqrt{R^2_{ODG} - R^2_{BG}} = Z$$

Therefore,

$$M_P = \frac{Z}{P_B}$$

To calculate the degrees of roll to the highest point of single tooth contact on the pinion, consider Figure 2.18. The distance XS is the sum of XT from Figure 2.18 and TS from Figure 2.15. The distance XT can be calculated using Figure 2.17:

$$XT = \sqrt{R^2_{TIFP} - R^2_{BP}}$$

From Figure 2.15 it is seen that the distance TS is a base pitch; therefore,

$$XS = \sqrt{R^2_{TIFP} - R^2_{BP}} + P_B$$

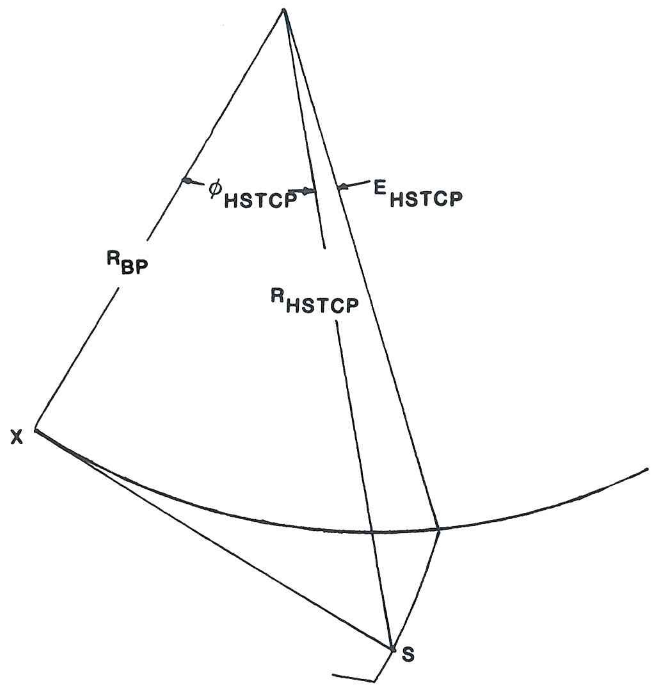


Figure 2.18 Degrees of roll to pinion highest single tooth contact diameter.

$$E_{HSTCP} = \tan \phi_{HSTCP}\left(\frac{180}{\pi}\right) = \frac{XS}{R_{BP}}\left(\frac{180}{\pi}\right)$$

$$= \frac{\sqrt{R_{TIFP}^2 - R_{BP}^2} + P_B}{R_{BP}}\left(\frac{180}{\pi}\right)$$

or

$$E_{HSTCP} = E_{TIFP} + \frac{2\pi}{N}$$

The involute curve changes very rapidly near the base diameter and more slowly at sections farther away from the base circle. This is illustrated in Figure 2.19, which shows the difference in length along the involute curve for equal increments taken on the base circle. The distance XY is far less than YZ. Since the

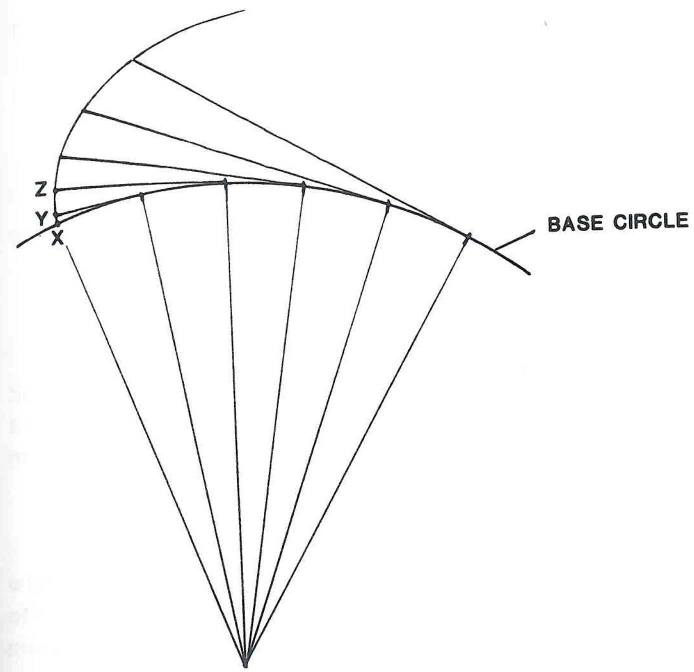


Figure 2.19 Involute curve properties.

involute is so sensitive near the base circle, the lowest point of contact on a gear tooth should be located well away from the base circle. As a rule of thumb the lowest point of contact on a gear tooth should be at least 9° of roll.

ROLLING AND SLIDING VELOCITIES

When involute gear teeth mesh, the action is not pure rolling as it would be when two friction disks are in contact, but a combination of rolling and sliding. Figure 2.20 shows a gear mesh with two base circles of equal size and the teeth meshing at the pitch point. Radii of curvature are drawn to the involutes from equal angular intervals on the base circle. It can be seen that arc XY on gear 2 will mesh with arc AB on gear 1 and that AB is longer than XY; therefore, the two profiles must slide past one another to make up the difference in length. The sliding velocity, which is usually expressed in feet per minute, at any point is calculated as follows:

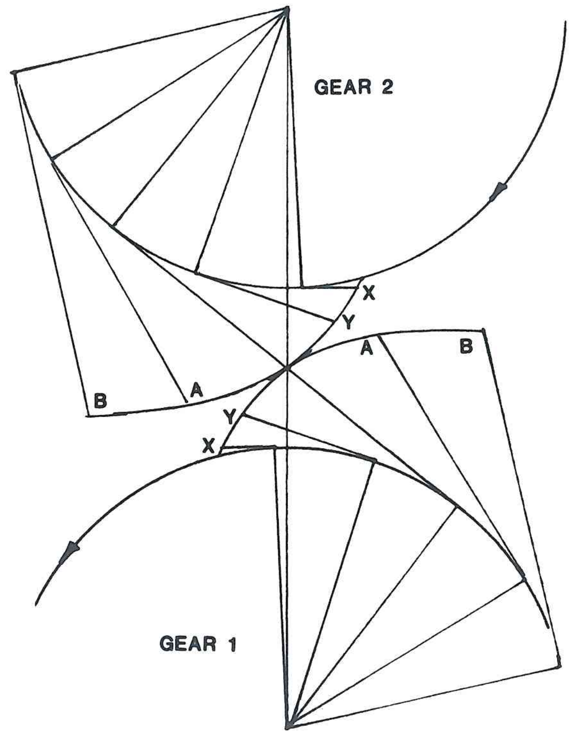


Figure 2.20 Relative sliding of gear teeth.

$$V_S = \frac{W_1 R_{C1} - W_2 R_{C2}}{12}$$

where

V_S = sliding velocity, fpm
W_1 = angular velocity of gear 1, rad/min
W_2 = angular velocity of gear 2, rad/min
R_{C1} = radius of curvature of gear 1, in.
R_{C2} = radius of curvature of gear 2, in.

From Figure 2.20 it can be seen that when point A on gear 1 and point Y on gear 2 mesh, R_{C1} will be larger than R_{C2} and since $W_1 = W_2$, V_S will be a positive number. As the meshing point nears the pitch point the difference in the radii of curvature lessens until at the pitch point the radii of curvature are equal and V_S is 0. When point A on gear 2 meshes with point Y on gear 1, R_{C1} will be smaller than R_{C2} and V_S will be negative. The significance of this is that as

the mesh goes through the pitch point the direction of sliding changes. There is always pure rolling at the pitch point. If the base circles were of unequal size at the pitch point, $W_1 R_{C1}$ would still equal $W_2 R_{C2}$ since

$$\frac{W_1}{W_2} = \frac{R_2}{R_1} = \frac{R_{C1}/\sin\phi}{R_{C2}/\sin\phi}$$

where

R_1 = pitch radius of gear 1, in.
R_2 = pitch radius of gear 2, in.
ϕ = pressure angle, deg

Sliding velocity is significant in that it affects the amount of heat generated in the gear mesh. Also, the fact that gear teeth undergo sliding as well as rolling must be appreciated.

Another significant velocity term is the sum or entraining velocity:

$$V_E = W_1 R_{C1} + W_2 R_{C2}$$

where V_E is the sum velocity in ips. The sum or entraining velocity is a measure of how quickly oil is being dragged into the conjunction between the two gear members.

The parameter generally used when expressing the speed of a gearset is the pitch line velocity:

$$V_T = \frac{W_P R_P(60)}{12} = \frac{W_G R_G(60)}{12}$$

$$= \frac{\pi D_P n_P}{12} = \frac{\pi D_G n_G}{12}$$

where

V_T = pitch line velocity, fpm
W_P = pinion angular velocity, rad/sec
W_G = gear angular velocity, rad/sec
R_P = pinion pitch radius, in.
R_G = gear pitch radius, in.
D_P = pinion pitch diameter, in.
D_G = gear pitch diameter, in.
n_P = pinion rpm
n_G = gear rpm

The pitch line velocity is a measure of the tangential or peripheral velocity of a gearset and a better indication of speed than the rpm. For instance, a 1-in. pitch diameter gear operating at 10,000 rpm has the same pitch line velocity as a 10-in. pitch diameter gear operating at 1000 rpm. Two meshing gears always have the same pitch line velocity.

American Gear Manufacturers Association (AGMA) Standards for enclosed drives consider units with pitch line velocities of 5000 fpm or more high speed. Gear units have been operated at pitch line velocities up to 50,000 fpm minute but applications over approximately 20,000 fpm require extremely careful analysis concerning lubrication, cooling, and centrifugal effects.

HELICAL GEARS

Figure 1.13 illustrates the difference between spur and helical gears. The tooth contact on spur gears is a straight line across the tooth and at any time either one or two teeth are in contact. The helical gear contact, because the teeth are at an angle to the axis of rotation, is a series of oblique lines with several teeth in contact simultaneously and the total length of contact varies as the teeth go through the mesh.

To understand the nature of the helical tooth, consider a base cylinder with a series of strings wrapped around it as shown in Figure 2.21. The start of each string is offset such that a line joining the string starts is at an angle ψ_B to the axis of rotation of the cylinder. The ends of each string when held taut and unwrapped from the base cylinder will define involutes and the surface defined by the string ends will be a helical involute gear tooth.

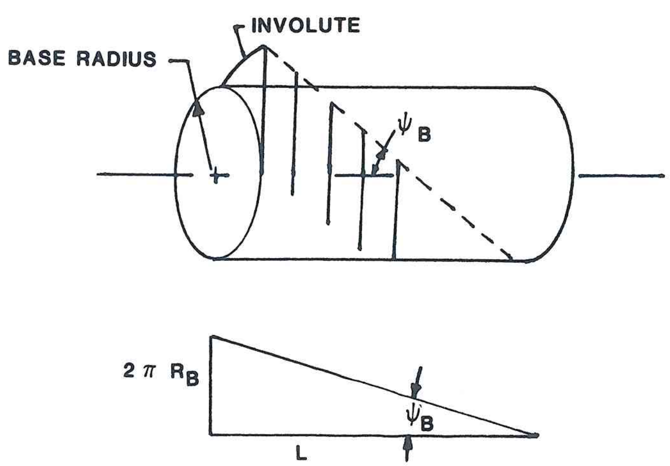


Figure 2.21 Helical gear base cylinder.

In one rotation of the base cylinder an axial length L of strings will be unwrapped. L is defined as the lead

$$L = \frac{2\pi R_B}{\tan \psi_B}$$

where

L = lead, in.
R_B = base radius, in.
ψ_B = base helix angle, deg

The helix angle along the tooth profile varies with the radius but the lead is a constant. Once the base radius and base helix angle are defined, the lead can be calculated and the helix angle at any radius R is known:

$$\tan \psi_R = \frac{2\pi R}{L}$$

where ψ_R is the helix angle at radius R, in degrees.

Transverse and Normal Planes

Figure 2.22 shows the relationship between the transverse and normal planes of a helical gear. The transverse plane ABCD is the plane of rotation, while the normal plane ABE is at right angles to the tooth. The normal and transverse planes are displaced from each other through the helix angle ψ. Following are the relationships between the normal and transverse pressure angles at any radius on the tooth illustrated at point B on Figure 2.22.

$$\tan \phi_N = \frac{AB}{AE}$$

$$\tan \phi_T = \frac{AB}{AD}$$

$$\cos \psi = \frac{AD}{AE}$$

$$AD \tan \phi_T = AE \tan \phi_N$$

Therefore,

$$\cos \psi \tan \phi_T = \tan \phi_N$$

Figure 2.23 shows a helical gear rotating about the axis XX. The teeth are inclided with relation to the axis of rotation the helix angle ψ. Usually, the helix angle at the operating pitch diameter is referred to. When the inclination of the

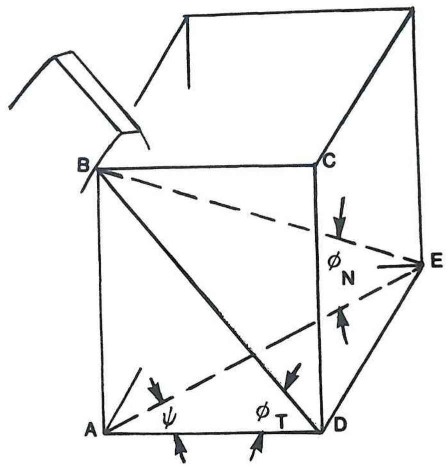


Figure 2.22 Normal and transverse planes.

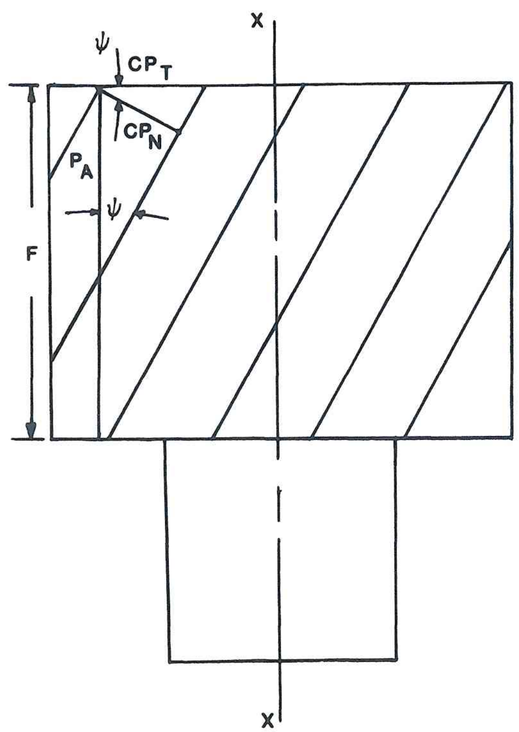


Figure 2.23 Normal and transverse pitches.

teeth is off to the right as shown in Figure 2.23, the gear helix is designated as right hand. When two external helical gears mesh, one must be right hand and the other left hand. When an external gear meshes with an internal gear they will both have the same hand of helix.

The circular pitch in the transverse plane CP_T has the following relationship with the circular pitch CP_N in the normal plane:

$$\cos \psi = \frac{CP_N}{CP_T}$$

The circular tooth thicknesses have the same relationship:

$$\cos \psi = \frac{CTTN}{CTTT}$$

where

CTTN = normal circular tooth thickness, in.
CTTT = transverse circular tooth thickness, in.

The normal and transverse diametral pitches have the relationship

$$DP_N \cos \psi = DP_T$$

The distance along the tooth axis from one tooth to another is called the axial pitch P_A, as shown in Figure 2.23. The ratio of face width F to axial pitch is called the face contact ratio or the helical overlap and is a measure analogous to the profile contact ratio for spur gears. The face contact ratio, designated as M_F, is

$$M_F = \frac{F}{P_A}$$

$$P_A = \frac{CP_N}{\sin \psi} = CP_T \frac{\cos \psi}{\sin \psi} = \frac{\pi}{DP_T \tan \psi}$$

Therefore,

$$M_F = F(DP_T) \frac{\tan \psi}{\pi}$$

The total contact of a helical gear mesh is therefore some combination of the profile and face contact ratios. Sometimes the sum of the two is called the total contact ratio and used as a measure of how much contact is achieved in a tooth mesh.

The actual total length of contact at any instant in a helical mesh is the sum of the length of the oblique lines of contact on each tooth in mesh and varies as the teeth go through mesh. A method of calculating the minimum and maximum length of the lines of contact was derived by E. J. Wellauer and presented to the Industrial Mathematics Society. The paper was entitled "The Nature of the Helical Gear Oblique Contact Line," and a small portion based on the original article is given below.

K_a and n_a are the whole number and fractional portion, respectively, of the face contact ratio. For example, if $M_F = F/P_A = 4.85$, then $K_a = 4.0$ and $n_a = 0.85$. K_r and n_r are the whole number and fractional portion, respectively, of the profile contact ratio. For example, if $M_P = Z/P_B = 1.32$, then $K_r = 1.0$ and $n_r = 0.32$.

If $(1 - n_r)/n_a \geqq 1$, then

$$L_{min} = \frac{(Z \cdot F/P_B) - n_r n_a P_A}{\cos \psi_B}$$

where

L_{min} = minimum total length of the oblique lines of contact, in.
Z = length of the line of action, in. (Figure 2.15)
P_B = base pitch, in.
P_A = axial pitch, in.
ψ_B = base helix angle, deg

If $(1 - n_r)/n_a < 1$, then

$$L_{min} = \frac{(Z \cdot F/P_B) - (1 - n_a)(1 - n_r)P_A}{\cos \psi_B}$$

An approximation used for calculating L_{min} in several AGMA Standards is

$$L_{min} = \frac{0.95\,(Z)F}{P_B \cos \psi_B}$$

To calculate the maximum total length of the lines of contact:

If $n_r \leqq n_a$, then

$$L_{max} = \frac{(Z \cdot F/P_B) + n_r(1 - n_a)P_A}{\cos \psi_B}$$

If $n_r > n_a$, then

$$L_{max} = \frac{(Z \cdot F/P_B) + n_a(1 - n_r)P_A}{\cos \psi_B}$$

INTERNAL GEARS

The involute form of internal gears (sometimes called ring gears) is the same as for external gears. The difference between the two lies in the fact that internal gears contact on the concave side of the involute rather than the convex. Also, the root diameter of an internal gear tooth is the largest diameter and the tips of the teeth are at the inside diameter, which is the smallest. Figure 2.24 shows internal gear tooth geometry. At any radius R, with pressure angle ϕ and base radius R_B, all the involutometry calculations will be essentially the same as those previously shown for external gear teeth.

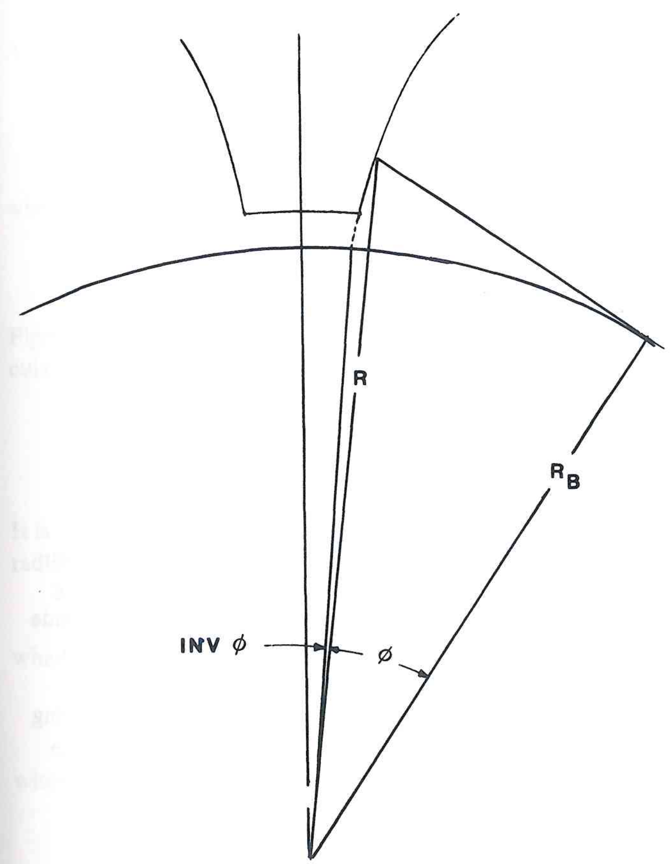


Figure 2.24 Internal gear geometry.

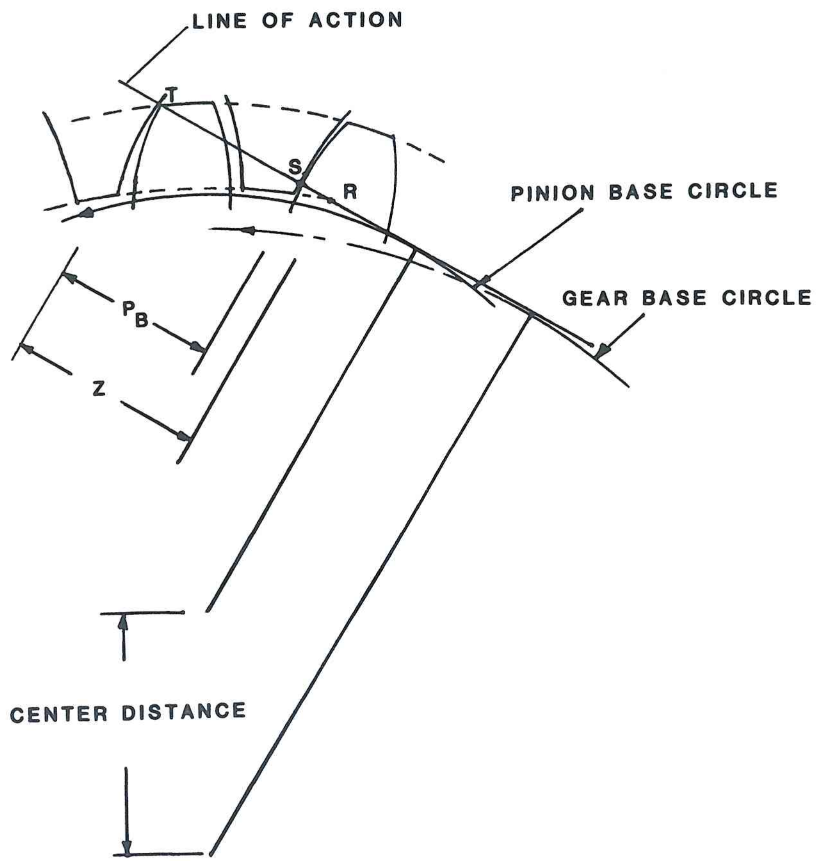


Figure 2.25 Internal gear mesh action.

To illustrate internal gear mesh calculations, let us derive the profile contact ratio for the situation shown in Figure 2.25 for an external pinion driving an internal gear. The start of contact is at point R, where the internal gear inside diameter crosses the line of action. Contact ends at point T, where the pinion outside diameter crosses the line of action. The total length of contact is the distance from R to T or Z as shown on Figure 2.25. One pair of teeth is meshing at point S on the figure and an adjacent pair at point T; therefore, the distance ST is a base pitch P_B. The profile contact ratio is

$$M_P = \frac{E_{TR}}{360°/N}$$

where

E_{TR} = degrees of roll to traverse the length of the line of action from point R to T
N = number of pinion teeth

E_{TR}, the total degrees of roll, is equal to the degrees of roll to the pinion outside diameter minus the degrees of roll to the lowest point of contact on the pinion (TIF, true involute form diameter).

$$E_{TR} = E_{ODP} - E_{TIFP}$$

where

E_{ODP} = degrees of roll to the pinion outside diameter
E_{TIFP} = degrees of roll to the pinion true involute form diameter

As shown previously (Figure 2.16),

$$E_{ODP} = \tan \phi_{ODP} = \left(\frac{180}{\pi}\right) \frac{\sqrt{R_{ODP}^2 - R_{BP}^2}}{R_{BP}} \left(\frac{180}{\pi}\right)$$

where

R_{ODP} = pinion outside radius, in.
R_{BP} = pinion base radius, in.

Figure 2.26 shows how the degrees of roll to the pinion form diameter is calculated:

$$E_{TIFP} = \tan \phi_{TIFP} = \left(\frac{180}{\pi}\right) \frac{\sqrt{R_{TIFP}^2 - R_{BP}^2}}{R_{BP}} \left(\frac{180}{\pi}\right)$$

It is more convenient to express E_{TIFP} in terms of the gear inside radius and base radius. From Figure 2.26,

$$XX = C \sin \phi_{PD}$$

where ϕ_{PD} is the pressure angle at the pitch diameter, in degrees.

$$\sqrt{R_{TIFP}^2 - R_{BP}^2} = \sqrt{R_{IDG}^2 - R_{BG}^2} - C \sin \phi_{PD}$$

where

R_{IDG} = gear inside radius, in.
R_{BG} = gear base radius, in.

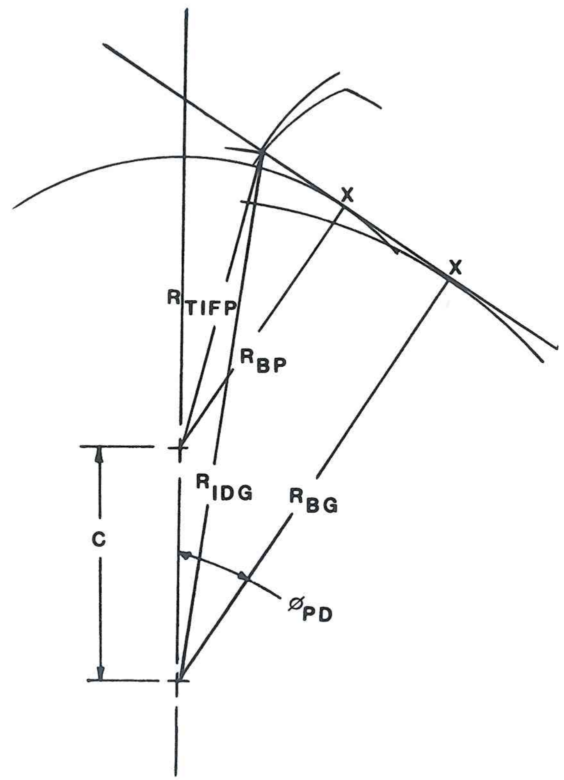


Figure 2.26 Internal gear mesh degrees of roll to pinion form diameter.

Therefore,

$$E_{TIFP} = \frac{\sqrt{R_{IDG}^2 - R_{BG}^2} - C \sin \phi_{PD}}{R_{BP}} \left(\frac{180}{\pi}\right)$$

and

$$M_P = \frac{\sqrt{R_{ODP}^2 - R_{BP}^2} + C \sin \phi_{PD} - \sqrt{R_{IDG}^2 - R_{BG}^2}}{R_{BP}} \quad \frac{180N}{\pi \cdot 360}$$

$$R_{BP} = \tfrac{1}{2} PD_P \cos \phi_{PD}$$

$$\frac{N}{PD_P} \pi = \frac{1}{CP}$$

where CP is the circular pitch in inches. Therefore,

$$M_P = \frac{\sqrt{R_{ODP}^2 - R_{BP}^2} + C \sin \phi_{PD} - \sqrt{R_{IDG}^2 - R_{BG}^2}}{CP \cos \phi_{PD}}$$

MEASUREMENT OVER BALLS OR WIRES

This subject is presented at this point not only because it is an important measurement in the manufacture of gear teeth but because it is a good illustration of the application of involutometry in the analysis of gear tooth geometry.

When cutting or grinding a gear tooth the machine operator will check the tooth thickness to determine when sufficient stock has been removed from the flank to bring the tooth to the required size. The drawing requirement may call for a tooth thickness at a given diameter. This is a difficult measurement to make directly; therefore, quite often an indirect measurement is used. Balls or wires (sometimes called pins) of a known diameter are placed in 180° opposite tooth spaces on the gear and an accurate micrometer measurement over the balls or wires is made.

The equations for calculating measurement over balls or wires will be derived first for an external spur gear with an even number of teeth. In this case two opposite tooth spaces will be in line. The analysis will then be extended to gears with odd numbers of teeth where the opposite tooth spaces are not in line and then internal gears and helical gears will be addressed.

Referring to Figure 2.27, for a spur gear with an even number of teeth the measurement over wires (MOW) is

$$MOW = 2R_2 + 2X$$

where

R_2 = radius to the center of the wire, in.
X = wire radius, in.

We are going to calculate the MOW for a gear with a known circular tooth thickness T at a known radius R_1.

$$R_2 = \frac{R_1 \cos \phi_1}{\cos \phi_2}$$

where

ϕ_1 = pressure angle at the radius R_1, deg
ϕ_2 = pressure angle at radius R_2, deg

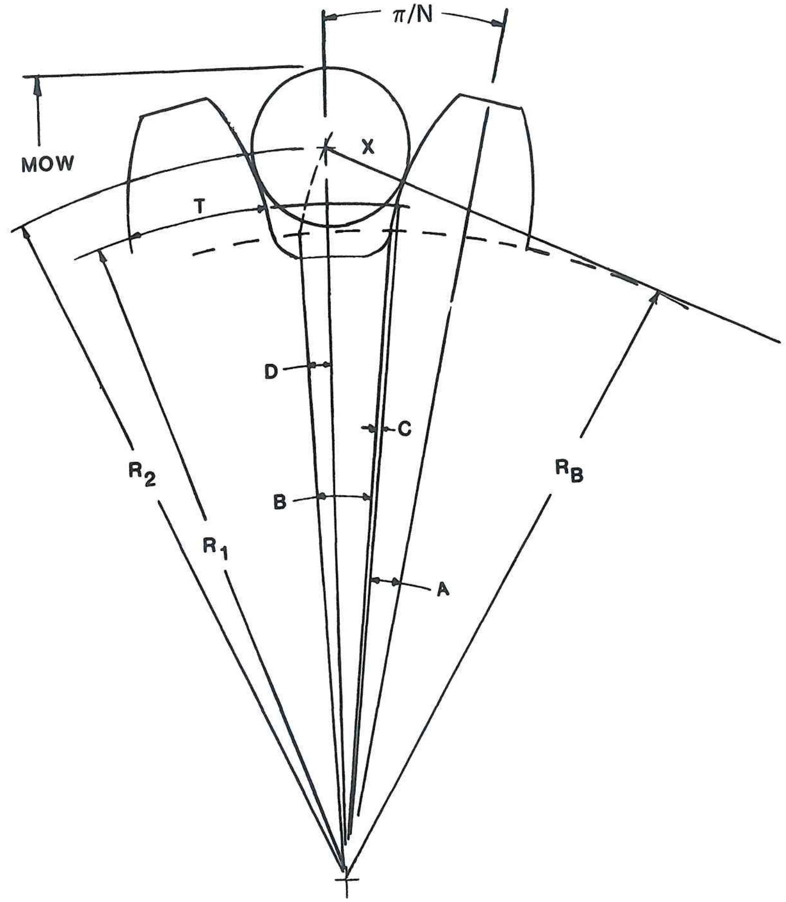


Figure 2.27 Measurement over wires of external spur gear.

The problem now is to calculate the angle ϕ_2. To accomplish this, an imaginary involute is drawn through the center of the wire as shown by the dashed profile in Figure 2.27. From this construction we can see that angle D is the involute of ϕ_2.

$$D = \text{Inv}\,\phi_2 = B + C + A - \frac{\pi}{N}$$

where

N = number of gear teeth
C = inv ϕ_1, which can be calculated knowing ϕ_1; Inv ϕ_1 = tan ϕ_1 - ϕ_1

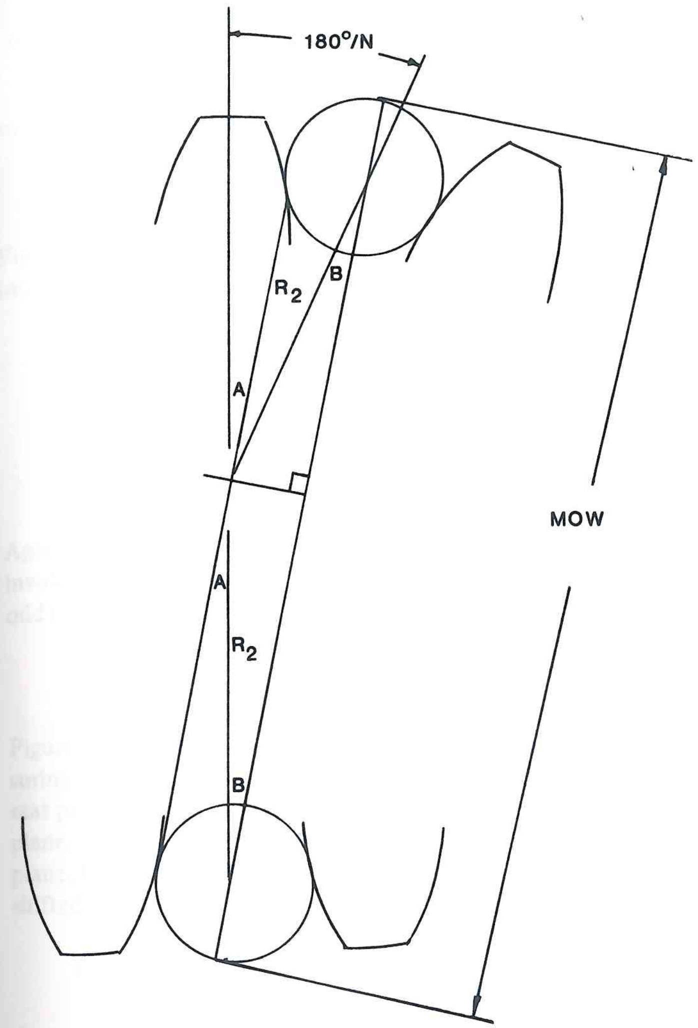


Figure 2.28 Measurement over wires for gear with odd number of teeth.

$$A = \frac{T}{2R_1}$$

$$B = \frac{X}{R_B}, \text{ where } R_B = \text{gear base radius}$$

$B = X/R_B$ because the circular distance between two involutes on the base circle is equal to the distance between normals to the involutes. In other words, the wire radius X is equal to the base pitch between the imaginary involute and the adjacent tooth involute. Knowing the involute of ϕ_2, the angle ϕ_2 can be calculated using the involute tables in the Appendix and then R_2 and the measurement over wires can be calculated.

When the gear has an odd number of teeth the situation is as shown in Figure 2.28 and the fact that the tooth spaces do not line up must be compensated for mathematically. In Figure 2.28:

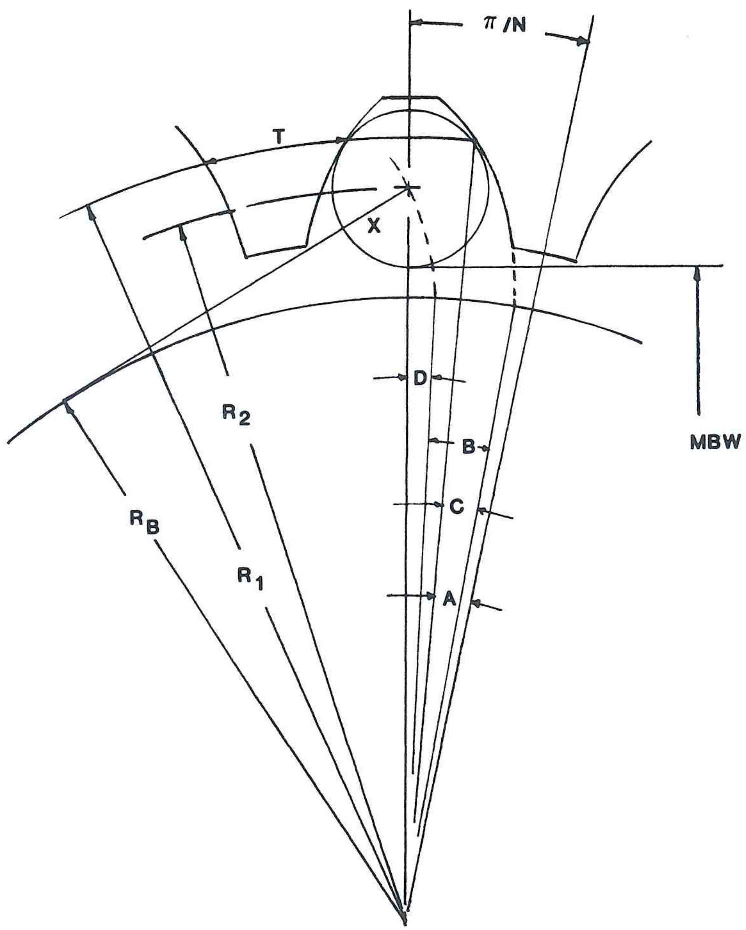


Figure 2.29 Measurement between wires of internal spur gear.

$$\measuredangle A = \measuredangle B = \frac{90^\circ}{N}$$

and

$$MOW = 2\left[R_2 \cos\left(\frac{90}{N}\right) + X\right]$$

Figure 2.29 illustrates the analysis for the measurement between wires for an integral gear with an even number of teeth:

$$MBW = 2R_2 - 2X$$

$$R_2 = R_1 \frac{\cos \phi_1}{\cos \phi_2}$$

$$\text{Inv } \phi_2 = D = \frac{\pi}{N} - B - A + C$$

Again an imaginary involute is drawn through the center of the wire and the involute ϕ_2 is calculated from which the angle ϕ_2 and R_2 can be derived. For odd numbers of teeth,

$$MBW = 2\left[R_2 \cos\left(\frac{90^\circ}{N}\right) - X\right]$$

Figure 2.30 illustrates a ball placed between two helical gear teeth. When measuring helical gears balls should be used rather than wires since the wires will not seat properly in the helices. The balls will contact the gear teeth in the normal plane, but the measurement over balls calculation must be made in the transverse plane. Figure 2.30 shows how the projection of the ball is mathematically shifted into the transverse plane and the equation for the involute ϕ_2 is

$$\text{Inv } \phi_2 = \frac{X}{R_B \cos \psi_B} + \text{Inv } \phi_1 + \frac{T}{2R_1} - \frac{\pi}{N}$$

for external gears and

$$\text{Inv } \phi_2 = - \frac{X}{R_B \cos \psi_B} + \text{Inv } \phi_1 - \frac{T}{2R_1} + \frac{\pi}{N}$$

for internal gears. It should be noted that the circular tooth thickness T is in the transverse plane. If a normal tooth thickness is given, it should be shifted to the transverse plane: $T = T_N/\cos \psi$.

To sum up, the measurement over or between balls or wires for gears with even numbers of teeth is

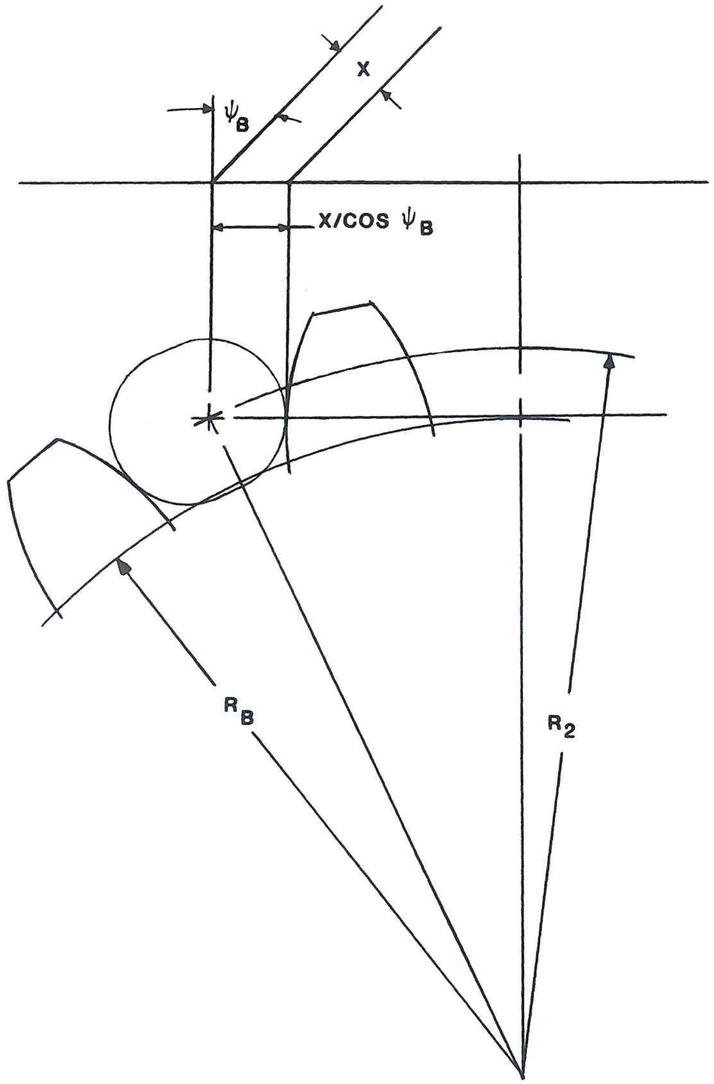


Figure 2.30 Measurement over balls of external helical gear.

$$MOW = 2(R_2 + X) \text{ for external gears}$$

$$MBW = 2(R_2 - X) \text{ for internal gears}$$

For gears with odd numbers of teeth,

$$MOW = 2\left[R_2 \cos\left(\frac{90}{N} + X\right)\right] \quad \text{for external gears}$$

$$MBW = 2\left[R_2 \cos\left(\frac{90}{N} - X\right)\right] \quad \text{for internal gears}$$

For all gears,

$$R_2 = R_1 \frac{\cos \phi_1}{\cos \phi_2}$$

To find $\cos \phi_2$,

$$\text{Inv}\,\phi_2 = \frac{X}{R_B \cos \psi_B} + \text{Inv}\,\phi_1 + \frac{T}{2R_1} - \frac{\pi}{N}$$

for external gears and

$$\text{Inv}\,\phi_2 = -\frac{X}{R_B \cos \psi_B} + \text{Inv}\,\phi_1 - \frac{T}{2R_1} + \frac{\pi}{N}$$

for internal gears. The cosine ϕ_2 is found from Inv ϕ_2 using involute tables. For spur gears, $\cos \psi_B = 0.0$.

When choosing ball or wire size a good estimate for the diameter is

$$D_{BALL} = 2X = \frac{1.728}{DP_N}$$

where

X = ball or wire radius, in.
DP_N = normal diametral pitch

Let us work through an example to illustrate the calculation for measurement over balls. Assume an external helical gear with the following dimensions:

Number of teeth	38
Normal diametral pitch	15.868103
Normal pressure angle	20.0°
Helix angle at pitch diameter	18.0°
Normal circular tooth thickness at pitch diameter, in.	0.0952

The transverse diametral pitch is

$$DP_T = 15.868103(\cos 18.0) = 15.091463$$

The transverse pressure angle is

$$\phi_T = \tan^{-1} \frac{\tan 20.0}{\cos 18.0} = 20.941896$$

The pitch diameter is

$$PD = \frac{38}{15.091463} = 2.517980$$

The base diameter is

$$BD = 2.517980(\cos 20.941896) = 2.351651$$

The lead is

$$L = \frac{\pi(2.517980)}{\tan 18} = 24.345915$$

The base helix angle is

$$\psi_B = \tan^{-1}\left(\frac{\pi(2.351651)}{24.345915}\right) = 16.880766°$$

The involute of the transverse pressure angle is

$$\text{Inv}\,\phi = \tan 20.941896 - 20.941896\left(\frac{\pi}{180}\right) = 0.017196$$

The transverse circular tooth thickness is

$$CTTT = \frac{0.0952}{\cos 18.0} = 0.100099$$

For a ball diameter of 0.125,

$$\text{Inv}\,\phi_2 = \frac{0.125/2}{(2.351651/2)\cos 16.880766} + 0.017196 + \frac{0.100099}{2.517980} - \frac{\pi}{38} = 0.029824$$

Using the involute tables (Appendix) yields

$$\phi_2 = 24.9599°$$

and

$$R_2 = \frac{(2.517980/2)\cos 20.941896}{\cos 24.9599} = \frac{2.593914}{2}$$

The measurement over balls for 0.125-in.-diameter balls is

$$MOB = 2\left(\frac{2.593914}{2} + \frac{0.125}{2}\right) = 2.7189$$

Measurement of Tooth Thickness by Calipers

Tooth thickness can be checked by measuring across several teeth with vernier calipers as shown on Figure 2.31. The calipers contact the teeth at points X and the line XX is tangent to the base circle. The arc AB along the base circle is equal to the length XX:

$$M = R_B\left(\frac{T}{R} + \frac{2\pi S}{N} + 2\,\text{Inv}\,\phi\right)$$

where

- M = caliper measurement, in.
- R_B = base circle radius, in.
- T = given transverse tooth thickness at a radius R, in.
- R = given radius, in.
- ϕ = given transverse pressure angle at radius R, in.
- S = number of tooth spaces between the contacting profiles
- N = number of teeth in the gear

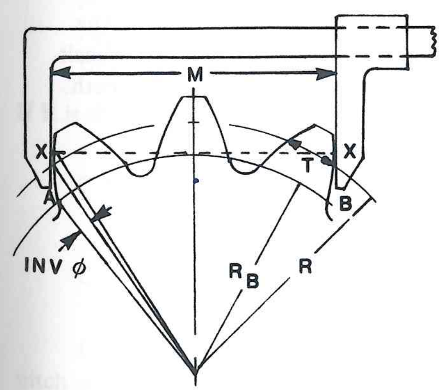


Figure 2.31 Tooth thickness measurement by vernier calipers.

For helical gears,

$$M = R_B \cos \psi_B \left(\frac{T}{R} + \frac{2\pi S}{N} + 2 \text{ Inv } \phi\right)$$

where ψ_B is the base helix angle in degrees.

Center Distance and Tooth Thickness

The center distance of a pair of meshing spur or helical gears is established by the location of the centers of the bearing bores locating the gear shafts. For the gears to mesh properly at a given center distance, the tooth thicknesses must be chosen such that the teeth will not bind under all operating conditions. The following variations in center distance must be taken into account when designing a gear set:

1. The center distance will vary due to tolerances in the bearing housings.
2. Clearance in the supporting bearings will affect the operating center distance.
3. Temperature variations during operation will change the operating center distance. At a minimum, expansion of the gear teeth must be taken into account. If dissimilar materials with varying coefficients of expansion are used in the gearbox, their thermal growths must be analyzed.

In order to accommodate all these variables, backlash is designed into the gear mesh. Backlash can be defined as the circular pitch minus the sum of the circular tooth thicknesses:

$$BL = CP - (T_P + T_G)$$

In most cases excessive backlash will not be harmful and is much more desirable than too little backlash, which can result in tight meshing and binding of the gears. In very high speed helical gearing it is important to have sufficient backlash to allow the air-oil mixture being pumped between the teeth to exit the mesh without becoming excessively churned and heated. The amount of backlash designed into a gear mesh will vary with the diametral pitch of the teeth. Following is a table of suggested backlash versus pitch.

Diametral pitch	Nominal backlash, (in.)
6	0.015
8 and 10	0.010
12 and 14	0.009
16	0.008
18	0.007
20	0.006

Excessive backlash may be detrimental if the transmitted load varies to the extent that the tooth can become unloaded and contact on the normally unloaded face. In this case the more backlash there is in the mesh, the greater freedom the teeth will have to rattle around and the greater dynamic load generated. There are also cases where gears are used as positioning devices and backlash is detrimental. In such designs special techniques such as adjustable center distance are used to achieve low backlash.

Center distance can be expressed mathematically in various ways:

$$C = \frac{PD_P + PD_G}{2} \quad \text{for external gears}$$

$$C = \frac{PD_G - PD_P}{2} \quad \text{for internal gears}$$

where

C = center distance, in.
PD_P = pinion pitch diameter, in.
PD_G = gear pitch diameter, in.

Since $PD_P/N_P = PD_G/N_G = DP_T$,

$$C = \frac{N_P + N_G}{2DP_T} \quad \text{for external gears}$$

$$C = \frac{N_G - N_P}{2DP_T} \quad \text{for internal gears}$$

where

N_P = number of pinion teeth
N_G = number of gear teeth
DP_T = transverse diametral pitch

If R is the gear ratio PD_G/PD_P,

$$C = \frac{PD_P(1 + R)}{2} \quad \text{for external gears}$$

$$C = \frac{PD_P(R - 1)}{2} \quad \text{for internal gears}$$

Let us look at a so-called standard spur gearset, where a standard diametral pitch cutter is used:

$$DP_T = 10.0$$
$$\phi_{T1} = 20^\circ$$
$$N_P = 20$$
$$N_G = 30$$
$$C_1 = \frac{20+30}{2(10)} = 2.5$$

If the backlash is 0.010,

$$T_{P1} = T_{G1} = \frac{CP}{2} - \frac{BL}{2} = \frac{\pi}{(10)2} - \frac{0.010}{2} = 0.152080$$

where

T_{P1} = pinion transverse circular tooth thickness at the pitch radius R_{P1}
T_{G1} = gear transverse circular tooth thickness at the pitch radius R_{G1}

Let us calculate at what center distance C_2 the mesh will have zero backlash. At this point of tight mesh or binding,

$$CP = T_{P2} + T_{G2} = \frac{2\pi R_{P2}}{N_P} = \frac{2\pi R_{G2}}{N_G}$$

where

CP = transverse circular pitch, in.
T_{P2} = pinion transverse circular tooth thickness at the tight mesh pitch radius R_{P2}, in.
T_{G2} = gear transverse circular tooth thickness at the tight mesh pitch radius R_{G2}, in.

As shown previously:

$$T_{P2} = 2R_{P2}\left(\frac{T_{P1}}{2R_{P1}} + \text{Inv}\,\phi_1 - \text{Inv}\,\phi_2\right)$$

$$T_{G2} = 2R_{G2}\left(\frac{T_{G1}}{2R_{G1}} + \text{Inv}\,\phi_1 - \text{Inv}\,\phi_2\right)$$

Also,

$$\frac{N_G}{N_P} = \frac{R_{G1}}{R_{P1}} = \frac{R_{G2}}{R_{P2}}$$

Combining the four equations above, we have

$$\text{Inv}\,\phi_2 = \frac{N_P(T_{P1} + T_{G1}) - 2R_{P1}\pi}{2R_{P1}(N_P + N_G)} + \text{Inv}\,\phi_1$$

$$\frac{\cos\phi_2}{\cos\phi_1} = \frac{R_{P1}}{R_{P2}} = \frac{R_{G1}}{R_{G2}}$$

and the tight mesh center distance $C_2 = C_1 \cos\phi_1/\cos\phi_2$. For the example above,

$$\text{Inv}\,\phi_2 = 0.012904$$
$$\phi_2 = 19.0910^\circ$$
$$C_2 = 2.4860$$

For internal gears the equation for T_{G2} is

$$T_{G2} = 2R_{G2}\left(\frac{T_{G1}}{2R_{G1}} - \text{Inv}\,\phi_1 + \text{Inv}\,\phi_2\right)$$

and

$$\text{Inv}\,\phi_2 = \frac{N_P(T_{P1} + T_{G1}) - 2R_{P1}\pi}{2R_{P1}(N_P - N_G)} + \text{Inv}\,\phi_1$$

In this case T_{G1} and T_{G2} are circular tooth thicknesses of the internal gear.

The situation is somewhat more complicated for helical gears since standard diametral pitch, pressure angle, and tooth thickness are defined in the normal plane, yet the calculations are carried out in the transverse plane. To work helical gear problems, all normal values must be transferred to the transverse plane prior to calculating. Quite often a series of gears is designed to achieve different ratios on the same center distance. Let us look at a helical gear example to illustrate the mathematics involved.

An electric motor operating at 3550 rpm drives a compressor. It is designed to operate the compressor at two different speeds, 33,897 and 31,842 rpm. With a 296-tooth gear driving, a 31-tooth pinion will achieve 33,897 rpm and a 33-tooth pinion will achieve 31,842 rpm. The idea is to use the same gear, housing, bearings, and so on, for both ratios, only changing the pinion to achieve either ratio. Also, it is desired to use 20 diametral pitch, 20° pressure angle cutters for all gears. The center distance is chosen by stress considerations as 8.4780 in. In order to encompass both the 31- and 33-tooth pinion designs, let us first calculate the gear geometry for a 32-tooth pinion design. The same gear will then mesh with the 31- and 33-tooth pinions.

The transverse diametral pitch of the 296 X 32 design is

$$DP_T = \frac{296+32}{2(8.4780)} = 19.34418495$$

The gear pitch diameter is

$$PD_G = \frac{296}{19.34418495} = 15.3017561$$

In order to use a standard 20 normal diametral pitch cutter the helix angle is

$$\psi = \cos^{-1} \frac{19.34418495}{20} = 14.71320405°$$

The gear lead is

$$L_G = \frac{\pi(15.3017561)}{\tan 14.71320405} = 183.0672333$$

For a normal pressure angle of 20° the transverse pressure angle is

$$\phi_T = \tan^{-1} \frac{\tan 20}{\cos 14.71320405} = 20.62180626°$$

For a standard gearset the pinion and gear outside diameter would be set by using a standard addendum of 1/DP; therefore, the gear outside diameter would be

$$OD_G = 15.3017561 + \frac{2}{20} = 15.402$$

When a large gear is meshing with a small pinion it is conventional to increase the pinion addendum and decrease the gear addendum, resulting in what is commonly called a long and short addendum design. In the example the pinion addendum is 0.0629 and the gear addendum is 0.0405. These values are arrived at using two criteria:

1. The degrees of roll to the form diameter of the pinion in the 296 X 33 mesh must be high enough to avoid undercutting of the pinion. Undercutting occurs when the gear tooth tip describes an arc through space that would cut through the active profile of the pinion. In other words, the trochoid generated by the gear tooth tip would interfere with the pinion involute above the pinion form diameter.
2. The addendums are varied such that the temperature rise in the mesh due to sliding is minimized. This subject is discussed in Chapter 3.

It should be noted that the pinion addendum is lengthened by the same amount the gear addendum is shortened. Because of the long and short addendum design standard tooth thicknesses cannot be used, since this would result in an imbalance of bending strength between the pinion and gear, the pinion being weakened. Assuming a backlash in the mesh of 0.006, the standard transverse circular tooth thicknesses for pinion and gear would be

$$CTTT_P = CTTT_G = \frac{CP}{2} - \frac{BL}{2} = \frac{\pi}{2(19.34418495)} - \frac{0.006}{2}$$

$$= 0.078203$$

The optimized transverse circular tooth thicknesses for the 296 X 32 mesh ar are:

$$CTTT_P = 0.09072$$

$$CTTT_G = 0.06569$$

Now let us look at the 296 X 31 tooth mesh. The transverse diametral pitch is

$$DP_T = \frac{296 + 31}{2(8.4780)} = 19.28520878$$

and the gear pitch diameter is

$$PD_G = \frac{296}{19.28520878} = 15.34855046$$

The helix angle at this diameter is

$$\psi = \tan^{-1} \left(\frac{\pi(15.34855046)}{183.0672333}\right) = 14.75623793°$$

and the transverse pressure angle is

$$\phi_T = \cos^{-1} \left(\frac{\cos 20.62180626(15.3017561)}{15.34855046}\right) = 21.08111697°$$

Knowing the gear transverse circular tooth thickness at the 15.3017561 diameter (0.06569), we can calculate the gear transverse circular tooth thickness at the 15.34855046 diameter:

$$CTTT_G = 0.0480391$$

and to achieve a backlash of 0.006 the pinion transverse circular tooth thickness is

$$CTTT_P = \frac{\pi}{19.28520878} - 0.0480391 - 0.006 = 0.10886192$$

Since we cannot optimize the addendums for both the 296 X 31 and 296 X 33 tooth meshes we will use the pinion outside diameter calculated for the 296 X 32 mesh 1.780 for the 31- and 33-tooth pinions. Figures 2.32 and 2.33 are computer printouts giving all the tooth geometry for the 296 X 31 and 296 X 33 meshes.

296 X 31 MESH

AMERICAN LOHMANN GEAR

	DRIVEN PINION	DRIVER EXTERNAL GEAR
NUMBER OF TEETH	31.0000000	296.0000000
HELIX ANGLE (DEG)	14.7562380	14.7562380
PITCH DIAMETER	1.6074495	15.3485505
RELATIVE ROLLING SPEED (RPM)	33897.0000000	3550.0236486
MESH TORQUE (IN-LBS)	316.0825442	3018.0784863
BENDING GEOMETRY FACTOR	0.5000000	0.5000000
BENDING STRESS (PSI)	19677.4648288	19677.4648288
BENDING LIFE (HOURS)	999999.0000000	999999.0000000
BENDING SAFETY FACTOR	2.2360604	2.2360604
COMPRESSIVE STRESS (PSI)	84528.9062052	84528.9062052
COMPRESSIVE LIFE (HOURS)	999999.0000000	999999.0000000
COMPRESSIVE SAFETY FACTOR	1.6089171	1.6089171
SLIDING VELOCITY AT TIP (FPM)	3728.8957744	-931.9806035
A.G.M.A. MATERIAL GRADE	1.0000000	1.0000000
ALTERNATING BENDING FACTOR	1.0000000	1.0000000
NUMBER OF MESHES PER REV	1.0000000	1.0000000
BASE HELIX ANGLE (DEG)	13.8076961	13.8076961
OUTSIDE DIAMETER	1.777- 1.780	15.380-15.383
PITCH DIAMETER	1.6074495	15.3485505
FORM DIAMETER	1.5757543	15.2158791
BASE DIAMETER	1.4998664	14.3213053
ROOT DIAMETER	1.521- 1.531	15.124-15.134
ROLL ANGLE-MAX OUTSIDE DIA	36.6168247	22.4672304
ROLL ANGLE-ROUND EDGE DIA	36.0681676	22.3643649
ROLL ANGLE-HIGH SINGLE TOOTH	30.0682702	21.7814020
ROLL ANGLE-PITCH DIA	22.0869009	22.0869009
ROLL ANGLE-LOW SINGLE TOOTH	25.0039227	21.2510143
ROLL ANGLE-FORM DIAMETER	18.4553682	20.5651859
TOP LAND THICKNESS	0.0264463	0.0347370
MAX CASE DEPTH	0.0241853	0.0261853
TRANSVERSE CIR TOOTH THICKNESS	0.1088630	0.0480387
NORMAL CIR TOOTH THICKNESS	0.1048- 0.1058	0.0460- 0.0470
NORMAL DIAMETRAL PITCH	19.9429632	19.9429632
NORMAL PRESSURE ANGLE	20.4441987	20.4441987
LEAD	19.1725861	183.0672733
ROUND EDGE RADIUS MAX	0.0050000	0.0050000
ROOT FILLET RADIUS MIN	0.0201191	0.0206911
WHOLE DEPTH CONSTANT	2.4000000	2.4000000
CLEARANCE AT TIP OF TOOTH	0.0209483	0.0209483
BALL DIAMETER	0.1250000	0.1250000
MEASUREMENT OVER BALLS	1.9000- 1.9018	15.5315-15.5341
BALL CONTACT DIAMETER	1.7152- 1.7169	15.3622-15.3647
DIM OVER TOP LAND	0.0600097	0.0742515

TRANSVERSE DIAMETRAL PITCH	19.2852[illegible]8
TRANSVERSE PRESSURE ANGLE(DEG)	21.0811130
CENTER DISTANCE	8.4780[illegible]0
PITCH LINE VELOCITY (FPM)	14264.8838538
MESH RATIO	9.5483871
RELATIVE HORSEPOWER PER MESH	170.0000[illegible]0
EFFECTIVE FACE WIDTH	1.6250000
STATIC TANGENTIAL LOAD (LBS)	393.2721197
DYNAMIC FACTOR	1.5349[illegible]1
ALIGNMENT FACTOR	1.3733637
MODIFIED TANGENTIAL LOAD (LBS)	829.0260[illegible]4
SURFACE FINISH	20.0000[illegible]0
FLASH TEMPERATURE RISE (DEG F)	72.9815383
PROFILE CONTACT RATIO	1.5640[illegible]3
FACE CONTACT RATIO	2.6274494
MIN CONTACT LENGTH	2.5137[illegible]4
MAXIMUM CONTACT LENGTH	2.7510[illegible]5
BACKLASH	0.0060[illegible]0
CIRCULAR PITCH	0.1629[illegible]7
LEAD ERROR (IN/IN)	0.0002000
BASE PITCH	0.1519[illegible]0
DEPTH TO POINT OF MAX SHEAR	-0.0027[illegible]4

Figure 2.32 A 296 X 31 tooth computer output sheet. (Courtesy of American Lohmann Corporation, Hillside, N.J.)

296 X 33 MESH

AMERICAN LOHMANN GEAR

	DRIVEN PINION	DRIVER EXTERNAL GEAR
NUMBER OF TEETH	33.0000000	296.0000000
HELIX ANGLE (DEG)	14.6704150	14.6704150
PITCH DIAMETER	1.7007538	15.2552462
RELATIVE ROLLING SPEED (RPM)	31842.0000000	3549.9527027
MESH TORQUE (IN-LBS)	296.8956096	2663.0636495
BENDING GEOMETRY FACTOR	0.5000000	0.5000000
BENDING STRESS (PSI)	19220.8571549	19220.8571549
BENDING LIFE (HOURS)	999999.0000000	999999.0000000
BENDING SAFETY FACTOR	2.2891799	2.2891799
COMPRESSIVE STRESS (PSI)	75092.5963251	75092.5963251
COMPRESSIVE LIFE (HOURS)	999999.0000000	999999.0000000
COMPRESSIVE SAFETY FACTOR	1.8110973	1.8110973
SLIDING VELOCITY AT TIP (FPM)	1861.7184171	-3335.9694645
A.G.M.A. MATERIAL GRADE	1.0000000	1.0000000
ALTERNATING BENDING FACTOR	1.0000000	1.0000000
NUMBER OF MESHES PER REV	1.0000000	1.0000000
BASE HELIX ANGLE (DEG)	13.8076960	13.8076960
OUTSIDE DIAMETER	1.777- 1.780	15.380-15.383
PITCH DIAMETER	1.7007538	15.2552462
FORM DIAMETER	1.6125344	15.1871940
BASE DIAMETER	1.5966320	14.3213052
ROOT DIAMETER	1.523- 1.533	15.126-15.136
ROLL ANGLE-MAX OUTSIDE DIA	28.2371869	22.4672316
ROLL ANGLE-ROUND EDGE DIA	27.5242396	22.3643661
ROLL ANGLE-HIGH SINGLE TOOTH	19.0158108	21.4391728
ROLL ANGLE-PITCH DIA	21.0268157	21.0268157
ROLL ANGLE-LOW SINGLE TOOTH	17.3280971	21.2510155
ROLL ANGLE-FORM DIAMETER	8.1067210	20.2229567
TOP LAND THICKNESS	0.0415479	0.0347377
MAX CASE DEPTH	0.0321538	0.0261538
TRANSVERSE CIR TOOTH THICKNESS	0.0731610	0.0827504
NORMAL CIR TOOTH THICKNESS	0.0703- 0.0713	0.0796- 0.0806
NORMAL DIAMETRAL PITCH	20.0570477	20.0570477
NORMAL PRESSURE ANGLE	19.5460270	19.5460270
LEAD	20.4095271	183.0672734
ROUND EDGE RADIUS MAX	0.0050000	0.0050000
ROOT FILLET RADIUS MIN	0.0241296	0.0197955
WHOLE DEPTH CONSTANT	2.4000000	2.4000000
CLEARANCE AT TIP OF TOOTH	0.0201918	0.0201918
BALL DIAMETER	0.1250000	0.1250000
MEASUREMENT OVER BALLS	1.9326- 1.9346	15.5315-15.5341
BALL CONTACT DIAMETER	1.7558- 1.7576	15.3622-15.3647
DIM OVER TOP LAND	0.0762863	0.0742512

TRANSVERSE DIAMETRAL PITCH	19.4031611
TRANSVERSE PRESSURE ANGLE(DEG)	20.1524[illegible]0
CENTER DISTANCE	8.4790[illegible]0
PITCH LINE VELOCITY (FPM)	14177.8838[illegible]90
MESH RATIO	8.9696[illegible]0
RELATIVE HORSEPOWER PER MESH	150.0000000
EFFECTIVE FACE WIDTH	1.6250[illegible]
STATIC TANGENTIAL LOAD (LBS)	349.1341[illegible]4
DYNAMIC FACTOR	1.5316706
ALIGNMENT FACTOR	1.5051[illegible]0
MODIFIED TANGENTIAL LOAD (LBS)	804.8660[illegible]6
SURFACE FINISH	20.0000000
FLASH TEMPERATURE RISE (DEG F)	78.9349[illegible]
PROFILE CONTACT RATIO	1.8454403
FACE CONTACT RATIO	2.6274494
MIN CONTACT LENGTH	3.0514[illegible]0
MAXIMUM CONTACT LENGTH	3.1498422
BACKLASH	0.0060[illegible]0
CIRCULAR PITCH	0.1619114
LEAD ERROR (IN/IN)	0.0002[illegible]
BASE PITCH	0.1519[illegible]0
DEPTH TO POINT OF MAX SHEAR	0.0026780

Figure 2.33 A 296 X 33 tooth computer output sheet. (Courtesy of American Lohmann Corporation, Hillside, N.J.)

ENGINEERING DRAWING FORMAT

The engineering drawing must contain sufficient information to define the component completely so that the manufacturing department can fabricate it and the quality control department can inspect it. There are several elements that should appear on the field of a gear drawing:

1. Gear blank features are usually shown in an end view and cross section, as illustrated in Figure 2.34. It is important to specify the reference surfaces that will locate the gear in the application. For instance, the gear in Figure 2.34 will be pressed onto a shaft; therefore, the surface C locates the gear in the assembly and the gear tooth geometry must be accurate with respect to this surface. Sides A and B must be parallel to each other according to the drawing, and surface C perpendicular to A and B. The surface finish is designated by the ⁄ symbol. The end view shows a gear tooth and calls out which face of the gear tooth is loaded. The X's indicate to how many decimal places a dimension is given.
2. A close-up view of the gear teeth as shown in Figure 2.35 defines the outside, pitch, form, and root diameters. It also calls out the roughness in the root area and in the area between the form diameter and the outside diameter: the active profile. The maximum undercut allowed in the root fillet

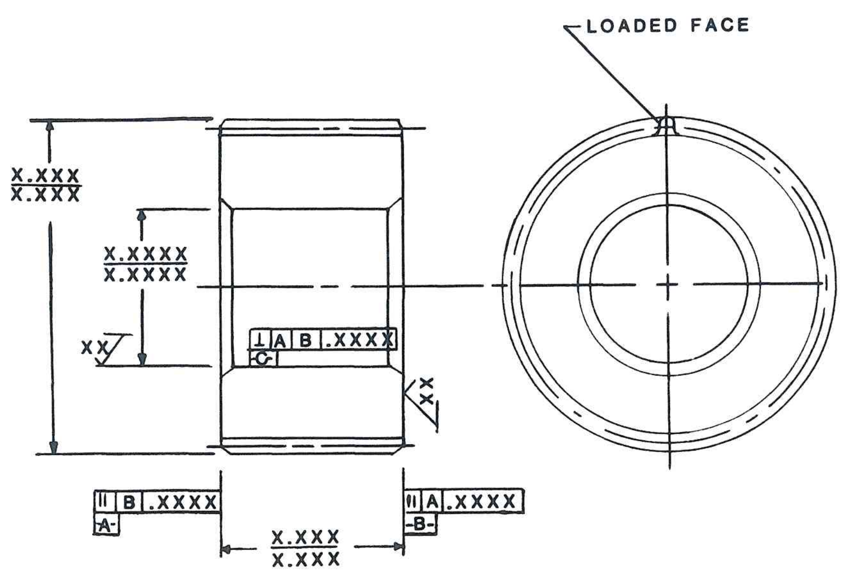


Figure 2.34 Gear blank dimensioning.

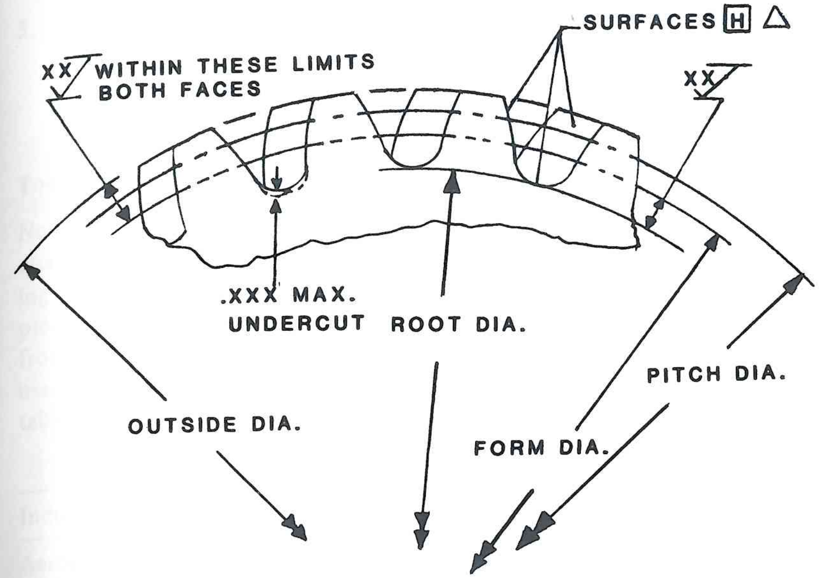


Figure 2.35 View of gear teeth.

area is defined. If the teeth are surface-hardened by processes such as carburizing or nitriding, the areas to be hardened are designated as surfaces [H] followed by a triangle, which refers to a note that defines the case hardness and depth. Note that in the illustration the top lands and tooth ends are not hardened. Some designers prefer to harden these areas and therefore would point to them in this view.

3. The tooth edges at the top land and the ends must be rounded, and Figure 2.36 illustrates how the radii of the tooth tips and edges are defined.
4. The gear material and its heat treatment must be specified and this is usually done in a block of data on the lower right-hand side of the drawing. As an illustration the callout for a carburized gear follows:

Material: AMS 6265 forging
Carburize surfaces [H]
Effective case depth of finished gear: 0.035 to 0.050
Case hardness: R_c 60 to 63
Core hardness: R_c 32 to 40
Per specification xxxx
Surface temper inspection per specification xxxx
Magnetic particle inspection per specification xxxx

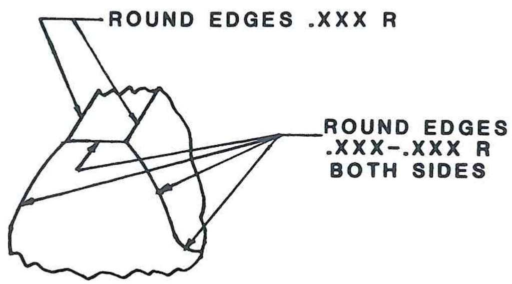


Figure 2.36 Round edge definition.

GEAR DATA		
METHOD OF MANUFACTURE		
NO. OF TEETH		
HELIX ANGLE	XX.XXXX°	
HAND OF HELIX		
LEAD OF HELIX	XXX.XXXX	
NORMAL DIAMETRAL PITCH	XX.XXXX	
NORMAL PRESSURE ANGLE	XX.XXXX°	
NORMAL CIRC. TOOTH THICK.	.XXXX/.XXXX	
PITCH DIAMETER	XX.XXXX	⊚ C .XXXX
ROOT DIAMETER	XX.XXX/XX.XXX	
FORM DIAMETER, MAX.	XX.XXXX	
WIRE OR BALL DIAMETER	.XXXX	
MEAS. OVER WIRE OR BALLS	XX.XXXX/XX.XXXX	
AGMA QUALITY NUMBER		
RUN OUT TOL.	.XXX	
PITCH TOL.	.XXXX	
PROFILE TOL.	SEE DIAGRAM	
LEAD TOL.	SEE DIAGRAM	
MATING GEAR PART NUMBER		
BACKLASH WITH MATING GEAR		

Figure 2.37 Gear data block.

5. Figure 2.37 is one form of a gear data block which is applicable to both spur and helical gears. In this format all data are given at the operating pitch diameter. Note that the pitch diameter must be concentric to surface C shown in Figure 2.34.

Tooth Tolerances

Note that the gear data form (Figure 2.37) has a line which calls out the AGMA quality number. AGMA Standard 390.03 [1] specifies quality numbers identifying specific tooth element tolerances. The higher the quality number, the more precise the gearing will be and the closer the tolerances. Quality numbers range from 3 to 15 and the standard contains a tabulation of many industrial and end use applications and suggested quality number ranges for each. The following table lists some sample applications:

Industry	Quality number
Aerospace engines	10-13
Agriculture	3-7
Automotive	10-11
Mining	5-8
Steel	5-6

In the industries cited above and in other applications, when high-speed drives are required or there are special considerations such as noise abatement, higher quality numbers may be called for. It should be noted that quality classes 13, 14, and 15 are extremely difficult to achieve and prior to requiring these classes there should be agreement between the manufacturer and user as to the method of inspection.

The majority of critical industrial applications in fields such as the process industries and turbomachinery will require gear units with elements that fall into the quality number range 10 to 13. Table 2.1 presents the tolerances for these classes. Following is a definition of each tooth tolerance element shown in Table 2.1:

1. *Runout tolerance.* The variation of the pitch diameter in a direction perpendicular to the axis of rotation with respect to a reference surface of revolution such as a bearing journal or a bore. The pitch diameter, being theoretical, must be indirectly measured and this can be done several ways. Two widely used methods are:
   a. Runout can be measured by indicating the position of a ball probe in successive teeth (see Figure 2.38).

Table 2.1 Pitch Gear Tolerances (in ten-thousandths of an inch)

AGMA QUALITY NUMBER	NORMAL DIAMETRAL PITCH	RUNOUT TOLERANCE									
		PITCH DIAMETER (INCHES)									
		3/4	1½	3	6	12	25	50	100	200	400
8	1/2					146.5	174.5	205.8	242.7	286.3	337.6
	1				88.8	104.8	124.8	147.2	173.6	204.7	241.4
	2			53.9	63.5	74.9	89.2	105.2	124.1	146.3	172.6
	4		32.7	38.5	45.4	53.6	63.8	75.2	88.7	104.6	123.4
	8	19.8	23.3	27.5	32.5	38.3	45.6	53.8	63.4	74.8	88.2
	12	16.3	19.2	22.6	26.7	31.5	37.5	44.2	52.1	61.5	72.5
	20	12.7	15.0	17.7	20.8	24.6	29.3	34.5	40.7	48.0	56.6
9	1/2					104.7	124.7	147.0	173.4	204.5	241.2
	1				63.5	74.8	89.1	105.1	124.0	146.2	172.4
	2			38.5	45.4	53.5	63.7	75.2	88.6	104.5	123.3
	4		23.3	27.5	32.4	38.3	45.6	53.7	63.4	74.7	88.1
	8	14.1	16.7	19.7	23.2	27.4	32.6	38.4	45.3	53.4	63.0
	12	11.6	13.7	16.2	19.1	22.5	26.8	31.6	37.2	43.9	51.8
	20	9.1	10.7	12.6	14.9	17.6	20.9	24.7	29.1	34.3	40.4
10	1/2					74.8	89.0	105.0	123.8	146.1	172.3
	1				45.3	53.5	63.7	75.1	88.5	104.4	123.2
	2			27.5	32.4	38.2	45.5	53.7	63.3	74.7	88.1
	4		16.7	19.6	23.2	27.3	32.5	38.4	45.3	53.4	63.0
	8	10.1	11.9	14.0	16.6	19.5	23.3	27.4	32.4	38.2	45.0
	12	8.3	9.8	11.5	13.6	16.1	19.1	22.6	26.6	31.4	37.0
	20	6.5	7.6	9.0	10.6	12.5	14.9	17.6	20.8	24.5	28.9
11	1/2					53.4	63.6	75.0	88.5	104.3	123.0
	1				32.4	38.2	45.5	53.6	63.2	74.6	88.0
	2			19.6	23.1	27.3	32.5	38.3	45.2	53.3	62.9
	4		11.9	14.0	16.6	19.5	23.2	27.4	32.3	38.1	45.0
	8	7.2	8.5	10.0	11.8	14.0	16.6	19.6	23.1	27.3	32.2
	12	5.9	7.0	8.2	9.7	11.5	13.7	16.1	19.0	22.4	26.4
	20	4.6	5.5	6.4	7.6	9.0	10.7	12.6	14.8	17.5	20.6
12	1/2					38.1	45.4	5.36	63.2	74.5	87.9
	1				23.1	27.3	32.5	38.3	45.2	53.3	62.8
	2			14.0	16.5	19.5	23.2	27.4	32.3	38.1	44.9
	4		8.5	10.0	11.8	13.9	16.6	19.6	23.1	27.2	32.1
	8	5.2	6.1	7.2	8.5	10.0	11.9	14.0	16.5	19.5	23.0
	12	4.2	5.0	5.9	6.9	8.2	9.8	11.5	13.6	16.0	18.9
	20	3.3	3.9	4.6	5.4	6.4	7.6	9.0	10.6	12.5	14.7

AGMA QUALITY NUMBER	NORMAL DIAMETRAL PITCH	PITCH TOLERANCE									
		PITCH DIAMETER (INCHES)									
		3/4	1½	3	6	12	25	50	100	200	400
8	1/2					19.0	21.7	24.5	27.7	31.3	35.4
	1				14.4	16.3	18.6	21.0	23.7	26.8	30.3
	2			10.9	12.3	14.0	15.9	18.0	20.3	23.0	26.0
	4		8.3	9.3	10.6	11.9	13.6	15.4	17.4	19.7	22.2
	8	6.3	7.1	8.0	9.0	10.2	11.7	13.2	14.9	16.8	19.0
	12	5.7	6.5	7.3	8.3	9.3	10.6	12.0	13.6	15.4	17.4
	20	5.1	5.8	6.5	7.4	8.3	9.5	10.7	12.1	13.7	15.5
9	1/2					13.4	15.3	17.3	19.5	22.1	24.9
	1				10.2	11.5	13.1	14.8	16.7	18.9	21.4
	2			7.7	8.7	9.8	11.2	12.7	14.3	16.2	18.3
	4		5.8	6.6	7.4	8.4	9.6	10.8	12.2	13.8	15.7
	8	4.4	5.0	5.6	6.4	7.2	8.2	9.3	10.5	11.9	13.4
	12	4.0	4.6	5.1	5.8	6.6	7.5	8.5	9.6	10.8	12.2
	20	3.6	4.1	4.6	5.2	5.9	6.7	7.6	8.5	9.7	10.9
10	1/2					9.4	10.8	12.2	13.7	15.5	17.6
	1				7.2	8.1	9.2	10.4	11.8	13.3	15.0
	2			5.4	6.1	6.9	7.9	8.9	10.1	11.4	12.9
	4		4.1	4.6	5.2	5.9	6.7	7.6	8.6	9.8	11.0
	8	3.1	3.5	4.0	4.5	5.1	5.8	6.5	7.4	8.3	9.4
	12	2.8	3.2	3.6	4.1	4.6	5.3	6.0	6.7	7.6	8.6
	20	2.5	2.9	3.2	3.7	4.1	4.7	5.3	6.0	6.8	7.7
11	1/2					6.6	7.6	8.6	9.7	10.9	12.4
	1				5.0	5.7	6.5	7.3	8.3	9.4	10.6
	2			3.8	4.3	4.9	5.6	6.3	7.1	8.0	9.1
	4		2.9	3.3	3.7	4.2	4.8	5.4	6.1	6.9	7.8
	8	2.2	2.5	2.8	3.2	3.6	4.1	4.6	5.2	5.9	6.6
	12	2.0	2.3	2.6	2.9	3.3	3.7	4.2	4.7	5.4	6.1
	20	1.8	2.0	2.3	2.6	2.9	3.3	3.7	4.2	4.8	5.4
12	1/2					4.7	5.3	6.0	6.8	7.7	8.7
	1				3.5	4.0	4.6	5.2	5.8	6.6	7.5
	2			2.7	3.0	3.4	3.9	4.4	5.0	5.6	6.4
	4		2.0	2.3	2.6	2.9	3.3	3.8	4.3	4.8	5.5
	8	1.5	1.7	2.0	2.2	2.5	2.9	3.2	3.7	4.1	4.7
	12	1.4	1.6	1.8	2.0	2.3	2.6	3.0	3.3	3.8	4.3
	20	1.3	1.4	1.6	1.8	2.0	2.3	2.6	3.0	3.4	3.8

AGMA QUALITY NUMBER	NORMAL DIAMETRAL PITCH	PROFILE TOLERANCE										LEAD TOLERANCE				
		PITCH DIAMETER (INCHES)										FACE WIDTH (INCHES)				
		3/4	1½	3	6	12	25	50	100	200	400	1 and Less	2	3	4	5
8	1/2					42.6	47.7	53.1	59.1	65.7	73.1					
	1				28.3	31.5	35.3	39.3	43.7	48.6	54.1					
	2			18.8	21.0	23.3	26.1	29.0	32.3	36.0	40.0					
	4		12.5	13.9	15.5	17.2	19.3	21.5	23.9	26.6	29.6	5	8	11	13	16
	8	8.3	9.3	10.3	11.5	12.8	14.3	15.9	17.7	19.7	21.9					
	12	7.0	7.8	8.6	9.6	10.7	12.0	13.3	14.8	16.5	18.4					
	20	5.6	6.2	6.9	7.7	8.6	9.6	10.7	11.9	13.2	14.7					
9	1/2					30.4	34.1	37.9	42.2	46.9	52.2					
	1				20.2	22.5	25.2	28.1	31.2	34.7	38.6					
	2			13.5	15.0	16.7	18.6	20.7	23.1	25.7	28.6					
	4		8.9	10.0	11.1	12.3	13.8	15.3	17.1	19.0	21.1	4	7	9	11	13
	8	5.9	6.6	7.4	8.2	9.1	10.2	11.4	12.6	14.1	15.6					
	12	5.0	5.5	6.2	6.9	7.6	8.6	9.5	10.6	11.8	13.1					
	20	4.0	4.4	4.9	5.5	6.1	6.8	7.6	8.5	9.4	10.5					
10	1/2					21.7	24.3	27.1	30.1	33.5	37.3					
	1				14.5	16.1	18.0	20.0	22.3	24.8	27.6					
	2			9.6	10.7	11.9	13.3	14.8	16.5	18.3	20.4					
	4		6.4	7.1	7.9	8.8	9.9	11.0	12.2	13.6	15.1	3	5	7	9	10
	8	4.2	4.7	5.3	5.9	6.5	7.3	8.1	9.0	10.0	11.2					
	12	3.6	4.0	4.4	4.9	5.5	6.1	6.8	7.6	8.4	9.4					
	20	2.9	3.2	3.5	3.9	4.4	4.9	5.4	6.1	6.7	7.5					
11	1/2					15.5	17.4	19.3	21.5	24.0	26.7					
	1				10.3	11.5	12.9	14.3	15.9	17.7	19.7					
	2			6.9	7.6	8.5	9.5	10.6	11.8	13.1	14.6					
	4		4.6	5.1	5.6	6.3	7.0	7.8	8.7	9.7	10.8	3	4	6	7	8
	8	3.0	3.4	3.8	4.2	4.6	5.2	5.8	6.4	7.2	8.0					
	12	2.5	2.8	3.1	3.5	3.9	4.4	4.9	5.4	6.0	6.7					
	20	2.0	2.3	2.5	2.8	3.1	3.5	3.9	4.3	4.8	5.4					
12	1/2					11.1	12.4	13.8	15.4	17.1	19.0					
	1				7.4	8.2	9.2	10.2	11.4	12.7	14.1					
	2			4.9	5.5	6.1	6.8	7.6	8.4	9.4	10.4					
	4		3.3	3.6	4.0	4.5	5.0	5.6	6.2	6.9	7.7	2	3	5	6	7
	8	2.2	2.4	2.7	3.0	3.3	3.7	4.1	4.6	5.1	5.7					
	12	1.8	2.0	2.2	2.5	2.8	3.1	3.5	3.9	4.3	4.8					
	20	1.5	1.6	1.8	2.0	2.2	2.5	2.8	3.1	3.4	3.8					

Source: AGMA Standard 390.03.

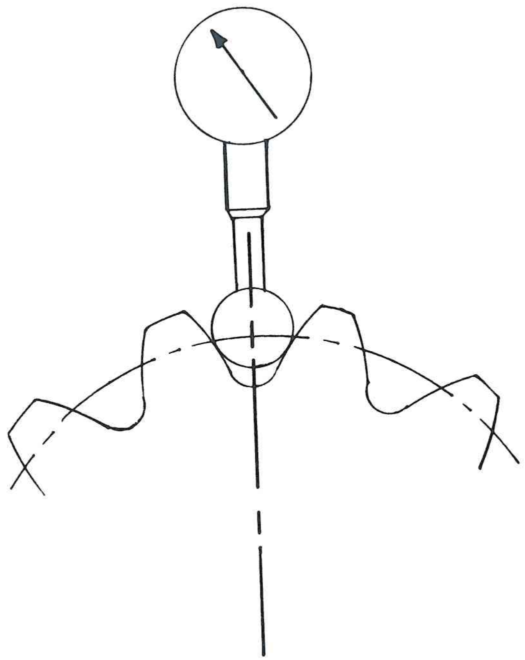


Figure 2.38 Single probe runout check.

b. a rolling check can be conducted meshing the gear to be inspected with a master gear of known accuracy on a fixture with a movable center distance. The variation of center distance is a measure of runout.

2. *Pitch tolerance*. The pitch is the theoretical distance between corresponding points on adjacent teeth. The variation from tooth to tooth can be measured using an instrument which employs a fixed finger and stop for consistent positioning on successive pairs of teeth, and a movable finger which displays pitch variations on a dial indicator or chart recorder (Figure 2.39).
3. *Profile tolerance*. The deviation from a true involute checked on an involute profile measuring instrument. In most cases, a modified involute is used; the drawing specification for involute modifications is discussed later in the chapter.
4. *Lead tolerance*. For a spur gear the lead inspection might be considered a check of the parallelism of the tooth with respect to the axis of rotation. The lead of a helical gear is the axial advance of a helix for one complete turn. Lead is checked by an instrument that advances a probe along the tooth surface, parallel to the axis, while the gear rotates in a specified, timed relation based on the lead.

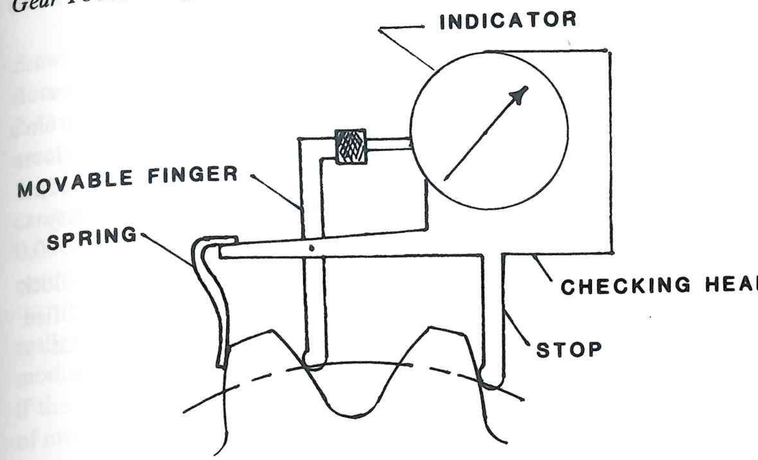


Figure 2.39 Tooth-to-tooth spacing check.

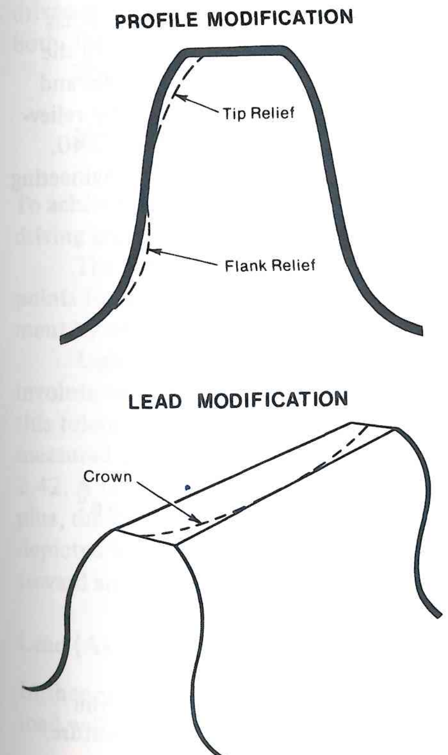


Figure 2.40 Tooth modifications.

In the data block (Figure 2.37) there are lines available for both the AGMA quality number and the specific tolerances mentioned above. It is possible that for a specific application the designer will not choose tolerances from a single quality number class. For instance, the designer may want to have a closer tolerance on tooth-to-tooth spacing that on profile. In such a case the individual tolerance on tooth-to-tooth spacing can be specified on the gear data block. Even when a single quality tolerances can be specified on the gear data block. Even when a single quality number is used, the tolerances can be placed on the block for reference.

When a modified involute or lead is required, a note in the gear data block will refer to a diagram on the drawing which defines the modification. Modified involute profiles and leads are used to attempt to compensate for deflections during operation and tooth errors. Figure 2.40 illustrates profile (involute) modifications and lead modification, sometimes called crowning.

Profile Modification

Tooth profiles are modified to avoid interference which can occur as the teeth enter into or leave the mesh. The interference is a result of deflection of the gear teeth, shafts, or gear casing due to the transmitted load or tooth discrepancies such as spacing or profile error. For instance, if a pinion tooth is misplaced from its theoretical position due to spacing error or because the previous tooth has deflected under load and enters into mesh too soon, the interference with the mating tooth will create a dynamic load which will increase tooth stresses and system noise and vibration levels. Such interference can be eliminated by relieving the pinion and gear tooth tips or flanks or both, as shown in Figure 2.40. Figure 2.41 illustrates how the profile modification is specified on the engineering

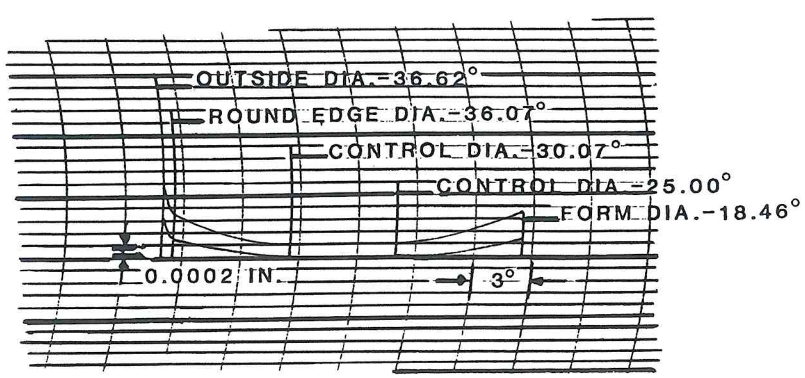


Figure 2.41 Modified involute diagram. The gear tooth profiles within the tolerance bands shall not depart from a smooth and gradual convex curvature.

drawing. The diagram provides tolerance bands for the chart that will be produced when the gear is inspected on an involute measuring instrument. Figure 2.41 presents a diagram for the 31-tooth pinion described in Figure 2.32. The amount of relief specified at the round edge diameter is 0.0003 to 0.0007. It is impractical to specify values at the outside diameter since the round edge radius cannot be closely controlled. The relief at the form diameter in this case is also 0.0003 to 0.0007. The tip and flank reliefs commence at the highest and lowest points of single tooth contact.

It should be recognized that the benefits from profile modification can be realized only if the teeth are accurately manufactured to tolerances less than the modifications specified. It would be pointless to have a 0.0005 tip modification if the tooth spacing were allowed to vary by 0.001. An estimate of the amount of modification required for a given application may be found in Ref. 2.

The modification at the first point of contact is given as:

$$\text{Modification} = \text{driving load (lb)} \times \frac{3.5 \times 10^{-7}}{\text{face width (in.)}}$$

To achieve this modification, material must be removed from the tip of the driven gear or the flank of the driving gear or both. If material is removed from both, the total modification is split between the two meshing teeth.

The modification at the last point of contact is given as:

$$\text{Modification} = \text{driving load (lb)} \times \frac{2.0 \times 10^{-7}}{\text{face width (in.)}}$$

To achieve this modification, material must be removed from the tip of the driving gear or the flank of the driven gear or both.

The foregoing estimates for profile modification are offered as starting points for a design. The final tooth modifications are arrived at through development by observing operating results.

Lightly loaded gears may not require profile modification. If a simple involute tolerance is called out on the gear data block, Figure 2.42 shows how this tolerance is to be interpreted. Assume a profile tolerance of 0.0008 in. The measured profile must fall within the checked area of the diagram in Figure 2.42. A true involute would be a straight line on the diagram. If the involute is plus, the line on the diagram will slant up toward the left. A minus involute is depicted by a dashed line on the diagram. In general, a plus involute tends toward an interference condition; therefore, the minus involute is more desirable.

Lead (Axial) Modifications

In theory, when gear teeth mesh the faces will be parallel to each other and the load will be distributed across the full face width. In practice, however, a

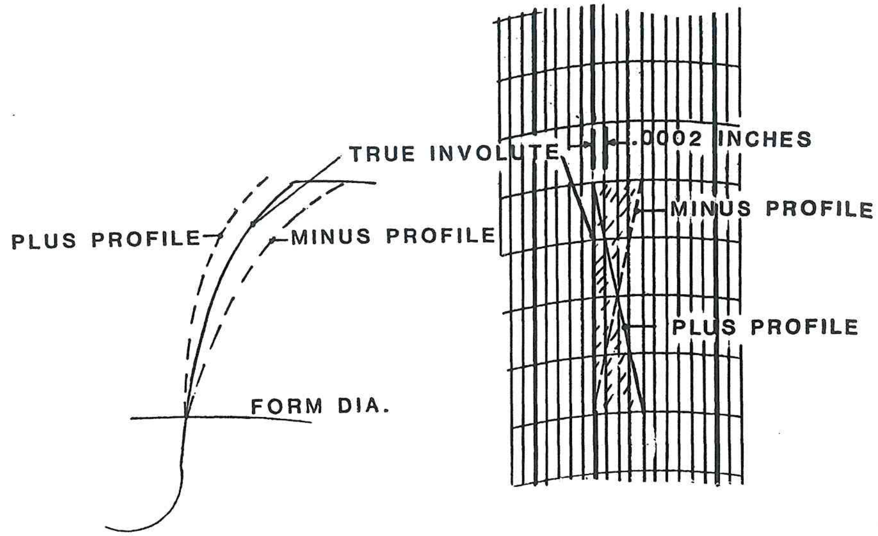


Figure 2.42 Interpretation of involute tolerances.

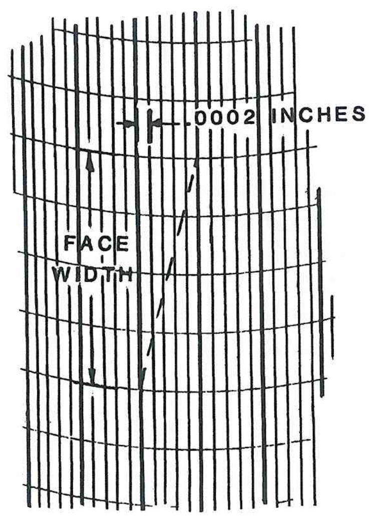


Figure 2.43 Lead diagram.

tolerance must be given. For spur gears it will be a tolerance on the parallelism of the tooth with the axis of rotation. For helical gears it is called a lead tolerance. Figure 2.43 shows how this tolerance is interpreted. The solid line is the theoretical trace and the dashed line is the measurement as recorded by a lead checking instrument. The diagram shows a 0.001-in. variation. If this is a right-hand helical gear, because the variation is off to the right, the measured helix angle is greater than the theoretical value.

Tooth faces tend not to be exactly parallel in operation not only because of tooth errors but also due to deflections of shafts, bearings, and casings. The load, therefore, may be concentrated on an end rather than distributed evenly across the face. To alleviate end loading, a lead modification, sometimes called a crown, is used. As shown in Figure 2.40, the crown relieves the tooth ends and avoids a heavy concentration of load in these areas. Figure 2.44 illustrates how a crowned tooth is specified on the engineering drawing. In this example the relief at either end of the face width is 0.0004 to 0.0008, blending smoothly into the flat at the center of the tooth. The amount of crowning generally is on the order of 0.001 in., but like the profile modification must be finally developed by observation of test results. In some cases deflections are such that only one end of the tooth need be crowned. On occasion two mating gears are designed with slightly different helix angles which become parallel as the system deflects under load.

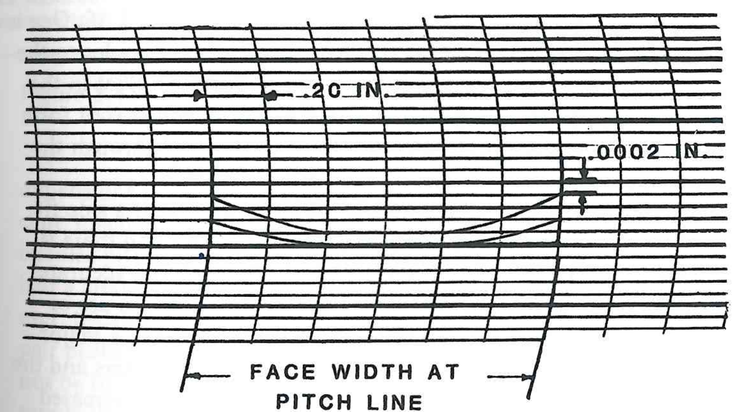


Figure 2.44 Crowned lead diagram. The lead contour within the tolerance bands shall not depart from a smooth and gradual convex curvature.

SPLINE DESIGN

Splines are used in mechanical systems to transmit torque and motion from one shaft to another. A spline connection consists of a set of external gear teeth arranged in a circle which fit into a corresponding set of internal gear teeth. Splines provide a strong, compact method of connection which can accommodate some misalignment.

In general use today are involute splines of 30° pressure angle with stub teeth. Stub teeth have short addendums of 1/2(diametral pitch) rather than the conventional 1/diametral pitch. Because of this the spline pitch is conventionally given as a fraction (e.g., 12/24, the numerator being the diametral pitch, which is the number of spline teeth per inch of pitch diameter and which controls the circular pitch and basic space width or tooth thickness. The denominator is known as the stub pitch and is always twice the numerator. The tooth addendum is 1/stub pitch.

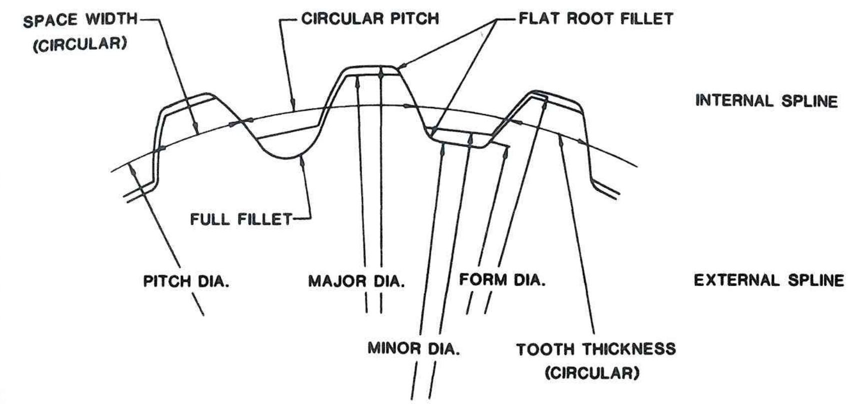
Spline teeth are of the involute form because of tooling advantages. A single hob or shaper cutter can generate all numbers of teeth of a given diametral pitch. Stub teeth and 30° pressure angles are used for ease of machining. The relatively high pressure angle increases tool life because the tool has more clearance behind the cutting edge. Also, higher cutting speeds are possible. The stub tooth is advantageous for broaching internal splines and for rolling of teeth.

Generally, splines are designed to ANSI Standard B92.1a [3]. In addition to 30° pressure angles the standard presents dimensioning systems for 37.5° and 45° pressure angle splines, which are sometimes known as serrations. Figure 2.45 taken from the standard shows spline tooth nomenclature and how spline data are presented on the engineering drawing.

Two root fillet configurations are possible, as shown in Figure 2.45. One is the flat root spline, in which fillets join the arcs of major or minor circles to the tooth sides. The other is the full fillet root spline, in which a single fillet in the general form of an arc joins the sides of adjacent teeth. The full fillet root form is stronger and should be used if appreciable torque is transmitted through the spline.

There are two types of fits possible with mating splines. One is a side fit where the mating members contact on the sides of the teeth only and there is clearance at the major diameters. When using a side fit spline if more accurate centralization of the shafts is desired, this can be accomplished by the use of shaft shoulders, as shown in Figure 2.46. It is also possible to have a major diameter fit where the mating members contact at their major diameters and the tooth sides act only as drivers. In this case the standard provides for increased clearance at the sides to ensure that all centering will be at the major diameters.

To be sure that two mating splines will fit together with minimum clearance, the concept of effective and actual tooth space and tooth thickness



DRAWING DATA			
INTERNAL INVOLUTE SPLINE DATA		EXTERNAL INVOLUTE SPLINE DATA	
FILLET ROOT SIDE FIT		FILLET ROOT SIDE FIT	
NUMBER OF TEETH	XX	NUMBER OF TEETH	XX
SPLINE PITCH	XX/XX	SPLINE PITCH	XX/XX
PRESSURE ANGLE	30°	PRESSURE ANGLE	30°
BASE DIAMETER	X.XXXXXX REF.	BASE DIAMETER	X.XXXXXX REF.
PITCH DIAMETER	X.XXXXXX REF.	PITCH DIAMETER	X.XXXXXX REF.
MAJOR DIAMETER	X.XXX MAX.	MAJOR DIAMETER	X.XXX/X.XXX
FORM DIAMETER	X.XXX	FORM DIAMETER	X.XXX
MINOR DIAMETER	X.XXX/X.XXX	MINOR DIAMETER	X.XXX MIN.
CIRCULAR SPACE WIDTH		CIRCULAR TOOTH THICKNESS	
MAX ACTUAL	X.XXXX	MAX EFFECTIVE	X.XXXX
MIN EFFECTIVE	X.XXXX	MIN ACTUAL	X.XXXX
MAX MEAS.BETW. PINS	X.XXXX REF.	MIN MEAS. OVER PINS	X.XXXX REF.
PIN DIAMETER	X.XXXX	PIN DIAMETER	X.XXXX

Figure 2.45 Spline nomenclature and drawing data.

dimensions is used in spline tooth dimension systems. To understand this concept, imagine an internal spline with each tooth space width exactly half a circular pitch and the mating external spline with each tooth thickness exactly half a circular pitch. It would seem that these splines would fit perfectly; however, because of such tooth errors as spacing, profile, out of round, and lead, the pair probably cannot be assembled. Because of these errors the spline teeth will not be in their theoretical locations on the pitch circle and at some point or points there will be interference between the internal and external teeth. To overcome this problem, all space widths of the internal spline must be widened by the amount of interference caused by tooth errors and all tooth thicknesses

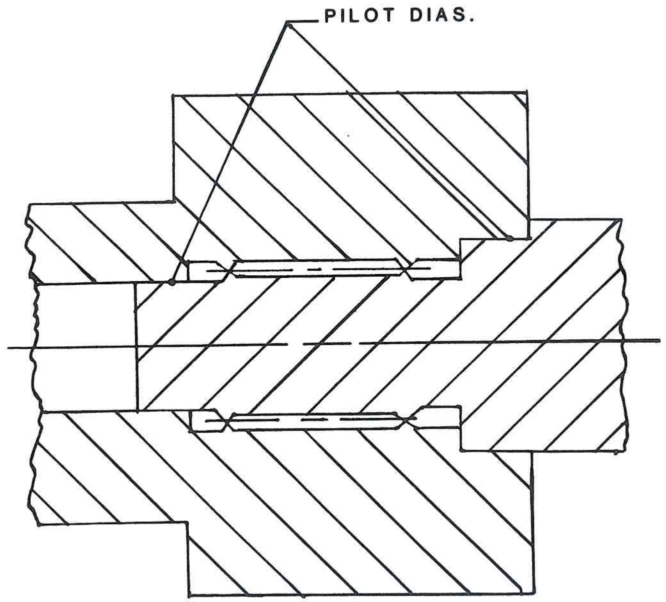


Figure 2.46 Piloted side fit spline.

of the external spline must be thinned. This concept leads to four dimensions for space width and tooth thickness:

Minimum effective space width = ½ circular pitch (ϕ = 30°)
Maximum effective space width
Minimum actual space width
Maximum actual space width
Maximum effective tooth thickness = ½ circular pitch (ϕ = 30°)
Minimum effective tooth thickness
Maximum actual tooth thickness
Minimum actual tooth thickness

The spline teeth are machined to the actual space width or tooth thickness dimensions which can be checked by the use of gages or measurements over pins. The effective dimensions are checked by gages. There are four machining tolerance classes set up for the effective and actual space widths and tooth thicknesses which result in varying degrees of clearance.

It must be remembered that the ability to assemble the spline is not the only criterion in critical applications. When significant loads are transmitted, or at high speed, the tooth geometry of the splined connection may have to be closely controlled and elements such as profile, lead, and surface texture specified.

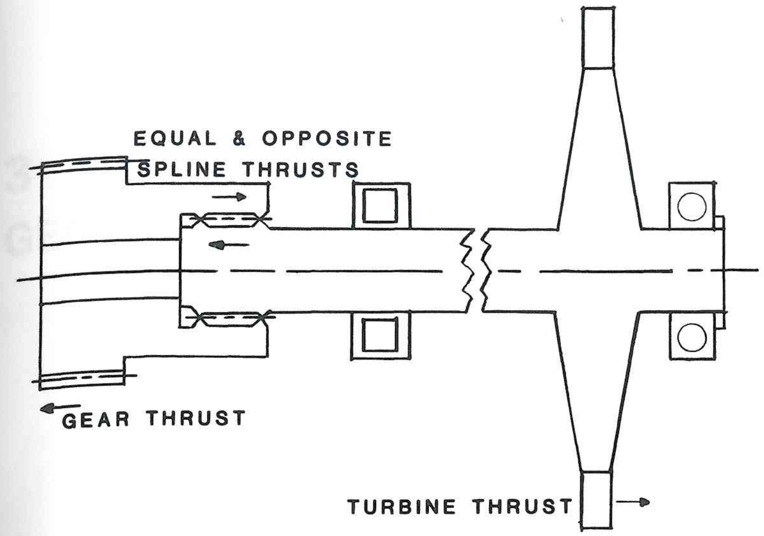


Figure 2.47 Helical spline shaft system.

In some cases where tooth bearing surface is important it may be desirable to use full-depth teeth. Full-depth splines would not use a 30° pressure angle since the teeth would be too pointed but would have a conventional 20° pressure angle or less.

There are applications where it is desirable to transmit thrust through a splined connection. For instance, a turbine wheel may be connected to a helical gear and the thrust of the wheel offset by the gear thrust. In such a case a helical tooth spline is effective. Figure 2.47 illustrates such a system. The spline helix angle is chosen such that the spline thrust exceeds the gear thrust and the shaft system locks up with the turbine shaft, bottoming out in the gear shaft shoulder. The net thrust in the system is then the turbine thrust minus the gear thrust, which is reacted by the ball bearing. The bearing loading, therefore, is greatly reduced from the case where the thrusts are not offset.

REFERENCES

1. AGMA Gear Handbook 390.03, Vol. 1, Gear Classification, Materials and Measuring Methods for Unassembled Gears, American Gear Manufacturers Association, Arlington, Va., January 1973.
2. Dudley, D. W., *Gear Handbook*, McGraw-Hill, New York, 1962, pp. 5–23.
3. ANSI Standard B92.1a, Involute Splines and Inspection, Society of Automotive Engineers, Warrendale, Pa., 1976.

3
GEARBOX RATING

The rating of a gearbox is determined by the loads the gearbox components are capable of transmitting. In some cases where a system is operating continuously at a uniform load such as an electric motor driving a fan, the loading is simple to predict and component analysis can be based on a continuous horsepower transmitted at steady speed. Some applications, however, experience variable loading such as high starting torque or shock loads and these conditions must be considered in the gearbox design.

Organizations such as the American Gear Manufacturers Association (AGMA) and the American Petroleum Institute (API) issue Standards that define gear rating procedures. AGMA Standard 420.04 [1] covers enclosed drives with pitch line velocities not exceeding 5000 fpm or pinion speeds not exceeding 3600 rpm. Higher-speed enclosed drives are covered by AGMA Standard 421.06 [2]. The general AGMA Standard for gear rating is 218.01 [4].

The rating methods used in these standards are discussed in this chapter. Before going into detail, an overview of the procedure one would use in rating a gearbox follows:

1. *Gear tooth rating*. The first step in determining a gearbox rating is to evaluate the tooth meshes. The classical gear tooth limitations that are calculated are the fatigue phenomena of breakage and pitting. Tooth breakage is analyzed by calculating the bending stress in the root fillet area and comparing it against a material strength rating. Pitting is analyzed by calculating the compressive stress at the tooth contact and comparing it against a material durability rating. A third gear tooth limitation encountered in high-speed gearing is instability of the lubricant film, allowing metal-to-metal contact leading to

scoring. The failure modes of tooth breakage, pitting, and scoring are described in Chapter 12. Their analysis is covered later in this chapter.

2. *Bearing rating*. Bearing ratings may be the limiting factor in determining the load a gearbox can transmit. A decision must be made as to the minimum acceptable L_{10} life desired for antifriction bearings or the maximum loading acceptable for journal bearings. The analysis and rating of bearings is presented in Chapter 4.
3. *Thermal rating*. Gear stresses or bearing lives usually determine the mechanical rating of a gearbox. In addition to the mechanical rating, gearboxes which do not use external cooling have a thermal capacity. This is defined in AGMA Standard 420.04 as the horsepower a unit will transmit continuously for 3 hr or more without exceeding a sump temperature of 200°F or a sump temperature rise of 100°F over ambient. If these thermal limits are exceeded, external cooling must be provided. Gearbox thermal ratings and lubrication systems are discussed in Chapter 5.
4. *Shaft rating*. Consideration must be given to gearbox components other than gears and bearings. Shafting, keyways, splines, and so on, must be analyzed to assure satisfactory performance under load. These machine elements are discussed later in the chapter.

The four points above are the obvious design details that must be addressed; however, there are many other details, such as housing and shaft deflections, critical speeds, and thermal expansion, of which the experienced gearbox designer is aware. Generally, in the procurement of a gearbox, gear ratings, bearing lives, and lubrication details are documented and the finer points of gearbox design are left to the manufacturer.

TOOTH LOADS

To calculate gear and shaft stresses and bearing lives, the gear tooth loads must be developed. Figure 3.1 illustrates the load diagram on a spur gear tooth. The gear torque is

$$T_q = \frac{63{,}025 \text{ hp}}{\text{rpm}}$$

where

T_q = torque, in.-lb
hp = horsepower
rpm = gear rotational speed

The total transmitted tooth load R acts normal to the involute profile. The component of R transmitting the torque is the tangential load:

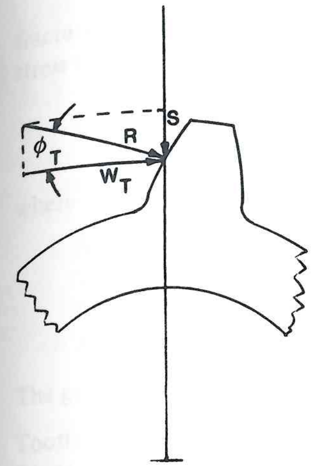


Figure 3.1 Spur gear tooth loads.

$$W_T = \frac{2T_q}{PD}$$

where

W_T = tangential load, lb
PD = gear pitch diameter, in.

The total transmitted load is

$$R = \frac{W_T}{\cos \phi_T}$$

where

R = total transmitted (resultant) force, lb
ϕ_T = transverse pressure angle, deg

As shown in Figure 3.1, the force R is resolved into the tangential load and a separating load:

$$S = W_T \tan \phi$$

where S is the separating load, in pounds. In the case of helical gears, the resultant force R is in the normal plane. To resolve R into a tangential and separating force, the geometry shown in Figure 3.2 is used. There is also a thrust force generated since the resultant force is at an angle ψ to the tangential force:

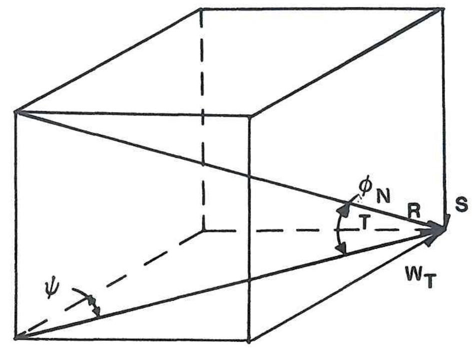


Figure 3.2 Helical gear tooth loads.

$$W_T = R \cos \phi_N \cos \psi$$
$$S = R \sin \phi_N = W_T \tan \phi_T$$
$$T = R \cos \phi_N \sin \psi = W_T \tan \psi$$

where

ψ = helix angle, deg
ϕ_N = normal pressure angle, deg
T = thrust load, lb

STRENGTH RATING

The strength rating of a gear tooth concerns itself with the bending stress (Figure 3.3) in the tooth fillet, where fatigue cracks initiate and propogate resulting in

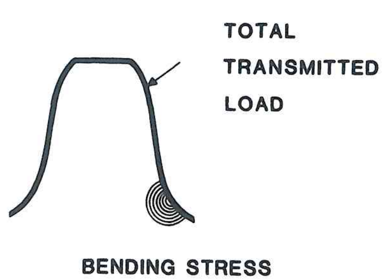


Figure 3.3 Bending stress criterion for strength rating.

fracture of teeth or portions of teeth. The fundamental equation for bending stress in a gear tooth is [4]

$$S_t = \frac{W_T P_d}{FJ}$$

where

S_t = tensile or bending stress, psi
W_T = transmitted tangential load, lb
P_d = transverse diametral pitch, in.$^{-1}$
J = geometry factor

The geometry factor J is an index of the following:

Tooth geometry in the root fillet area
Stress concentration in the root fillet
Load sharing between teeth
The position at which the most damaging load is applied

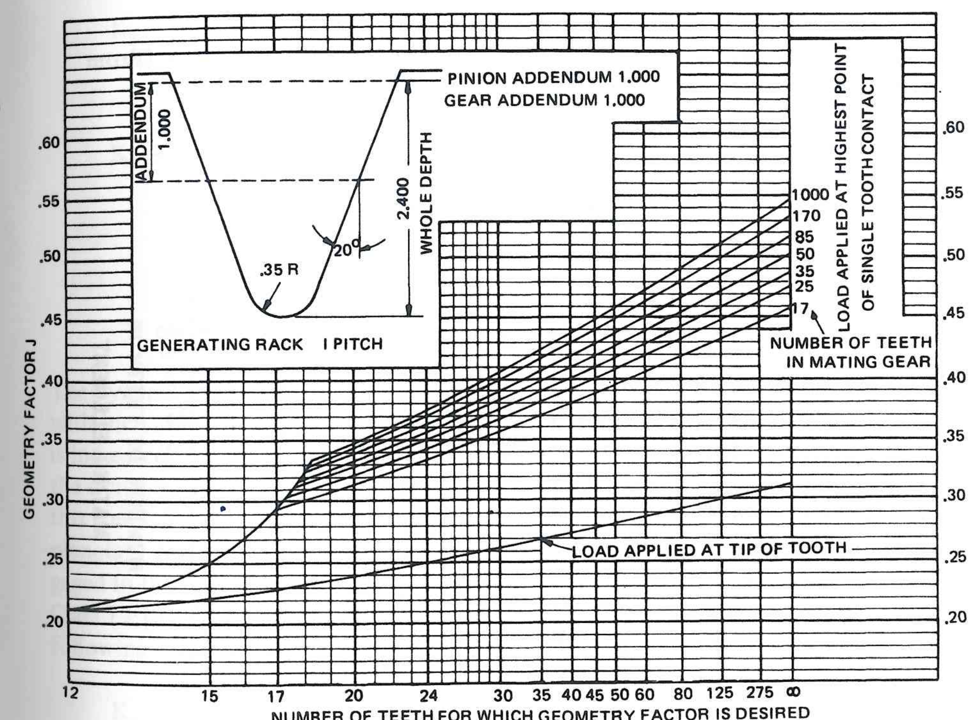


Figure 3.4a Geometry factors, 20° spur gears, standard addendum (From Ref. 5.)

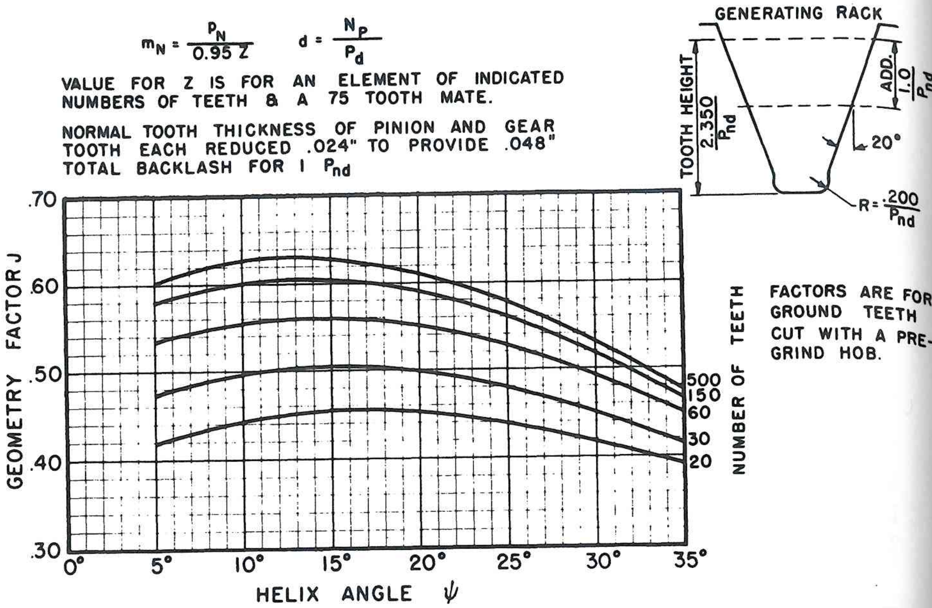


Figure 3.4b Geometry factors, 20° helical gears, standard addendum. (From Ref. 5.)

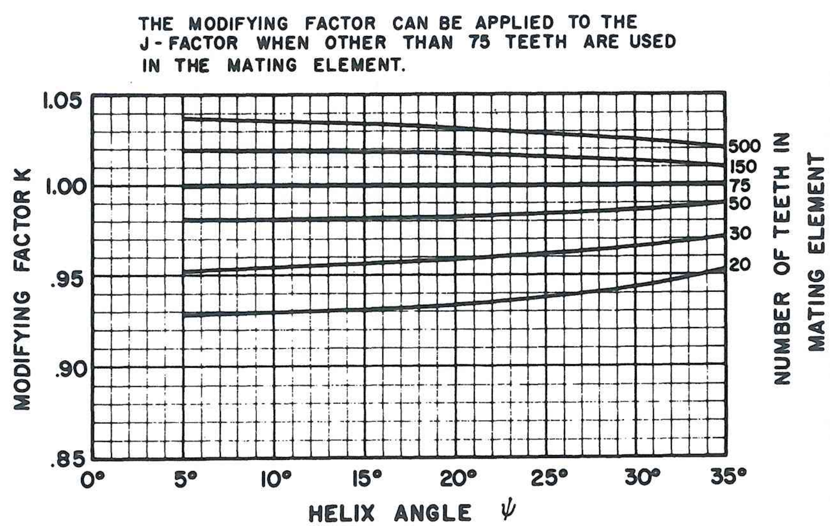


Figure 3.4c Modifying factor for helical gear geometry factors. (From Ref. 5.)

Geometry factors may be arrived at by graphical layout of the tooth form or computer analysis of the graphical procedure. Reference 5 presents a method for calculating the geometry factor. Figures 3.4a, 3.4b, and 3.4c present geometry factors for 20° pressure angle spur and helical gears. The geometry factor is strongly dependent on the cutting tool geometry and these figures are for hobbed gears. In general, spur gear geometry factors vary from approximately 0.35 to 0.45 and helical gear geometry factors have values from approximately 0.4 to 0.6.

BENDING STRESS RATING

In the AGMA rating system the basic bending stress is modified by several factors that deal with the characteristics of a specific application:

$$S_t = \frac{W_T K_a}{K_v} \frac{P_d}{F} \frac{K_s K_m}{J} \tag{3.1}$$

where

K_a = application factor. This factor takes into account the roughness or smoothness of the driving and driven equipment. When no overloads are anticipated K_a may be taken as 1.0. For very rough operation K_a may be 2.25 or greater.

K_v = dynamic factor. The dynamic factor represents the ratio between the maximum dynamic load on the gear teeth and the static calculated load. Gear teeth generate dynamic loads due to component geometry errors which result in gear accelerations and decelerations. Although the dynamic factor is used as a multiplier in the stress equation, the dynamic load is actually an incremental force which adds to the tangential force.

The dynamic factor increases with increasing pitch line velocity and decreases with increasing tooth accuracy and increased tooth loading. As the tooth loading is increased, the tooth deflections tend to overshadow tooth geometry errors and the dynamic load is a smaller percentage of the total load. Figure 3.5 illustrates this trend. The data shown were developed from a test conducted on a helical planetary gearset transmitting 1100 hp at 21,000 rpm-in. The sun gear was strain gaged in the root to measure tooth loading. Gear quality was AGMA Quality Class 12 [6]. For gears of lower quality classes operating at lower speeds, the following estimates can be used for dynamic factors [4]:

$$K_v = \left(\frac{92}{92 + PLV}\right)^{0.25}$$

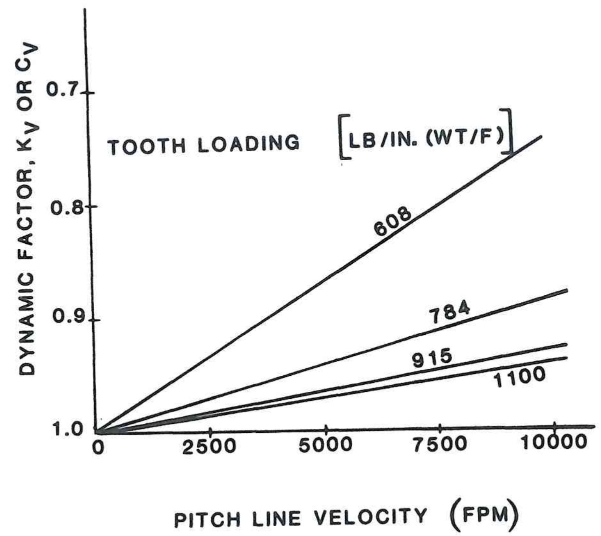


Figure 3.5 Dynamic factors for accurate gearing.

where PLV is the pitch line velocity in fpm, for AGMA Quality Class 11 gearing operating at pitch line velocities less than 8000 fpm with rigid accurate mountings;

$$K_v = \left(\frac{84}{84 + \text{PLV}}\right)^{0.4}$$

for AGMA Quality Class 10 gearing operating at pitch line velocities of less than 6000 fpm and

$$K_v = \left(\frac{70.7}{70.7 + \text{PLV}}\right)^{0.63}$$

for AGMA Quality Class 8 gearing operating at pitch line velocities of less than 5000 fpm.

There are analytical methods for calculating dynamic loads [7]. The tooth stiffness and mass are determined, and assuming the magnitude of tooth errors, an estimate of the dynamic load can be arrived at.

K_s = size factor. The size factor reflects nonuniformity of material properties which become more prevalent as the size of a gear increases; however, standard size factors have not yet been established and K_s is usually taken as 1.0.

Table 3.1 Allowable Bending Stress S_{at} for Gear Steels

Material hardness	S_{at} (psi)
180 Bhn	25,000–33,000
300 Bhn	36,000–47,000
400 Bhn	42,000–56,000
Carburized R_c 55	55,000–65,000
Carburized R_c 60	55,000–70,000
Nitrided R_c 60	38,000–48,000

Source: Ref. 4.

K_m = load distribution factor. The load distribution factor accounts for inaccuracies in the bearing bore locations leading to misalignment of the axes of rotation, alignment errors due to gear tooth inaccuracies, and deflections due to load or thermal distortion. For face widths less than 2.0 in., accurate gears and mountings and stiff housings a K_m as low as 1.1 may be used. In cases where poor alignment is anticipated K_m may equal 2.0 or more.

The relation of calculated bending stress to the allowable stress of the material is [4]

$$S_t \leqq \frac{S_{at}K_l}{K_tK_r}$$

where

S_{at} = allowable material stress, psi (see Table 3.1)
K_l = life factor

Table 3.2 Life Factor—K_l

Number of cycles	160 Bhn	250 Bhn	400 Bhn	Case carb.
Up to 1000	1.6	2.4	3.4	2.7
10,000	1.4	1.9	2.4	2.1
100,000	1.25	1.5	1.7	1.6
1 million	1.1	1.1	1.2	1.2
10 million	1.0	1.0	1.0	1.0

Source: Ref. 4.