REEDS MARINE ENGINEERING AND TECHNOLOGY

BASIC ELECTROTECHNOLOGY
FOR MARINE ENGINEERS

6

REEDS MARINE ENGINEERING AND TECHNOLOGY

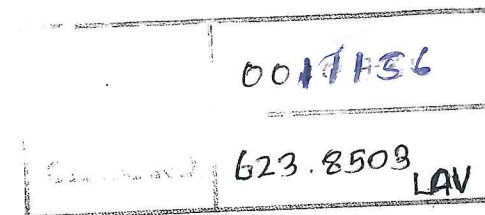# BASIC ELECTROTECHNOLOGY FOR MARINE ENGINEERS

Revised by Christopher Lavers
Edmund G R Kraal
Stanley Buyers

ADLARD COLES NAUTICAL

BLOOMSBURY

LONDON · NEW DELHI · NEW YORK · SYDNEY

CONTENTS

PREFACE

This book is intended to cover the basic theory underpinning applied work detailed in various syllabi for Electrotechnology written examinations, including the Maritime Coastguard Agency (www.dft.gov.uk/mca) Department of Transport Examinations for Class 1 and Class 2 Engineers (GENERAL ENGINEERING SCIENCE II, STCW III/2 Second Engineer, and STCW III/2 Chief Engineer) but is designed to be much more than an examination focused book. It is also suitable for Marine Engineering students completing Electrical and Electronic Engineering Principles, Electrical Technology and Further Electrical Principles units of the Level 3 Business and Technician Education Council (BTEC) Nationals specifications in Engineering, as well as various International Naval, Coastguard and Marine Science Degree students, such as those taught at Britannia Royal Naval College, Dartmouth, Devon.

As third editor of the Basic Electrotechnology Volume 6, and the subsequent Advanced Eectrotechnology Volume 7, now both nearing their important half-century way-marks, I am conscious of inheriting a valuable global resource for engineers and marine engineers, specifically building upon the splendid work of both EDMUND G R KRAAL CEng, DFH (Hons), MIEE, MIMarE and STANLEY BUYERS BEd, TEng, MIELecIE.

Sir Isaac Newton stated: *'If I have seen further, it is by standing on the shoulders of giants'*, echoing this sentiment I have been careful not to delete many vital principles – even if they have temporarily fallen out of favour with current examinations' syllabi, where appropriate I have included some detail and quotes from several pioneers of Electrotechnology. This marine engineering series was created to emphasise the engineering and applied physics ideas starting from first principles, and with reference to numerous illustrations, to provide worked examples within the text, supplemented with many problems for students to work through on their own to develop a deep understanding of the practical outworking of these key engineering concepts. Typical examination questions at the end of each chapter, and at the end of the book, provide students with further opportunities to test themselves thoroughly against a high standard of questions before attempting the examinations themselves. Fully worked out step-by-step solutions are given to each and every problem which is especially useful to a marine engineer at sea without recourse to a college tutor. The methodical approach taken throughout the book, with questions followed by detailed logical answers, lends itself to the student generating their *own* questions and using the laid out established methods to test how well they have understood the process. The student thereby begins to take ownership of his own learning – the experiential doing – partly, I anticipate, by trial and error! And it is the author's hope that among those reading and

studying these subjects there will perhaps be a new Tesla, Faraday or Edison capable of lighting up the world with a new generation of electrical devices.

The revolution in global communications within the past two decades, especially for those at sea, and the massive increase in digital storage capacity provide additional tools to this generation of engineers which was not possible in the past, yet these do not provide substitute for *understanding* and applying these key principles. It is also a sobering note that this generation is both more *reliant* than ever on the use of electromagnetic devices and systems and at the same time is most *vulnerable* to their failure. This has always been well understood by the marine engineer, often far from land with the potential failure of vital onboard systems, but is one that has only recently begun to be fully appreciated more widely. We now rely on a multitude of global systems – not just those on land, but sea, air and space – which are especially vulnerable to EMP or the caprices of solar weather, which has the real capability, in its most severe form, to remove, if unprepared, much of the electrical infrastructure that has been so painstakingly developed from the early work by Faraday and Tesla in only a few hours. Marine and terrestrial engineers alike would be well advised to consider the implications of another such 'Carrington Event' solar storm of the magnitude experienced in 1859 which resulted in disruption at the infancy of electrical devices, and even the electrocution of telegraphy operators – that first Victorian Internet.

I, like my predecessors, would like to acknowledge the constructive comments made over the past 19 years by colleagues, officers and students of various nationalities educated and trained at the Britannia Royal Naval College, Dartmouth, Devon, UK and of course the editorial services team at Newgen.

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THE S.I. SYSTEM

Prefixes, Symbols, Multiples and Submultiples

Prefix	Symbol	Units multiplying factor
tera	T	$\times 10^{12}$
giga	G	$\times 10^{9}$
mega	M	$\times 10^{6}$
kilo	k	$\times 10^{3}$
milli	m	$\times 10^{-3}$
micro	µ	$\times 10^{-6}$
nano	n	$\times 10^{-9}$
pico	p	$\times 10^{-12}$

Examples 1 megawatt (MW) = 1×10^{3} kilowatts (kW)

$= 1 \times 10^{6}$ watts (W)

1 kilovolt (kV) $= 1 \times 10^{3}$ volts (V)

1 milliampere (mA) $= 1 \times 10^{-3}$ ampere (A)

1 microfarad (µF) $= 1 \times 10^{-6}$ farad (F)

Physical Quantities (Electrical), Symbols and Units

The table has been compiled from recommendations in B.S. 1991 and the List of Symbols and Abbreviations issued by the I.E.E.

Quantity	Symbol	Unit	Abbreviation of unit after numerical value
Force	F	newton	N
Work		joule	J
or	W	or	
Energy		newton metre	Nm
Torque	T	newton metre	Nm
Power	P	watt	W
Time	t	second	s
Angular velocity	ω (omega)	radians per second	rad/s
Speed	N	revolutions per minute	rev/min
	n	revolutions per second	rev/s
Electric charge	Q	coulomb	C
Potential difference (P.D.)	V	volt	V
Electromotive force (e.m.f.)	E	volt	V
Current	I	ampere	A
Resistance	R	ohm	Ω (omega)
Resistivity (specific resistance)	ρ (rho)	Ohm-metre	Ωm
Conductance	G	siemens	S
Magnetomotive force (m.m.f.)	F	ampere-turn	At
Magnetic field strength	H	ampere-turn per metre or	At/m

Quantity	Symbol	Unit	Abbreviation of unit after numerical value
Magnetic flux	Φ (phi)	weber	Wb
Magnetic flux density	B	tesla	T
Reluctance	S	ampere-turn or ampere per weber	At/Wb or A/Wb
Absolute permeability of free space	μ_0	henry per metre	H/m
Absolute permeability	μ (mu)	henry per metre	H/m
Relative permeability	μ_r	–	–
Self-inductance	L	henry	H
Mutual inductance	M	henry	H
Reactance	X	ohm	Ω
Impedance	Z	ohm	Ω
Frequency	f	hertz	Hz
Capacitance	C	farad	F
Absolute permittivity of free space	$\in_0$ (epsilon)	farad per metre	F/m
Absolute permittivity	$\in$	farad per metre	F/m
Relative permittivity (dielectric constant, specific inductive capacity)	$\in_r$	–	
Electric field strength, electric force	E	volt per metre	V/m
Electric flux	ψ (psi)	coulomb	C
Electric flux density, electric displacement	D	coulomb per square metre	C/m²
Active power	P	watt	W
Reactive power	Q	volt amperes reactive	VAr
Apparent power	S	volt ampere	VA
Phase difference	ϕ (phi)	degree	°
Power factor (p.f.)	$\cos \phi$	–	–

1

FUNDAMENTAL ELECTRICAL THEORY TERMS AND LAWS

I shall make electricity so cheap that only the rich can afford to burn candles.

Thomas Edison

Electron Theory

The nature of electricity

To enable the student engineer to achieve practice with relevant problems and to appreciate fundamental concepts, a start is made in this chapter with an introduction to the subatomic nature of electricity before considering basic circuit theory and relevant calculations. A more detailed explanation is developed as needed in later chapters but it is hoped that the student will understand from the start that electronics and electrical engineering are *related* and that the nature of electricity and many electrical phenomena all have their origin in atomic structure.

The structure of the atom

It is now accepted that the transmission of electricity is due to a flow of *electrons* and as there is no observable indication of such a flow in a conductor, we must accept the classical atomic theory on the structure of matter and the effects of electron movement and their rearrangement. Matter is defined as anything that occupies space; it may be in solid, liquid or gas form, but basically consists of *molecules* of a substance. A molecule is the smallest particle of a substance that exists by itself. Molecules have the properties of the substance which they form but are themselves made of groups of *atoms*. For example, a molecule of water, written H_2O, consists of 2 atoms of hydrogen and one of oxygen. The atom is defined as the smallest particle that can enter into chemical action, but is itself a complex structure consisting of subatomic particles. A substance containing only atoms with the same identical properties is called an *element*, but one containing atoms of different properties is called a *compound*. All atoms of a given element are identical and atoms of different elements differ only in the number and arrangement of the subatomic particles contained therein. Subatomic particles can be *charged* or *uncharged*. Reference to charge will be made throughout this volume, but at this stage, it is stated that electricity in its smallest form consists of charges and these are of 2 kinds only, positive (+ve) and negative (−ve). Like charges repel each other and unlike charges attract each other. Generally the space in which a physical force exists between charges is referred to as an *Electric Field*. A more detailed consideration will however be made in Chapter 8 when dealing with electrostatics.

According to the theory, proposed by scientists like Rutherford and Bohr (1911), tested by experiment (1909), each atom has a core or *nucleus* surrounded by *orbital electrons*. The nucleus consists of tiny masses of positively charged subatomic particles or *protons*, and *neutrons* which have no charge. The main purpose of the neutrons is to 'fix' or 'cement' the positively charged protons together within the nucleus. In a normal stable atom the number of protons is equal to the number of orbiting electrons. The number of protons in an atom determines the *atomic number* (Z) of the element and thus the number of negatively charged orbital electrons. An electron has a mass of 9.04×10^{-28} g and has a charge of 1.6×10^{-19} Coulomb. A proton has a mass 1850 times greater than that of an electron while a neutron has a mass slightly more than a proton. The concept of the atom is shown in figure 1.1.

The negatively charged electrons are considered to spin about an axis and to revolve around the nucleus similar to the structure of a miniature 'solar system'. The nucleus represents the 'sun' and the electrons represent the 'planets'. Under normal conditions an atom is said to be stable or unexcited. The planetary electrons together neutralise
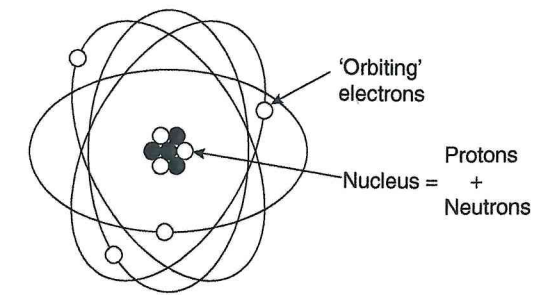
▲ Figure 1.1

the positively charged protons in the nucleus, so a complete atom itself has no electrical charge. Figure 1.2 shows examples of atomic structure for different elements but these illustrations are drawn in 1 plane only. The simplest atom is that of the element hydrogen, consisting of a nucleus with 1 proton (having a +ve charge) around which orbits 1 electron. The electron with its −ve charge neutralises that of the proton. In the diagrams, electrons are denoted by circles, with their charges shown, and are considered to move on dotted orbits. The nucleus is shown with a full circle, has a net positive charge attributed to the protons and these are shown by + marked circles. Neutrons are shown by small circles with no charge sign.

The next element considered is helium which has 2 planetary electrons and a nucleus consisting of 2 protons and 2 neutrons. The electrons of most atoms are associated with the nucleus in a definite manner, i.e. the electrons are in groups or *shells*, such that the planetary path of each shell is different. This is shown if an oxygen atom is considered, oxygen has a nucleus of 8 protons and 8 neutrons. The planetary electrons are 8 in 2 orbits or shells – 6 in the outer shell and 2 in the inner shell. For any one atom, the electrons in the first shell may be less than, but never more than 2 electrons and no more than 8 in the second shell.

▲ Figure 1.2
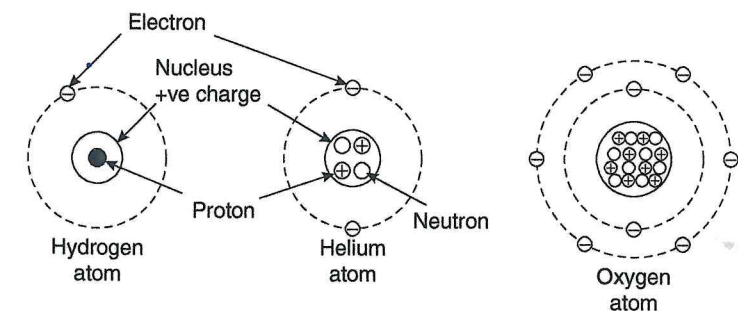
Lithium atom
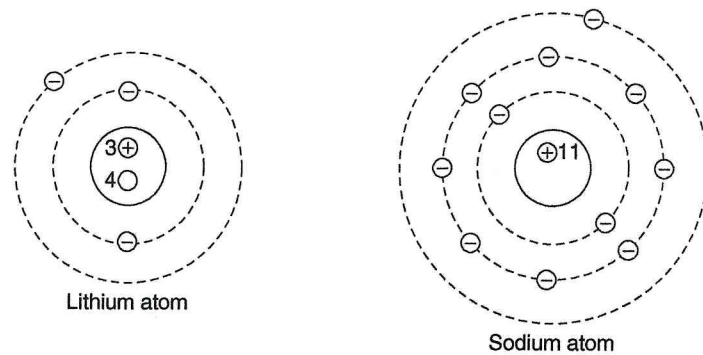
Sodium atom

▲ Figure 1.3

Figure 1.3 represents the atomic structure of 2 metals: lithium and sodium. In each case, and if other metals are considered, it will be seen that all metals have 1 or 2 electrons in the outermost shell, which is considered the reason metals have good electrical conducting properties. It is suggested that for metals in their normal crystalline state, the atoms are arranged so their outermost electrons are partially screened from the +ve attractive force of the nucleus and are not so strongly bound. Thus electrons can move relatively freely between one atom and its neighbours. Such outer orbital electrons, or *mobile* or *valence electrons* move randomly from one atom to another atom and constitute a 'pool' of moving negative charges, which helps to explain the transmission of electricity or current in a circuit. Note that 'valency' is a chemical term of which mention will be made later.

Current as electron movement

Current, according to electron theory, is due to movement of electrons from one atom to the next, each electron carrying a −ve charge. As mobile electrons move randomly between atoms, transfer of charge, and thus electricity in a particular direction, fails to occur and no current flows. If an electrical force, in the form of an electromotive force (e.m.f.) or potential difference (P.D.) is applied across a good conductor then mobile electrons are forced to move towards the higher potential or +ve terminal. The required electrical force produced by a battery or generator can be thought of as a pump moving electrons round a circuit. A 'stream' or movement of electrons is said to constitute an electric current but, there is a key difference between the direction of conventional current flow and actual electron flow. If a length of wire is connected to 2 terminals, between which an e.m.f. or P.D. exists, a current will flow from the +ve terminal through the wire to the −ve terminal. However, electron flow will be from

the −ve terminal to the +ve terminal. This fundamental difference between conventional current and electron flow must always be remembered and is illustrated in figures 1.4a and 1.4b. This distinction arises from a practical lack of understanding in the earliest days that electrons were the mobile charge carriers. It is noted that the electrical generator or battery, which maintains the e.m.f. or P.D. between the ends of a conductor, does not itself *make* electricity but merely causes movement of electrons which are present in the circuit.

Free electrons in random motion

Current

Electrons

No applied P.D.
(a)

With applied P.D.
(b)
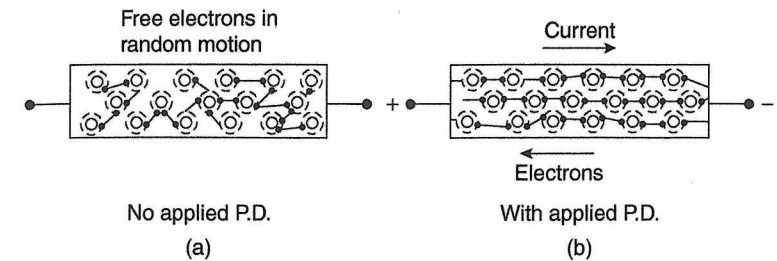
▲ Figure 1.4

Ionisation

An atom can lose or gain electrons as the result of a disturbing action or force. It then becomes electrically unbalanced having acquired charge and is called an *ion*. An atom minus an electron, exhibits a +ve charge and is a +ve ion. Atoms which gain an electron, exhibit a −ve charge and are −ve ions. When an electron is made to leave a parent atom by application of a force, for example, due to an electric field, or the application of heat or light, it can acquire sufficient energy to detach further electrons from other atoms with which it collides. Such action causes these secondary atoms to become +ve ions and, if electrons leave these atoms faster than they are regained, the state of ionisation continues. Electronic devices such as the fluorescent lamp or Cathode Ray Tube depend on ionisation to operate.

The electric circuit

A circuit is defined as a path taken by an electric *current*. A current flows through a circuit if (1) a source of electrical energy such as a battery or generator is connected, and (2) the circuit is continuous or conducting along its complete length. Figure 1.5 represents a simple circuit in which a current flows. It shows a source, from which energy is transmitted in the current, the conducting path or cable along which the

current flows and the 'load'. The load is the point where energy is released or work is to be done by the flowing current.

The conditions of figure 1.5 are better represented by a circuit diagram as in figure 1.6, which illustrates the energy source as a chemical cell, the conducting path as the leads or wires and the load. A switch is shown as a vital link which, when opened, interrupts a circuit's continuity and thus stops the current flowing.

Consideration of the simple circuit introduces more fundamental terms and the practical units used in electrical engineering. Flow of electricity or current is the result of pressure built up within the energy source which manifests itself, at the circuit connecting points or terminals, as a pressure difference. One terminal, called the positive, is viewed as being at a higher pressure or potential than the other terminal, called the negative. A P.D. exists between these terminals. The current direction is from the positive (+ve) terminal through the circuit *external to the energy source*, back to the
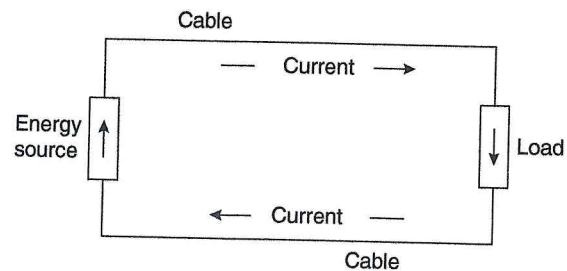
▲ Figure 1.5
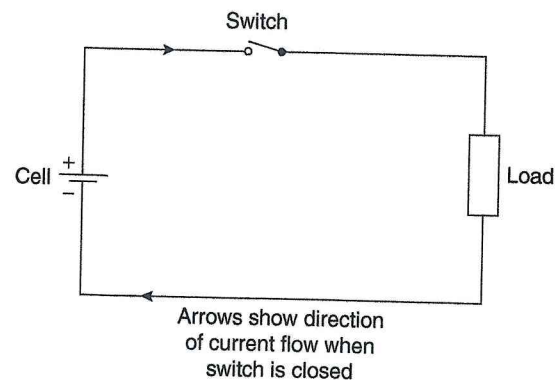
▲ Figure 1.6

negative (−ve) terminal and then through the source to the +ve terminal. Thus for the load, conventional current is from +ve to −ve terminal, but for the energy source in a cell, battery or generator, electrical current is from the −ve to the +ve terminal!

The electrical pressure generated by an energy source is termed the *electromotive force* (e.m.f.). The symbol used, E, or e.m.f. is measured as a *voltage*. The unit is the *Volt*, which is defined later, but any voltage value is represented by the letter V attached to a numerical value. Thus a voltage of two hundred and twenty volts is written as 220V. For reasons to be explained when the maths of a circuit is considered, the whole generated e.m.f. of a cell, battery or generator doesn't appear at the terminals when current flows. The P.D. across the terminals is measured in terms of the potential or voltage dropped round the external circuit. The symbol used for the terminal P.D. is V and is measured as a voltage, i.e. in volts.

Circuit Laws

1. For any circuit, current strength is found to be proportional to the voltage applied *across* its ends. Current strength is denoted by the symbol I and is measured in *Amperes*. The ampere is defined later by considering the electromagnetic effect of current flow, but any current value can be represented by the letter A added to the numerical value. Thus two hundred amperes is 200 A.

Any electrical circuit is found to *oppose* the current flow. This opposition is termed the resistance of a circuit and is given the symbol R. The unit of resistance is the *Ohm*, but any value is represented by the Greek letter capital Ω (Omega) added to the numerical value. Thus one hundred ohms may be written 100Ω. The ohm is defined in terms of the volt and ampere so: a resistor has a value of 1 ohm's resistance, if a current of 1 ampere passes through it when a P.D. of 1 volt is applied across its ends. An alternative definition is given in Chapter 2.

2. The current in a circuit, for a constant voltage, is found to vary inversely with resistance, i.e. the greater the resistance, the smaller the current and *vice versa*.

Ohm's law

The relationships stated above, are summarised by the first law of an electrical circuit, which is called Ohm's law and is expressed thus: the current in a circuit is directly

proportional to the voltage and inversely proportional to the resistance. This can be written as:

$$\text{Current} = \frac{\text{Voltage}}{\text{Resistance}}$$

$$\text{or } I \text{ (amperes)} = \frac{V \text{ (volts)}}{R \text{ (ohms)}} \quad \text{or} \quad \frac{E}{R}$$

Other forms are $V = IR$ or $R = \dfrac{V}{I}$

When using Ohm's law formulae it is essential to pay due regard to the magnitudes of the units used. Reference should be made to the appropriate table of conversions.

Example 1.1. An e.m.f. of 6V is applied across a 300Ω resistor. Find the current which will flow.

$$I = \frac{E}{R} = \frac{6}{300} = 0.02\,\text{A} = 20\,\text{mA}.$$

Example 1.2. A current of 20mA passes through a 30kΩ resistor. Find the voltage drop across the ends of the resistor.

$$V = IR = (20 \times 10^{-3}) \times (30 \times 10^{3}) = 600\text{V}.$$

Series and parallel circuits

Study of the electrical circuit shows that in its simplest form it may be built up as (1) a series circuit or (2) a parallel circuit. Resistance is considered to be concentrated in a resistor, or in more than 1 resistor; while connecting leads are assumed to have negligible resistance, unless a definite resistance value for these is stated. Similarly the cell, battery or generator is assumed to have no resistance unless otherwise stated.

Figure 1.7 shows a series circuit. Only one current path is possible and the same current passes through all the resistors. The current is thus common for such a circuit but the applied potential drops progressively as current flows along the circuit.
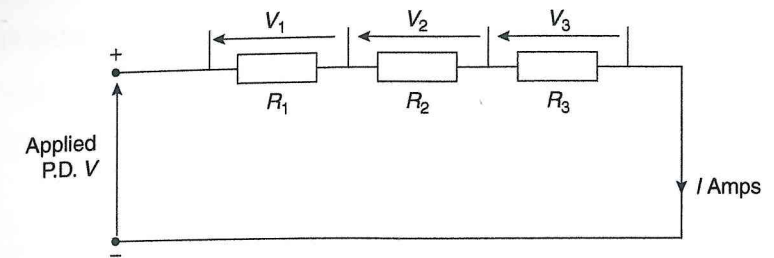
▲ Figure 1.7
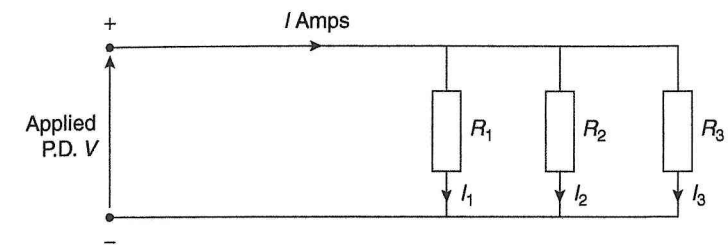
▲ Figure 1.8

Figure 1.8 shows a parallel circuit. Here the main current is made up of 3 branch currents, but the applied P.D. is the same or common for all 3 branches. At any junction point there is no current accumulation, i.e. the total current entering a point is the same as the total current leaving the point. Simple laws based on voltage conditions for the series circuit and current conditions for the parallel circuit allow the solution of problems for such simple circuits, and also those of more complicated series–parallel arrangements or electrical networks.

Kirchhoff's laws

(1) VOLTAGE LAW. The sum of the potential or voltage drops taken round a circuit must equal to the applied P.D. Thus for figure 1.7:

$$V_1 + V_2 + V_3 = V$$

(2) CURRENT LAW. The current flowing away from a junction point in a circuit must equal the current flowing into that point. Thus for figure 1.8:

$$I_1 + I_2 + I_3 = I$$

The above laws help deduce simple formulae for series and parallel circuits in terms of the equivalent resistances of the circuits.

THE SERIES CIRCUIT. For figure 1.7, let I amperes be the common current flowing round a circuit. Then from Ohm's law, the voltage dropped across resistor R_1 is V_1 volts $= IR_1$. Similarly the voltage dropped across R_2 is $V_2 = IR_2$ etc. If R is taken as the equivalent resistance of the whole circuit then as V is the applied voltage and it will be dropped over this equivalent resistance, we can write $V = IR$.

Using Kirchhoff's voltage law then

$$V = V_1 + V_2 + V_3$$

$$\text{or } IR = IR_1 + IR_2 + IR_3 = I\,(R_1 + R_2 + R_3)$$

$$\therefore \; R = R_1 + R_2 + R_3$$

THE PARALLEL CIRCUIT. For figure 1.8, let V volts be the common voltage applied to all the parallel branches and with a total main current of I amperes. Voltage V would also cause a current of I amperes through an equivalent circuit of resistance R ohms.

Thus $I = V/R$ and using Kirchhoff's current law then

$$I = I_1 + I_2 + I_3$$

But for branch 1 $V = I_1 R_1$ or $I_1 = \dfrac{V}{R_1}$

And similarly for branch 2 $I_2 = \dfrac{V}{R_2}$ etc.

Thus $I = I_1 + I_2 + I_3$ can be written as:

$$\frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

$$\therefore \; \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Note. The reciprocal of resistance is often referred to as *Conductance*, symbol $G = 1/R$.

The unit is the *Siemens*; the symbol S appended to the numerical value.

So for a parallel circuit $G = G_1 + G_2 + G_3$ etc.

Example 1.3. Three resistors of values 2, 4 and 8 ohms are connected in series across a supply of 42 volts. Find the current taken from the supply and the voltage dropped across each resistor.

$$\text{Here } R = R_1 + R_2 + R_3 = 2 + 4 + 8 = 14\Omega$$

$$\text{So supply current } I = V/R = \frac{42}{14} = 3\text{A}$$

Voltage dropped across 2Ω resistor $= 3 \times 2 = 6\text{V}$

Voltage dropped across 4Ω resistor $= 3 \times 4 = 12\text{V}$

Voltage dropped across 8Ω resistor $= 3 \times 8 = 24\text{V}$

Check. $6\text{V} + 12\text{V} + 24\text{V} = 42\text{V}$ (the applied voltage).

Example 1.4. The above resistors are connected in parallel across the same supply voltage. Find the total current and the current in each branch.

$$\text{Here } \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} = \frac{7}{8} = 0.875\text{S}$$

$$\text{or } R = \frac{8}{7} = 1.14\,\Omega \text{ and } I = \frac{42}{1.14} = 36.75\text{A}$$

$$\text{The current in Branch 1} = \frac{42}{2} = 21\text{A}$$

$$\text{The current in Branch 2} = \frac{42}{4} = 10.5\text{A}$$

$$\text{The current in Branch 3} = \frac{42}{8} = 5.25\text{A}$$

Check. 21A + 10.5A + 5.25A = 36.75A (the total supply current)

Internal resistance of a supply source

Until now the energy supply source was considered as having negligible resistance. In practice, a cell, battery or generator has an internal resistance which results in an internal voltage drop when current is supplied. Thus the e.m.f. generated appears at the supply terminals, shown as AB in figure 1.9, *only* when the circuit switch is open, i.e. the cell is on 'open circuit' (O.C.). When current I is supplied, an internal voltage drop of IR_i occurs, R_i being the internal cell resistance. The P.D. V at the energy source terminals is thus $E - IR_i$. V is less than the generated e.m.f. E by the P.D. required to drive the current through the cell resistance.
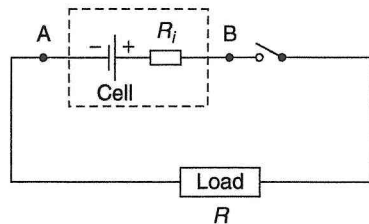
▲ Figure 1.9

Electromotive force (e.m.f.) and terminal P.D. or voltage

In figure 1.9, R_i represents the internal cell resistance as shown external to the cell itself. This is diagrammatic only; sometimes this resistance is not shown, written only as a figure beside the cell e.m.f. However, internal resistance must be considered and gives rise to the difference between e.m.f. E and P.D. V. On O.C. terminal, P.D. of a source equals the e.m.f. generated; but 'on load', i.e. when current is supplied, terminal P.D. equals the e.m.f. minus the internal voltage drop. This is summarised mathematically thus:

On O.C. $V = E$

On Load $V = E - IR_i$

We also deduce that since $V = IR$ where R is the load resistance, then

$IR = E - IR_i$ and $E = IR + IR_i$ or $E = I(R + R_i)$

Expressed another way:

On O.C.   Cell terminal voltage $V = E$

On Load   Cell terminal voltage $V = E - IR_i$

also   Cell terminal voltage $V = IR$

Problems are treated as a simple series circuit, if E is used as the circuit voltage and R_i included in the series resistance.

Example 1.5. A battery of e.m.f. 42V and internal resistance 7Ω supplies the series circuit of Example 1.3, i.e. 3 resistors of 2Ω, 4Ω and 8Ω in series (figure 1.10). Find the current and terminal voltage and by how much the cell voltage 'sits' or 'drops down' when supplying the load.

Note. It is appropriate here to explain that a battery is an arrangement of more than 1 cell. The methods of connecting cells is discussed in Chapter 2; here a battery is considered arrangement of cells in series. Thus a battery e.m.f. is the sum of its cell e.m.f.s and the battery internal resistance is the sum of the cell internal resistances.
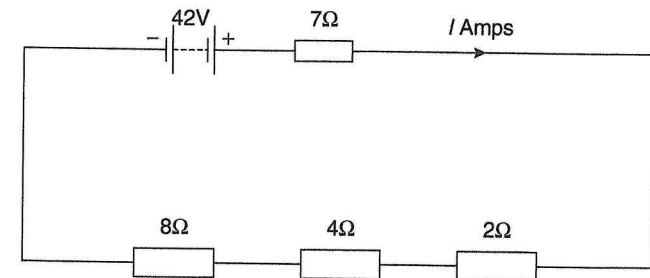
▲ Figure 1.10

External resistance of the circuit $R = 2 + 4 + 8 = 14Ω$

Battery resistance $= 7Ω$

Total resistance of circuit $= 14 + 7 = 21Ω$

Circuit current $I = E/R = 42/21 = 2A$

Terminal voltage $V = IR = 2 \times 14 = 28V$

Voltage drop in cell $IR_i = 2 \times 7 = 14V$

Check. Terminal voltage $V = E - IR_i = 42 - 14 = 28V$

The 'Lost volts' or Cell voltage 'drops down' by 14 volts.

THE SERIES–PARALLEL CIRCUIT. In practice many circuits are built up from series and parallel groups of resistors. Solution of these associated problems, though not simple, follows a logical sequence of operations based on the methods used for simple series and parallel resistor arrangements. It is strongly urged that the idea of 'practice makes perfect' is essential and the reader should work through sufficient appropriate problems until proficiency is achieved. A solution method for a particular problem will become clear once its form is recognised and time and labour with real engineering problems will then be saved.

Figure 1.11a shows a simple series–parallel circuit, consisting of a series circuit made up of 2 sections, each comprising a group of resistors in parallel. As the main circuit or supply current may need to be found along with the current in each resistor, solution is only obtained by simplifying the problem. It is noted that the parallel groups (or banks of resistors) are called sections A and B respectively. The voltage dropped across these sections is unknown and as the voltages are essential, the procedure is set out below.

The circuit is simplified by finding the *equivalent resistance* values R_A and R_B of the parallel banks from

$$\frac{1}{R_A} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \quad \text{and} \quad \frac{1}{R_B} = \frac{1}{R_4} + \frac{1}{R_5} + \frac{1}{R_6}$$

Once R_A and R_B are found the total supply current can be obtained as is shown in figure 1.11b. The equivalent circuit is now of the simple series type.

The supply current is

$$I = \frac{V}{R_A + R_B} \quad \text{and the voltage drops across groups A and B are respectively}$$
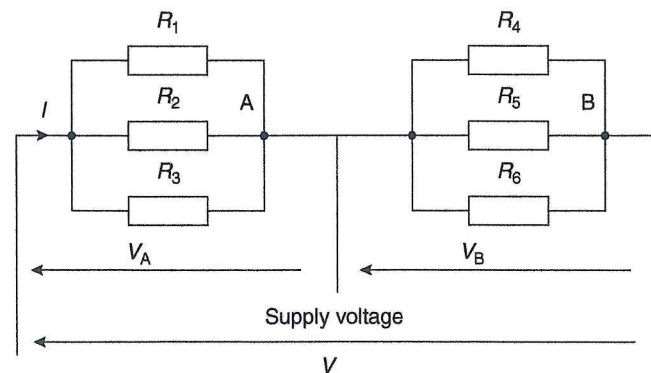
$V_A = IR_A$ and $V_B = IR_B$, and substituting for current I,

$$\text{or } V_A = \left(\frac{V}{R_A + R_B}\right) R_A \quad \text{and} \quad V_B = \left(\frac{V}{R_A + R_B}\right) R_B$$
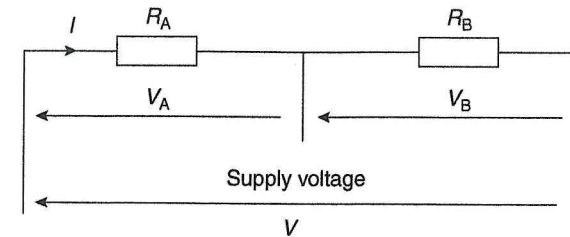
Once V_A and V_B are known, individual currents in each resistor are found by reverting to the original circuit.

$$\text{Thus } I_1 = V_A/R_1 \quad \text{and} \quad I_2 = V_A/R_2 \text{ Also } I_4 = V_B/R_4 \text{ etc.}$$

The above method is only given to explain the solution of Example 1.6 and illustrate a step-by-step procedure. There is no short-cut for problems of the series–parallel type. The reader should work only with the data given and not make any assumptions. Methods of solution using proportions for currents or voltages across parallel or series circuit sections are discouraged, as in practice resistance ratios are rarely simple, while adhering to and following simple, but sometimes more tedious methods will result in the correct answer.

Example 1.6. A circuit is built up from 5 resistors. Resistors of values 4Ω, 6Ω and 8Ω are connected in parallel to form a group, while resistors of 3Ω and 6Ω are connected in parallel to form another group (figure 1.12). The 2 parallel groups of resistors are connected in series across a 10V supply. Find the voltage dropped across each parallel group, the main supply current and the current in each resistor.

Let R_A be the equivalent of the first group.

$$\text{Then } \frac{1}{R_A} = \frac{1}{4} + \frac{1}{6} + \frac{1}{8} = \frac{13}{24} = 0.54S$$

$$\text{or } R_A = \frac{24}{13} = 1.85\Omega$$
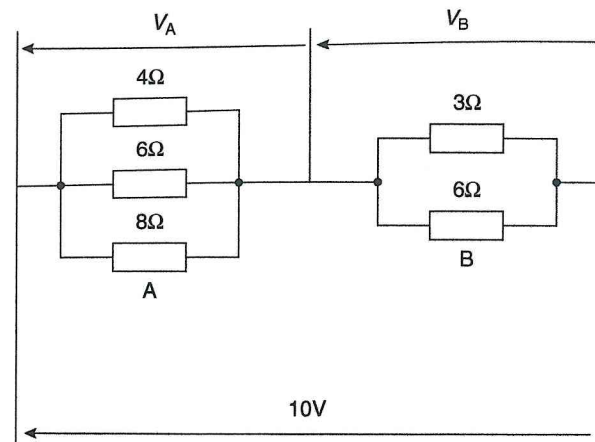
▲ Figure 1.12

Similarly let R_B be the equivalent resistance of the second group.

$$\text{Then } \frac{1}{R_B} = \frac{1}{3} + \frac{1}{6} = \frac{3}{6} = 0.5S$$

$$\text{or } R_B = \frac{6}{3} = 2\Omega.$$

For the equivalent series circuit, total resistance is R

$$\text{or } R = R_A + R_B = 1.85 + 2 = 3.85\Omega$$

Main supply current $I = \dfrac{V}{R} = \dfrac{10}{3.85} = 2.6A$

Voltage drop across R_A or the first parallel group $= 1.85 \times 2.6$

$$= 4.8V$$

Voltage drop across R_B or the second parallel group $= 2 \times 2.6$

$$= 5.2V$$

Check. Total supply voltage is $(4.8 + 5.2) = 10V$.

Current in 4Ω resistor $= \dfrac{4.8}{4} = 1.2A$

Current in 6Ω resistor $= \dfrac{4.8}{6} = 0.8A$

Current in 8Ω resistor $= \dfrac{4.8}{8} = 0.6A$

Check. Total current is $(1.2 + 0.8 + 0.6) = 2.6A$

Similarly: Current in 3Ω resistor $= \dfrac{5.2}{3} = 1.73A$

Similarly: Current in 6Ω resistor $= \dfrac{5.2}{6} = 0.87A$

Check. Total current is $(1.73 + 0.87) = 2.6A$.

Example 1.7. A battery of e.m.f. 42V and internal resistance 7Ω feeds a circuit consisting of 3 resistors connected in parallel. The resistors have values of 2Ω, 4Ω and 8Ω. Find the battery current, the battery terminal voltage and the current in each resistor (figure 1.13).

Let R be the equivalent resistance of the parallel-connected load.
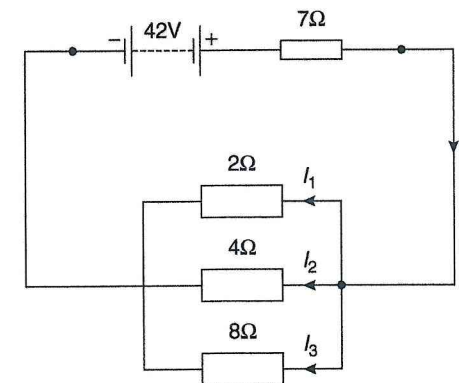
▲ Figure 1.13

$$\text{Then } \frac{1}{R} = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} = \frac{7}{8} = 0.875S \text{ and}$$

$$R = \frac{8}{7} = 1.14\Omega$$

The circuit can now be considered to have a total resistance of 8.14Ω made up from 1.14Ω and 7Ω in series.

The battery current I is given by $\dfrac{42}{8.14} = 5.16A$

The terminal voltage will be $5.16 \times 1.14 = 5.88V$

or the terminal voltage will be $42 - (7 \times 5.16) = 42 - 36.12 = 5.88V$

Current I_1 in 2Ω resistor is $\dfrac{5.88}{2} = 2.94A$

Current I_2 in 4Ω resistor is $\dfrac{5.88}{4} = 1.47A$

Current I_3 in 4Ω resistor is $\dfrac{5.88}{8} = 0.74A$

Check. Total current $I = 5.16A$.

Ammeters and Voltmeters

These are the main instruments used for electrical work and figure 1.14 shows how they are connected in a circuit. Ammeters are used to measure current and voltmeters for measuring P.D. or voltage. Both instruments operate on the same principle, but ammeters must have very low resistance as they are in series with the load and must not result in appreciable voltage drop. Voltmeters on the other hand must be of high resistance, as they may be connected across points which could be at a high P.D. For most circuit purposes, an ammeter is considered to have negligible resistance and a voltmeter to have infinite resistance, i.e. it draws no current.

In figure 1.14 a generator is shown as the energy source, S may be a single-pole or double-pole switch, as is shown here, and R is the load resistance. As a practical
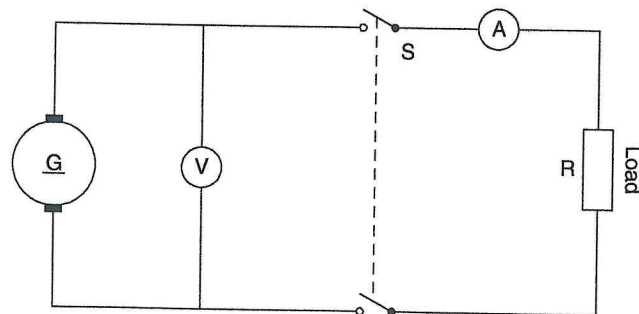
▲ Figure 1.14

example, the generator may have an internal resistance of 0.02Ω, the cable leads may have a total resistance of 0.03Ω, and R may have a value of 5Ω. If the generator is set to 220V on O.C., i.e. with the switch open, when the switch is closed a current of

$$\frac{220}{5 + 0.02 + 0.0.3} = \frac{220}{5.05} = 43.56A \text{ will flow round the circuit.}$$

The terminal voltage of the generator will 'sit down' to $220 - (43.56 \times 0.02)$ volts = $220 - 0.87 = 219.13V$. This would be shown by a voltmeter, while an ammeter would show 43.56A. If the voltmeter was disconnected and then connected directly across R it would indicate $219.13 - (43.56 \times 0.03)$ volts = $219.13 - 1.3 = 217.83V$ or voltage across R = IR = $43.56 \times 5 = 217.83V$. The voltage drop in the cables will be 1.3V. It is seen that the example of a simple distribution system has been worked as a simple series circuit and that the instruments perform their required functions. The ammeter shows the series circuit current, while the voltmeter indicates the potential drop across any chosen part of a circuit. It can also record the e.m.f. built up by a generator when the switch is opened, as this is the only condition when the e.m.f. appears at the terminals of the energy source.

Range of extension of ammeters and voltmeters

For practical work it may not be possible to pass all the circuit current through the ammeter. It may be difficult to construct a suitable instrument because of size or other limitations, and to introduce a certain degree of standardisation, it may be easier to use the ammeter with a *shunt* to measure the circuit current. Before considering applications of a shunt it is appropriate to point out that there are various types of electrical measuring instruments, described by their 'movements'. Such 'movements' utilise different operating forces and a shunt is normally used with the 'moving-coil' type only since this is constructed to the highest accuracy and sensitivity and is ideal for working with various types of transducer. Transducers are devices which generate mechanical or electrical outputs for measured quantities. It is assumed in this chapter, that a moving-coil ammeter or voltmeter is being considered.

A shunt is a specially constructed resistor of low ohmic value and, to make an ammeter capable of measuring a current greater than that which can be passed through it, a parallel arrangement of the ammeter and shunt is used. The ammeter is designed to carry a definite but small proportion of the main current and the rest of the current is forced to bypass the ammeter through the shunt, which is accurately made and set to a definite resistance value. It is calibrated with the ammeter instrument and must always be used with it. The calibrated leads between instrument and shunt form part

R_M includes resistance of instrument and leads
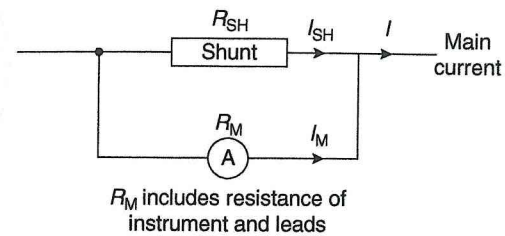
▲ Figure 1.15

of the arrangement and must not be cut or substituted for by other pieces of copper wire! Figure 1.15 shows the normal arrangement of instrument and shunt and the example shows the form of calculation necessary. It is seen that the calculation follows the pattern set for parallel-resistance circuits.

Example 1.8. Calculate the resistance of a shunt required to operate with a moving-coil milliammeter, which gives full-scale deflection (f.s.d.) for a current of 15mA and which has a resistance of 5Ω. (*Note.* 5Ω is taken to include the resistance of the connecting leads, as no specific mention of lead resistance has been made.) The combination of meter and shunt is required to read currents up to 100A.

Voltage drop across instrument when giving f.s.d. = current causing f.s.d. × resistance of instrument circuit

$$= I_M \times R_M = (15 \times 10^{-3}) \times 5 = 75 \times 10^{-3} \text{ volts}$$

$$= 0.075\text{V or } 75\text{mV}$$

Now the voltage drop across the instrument is the same as the voltage drop across the shunt.

Thus $I_{SH} \times R_{SH} = 0.075$ volts.

But the shunt current I_{SH} will be 100 − meter current

$$= 100 - 0.015 = 99.985\text{A}$$

So $R_{SH} = \dfrac{75 \times 10^{-3}}{99.985}\Omega$

$$= 0.000751\Omega$$

It is important to note the low resistance value of the shunt which is designed to carry the current without 'heating up'. The shunt is usually mounted behind the ammeter in the main current circuit. The 'light' calibrated leads are coiled to take up any 'slack' and

then brought up to the instrument. The ammeter may be marked 0–100 amperes, but in actual fact only a tiny current, some 15 mA, passes through the instrument, while the larger proportion of the current passes through the shunt. The reason for always using the instrument with its own calibrated shunt and leads is hopefully obvious!

To measure voltages higher than that for which the instrument movement is designed a *series* or *range* resistor must be used. This resistor is designed to drop excess voltage and dissipates some heat. It consists of special fine-gauge wire wound on a porcelain spool or on a mica card, and is mounted inside a ventilated case. The arrangement may be mounted behind a switchboard, if not contained in the instrument's case. Thin leads for carrying the small instrument current connect the range resistor unit and the instrument to the main supply terminals, usually through fuses. The voltmeter may be scaled 0–250 volts, but in fact only 0.075V may be dropped across it, when f.s.d. occurs (see Example 1.8). By far the biggest voltage drop occurs across the range resistor, and it should be noted that it always has a high ohmic value: thousands of ohms. This fact should be noted. Figure 1.16 shows the arrangement and Example 1.9 shows how the value of an appropriate range resistor is calculated.

Voltage to be measured

Range or series resistor R_{SE}
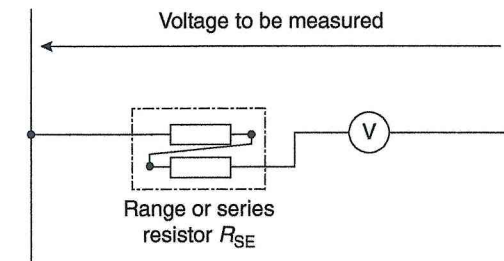
▲ Figure 1.16

Example 1.9. Calculate the resistance of the range resistor required to be placed in series with the instrument of Example 1.8 to make it into a voltmeter reading 0–250V. (The instrument has a resistance of 5Ω and gives f.s.d. with a current of 15mA.)

The current through the complete voltmeter circuit must be limited to 15mA, otherwise the instrument will 'burn out'. Resistance of the voltmeter circuit will be

$$\frac{250}{15 \times 10^{-3}} = 16.667 \times 10^3 \, \Omega = 16.667\text{k}\Omega$$

The instrument has a resistance of 5Ω, so the series or range resistor R_{SE} must have a value of (16 667 − 5) ohms = 16 662Ω.

The actual 'movement' or working unit of an ammeter or voltmeter is much the same and it is the use of a shunt or range resistor which determines whether current or voltage is measured. Multi-purpose portable test instruments are available which can make a range of measurements. A range switch, or *range multiplier* is provided to make the appropriate connection of shunt or range resistors.

Consider an instrument movement in which 15mA at a P.D. of 75mV gives f.s.d. Its resistance $= \dfrac{75 \times 10^{-3}}{15 \times 10^{-3}} = 5\Omega$.

If a voltage range 0–15V is required, the instrument circuit resistance must be $= \dfrac{15}{15 \times 10^{-3}} = 1000\Omega$ and a range resistor of $1000 - 5 = 995\Omega$ must be switched in.

Similarly if a voltage range of 0–150V is required the range resistor must be $\dfrac{150}{15 \times 10^{-3}} - 5 = 10000 - 5 = 9995\Omega$

If a current range 0–5A is required, a shunt is used whose value can be obtained thus:

P.D. across shunt = P.D. across instrument movement for f.s.d., or

P.D. across shunt = 75mV = 75×10^{-3} volts

The current through the shunt = $5 - 0.015 = 4.985$A and the resistance of the shunt would be

$$\frac{75 \times 10^{-3}}{4.985} = 0.015\,05\Omega$$

Instrument sensitivity

The term is used to consider the suitability of a measuring instrument for a particular purpose. If, for example, a voltmeter is badly constructed so that it requires a relatively large current for f.s.d., then the overall circuit current will be badly affected when the instrument is connected across any particular part of a circuit. This is very important for electronic circuitry. Consider a component of resistance value 1kΩ forming part of a series circuit drawing 1mA. A voltmeter of resistance 5kΩ connected across such a component will lower the resistance of the parallel arrangement to 0.803kΩ.

$$\text{Note } \frac{1}{R} = \frac{1}{5} + \frac{1}{1} = \frac{6}{5} = 1.25 \quad \text{or} \quad R = \frac{5}{6} = 0.803\text{k}\Omega.$$

Accordingly the circuit current will rise appreciably and the overall circuit conditions would change – an unwanted effect. The higher the resistance value of a voltmeter, the less the effect and voltmeters are therefore given a 'sensitivity' figure of ohms per volt. Thus a meter rated at 20kΩ/v will require a current of $\dfrac{1}{20 \times 10^3}$ amperes or $\dfrac{1}{20}$ milliamperes or $\dfrac{1}{20} \times 10^{-3} = 50\mu$A for f.s.d. and the range resistor required is calculated on this basis. Such a voltmeter connected across the component of the example would have little effect on the circuit current and should be the instrument used.

Practice Examples

1.1. A circuit is made up from 4 resistors of value 2Ω, 4Ω, 5Ω and 10Ω connected in parallel. If the current is 8.6A, find the voltage drop across the arrangement and the current in each resistor (1 decimal place).

1.2. One resistor group consists of 4Ω, 6Ω and 8Ω connected in parallel and a second group consists of 3Ω and 6Ω in parallel. The 2 groups are connected in series across a 24V supply. Calculate (a) the circuit current, (b) the P.D. across each group and (c) the current in each resistor (all 2 decimal places).

1.3. If the resistor arrangement of Q1.1 is connected to a 12V battery of internal resistance 0.65Ω, find the circuit current and the battery terminal voltage. Find also, the current in the 5Ω resistor (all 1 decimal place).

1.4. A moving-coil instrument has a resistance of 10Ω and requires a current of 15mA to give an f.s.d.. Calculate the resistance value of the resistor necessary to enable it to be used to measure (a) currents up to 25A (3 decimal places) and (b) voltages up to 500V (5 significant figures).

1.5. Two resistors of 60kΩ and 40kΩ value are connected in series across a 240V supply and a voltmeter having a resistance value of 40kΩ is connected across the 40kΩ resistor. What is the reading on the voltmeter (2 significant figures)?

1.6. When a 10Ω resistor is connected across a battery, the current is measured to be 0.18A. If similarly tested with a 25Ω resistor, the current is measured to be 0.08A. Find the e.m.f. of the battery (2 decimal places) and its internal resistance (1 significant figure). (Hint: consider this as 2 linear equations for E.) Neglect the resistance of the ammeter used to measure the current.

1.7. Two groups of resistors A and B are connected in series. Group A consists of 4 resistors of values 2Ω, 4Ω, 6Ω and 8Ω connected in parallel and group B consists of 2 resistors of values 10Ω and 15Ω in parallel. If the current in the 4Ω resistor is 1.5A, calculate (a) the current in each of the remaining resistors, (b) the supply voltage and (c) the voltage drop across the groups A and B (all 2 decimal places).

1.8. The voltage of a D.C. generator, when supplying a current of 75A to a load, is measured to be 108.8V at the switchboard. At the load, the voltage recorded is 105V and when the load is switched off the voltage rises to 110V. Find the internal resistance of the generator (3 decimal places), the resistance of the supply cables (1 decimal place) and estimate the fault current if a 'short-circuit' of negligible resistance occurred at the load terminals (3 significant figures).

1.9. The ammeter on a switchboard, scaled 0–300A is accidentally damaged. The associated shunt is marked 300A, 150mV. A small ammeter, scaled 0–1A with a resistance of 0.12Ω, is available, and might be used. Find if such an arrangement is possible, and if so, how it could be achieved using surplus resistors which are available?

1.10. Five resistors AB, BC, CD, DE and EA are connected to form a closed ring ABCDEA. A supply of 90V is connected across AD, A being positive. The following is known about the resistors: AB is 10Ω, BC is of unknown value R_1 ohms, CD is of unknown value R_2 ohms. DE is 6Ω and EA is 9Ω. A high-resistance voltmeter (taking negligible current) when connected across BE reads 34V with B positive and when connected across CE reads 6V with E positive. Find the values of R_1 and R_2, the current in branch ABCD and the main supply current (all 1 significant figure).
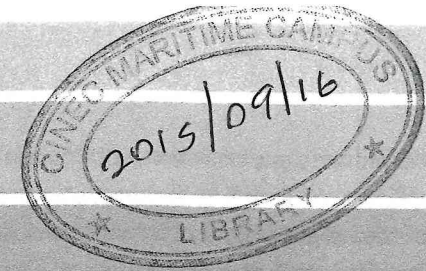
2

THE ELECTRIC CIRCUIT: UNITS

Wisdom is like electricity. There is no permanently wise man, but men capable of wisdom, who, being put into certain company, or other favourable conditions, become wise for a short time, as glasses rubbed acquire electric power for a while.

Ralph Waldo Emerson

All engineering studies stress the need for units and some of these will be introduced when mechanics and heat topics are covered. Units allow measurements to be taken and calculations to be made, and these are essential for comparison of experimental measurements with theoretical predictions and in the derivation of formulae from theory. In Chapter 1, the ampere, volt and ohm were considered, and although these units are yet to be defined, their importance in relation to basic electric circuits will be appreciated. The engineering student will recognise these units as being among those in daily use. Electrotechnology uses the same range of units. Modern engineering technology is fortunately based on the universal adoption of SI units and, several of these were encountered earlier in Chapter 1.

Before proceeding with any further study of SI system units, it is useful to introduce a historical note and consider the situation of engineering units as they developed. Towards the end of the nineteenth century 2 unit systems emerged in engineering; the British or foot–pound–second (fps) system and a metric or centimetre–gramme–second (cgs) system. The British or *Imperial system* had little merit since all units of the same kind, such as those of length, area, volume, etc., bore no relation to each other; there were also other units such as the calorie and horsepower which were arbitrarily and sometimes differently defined! The European metric system, however, was first

devised as a benefit to industry and commerce but physicists soon realised its benefits and it was adopted prior to 1870 in British scientific and technical circles. In 1873 the British Association for the Advancement of Science (BAAS) selected the centimetre and gramme as basic units of length and mass for physical purposes. Measurement of other quantities called for a base time unit and adoption of the second gave the cgs system.

The metric system, in cgs form, was adopted for electrical engineering in the early days of development. The system had the benefit that all the same kind of quantities are multiples of 10 and it was readily accepted internationally. The sizes of the absolute unit of the centimetre and the gramme gave rise to difficulties for the size of the desired electrical units which were either too large or too small for practical working. Use of these absolute units for key engineering formulae also proved difficult and so more workable units had to be defined. Such practical units include the volt, ampere and ohm. In about 1900, practical measurement in metric units began to be based on the Metre, Kilogramme and the Second (MKS) and the mentioned electrical units, constituting the unrationalised MKS system.

The next development came from a fact, repeatedly pointed out over the first half of the twentieth century, that a system of units could be devised to make the practical units of volt, ampere and ohm the absolute units of such a system. A more workable unit system known as the rationalised MKS system was recommended by the International Electrotechnical Commission of 1950. The change to the MKS necessitated revision of many reference and textbooks. For the student the new units made learning easier and formulae more manageable.

Prior to 1970, conditions existed when both older Imperial and newer rationalised MKS systems of units were in use simultaneously. The latest extension of metric units into all branches of business and industry enabled engineering to develop the SI system, the units of which are used throughout this volume. From an electrical viewpoint, the SI system is a rationalised MKS system with units in all the other fields of measurement being fully metricated. Further refining developments of the MKS standards included adoption of atomic definitions for both distance and time, but for the practical marine engineering student this does not concern topics covered in this volume.

The SI System

All measurement is in comparison with some standard or unit. The 3 fundamental units are those of length, mass and time. In the SI system the metre is taken as the fundamental

unit of length (s), the kilogramme as the unit of mass (m) and the second as the unit of time (t). From these fundamental units are derived all other units, which may further be classified as mechanical or electrical units. Thus Force is a derived mechanical unit involving a fundamental unit and a derived unit, i.e. mass and acceleration. For the SI system, a unit of force, or *Newton,* is introduced. Velocity is a derived unit involving distance and time, as is acceleration involving velocity and time. Both velocity and acceleration are mechanical units. The ampere is a derived unit involving force and length but it is used as a fundamental electrical unit. Other electrical units are the volt and the ohm which are derived units. The *Joule* and *Watt,* although used mostly in the past in connection with electrical engineering, are both derived from mechanical relationships and will be defined later.

Once units are recognised and understood, the reader should consider them as general engineering units rather than mechanical or electrical units. This applies especially to the units of work and power. Both mechanical and electrical engineering fields are concerned with common appliances and associated problems and using appropriate units, where a correct understanding of the magnitudes of the quantities involved, is essential to the engineer.

Mechanical units

Notwithstanding my earlier comment about atomic standards the fundamental units here require little definition as they are *accepted* working standards. The metre is the absolute standard, and is taken as the distance between 2 set marks on a certain metal bar. Similarly the kilogramme is the mass of an accepted 'standard' of a chosen metal. The time unit is the second, defined as $\frac{1}{86400}$ of a mean solar day.

Most of the principal SI derived units have already been introduced to the engineering student but revision is made only to extend use into electrical engineering.

Unit of force

Force can be defined as that which tends to cause an object to move, to change its motion or to keep the object at rest. The symbol for force is F but any value of newtons can be represented by the letter N after the numerical value.

THE NEWTON. This is the force needed to accelerate a mass of 1 kilogramme at a rate of 1 metre/second^2 (i.e. F = ma N). However, if the acceleration of the mass is due to gravity

alone, the rate of acceleration is a constant 9.81 ms^{-2}. This acceleration due to gravity is given the symbol 'g',

$$\therefore \ F = mg \ N.$$

The force due to gravitation is referred to as *weight*. A mass of 1kg has a weight of 1kg × 9.81 ms^{-2} = 9.81N

$$\text{i.e.} \ \ 1\,kg = 9.81N$$

Unit of work and energy

THE JOULE. This is defined as the *work* done or *energy* stored when a force of 1 newton acts through a distance of 1 metre in the direction of the force. The symbol for work or energy is *W* but any value in joules can be represented by the letter J after the numerical value.

From the definition, it follows that a force of *F* newtons, acting through a distance of *s* metres, does *F* × *s* newton metres of work or *F* × *s* joules.

$$\text{Hence:} \ W \ (\text{joules}) = F \ (\text{newtons}) \times s \ (\text{metres})$$

Unit of power

THE WATT. *Power* is the rate at which work is done or energy is converted and its unit is the watt. A watt is the power resulting, when a joule of energy is expended in a second. The symbol for power or rate of doing work is *P* but any value in watts can be represented by the letter *W* after the numerical value.

The definition can be more generally written as

$$P \ (\text{watts}) = \frac{W \ (\text{joules})}{t \ (\text{seconds})}$$

The joule and the watt were originally used in electrical engineering and are encountered throughout electrical problems. Example 2.1 is set out here to introduce electro/mechanical relationships.

Example 2.1. An electrical pump is required to lift 1200 litres of water through 10 metres in 6 minutes. Calculate the work done in joules and the pump's power rating. Assume 1 litre of water has a mass of 1 kilogramme.

Work done = force of gravity × distance lifted

$$\text{Thus } W = Fs = mgs$$
$$= (1200 \times 9.81) \times 10$$
$$= 12 \times 9.81 \times 10^3 \text{ newton metres}$$
$$= 117.72 \times 10^3 = 117\,720Nm \text{ or } 117\,720J$$
$$\text{Power} = \frac{\text{work done}}{\text{time taken}} = \frac{117\,720}{6 \times 60} = 327W$$

Pump power rating will be 327 watts.

Note. In this problem no account has been made of machine *efficiency*. This will be introduced later, but here, the practical rating figure of the electric motor driving the pump will be larger.

Electrical units

The same fundamental units are used as for the mechanical units namely: the metre, kilogramme and second. The primary derived unit is the ampere, which is the basic electrical unit of current and is a fourth fundamental unit. Before considering the ampere's definition, we will describe 2 related effects, which are observed when a current flows in a circuit.

(1) If the circuit's resistance is concentrated in a short length of conductive wire, a temperature rise of the wire is noted, showing a conversion of electrical energy into heat energy.

(2) If the circuit is supplied through 2 wires laid together, when the current is large and the wires flexible, a mechanical effect is observed. When a current is switched on, the wires are observed to move and this electromagnetic effect is used to define the ampere for the SI system. The factors governing the magnitude and direction of the force on the wires is described in the chapter on Electromagnetism.

Unit of current

THE AMPERE. This is the current which, when maintained in each of 2 infinitely long, straight, parallel conductors placed in a vacuum and separated by a distance of 1 metre between the wires' centres, produces on each conductor a force of 2 × 10^{-7} newtons per metre length of the conductor.

As stated in Chapter 1, the symbol for current is *I* and any value in amperes is represented by the letter A after the numerical value. The reader is reminded that practical circuit

currents range from only a few microamperes to several thousand amperes. Attention is drawn to the Table of Prefixes of Magnitudes given at the front of this book. Full consideration *must be* given to correct use of the abbreviation which follows the numerical value.

When a current flows for a set period of time, a quantity of electricity or current is conveyed round the circuit. The quantity which passes can be shown to be related to the work done in the circuit, but before this relationship is considered further, electricity must be defined in terms of current and time.

Unit of quantity

THE COULOMB. The usual unit – is sometimes called the ampere second. For practical every day use a larger unit, in electrical engineering, the *Ampere hour*, is used in connection with the capacity of batteries for accumulator charging.

The symbol for the quantity of electricity is Q and any value in coulombs can be represented by the letter C after the numerical value. Similarly any value in ampere hours may be represented by the letters Ah after the numerical value. As the quantity of electricity which is conveyed round a circuit varies with the strength of the flow of electricity and with time, a simple definition for the coulomb can be given:

A coulomb is the quantity of electricity conveyed by a steady current of 1 ampere flowing for a time of 1 second.

Thus Q (coulombs) = I (amperes) $\times t$ (seconds)

or Q (ampere hours) = I (amperes) $\times t$ (hours)

Thus, the following can be deduced:

1 ampere hour = 1 ampere $\times$ 1 hour

$\qquad$ = 1 ampere $\times$ 3600 seconds

$\qquad$ = 3600 ampere seconds

$\qquad$ = 3600 coulombs

Thus 1A h = 3600C.

Example 2.2. Consider Example 1.5, where a battery of e.m.f. 42V and internal resistance 7Ω is used to supply a circuit of 3 resistors 2Ω, 4Ω and 8Ω in series. If the current is switched on for 30 minutes, find the quantity of electricity which would have been transferred (1 decimal place).

Total resistance of circuit = 7 + 2 + 4 + 8 = 21Ω

Circuit current $= \dfrac{V}{R} = \dfrac{42}{21} = 2A$

Quantity of electricity = current $\times$ time in seconds

$$= 2 \times 30 \times 60 = 3600C$$

or Quantity of electricity = current $\times$ time in hours

$$= 2 \times 30/60 = 1.0A\ h.$$

Flow of an electric current results in energy being expended. This energy may appear as the work done by rotation of an electric motor, the heating of a furnace element or that needed for electrolytic dissociation of a salt solution. However, the relation between conveying a quantity of electricity round a circuit by an applied voltage and the resulting work done helps derive the units of voltage and resistance in terms of the coulomb and the joule defined previously.

Unit of voltage

THE VOLT. This is the unit of e.m.f. and P.D. and is defined as the e.m.f. applied, or the P.D. available between 2 points in a circuit, if 1 joule of work is to be done when 1 coulomb of electricity passes between the 2 points.

As stated in Chapter 1, the voltage symbol or e.m.f. is V and any value in volts is represented by the letter V after the numerical value. The reader should again refer to the Table of Prefixes of Magnitudes, and the correct use of the abbreviations.

From the definition above it is stated that the work done by the electric circuit equals the voltage applied across that part of the circuit times the quantity of electricity conveyed.

$\qquad$ • Thus: W (joules) = V (volts) $\times Q$ (coulombs)

$\qquad$ or W (joules) = V (volts) $\times I$ (amperes) $\times t$ (seconds)

$\qquad\qquad W = VIt = I^2Rt$

Example 2.3. Consider Example 2.2. A battery of e.m.f. 42V and internal resistance 7Ω is used to supply a circuit of 3 resistors, 2Ω, 4Ω and 8Ω in series. If the current is switched on for 30 minutes, find the energy converted (as heat) in Joules by each resistor and within the battery itself.

Circuit current was found to be 2 amperes.

Using form $W = I^2Rt$ then energy converted in each resistor will be:

2 ohm resistor $= 2^2 \times 2 \times 30 \times 60 = 14\,400$ joules

4 ohm resistor $= 2^2 \times 4 \times 30 \times 60 = 28\,800$ joules

8 ohm resistor $= 2^2 \times 8 \times 30 \times 60 = 57\,600$ joules

7 ohm battery $= 2^2 \times 7 \times 30 \times 60 = 50\,400$ joules

Total energy converted by the circuit $=$

$14\,400 + 28\,800 + 57\,600 + 50\,400 = 151\,200$ joules

Check. The total energy converted by the entire circuit may be found from $W = VIt$ joules

$= 42 \times 2 \times 30 \times 60 = 151.2$kJ.

The definitions of Power and Energy have already been considered, but it is useful to emphasise the key points, namely that power is the *rate* at which work is done and its unit is the Watt. Thus: $P = W/t$ or $W = Pt$

From $W = VIt$ it follows that the rate of work $P = VI$ or P (watts) $= V$ (volts) $\times I$ (amperes).

The above is a most important relationship, which can also be expressed in the following forms:

$$P = I^2R \quad \text{or} \quad P = \frac{V^2}{R}$$

The student's attention is drawn to the following facts.

As Energy $=$ Power $\times$ time   1 Joule $=$ 1 Watt second

Now a joule is a tiny unit of energy and for practical purposes a larger electrical unit of energy is needed. This unit is the *kilowatt-hour*, abbreviated to *kW h* and is the commercial unit of electricity or 'unit'.

As 1 kilowatt-hour $=$ 1 kilowatt $\times$ 1 hour

$= 1000$ watts $\times$ 3600 seconds

So 1 kilowatt-hour $= 3\,600\,000$ joules (3.6MJ)

Example 2.4. A 220V electric fire is rated at 2kW. Find the current taken when the fire is

Current taken $= \dfrac{W}{V} \dfrac{2 \times 1000}{220} = 9.09$A

Electricity used $= 2 \times 5 = 10$ kW h $= 10$ units

Cost $= 10 \times 6 = 60$p.

Unit of resistance

THE OHM. The ohm was defined in Chapter 1 as the unit of resistance in terms of the volt and ampere. A resistor has a value of 1 ohm resistance if 1 ampere passes through it when a P.D. of 1 volt is applied across its ends. Now that the fundamental relationships between the ampere, volt, joule and watt are defined, it is possible to further define the ohm in terms of the power or energy dissipated. Thus the ohm is defined as: that resistance which when 1 ampere passes through it dissipates energy at the rate of 1 joule per second (i.e. 1 watt).

Alternatively, the ohm is that resistance in which a current of 1 ampere flowing for 1 second generates 1 joule of energy.

For a resistor the energy produced by current flow appears as heat, and the following is of importance.

Since $P = VI$ and $V = IR$ then $P = (IR)I$

or $P = I^2R$ as seen earlier.

Power dissipated in a resistor is thus proportional to the current *squared*. If the current is doubled by raising the voltage, the power dissipation will be 4 times as large. Temperature rises in similar proportion and assuming the resistor is capable of carrying its normal current, with little capacity for working at a higher temperatures, a 'burn-out' will occur. The same limitations apply to cables, electrical machines and switchgear. Electrical equipment is assigned a rating which, on full load, enables it to operate with a safe temperature rise. Increase of the normal rated current, due to overloading or an 'overvoltage', results in a temperature rise proportional to the new current squared. The total temperature will rise rapidly as the over current occurs and if this is maintained circuit damage will result. Damage to electrical insulating materials can occur because of sustained overloads, and overheating is often the main cause of electrical machine failure.

Example 2.5. A hotplate of a ship's electric galley is fitted with a control marked High, Medium and Low. The heating element consists of 2 equal sections, connected in parallel for High, and in series for Low. Only one section is used for Medium. If the plate

$$\text{Load current} = W/V = \frac{93 \times 60}{110}$$

$$= 50.73A, \text{ i.e. } 51A$$

or alternatively

$$\text{Load current} = \frac{5610}{110} = 51A \quad \text{(total power/voltage)}.$$

Example 2.7. A pump is required to lift 12 tonnes of water through 10m in 2 minutes. Calculate the power needed to drive the pump, and the current taken if driven by a 220V motor and the cost of pumping is 5p per kW h unit. Assume the pump efficiency is 60% and the driving motor efficiency is 85%.

Work to be done = Force opposing gravity × distance lifted. Note 1 tonne = 10^3 kg

Thus work to be done = $(12 \times 10^3 \times 9.81) \times 10$ newton metres

$$= 117.72 \times 10^4 \text{ Nm}$$

Also 1 177 200 Nm = 1177.2kJ

This is the pump output. The input will be greater, i.e.

$$1177.2 \times \frac{100}{60} = 1962kJ$$

As pumping is achieved in 2 minutes or 120 seconds, the power input during this

$$\text{time} = \frac{1\,962\,000}{120} = 16350W \text{ Thus power required to drive the pump supplied by the}$$

motor is 16.35kW.

The motor's output power rating must be 16.35kW and the input power will be

$$16.35 \times \frac{100}{85} = 19.24kW$$

$$\text{Current taken by motor} = P/V = \frac{19\,240}{220} = \frac{962}{11} = 87.45A$$

$$\text{Energy used} = \text{Power} \times \text{time} = 19.24 \times \frac{2}{60} = \frac{19.24}{30} = 0.641kW \text{ h}$$

Cost 0.641 × 5 = 3.2p.

winding gear has an efficiency of 60%. Calculate the motor's power rating and also the current taken from the 220V ship's mains to 2 decimal places.

Work done by the winch = 500 × 9.81 × 22 newton metres

$$= 107\,910 \text{ Nm or } 107\,910J$$

This is the winch output or the winding gear's output. The input to the winding gear will be

$$107\,910 \times \frac{100}{60} = 179\,850J$$

The input to the winding gear will also be the motor output = 179 850J.

As the lifting is done in 22 seconds, the motor will give out power during this time.

$$= \frac{179\,850}{22} = 8175W$$

For a motor output of 8175W, the input power will be

$$8175 \times \frac{100}{86} = 9505.8W$$

Thus power rating of motor is 9.51kW

$$\text{Input or motor current} = \frac{9506}{220} = 43.21A$$

Example 2.9. A storage battery is provided for emergency use onboard a ship. The battery is arranged to supply essential services such as lighting during the period of time taken to start-up a 'standby' generator. The principal load supplied by the battery is the 'emergency' motor for an electric–hydraulic steering gear. This motor is rated at 220V, 15kW and has an efficiency of 88%. The battery must have a capacity sufficient to operate this motor and an additional lighting load of 20 60W lamps for a period of 30 minutes. Estimate the size of the battery and also its discharge current (2 decimal places).

Output of motor = 15kW

$$\text{Input to motor} = 15 \times \frac{100}{88} = \frac{187.5}{11} = 17.045kW$$

$$\text{Input current to motor} = W/V = \frac{17\,045}{220} = \frac{852.3}{11} = 77.48A$$

Lighting load $= 20 \times 60 = 1200W$

Lighting current $= W/V = \dfrac{1200}{220} = \dfrac{60}{11} = 5.45A$

Total current $= 77.48 + 5.45 = 82.93A$

or Discharge current $= 82.93A$

Size of battery (Ah) $= 82.93 \times \dfrac{30}{60} = 41.47A$ h.

▲ Figure 2.1

Grouping of Cells

Ohm's law states that the current in a circuit can be increased by raising the P.D. applied across a circuit or by decreasing circuit resistance. If the supply source is a generator, the applied P.D. can be varied by controlling the e.m.f. generated in the machine, but if a battery is the source of energy applied voltage cannot be varied easily. As a battery consists of a group of cells and as the e.m.f. of any cell is fixed, determined by its chemical composition, then a larger e.m.f. or a greater current can only be obtained by correct arrangement of the cells connected in either series, parallel or series–parallel arrangements.

Series connection

For this arrangement the –ve terminal of a cell is connected to the +ve terminal of an adjacent cell as shown in figure 2.1a. The arrangement is more simply depicted in figure 2.1b with a battery of 3 cells in series shown.

From Kirchhoff's voltage law, the e.m.f. of the source is equal to the sum of e.m.f.s taken round the circuit and for a battery of n cells in series, the e.m.f. $=$ e.m.f. of 1 cell $\times n$. Since this is a series circuit, the current in any 1 cell is the circuit current. The internal resistances of the cells are also in series and should be deducted as for resistances in a series circuit. This point is illustrated in figure 2.2.

Example 2.10. A battery consists of 4 cells in series, each of e.m.f. 1.5V and internal resistance 0.6Ω. Find the current flowing, if the battery is connected to 2 resistors of 2Ω
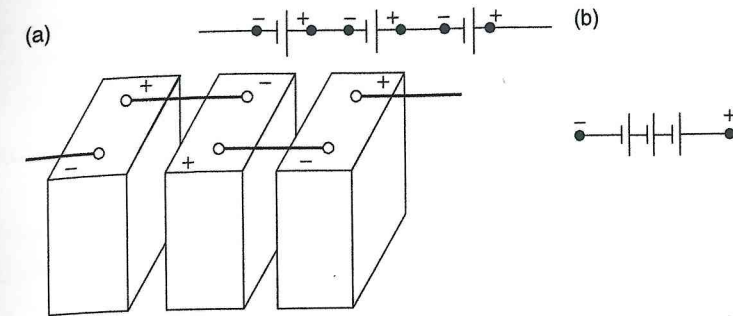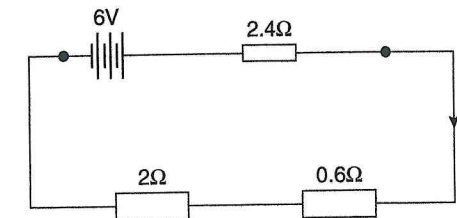
▲ Figure 2.2

Total battery e.m.f. $= 4 \times 1.5 = 6V$

Total battery internal resistance $= 4 \times 0.6 = 2.4Ω$

Total circuit resistance $= 2.4 + 2 + 0.6 = 5Ω$

So circuit current $= \dfrac{V}{R} = \dfrac{6}{5} = 1.2A$

Other values of interest would be

Battery terminal voltage $= iR = 1.2 \times 2.6 = 3.12V$, or

Battery terminal voltage $= V - iR_i = 6 - (1.2 \times 2.4)$

$$= 6 - 2.88$$

$$= 3.12V$$

Voltage drop across each resistance $= 1.2 \times 2 = 2.4V$

and $1.2 \times 0.6 = 0.72V$

Parallel connection

For this arrangement, the +ve terminals of all the cells are connected together as are all the −ve terminals. The arrangement is shown in figure 2.3.

From Kirchhoff's current law, the total current is the sum of the currents in each branch. The total current from the battery is equal to the sum of the currents available from each cell. For correct working, the e.m.f. of each cell should be the same and ideally have the same internal resistance. If n cells are in parallel, the total current is n times that given by 1 cell, but the battery e.m.f. is that of any one cell.

Battery internal resistance is obtained from the parallel-resistance formula, i.e. it is $\frac{1}{n}$ th of a cell's resistance. Battery resistance once determined is added to the external resistance to give the total circuit resistance as seen in Example 2.11.

Example 2.11. A battery consists of 4 cells in *parallel*, each of e.m.f. 1.5V and internal resistance 0.6Ω. Find the total current flowing if the battery is connected to a resistance of 2.6Ω in series for the arrangement shown in figure 2.4 (3 decimal places).

E.m.f. of battery = e.m.f. of 1 cell = 1.5V

Since $1/Ri_{TOTAL} = 1/Ri_1 + 1/Ri_2 + 1/Ri_3 + 1/Ri_4 = 4/0.6Ω$

Internal resistance of battery $= \frac{0.6}{4} = 0.15Ω$

Total resistance of circuit = 2.6 + 0.15 = 2.75Ω

Current $= \frac{V}{R} = \frac{1.5}{2.75} = 0.545A$. Other information would be
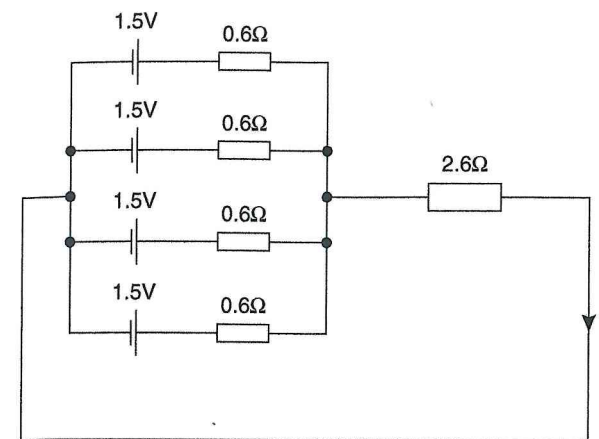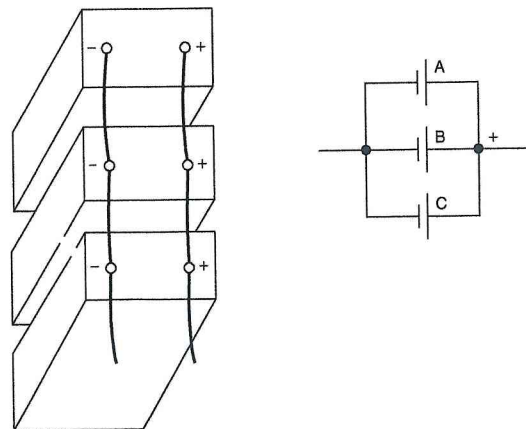
▲ Figure 2.4

Terminal voltage = 0.545 × 2.6 = 1.418V

or Terminal voltage = 1.5 − (0.545 × 0.15) = 1.5 − 0.082

$$= 1.418V$$

Current of 1 cell $= \frac{0.545}{4} = 0.136A.$

Series–parallel connection

To build this arrangement several cells connected in series are then connected in parallel with a similar number of cells in series. The arrangement is as shown (figure 2.5) and provides respectively both increased voltage and current. Cells in series provide an increased e.m.f., while parallel banks of cells supply an increased current.
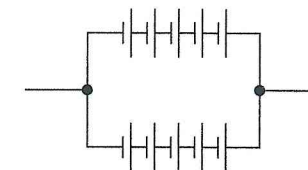
▲ Figure 2.5

The procedure for solving problems follows the same sequential approach already

Example 2.12 (a). Ten cells each of internal resistance 3Ω and e.m.f. 2V are connected in 2 parallel banks of 5 series cells per bank. They are then connected to an external load resistance of 20Ω. Find the load current and the P.D. across the battery terminals (3 decimal places). The arrangement is shown in figure 2.6a.

E.m.f. of a bank = $5 \times 2 = 10V$ = battery e.m.f.

Resistance of 1 bank = $5 \times 3 = 15Ω$

Resistance of battery $= \dfrac{15}{2} = 7.5Ω$ (as $1/R = 1/15 + 1/15$, 2 banks in parallel)

Total circuit resistance = $7.5 + 20 = 27.5Ω$

Circuit or load current $= \dfrac{10}{27.5} = 0.364A$

P.D. or terminal voltage = $0.364 \times 20 = 7.28V$

Current per cell = iR = current of 1 bank $= \dfrac{0.364}{2} = 0.182A$.

Example 2.12 (b). If the battery is rearranged with 5 banks of 2 cells in each, find the new current and voltage (3 decimal places). The arrangement is shown in figure 2.6b.

E.m.f. of a bank = $2 \times 2 = 4V$ = battery e.m.f.

Internal series resistance of a 1 bank = $2 \times 3 = 6Ω$

Internal resistance of 5 banks in parallel battery $= \dfrac{6}{5} = 1.2Ω$
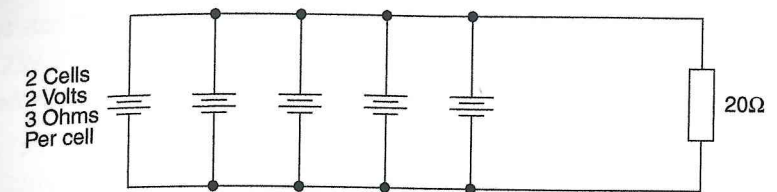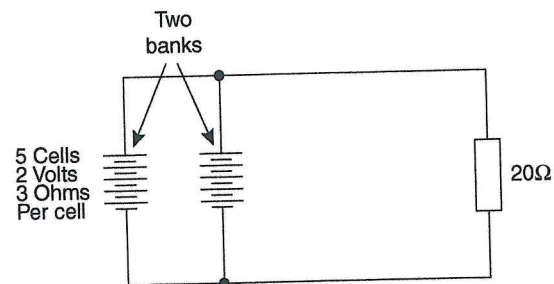
▲ Figure 2.6b

Total circuit resistance = external resistance + internal resistance = $20 + 1.2 = 21.2Ω$

Circuit or load current $= \dfrac{V}{R} = \dfrac{4}{21.2} = 0.188A$

Terminal voltage = $0.188 \times 20 = 3.77V$

(or $V - iR_i$) $4 - 1.2 \times 0.188 = 4 - 0.23 = 3.77V$

Current/cell = current of 1 bank $= \dfrac{0.188}{5} = 0.0376A$.

Maximum power transfer condition for a loaded circuit

Consider the simple circuit of figure 1.5, Chapter 1. If I was the circuit current then the power P supplied to the external load R will be given by I^2R, where $I = \dfrac{E}{R + R_i}$, and R_i is the internal resistance of the supply source, so:

$$P = \left(\dfrac{E}{R + R_i}\right)^2 R = \left[\dfrac{E^2R}{(R + R_i)^2}\right] \text{ or } P(R) = U(R)/V(R) \quad \text{where} \quad U(R) \quad \text{and} \quad V(R) \quad \text{represent}$$

respectively the numerator and denominator as functions of R.

Differentiating with respect to R, the maximum power condition is obtained when

$$\dfrac{d}{dR}\left[\dfrac{E^2R}{(R + R_i)^2}\right] = 0$$

Differentiation using standard methods results in a maximum for P when $R = R_i$, i.e. when the load resistance is equal to the internal resistance of the supply source.

The efficiency (as a percentage) of the supply for the condition $R = R_i$ is:

$$\dfrac{\text{Output Power}}{} \times 100 \quad \text{or} \quad \eta = \dfrac{I^2R}{} \times 100$$

But as $R = R_i$

$$\therefore \eta = \frac{I^2 R}{I^2 2R} \times 100 = \frac{1}{2} \times 100 = 50\% \text{ at maximum power transfer.}$$

Practice Examples

2.1. An electric hoist must lift a load of 2 tonnes to a height of 30m. The cage has a mass of 0.25 tonnes and the lifting operation must take 1.5 minutes. If the 220V motor is metered to take a current of 50A, find the installation's *percentage* efficiency (1 decimal place).

2.2. Thirty cells each having an e.m.f. of 2.2V and an internal resistance of 0.3Ω are connected to give a supply e.m.f. of 22V. If the arrangement is then connected to three 20V, 10W lamps in parallel, calculate (a) the battery terminal voltage (2 decimal places), (b) the current taken by each lamp (3 decimal places) and (c) the power wasted in each cell (3 decimal places).

2.3. A pump delivers 12 700 litres of water per hour into a boiler working at 15 bars pressure. The pump which is 82% efficient is driven by a 220V motor, having an efficiency of 89%. Calculate the current taken by the motor (1 decimal place). Assume 1 litre of water has a mass of 1kg and 1 bar = 10^5N/m².

2.4. A resistor of 5Ω is connected to a battery made up of 4 similar cells in series. Each cell has an e.m.f. of 2.2V and the current which flows is 1.4A. If the cells were connected in parallel, find the current which would flow through the 5Ω resistor (2 decimal places).

2.5. A 5-tonne cargo winch is required to lift a load of 5 tonnes at 36.5m/min. Calculate the power rating of the 220V driving motor if the efficiency of the winch gearing is 75% and that of the motor is 85%. Calculate also the current taken from the ship's 220V mains (1 decimal place).

2.6. A 220V diesel-driven generator is required to supply the following on full load: (a) lighting load comprising one hundred 100W and two hundred 60W lamps, (b) a heating load of 25kW, (c) miscellaneous small loads draw a current of 30A. Calculate the required power output of the diesel engine when the generator supplies all the loads at the same time (1 decimal place). Assume a generator efficiency of 85%.

2.7. ... similar correctly connected dry cells in series. The

resistor the current is metered to be 0.4A and the battery terminal voltage as 4.23V. If one of the cells of the battery is reversed and the circuit made up as before, estimate the new current value (3 decimal places).

2.8. A 150W, 100V lamp is to be connected in series with a 40W, 110V lamp across a 230V supply. The lamps are required to operate at their rated power values. Determine the values of suitable resistors to be used with the lamps and make a sketch showing how they should be connected (1 decimal place).

2.9. A resistor of 0.525Ω is connected to the terminals of a battery consisting of 4 cells, each of e.m.f. 1.46V joined in parallel. The circuit current is found to be 0.8A. Find the internal resistance of each cell (1 decimal place).

2.10. Twelve cells, each of e.m.f. 1.5V and internal resistance 0.225Ω, are arranged 4 in series per row or bank, with 3 banks in parallel. This battery is connected to a load consisting of a series–parallel resistor arrangement, made up of a 2Ω resistor connected in parallel with a 3Ω resistor, which are in turn connected in series with a 2.5Ω resistor. Find the battery terminal voltage, the power ratings of the resistors (all these 1 decimal place) and the energy converted into heat in the complete circuit which is switched on for 1 hour (3 significant figures).

3

CONDUCTORS, INSULATORS AND SEMICONDUCTORS

Nothing tends so much to the advancement of knowledge as the application of a new instrument.

Sir Humphry Davy

In this chapter we will look at the experimental differences between conductors, insulators and semiconductors. The reasons why certain materials are good conductors of electricity, while others are not, will be discussed at the electronic level. A substance which freely allows the passage of electricity is classed as a conductor, for example, metals, certain arrangements of carbon and some liquids – chiefly solution of salts, acids or alkalis. An insulator can be defined as a substance which prevents the free passage of electricity. Examples are rubber, porcelain, slate, mica, some organic materials and some liquids – notably oils. A semiconductor is defined as a material with properties between that of a conductor and an insulator due to controllable manufacturing processes to create precise atomic layered fabrication.

Resistance of a Conductor

Variation of conductor resistance with dimensions and material

and with identical cross-sectional area, are measured for resistance, their ohmic values vary in direct proportion to their lengths. Again if coils of wire of the same material and length, but of *different cross-section* are measured, their resistance values will vary in inverse proportion to the areas of the wires of which they are wound.

A similar series of comparative measurements with coils of wire of the same length and cross-sectional area but different material show that resistance values vary with the conductor material.

The basic tests described above, indicate that the resistance of a conductor or resistor can be changed by varying its dimensions or the material, and the relation of these factors to the actual conductor resistance will now be examined in detail.

1. DIMENSIONS. 1 (a). Resistance of a conductor is proportional to its length, for example, the conductor resistance of a 100m length of cable will be double that of a 50m length of the same cable. This can be shown as follows:

Let R_A ohms = the resistance of a 50m length. Then two 50m sections in series will have a resistance of R ohms

Hence $R = R_A + R_A = 2R_A$

But the length has been doubled

So $2 \times$ Length = $2 \times$ Resistance of original length. Summarising:

Resistance is proportional to Length or $R \propto l$.

1 (b). Resistance of a conductor is inversely proportional to its area, for example, the conductor resistance of a 1mm² cross-sectional area cable will be twice that of the same length of cable of the same conductor material, but of 2mm² or twice the cross-sectional area. This can also be shown thus:

Let the resistance of 1mm² cable be R_A and suppose an identical cable to be connected in parallel with it. The resistance of the combination will be R ohms.

$$\text{Hence } \frac{1}{R} = \frac{1}{R_A} + \frac{1}{R_A} = \frac{2}{R_A} \quad \text{or} \quad R = \frac{R_A}{2}$$

Thus the resistance of the combination is *half* the original cable resistance, but the area of the combination is twice that of the original cable

From the above, it follows that doubling the area halves the resistance of a conductor of the same length and material. Thus:

Resistance is inversely proportional to Area or $R \propto \dfrac{1}{A}$

2. MATERIAL. The resistance of a conductor depends upon the material from which it is made, for example, the resistance of a length of iron wire is nearly 7 times greater than the resistance of a piece of copper wire of identical dimensions, i.e. same length and cross-sectional area. If resistance varies with the material, we must define a property to compare resistance values with standard dimensions of the conductor. The term *resistivity or specific resistance* (symbol ρ – the Greek letter rho), is introduced, and measured in ohm-metres or microhm-millimetres.

The Resistivity or Specific Resistance of a material is the resistance measured between the opposite faces of a cube of unit dimensions. Thus for the diagram (figure 3.1) a cube of pure copper of sides 1 metre is taken and the resistance measured between the faces as shown by the arrows, the resistivity would be measured as $1.725 \times 10^{-8}\,\Omega m$ or $17.25\mu\Omega mm$.

Material temperature is recorded at the time the test is made and is often specified with the resistivity figure. Thus ρ for copper is given as 1.725×10^{-8} ohm-metre at 20°C. The reasons for specifying the temperature will be explained shortly.

As R (Resistance) $\propto 1$ (length) and $R \propto \dfrac{l}{A}$ (Area). So $R \propto \dfrac{l}{A}$ or $R = k\dfrac{l}{A}$ or where k is a constant. If ρ is taken as this constant k, then the foregoing can be written as:

$$R = \frac{\rho l}{A}$$

Problems involving resistivity are solved by use of the above expression, but it is essential to note the units used for the resistivity. If l and A are not in these units, they must be converted as shown (Example 3.2.)
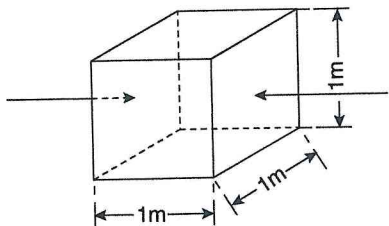
Example 3.1. If 5m of manganin resistance wire, 0.1mm diameter, have a resistance of 267.5 ohms, find the resistivity of the material (2 significant figures).

$$A = \frac{\pi \times d^2}{4} = \frac{3.14 \times (1 \times 10^{-4})^2}{4}$$

$$= 0.785 \times 1 \times 10^{-8}\,m^2$$

So $l = 5m$

Since $R = \dfrac{\rho l}{A}$ then $\rho = \dfrac{RA}{l} = \dfrac{267.5 \times 0.785 \times 1 \times 10^{-8}}{5}$

or $\rho = \dfrac{535 \times 0.785 \times 10^{-8}}{10}$

$$= 41.998 \times 10^{-8} \text{ ohm-metre or } 42 \times 10^{-8}\,\Omega m$$

$$= 420 \times 10^{-9} = 420 \times 10^{-6} \times 10^{-3}$$

$$= 420\mu\Omega mm.$$

Example 3.2. Find the length of wire required to make a 10Ω resistor, if the diameter is 1mm and the resistivity is 450$\mu\Omega$ mm (2 decimal places).

Here $R = 10\Omega$. $\rho = 450 \times 10^{-6} \times 10^{-3}$

$$= 45 \times 10^{-8} \text{ ohm-metre}$$

$d = 1mm = 1 \times 10^{-3}$ metre

Since $R = \dfrac{\rho l}{A}$ hence $l = \dfrac{RA}{\rho}$

or $l = \dfrac{10 \times \pi \times (1.0 \times 10^{-3})^2}{45 \times 10^{-8} \times 4}$ metres

$\therefore l = 17.44m$.

Occasionally a problem involving the formula $R = \dfrac{\rho l}{A}$ can be worked by a method of proportion.

This makes for easier working than finding the resistivity values and resubstituting in the formula to obtain the answer. This is illustrated by the following example.

Example 3.3. If the resistance of 1.6km of copper wire of 0.5mm diameter is 170Ω, calculate the resistance of 1km of iron wire of 1.0mm diameter, assuming that the resistivity of iron is 7 times that of copper (3 significant figures).

Resistance of 1600m of copper wire 0.5mm diameter is 170 ohms, then resistance of 1000m of copper wire 0.5mm diameter is

$$\frac{170 \times 1000}{1600 \times 4} \text{ ohms}$$

and the resistance of 1000m of copper wire 1.0mm diameter is

$$\frac{170 \times 1000}{1600} \times \frac{1}{4} \text{ ohms}$$

Since Area $= \frac{\pi d^2}{4}$, it follows that wire of twice the diameter will have an area 4 times as great and the resistance therefore will be reduced by a factor of 4.

Resistance of 1000m of copper wire 1.0mm in diameter is

$$\frac{170 \times 1000}{1600 \times 4} \text{ ohms}$$

So resistance of 1000m of iron wire 1.0mm diameter

$$= \frac{170 \times 1000 \times 7}{1600 \times 4} = 186\Omega$$

Alternatively as $R_1 = \rho_1 l_1/A_1$ for the copper and $R_2 = \rho_2 l_2/A_2$ for iron dividing one by the other will give the ratio: $R_2/R_1 = \rho_2 l_2 A_1 /\rho_1 l_1 A$ and using the ratio $\rho_2/\rho_1 = 7$ will arrive at the same answer.

Variation of conductor resistance with temperature

Most conductors resistance changes when their temperature changes. Usually this change or variation follows a straight-line relation as is shown in figure 3.2. If the resistance of a resistor is measured each time its temperature is altered and the results plotted, a graph such as (1) or (2) or (3) is obtained. These graphs cover the main types of conductor and show that:

(1) For pure metals, resistance increases continuously with temperature.

(2) For certain metal alloys used for making resistors, such as Manganin or Constantan, the graph is horizontal, i.e. resistance is largely unaffected by temperature.

(3) For certain partial conductors, such as carbon, resistance is found to *decrease* with temperature.

TEMPERATURE COEFFICIENT OF RESISTANCE. Because of the straight-line relationship between resistance and temperature, illustrated in figure 3.2, a simple law is possible and an equation can be deduced, which within normal ranges of temperature, allows the resistance R of a conductor at any temperature T to be obtained in terms of the resistance R_0 at 0°C and a coefficient (symbol α – the Greek letter alpha), known as the *temperature coefficient* of the conductor material. The appropriate equation is:

$$R = R_0(1+\alpha T) \text{ where } T \text{ is the temperature in } °C.$$

For the diagram (figure 3.3) the graph for a pure metal (copper) is illustrated and enlarged to show this formula for metals, alloys and carbon, which cuts the R axis to give a value of R_0, the resistance at 0°C. If the resistance has a value R_0 at 0°C then at 1°C it will be increased by a small amount x. The fraction $\frac{x}{R_0}$ is taken as the temperature coefficient α of the metal or $\frac{x}{R_0} = \alpha$ and $x = \alpha R_0$.
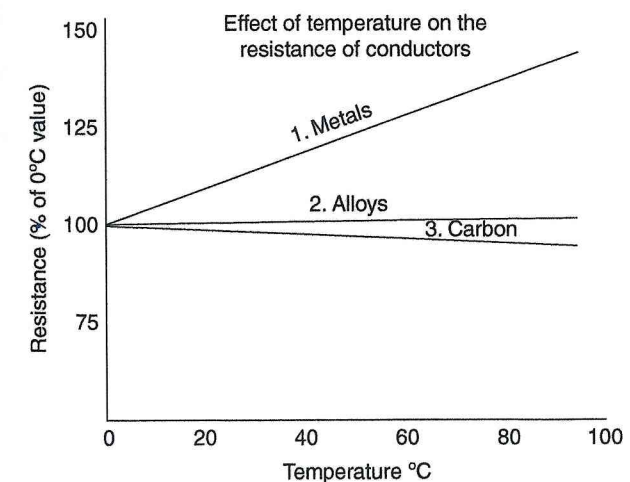
▲ Figure 3.2

α is thus the fraction of the resistance at 0°C by which the resistance increases for 1°C rise in temperature.

If x = increase in resistance for 1° rise in temperature, then

xT = increase in resistance for $T°$ rise in temperature

Thus $R = R_0 + xT = R_0 + R_0\alpha T$

or $R = R_0(1 + \alpha T)$

For copper, α has a value $\dfrac{1}{234.5} = 0.004\,265/°C$.

The temperature coefficient of resistance α, is usually based on average conditions obtaining from 0°C to 100°C and is thus the ratio of the increase in resistance per °C rise in temperature to the resistance at 0°C. Alternatively, the temperature coefficient of resistance is defined as the increase of resistance of 1 ohm at 0°C for 1°C rise of temperature.

If the graph (figure 3.3) rises as temperature increases, the material will have a 'positive' temperature coefficient of resistance; whereas if the graph falls, the material has a

Note:
$\propto$ Taken as $\dfrac{X}{R_0}$
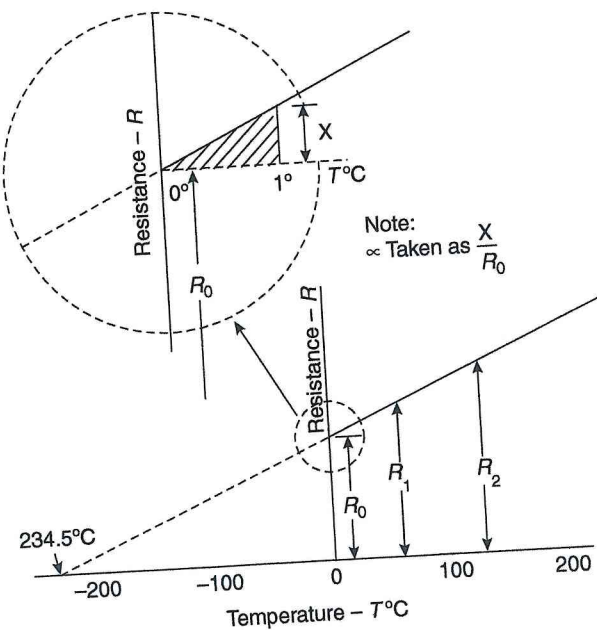
'negative' temperature coefficient. Usually for practical work and problems, the ohmic value as given for a resistor will be at a temperature other than 0°C and the following expression will help facilitate obtaining the resistance value at any temperature, if its resistance value is given for any other temperature condition.

Let R_1 be the resistance at a temperature T_1°C and R_2 be the resistance at a higher temperature T_2°C.

Then $R_2 = R_0(1 + \alpha T_2)$ and $R_1 = R_0(1 + \alpha T_1)$

Dividing, we have

$$\frac{R_2}{R_1} = \frac{R_0(1+\alpha T_2)}{R_0(1+\alpha T_1)} \quad \text{or} \quad R_2 = R_1\frac{(1+\alpha T_2)}{(1+\alpha T_1)}$$

Example 3.4. The cold resistance of a coil of wire is 20Ω at 15°C. It is heated to give a resistance of 23Ω. Find its temperature rise, if the temperature coefficient of the resistance material is 0.0042 per °C (1 decimal place).

Using the expression given above.

$$\frac{23}{20} = \frac{R_0[1+(0.0042\times T_2)]}{R_0[1+(0.0042\times 15)]}$$

Thus $1.15 \times 1.063 = 1 + 0.0042 \times T_2$

or $1.222\,45 - 1 = 0.0042 \times T_2$

and $0.223 = 0.0042 \times T_2$ or $T_2 = 53.1°C$

So the temperature rise is 38.1°C.

Example 3.5. The filament of a 230V lamp takes a current of 0.261A when working at its normal temperature of 2000°C. The temperature coefficient of the Tungsten filament material can be taken as 0.005/°C at 0°C. Find the current which flows at the instant of switching on the supply to the cold lamp, which can be considered to be at a room temperature of 20°C (1 decimal place).

Resistance of lamp (hot) $= V/I = \dfrac{230}{0.261} = 882Ω$ at 2000°C

Here $R_2 = R_0(1 + \alpha T_2)$.

So $R_0 = \dfrac{882}{[1 + (0.005 \times 2000)]} = \dfrac{882}{11} = 80.2\,\Omega$

Again

$R_{20} = 80.2\,[1 + (0.005 \times 20)]$

or $R_{20} = 80.2\,(1 + 0.1) = 80.2 \times 1.1 = 88.22\,\Omega$

and current taken at switching on when cold $= \dfrac{V}{R} = \dfrac{230}{88.22} = 2.6\text{A}.$

This example shows how resistance change with temperature affects practical working conditions and this should be taken into account. The 'tripping' of a circuit-breaker or 'blowing' of a main-fuse could mean loss of supply to a large lighting or resistance load. Before any attempt is made to restore the supply, sections of the load must be isolated, so that when the main switching on takes place, only part of the load is applied to the supply and this load is then gradually increased to its full value, by closing the individual circuit switches. The reason for 'shedding load' is related to the fact that the lamps will have cooled when the supply was off and when supply is restored, a current of 7–8 times the full-load value will be taken as a surge! This current will fall as the lamps heat up and may only last for a few milliseconds, but it could be sufficient to re-trip the circuit-breaker or blow the main-fuse. Thus a fault condition may be suspected, but in fact the cause of the current surge can be explained and appropriate action taken.

Resistance of an Insulator

Variation of insulation resistance with dimensions and material

An insulator is defined as a substance which prevents the free flow of electrons. In electrical devices, machines and cables, insulation helps confine electricity flow to the required circuit and to prevent any current from taking 'leakage paths'. Leakage currents are minimised by making the resistance of their paths as large as possible. Thus material with a high resistivity (ρ) is used for the insulation, the length of the leakage path is kept as large as possible and the area as small as possible. It should be realised that insulation allows some current to pass and that by measuring this current

the insulation quality can be judged. Instruments such as high-resistance ohmmeters or insulation testers are available such as the 5kV and 15kV 'Megohmmeter' available from Megger.

The simple diagram of figure 3.4 shows the path of the leakage currents in the cables forming part of the circuit feeding a load and it will be noted that the leakage currents flow radially from or towards the conductors of the cable. Cables can be looked upon as steam pipes radiating heat, the longer the pipe the more the heat radiated, i.e. the greater its radiating area. Similarly for the cables, the thickness of the insulation, shown as t is actually the length l in the formula $R = \dfrac{\rho l}{A}$. while area A is given by the curved surface of the insulation. The larger this surface the easier it is for leakage current to flow.

Cable insulation resistance is measured between the core and sheath, or 'earth' and is given by an approximation of the formula $R = \dfrac{\rho l}{A}.$

Here ρ would have a very high value; for vulcanised rubber it is $10^{15}\,\Omega\text{m}$ or $10^{9}\,\text{M}\Omega\text{m}$. l will be the insulation thickness t and surface area a are proportional to the cable length. Thus if the insulation resistance of 100m of cable was measured as 180MΩ, then 200m of the same cable would have a resistance value of 90MΩ. The key point is that cable-conductor resistance is doubled for double the length, but insulation resistance is halved. Doubling the length has doubled the area of the leakage paths and since

$r \propto \dfrac{1}{A}$, if a is doubled r is actually halved.
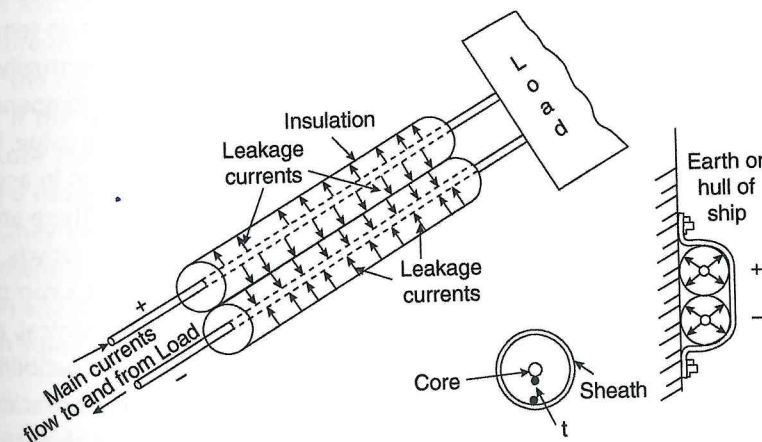
▲ Figure 3.4

It must be understood why a large electrical cable installation or a machine when tested for insulation resistance may give a low figure, while the value obtained for a small installation or machine may be much larger. Insulation resistance is also affected by other factors, besides the size of the installation or machine. Site conditions such as temperature, humidity, cleanliness together with age must be taken into account and the resistance value means little unless compared with that obtained for a comparable new installation or machine. Acceptable insulation-resistance values for installations and machines are set out in the appropriate Institution of Electrical Engineers (IEE) Regulations and the points made above are stressed to show that test results must be treated with due consideration. Conductor-resistance measurements are more straightforward, although here again, special testing techniques should be employed depending on the type of resistor or apparatus being measured.

Variation of insulation resistance with temperature

For electrical devices, machines and cables, the permitted working temperature and thus the equipment's current-carrying capacity is limited largely by the restrictions imposed by the insulation. Insulation is usually made from cotton, silk, rubber or plastics, and as a general rule, if they are subjected to excessive temperatures their electrical and mechanical properties are reduced. Even if insulation such as mica or porcelain are not damaged by excess temperatures, it is seen from the attached graph (figure 3.5) that, like the partial conductor carbon, the insulation resistance falls with temperature rise, but here the relationship is not a straight line. The graph follows a logarithmic law and so insulation resistance falls rapidly as temperature rises. An increasing leakage current flows through the insulation as its temperature rises and the current generates more internal heat which may eventually cause a sudden catastrophic 'breakdown' of the insulation. The allowable temperature rise for any electrical equipment, which gives a safe insulation-resistance value, has been determined by experience and the power rating of appliances is set in accordance with accepted specifications. Lloyds has its set maritime standards. There are several standards which cover marine equipment, including EN 60533, Lloyds Register specification N°1 and the most common standard IEC 945. IEC 945 is a complete test standard for the marine environment taking into account the mechanical, electrical and environmental conditions as well as the ElectroMagnetic Compatibility (EMC) phenomena. Sections 9 and 10 of IEC 945 include EMC requirements, covering a wide frequency range due to the breadth of equipment found on board ships. Section 9 covers EMC emissions, which take into account the protection of the ship's essential navigation and radio systems as well as the consideration that ships normally operate
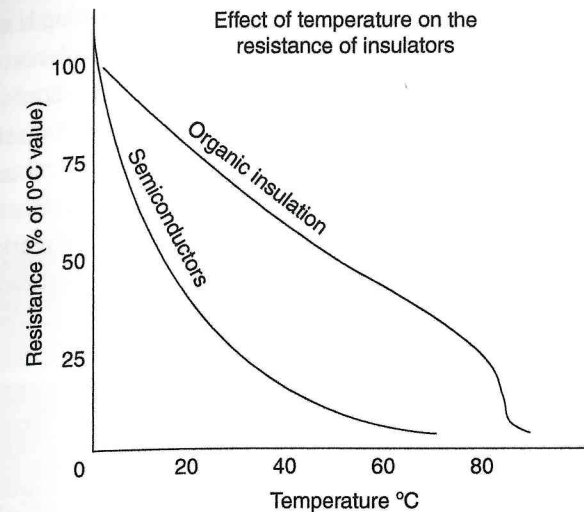
▲ Figure 3.5

at sea and must not interfere with land based radio communications. Section 10 is an EMC immunity section, designed to suit the marine environment. Although IEC 945 and EN 60533 are comprehensive test specifications (in some cases the Lloyds Register Spec. N°1 is applicable for the ships underwriters, and may be required), this is normally part of the contractual agreement when supplying equipment integral to a particular ship.

For example, BS Specifications or Lloyd's Regulations may specify a working temperature rise of 50°C for a particular motor when performing a certain duty. This would be when it developed its rated output in an ambient or room temperature of 30°C. Thus a total temperature of 80°C will be allowed. This figure varies for the type of insulation with which the machine is constructed, but for the example, if the same motor is to work in an ambient of 50°C, then the allowable temperature rise will be reduced to 30°C. The motor will now only be capable of giving a reduced output and would have to be derated. Alternatively a larger machine may be used, if the full original power output is still required. Derating of equipment is necessary to ensure a maximum *safe* working temperature for the insulation and for this condition, the insulation resistance will reach an acceptable minimum value.

Since insulation-resistance values change as the temperature of the equipment alters, and it is also affected by other load factors such as: size of installation, humidity, cleanliness, age and site conditions, then a true indication as to the state of the installation or machine can only be gained by reference to a record or log of

readings, built up over time. It should be accepted that such a log is essential for large electrical installations. Many ships are now fitted with insulation-resistance indicators which record leakage current and the insulation resistance. Such indicators assist the keeping of a log which show comparative readings for the same temperature rise taken when the installation or machine was new, dry and clean. The difference between the readings helps assess the state of the equipment at the time of checking, and if an improvement in readings is deemed essential for safe working, appropriate arrangements can be made for cleaning, drying out or a more thorough inspection and overhaul.

Resistance of a Semiconductor

Electronic devices utilising semiconductor materials will be considered in some depth in Volume 7. A semiconductor can be described as a material which, for given dimensions, has a resistance value between that of a conductor and an insulator of the same dimensions. The main use of semiconductor materials is in solid-state devices such as rectifier diodes and transistors, but here we consider the resistance/temperature property in relation to the *thermistor* which is an electrical resistor whose resistance is greatly reduced by heating, and is used for measurement and control.

Variation of semiconductor resistance with temperature

Semiconductor materials have resistance values which change when heated. Germanium and silicon, as typical examples, both have negative temperature coefficients which are not constant, but increase as the materials are heated. The relationship of resistance with temperature is an inverse variation and gives a graph similar to that shown (figure 3.5). It is observed that as a semiconductor material is heated, its resistance *falls* and if this material is used as a resistor the current passed will *increase* as the material heats up. The semiconductor, when used in this way, is known as a thermistor. It was adapted for use as a measuring or regulating device and developed for marine work as the detecting element of an electrical temperature-indicating instrument. The original thermometer head consisted of a coil of platinum wire which, when heated, altered the resistance of an indicator circuit so the circuit could be calibrated to indicate temperature. A thermistor element, being more robust, and of smaller dimensions than a platinum wire, gives a greater resistance change for a given temperature change and is thus more sensitive and accurate.

The device can be used as a regulator since it can alter the operating current to a controlling circuit when its temperature changes. Thus if a thermistor is placed in the windings of an electric motor, any overheating adjacent to it will result in the thermistor-circuit current increasing until the connected motor protective device is operated. Such thermistor operated units are used for marine applications in conjunction with motor starters but must be correctly located and connected.

Heat and Electrical Energy

Energy exists in various forms, the mechanical, electrical, thermal and chemical forms are those most commonly used in the modern marine industry and the work done when energy is expended is put to use in various ways. Although the term 'energy expended' is commonly used, it should be remembered that energy cannot be destroyed or lost, it can only be changed from one form to another and the convertibility between mechanical and electrical energy is seen in a machine like the electrical generator or electric motor. For the former, mechanical energy is passed in at the shaft and electrical energy obtained and utilised in a circuit connected across the machine terminals. Electrical energy may be converted into heat, light or mechanical energy. For the electric motor electrical energy is passed in at the terminals and mechanical energy is passed out at the shaft.

Relation between mechanical and heat energy

The fact that heat is a form of energy is obvious to the practical engineer, who is aware of the dangers associated with a 'hot bearing', 'slipping belt' or 'clutch'. In these instances mechanical energy is made available by a prime-mover and converted into unwanted heat through the mechanism of friction. If this energy conversion process continues unchecked, the temperature of associated machine parts may rise to dangerous levels, when a 'burn-out' or fire may result. Examples have been given to show that an elementary deduction can be made showing that the heat energy produced is proportional to the mechanical energy being expended.

SPECIFIC HEAT CAPACITY. This is found by a simple mechanical test. The apparatus consists of a hollow brass cylinder, rotated by a belt drive. The cylinder is filled with a known amount of water and rotated against a friction surface applied with known tension. By simple calculation, the work put in at the drive pulley is related to the heat produced at the cylinder. James Joule, an English scientist, by careful experimentation

showed that 4.187 joules of work is required to produce sufficient heat to raise the temperature of 1 gramme of water by 1 degree Celsius (or 1 Kelvin). In SI units, if the mass of water is taken as 1 kilogramme it follows that 4187 joules (4200J approx.) is required. The joule is also an SI unit of heat, and this constant of 4200 is taken into account by introducing the term *specific heat capacity*, defined as the quantity of heat needed to raise unit mass of a material through a temperature interval of 1 degree Celsius or 1 Kelvin. Different materials require different amounts of heat to produce the same temperature rise on the same mass. The units of specific heat capacity (symbol c), are heat units per unit mass per unit temperature. As, for SI units, the most convenient unit of mass is the kilogramme so the kilojoule will be the appropriate size of heat unit to give specific heat capacity in kilojoules per kilogramme per Kelvin or kJ/kgK. In terms of the Celsius temperature scale, this will be in kJ/kg°C. Because the relation between energy and heat is readily determined for water it is taken as 4200 joules, and follows that the specific heat capacity value for water would be in 4.2kJ/kg°C. The values for other materials are also determined by experiment and compiled into tables of physical constants (e.g. Tables of Physical and Chemical Constants by *Kaye & Laby*). The following examples illustrate conversion from mechanical to heat units which involves use of differing specific heat capacity values.

Example 3.6. A motor brake-test rig consists of a water-cooled, cast-iron pulley and a fixed frame made to carry 2 spring balances to which are fastened the ends of a rope which passes round the pulley. Both spring balances hang from screwed rods which are arranged to be adjustable to alter the tension on the rope. Tests made on a small motor running at a full-load speed of 750 rev/min give the following readings. Spring balances 16.89kg and 0.55kg. The pulley is hollow 102mm long, 380mm in diameter (these are outside dimensions). It has an average wall thickness of 6.4mm. It has a mass of 2.72kg and is designed to be half-filled with water. Estimate the output power of the motor being tested and the time for which the motor can be tested before the water starts to boil. The temperature of the pulley and water is 15°C at the start of the test and the rope diameter is 25mm. Take the specific heat capacities of water and cast iron as 4.2 and 0.42kJ/kg°C respectively.

Power output of the motor is given by the expression $\dfrac{2\pi NT}{60}$

Effective load on brake = (16.89 − 0.55) = 16.34kg

Hence restraining force F = 16.34 × 9.81 newtons

Effective radius $= \dfrac{380 + 25}{2} = 202.5$mm

Output power $= 2 \times \pi \times \dfrac{750}{60} \times (16.34 \times 9.81 \times 202.5 \times 10^{-3})$ W

$= 2560\text{W} = 2.56\text{kW}$

Heat energy available per minute = 2.56 × 60 = 153.6kJ

Volume of water $= \dfrac{\pi d^2}{4} \times \text{width} \times \dfrac{1}{2}$

$= \dfrac{\pi}{4}(380 - 12.8)^2 \times (102 - 12.8) \times \dfrac{1}{2}$ mm^3

$= 4.72 \times 10^{-3}$ m^3

Since 10^3kg is the mass of 1 cubic metre of water

Then mass of water $= 4.72 \times 10^{-3} \times 10^3 = 4.72$kg

Mass of pulley = 2.72kg

Temp rise/min $= \dfrac{153.6}{(4.72 \times 4.2) + (2.72 \times 0.42)}$

$= \dfrac{153.6}{20.97} = 7.3°\text{C}$

Allowable temperature rise = 85°C

Heating time $= \dfrac{85}{7.3} = 11.7 = 12$ min (approx.).

Relation between electrical and heat energy

In Chapter 2 mention was made of the associated effect noted when current flowed in a circuit, being the temperature rise in any part of the circuit, where resistance was concentrated. As energy cannot be destroyed, this is another example of energy conversion from one form to another, and a simple test can examine the relation between heat and electrical energy. Such a test will determine a material's specific heat capacity by an electrical method and, as water is a convenient substance, an appropriate experiment is described.

The experimental apparatus consists of a glass flow-tube surrounded by a glass water-jacket – spaced a little distance from it which is completely sealed so the space

between the flow-tube and jacket is a vacuum. A heating wire runs along the tube and thermometers are placed at either end of the tube. Water, the specific heat capacity of which is to be found, flows steadily through the tube and is heated electrically by a known current passing through the wire. After a period the inlet and outlet temperatures of the water become constant and this difference in temperature is noted. The constant rate of flow of the water is measured and so the mass of liquid being heated in a given time is found. The voltage drop across the heater is also measured and the quantity of heat absorbed by the water equals the electrical energy expended by the heating element:

$$VIt = mcT$$

where m is the mass of liquid, c the specific heat capacity and T the temperature rise.

The value of c is found to be 4.2kJ/kg°C. It is noted that the specific heat capacity of water, determined either by mechanical or electrical means, is the same. Different forms of apparatus have been developed to find the c values of various materials, the electrical method is favoured because of the accuracy with which control of the test can be made and measurements taken.

The following examples show how the specific heat capacity value is used in electrical problems.

Example 3.7. A brass calorimeter was found to have a mass of 67g. It was filled with water when the new mass was 131.7g. The temperature of water and container was 18°C. A heater coil immersed in the calorimeter is suitably lagged to minimise heat loss. Find the time taken to heat the water and calorimeter to a temperature of 33°C, if the heating is done by passing a current of 2A through the coil, the voltage drop across the coil is 7.5V. Take the specific heat capacity of brass as 0.39kJ/kg°C and that of water as 4.2kJ/kg°C.

Mass of water = 131.7 − 67 = 64.7g or 0.0647kg

Mass of calorimeter = 67g = 0.067kg

Temperature rise of water and calorimeter = 33 − 18 = 15°C

Heat required by water (mcT) = 0.0647 × 4.2 × 15kJ

Heat required by calorimeter (mcT) = 0.067 × 0.39 × 15kJ

Total amount of heat required

= 15 [(0.0647 × 4.2) + (0.067 × 0.39)]kJ

Input power = $IV = 2 \times 7.5 = 15W$

∴ Time taken to produce the temperature rise

$$\frac{15 \times 297.8}{15} \text{ seconds} = 4.96 \text{ minutes}$$

$$= 5 \text{ min (approx.).}$$

Example 3.8. A 220V electric kettle has an efficiency of 90%. Calculate the resistance of the heater coil and the current needed to raise the temperature of 1 litre of water from 15°C to boiling point in 9 minutes. Take a litre of water to have a mass of 1 kilogramme and the specific heat capacity as 4.2kJ/kg°C.

Since no information is given about the kettle, the effect of heating it is neglected.

Heat received by water (mcT) = 1 × (100 − 15) × 4.2

$$= 357kJ$$

Heat energy put out by the heater (kettle is only 90% efficient)

$$\frac{357}{0.9} \text{ KJ}$$

Time of heating = 9 × 60 seconds

Power rating of heater coil $= \dfrac{357 \times 10^3}{0.9 \times 9 \times 60}$

$$= 735W$$

Heater current $= \dfrac{W}{V} = \dfrac{735}{220} = 3.34A$

Heater resistance $= \dfrac{V}{J} = \dfrac{220}{3.34} = 65.9\,\Omega$

Example 3.9. A 120W electric soldering-iron is plugged into the 120V ship's mains for 5 minutes, the ambient temperature being 15°C. The copper mass is 0.133kg and 50% of the heat generated is assumed to be lost in radiation and heating other parts of the iron. Find whether the iron reaches the working temperature in the time specified. Take the specific heat capacity of copper as 0.39kJ/kg°C and the temperature of melting solder as 310°C.

Heat required by the iron = mcT = 0.133 × 0.39 × (310 − 15)

Heat produced by the element $= \dfrac{15.34}{0.5} = 30.68$kJ

$$= 30\,680\text{J}$$

Power rating of the element $= 120$W

Time for 30 680 joules to be expended

$$= \dfrac{30\,680}{120 \times 60} \text{ minutes}$$

$$= 4.27 \text{ min.}$$

Since only 4.27 min are required to achieve working temperature, then the time of 5 min as specified will be sufficient.

Example 3.10. A resistance unit has 500 turns of nickel–chrome wire, 0.5mm diameter. It is wound on a former 30 × 100mm and its resistivity is 1060$\mu\Omega$ mm at 15°C. At 100°C, its resistance is 2% greater than at 0°C. Determine the current taken at a temperature of 300°C, when the resistance is connected across a 250V supply.

Length of a turn $= (2 \times 30) + (2 \times 100) = 260$mm

No of turns $= 500$

Total length of wire $= 260 \times 500 = 130\,000$mm $= 130$m

Resistance at 15°C $= \rho \dfrac{l}{A}$

$$= \dfrac{1060 \times 10^{-9} \times 130}{\dfrac{\pi \, (0.5 \times 10^{-3})^2}{4}}$$

or $R = \dfrac{106 \times 13 \times 10^{-7} \times 4}{\pi \times 0.25 \times 10^{-6}}$

$$= 702\Omega$$

Also since

$R = R_0 \, (1 + \alpha T)$ or $1.02 = (1 + \alpha 100)$

$$1.20 - 1$$

Similarly $R_1 = R_0 \, (1 + \alpha T_1)$ and $R_2 = R_0 \, (1 + \alpha T_2)$

or $R_2 = R_1 \dfrac{(1 + \alpha T_2)}{(1 + \alpha T_1)}$

where $R_1 = 702\Omega$ at 15°C.

Giving $R_2 = 702 \dfrac{1 + (300 \times 0.0002)}{1 + (15 \times 0.0002)} = \dfrac{702 \, (1 + 0.06)}{(1 + 0.003)}$

$$= \dfrac{702 \times 1.06}{1.003} = 742\Omega$$

Current taken $= \dfrac{250}{742} = 0.337$A

Atomic theory of conduction

In Chapter 1 reference was made to the fact that different atomic arrangements result in matter being in solid, liquid or gaseous states. Returning to atomic physics, let us consider in more detail the electron or *quantum shell*, which are considered to be concentric and to be 7 in number. They are identified by the letters: K, L, M, N, O, P and Q, the K shell being closest to the nucleus. The appropriate numbers of electrons in each shell are found to be: 2, 8, 18, 32, 18, 13 and 2 respectively. Figure 3.6 illustrates the arrangement of shells in a copper atom (Z = 29) the most common metal in electrical work.

Quantum theory postulates that shells are identified by specific potential energy levels relating to the orbital distance from the nucleus. These levels are important factors in the conduction process and an understanding is needed to distinguish between conductors and insulators.
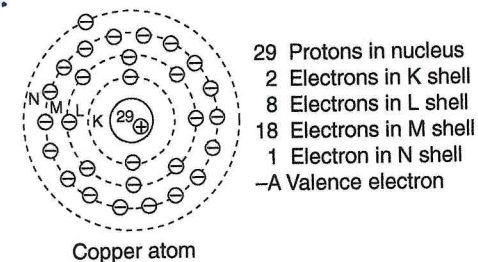
29 Protons in nucleus
2 Electrons in K shell
8 Electrons in L shell
18 Electrons in M shell
1 Electron in N shell
–A Valence electron

Copper atom

▲ Figure 3.6

Energy levels

When an atom is excited by an external source such as: heat, light or some other form, energy is acquired which can move an electron from an inner to an outer shell. Electrons only exist in the definite energy levels of a shell and cannot exist with energies between these levels. A sudden change in energy level can however result in movement from one shell to another. Electrons with low energy lie in the inner shells, i.e. nearer the nucleus. If the movement or jump is made to a lower or inner shell, the energy given out is **emitted** as radiation. When all the inner shells are filled, additional electrons can only exist in the outer shells and increase until the total −ve charge of the electrons equals the +ve charge of the nucleus, i.e. when the atom becomes stable. The unit for measuring electron energy level is the electron-volt (eV).

Energy bands

A number of fundamental laws have been determined by research into atomic behaviour. One law states that not more than 2 electrons can be in an identical energy state and these electrons must have opposing directions of *spin* (as proposed by Wolfgang Pauli 1925). Thus other electrons, while within the energy range of the shell, will occupy orbits at progressively higher energy levels. Consider a single isolated atom of copper (Z = 29), figure 3.6, there are 2, 8 and 18 electrons respectively in the K, L and M shells. These are filled with electrons which are very nearly, for each shell, at the same energy level. There is also one outer or *Valence* electron in the N shell with its higher energy value. The energy levels are sharp and are depicted (figure 3.7a). Consider several adjacent copper atoms whose electrons, if excited, can be given *slightly* higher energy levels. The original atom is now affected by the electric fields of the nuclei of nearby atoms as well as of its own. The shell electrons are in close proximity but are at slightly different levels. The definite shell energy lines of figure 3.7a are now broadened to a band, shown by 2 lines spaced from each other (figure 3.7b).

It is seen that the K, L and M bands, though broadened, represent *filled* shells with no free electrons and play no part in electrical conduction. The partially filled valence band indicates the presence of free electrons with an ability to move into the unfilled bands. The unfilled bands constitute a *conduction* band as these electrons are far from the nucleus and can move readily. Thus for figure 3.8a the conduction condition is shown for a conductor where the upper valency and conduction bands overlap.

For an insulator (figure 3.8b) the lower band is filled and far from the upper band, so
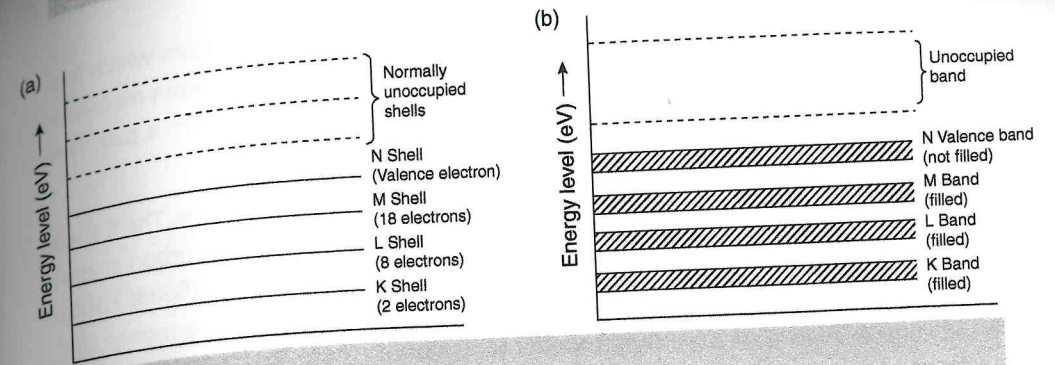
▲ Figure 3.7
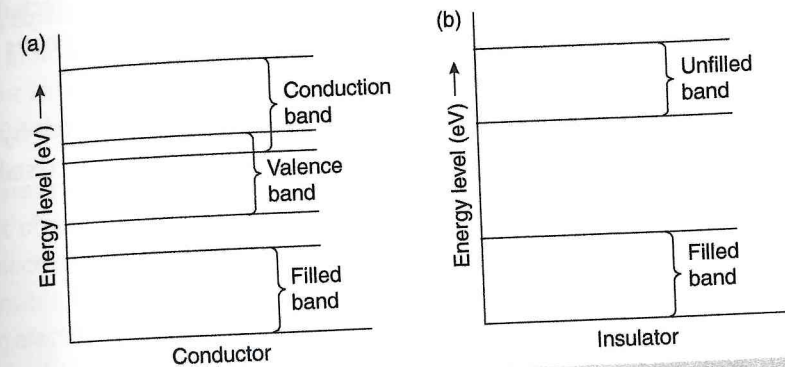
▲ Figure 3.8

gap is small and will only be achieved by application of so high a P.D. that the material would be destroyed. Such movement will manifest itself in a spark or 'puncture' of the insulating material which results in 'tracking', a burnt conducting path or area.

Crystal lattice

Some substances, including metals, in their smallest elemental form consist of many *crystals* joined together. Crystals are built up from a regular structure of atoms which repeats continuously to form a *lattice*. The simplest crystal to examine is that of carbon but the term *covalent bonding* is introduced to allow a full understanding of the lattice concept. The idea of a covalent bond is simply shown if a molecule of hydrogen, which though not a metal, exhibits many similar characteristics in chemical reactions.

One way in which an atom may combine with other atoms and create a change in the number of electrons in its valence shell is by covalent bonding, the *sharing* of a

A powerful bond between these atoms results. Consider a hydrogen atom which has only 1 valence electron. The K shell electrons should be 2 thus a hydrogen molecule contains 2 atoms with the nuclei linked by the valence electrons to form a pair. The arrangement is illustrated in figure 3.9.

Now consider the carbon atom, shown by the first diagram of figure 3.10a. The K shell is full but the L shell is incomplete, containing only 4 electrons. For a carbon crystal lattice, coupling occurs between the outer shells of neighbouring atoms (figure 3.10b). An atom is now produced which shares 8 electrons in the L shell. This means that the shell is full having its maximum number of electrons. There are therefore no free electrons and pure carbon is a poor conductor of electricity. The carbon brushes used in electrical machines are the result of processing to produce certain characteristics. A controlled heat treatment results in grades of natural graphite or amorphous carbon which improves conductivity.

In crystals, atoms are arranged in an orderly geometrical pattern and all pure crystals of the same element have identical lattice structure. For carbon or diamond crystals, each
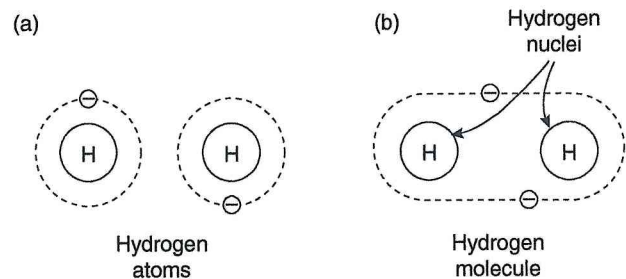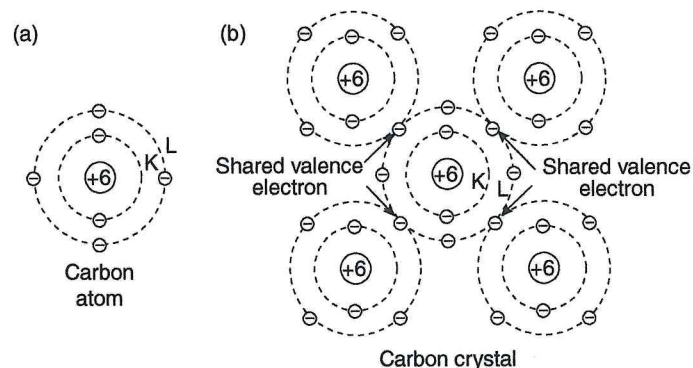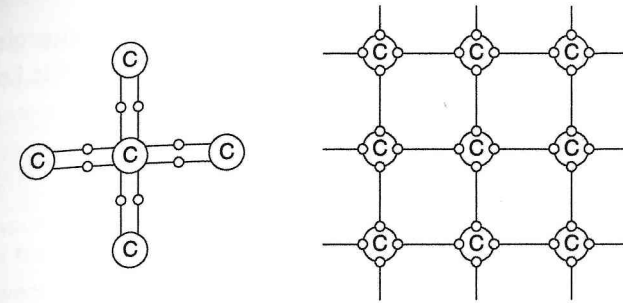
▲ Figure 3.11

atom is equidistant from 4 neighbours with each valence electron shared between the parent atom and 1 neighbouring atom. Two electrons, 1 from each neighbour, form a covalent bond and the arrangement is represented by the diagram of figure 3.11a, or figure 3.11b which depicts the crystal structure.

There are several basic crystal patterns and for metals it is generally believed that these consist of a lattice of +ve ions through which a cloud of negative electrons move, thus electrons are the valency electrons of the metal. The crystalline form of metals is responsible for their strength, elasticity and relative resistance to chemical reactions. Strong electrical bonds are provided by the mutual attractive forces between atoms and considerable work must be done to distort the equilibrium arrangement, irrespective of whether the metal is in tension or compression.

Conductivity

As described previously, atoms make up a crystal lattice and in these atoms the outermost electrons are influenced by adjacent atoms in the lattice, the result being that valence electrons move continuously atom to atom randomly. When a P.D. is applied to a conductor, these electrons pass through the crystal towards the positive potential. This action, representing an opposition to the passage of current, is termed conductor resistance. It is known that all atoms vibrate about a mean position, the amplitude of which is dependent on temperature. The magnitude of these vibrations are also known to increase with temperature until the melting point of the material is reached when the crystal lattice breaks up into either individual or groups of atoms. Electronic theory considers that since the vibrations increase with temperature, extra force is necessary to move free electrons through the lattice and so the resistance of a pure metal rises with temperature rise. It should be noted that since extra force is needed to move the electrons extra energy is absorbed into the conductor and appears as heat. By similar

▲ Figure 3.9

▲ Figure 3.10

explanation resistance is found to decrease as temperature falls, the relationship being linear down to very low temperature values. Extrapolation of this linear law shows that as the temperature approaches absolute zero, conductor resistance reduces to a negligible value and hence conductivity rises to an immense value. This condition brings about 'superconductivity' which is at present, being actively investigated by practical research in the electrical power industry for potential room temperature applications and as high power magnets for medical imaging.

Metallic conduction

Consider a short length of copper wire connected to 2 terminals or electrodes at different potentials. One electrode is given a +ve charge (deficient in electrons and the other a −ve charge (with an excess of electrons). The +ve electrode attracts electrons from the nearest copper atoms of the wire and these atoms will be left with a +ve charge, i.e. will become +ve ions. These ions attract the mobile electrons from the next atoms down the wire which in turn are ionised to attract electrons from the adjacent atoms further down the line and this process continues until the atoms at the other end of the wire become −ve ions. These ions now accept electrons from the other electrode which being negatively charged carries an excess of electrons. It is assumed that this 'jumping' action of electrons is so fast as to be almost instantaneous. It is noted that as an electron leaves an atom it may lead to formation of a *hole*. This term is often used to explain semiconductor operation. When an electron with its −ve charge leaves an atom, a gap or space is left in the atom's structure and so the atom now acquires a +ve charge. This gap or hole can be credited with this +ve charge and we now visualise a 'hole' as an entity in itself having a +ve charge. Any other mobile electron can be captured by the hole to neutralise its charge and make the original atom stable since its structure is now complete. As a result of this action, an electron has left a neighbouring atom and a hole is created there. This action continues between adjacent atoms and, as we consider metallic conduction, under the application of a P.D., electrons move towards the +ve terminal and holes move towards the −ve terminal, i.e. in the same direction as the current. The similarity with a row of occupied seats is often used. If a person (electron) is induced to leave and everyone is asked to 'move up one seat', the unoccupied space or hole can be reasoned to move down in the direction opposite to the movement of the people. The concept of a hole is useful, it is often referred to as a 'current carrier' and is considered later with electronic theory.

The conduction action described above has been treated in terms of a few atoms. It is estimated that in a cubic millimetre of copper there are as many as 10^{20} atoms so, even

if we consider a wire 1/10 mm in diameter, it is obvious that the number of mobile electrons and holes available for conduction is considerable!

Liquid conduction

Unlike a metal the molecules in a liquid are less closely packed and have little or no cohesion between them. There is no crystal lattice and the electrostatic bonding between atoms is also very weak. Electrical conduction depends on the nature of the liquid and also whether it is capable of electrolytic dissociation. For the passage of current ionic action is involved and some basic explanation of this will assist in the appreciation of the conduction process. The theory involved is detailed in Chapter 4.

Gaseous conduction

Consider a container evacuated completely and filled with a gas at normal atmospheric pressure. The gas consists of molecules of a simple structure, mainly stable, except for the occasional one which is *ionised*. Ionisation means the existence of some free electrons and molecules which have lost an electron and acquired a +ve charge. Ionisation is the result of some action by light, cosmic rays or radioactivity. Gas molecules are known to oscillate at high speed in random directions resulting in frequent collisions. The distance between gas molecules is small and the spaces between collisions are small but if the gas pressure is reduced the distance between collisions becomes larger. When the free electrons move through the gas and meet an ionised molecule, an immediate recombination results and stability is restored. With the reduction of gas pressure, the +ve and −ve charged molecules and electrons travel a very longer distance before being 'normalised', a distance known as the 'mean free path'. As it increases, the period during which ionisation continues becomes longer. Continued reduction of gas pressure increases the free path until this approximates to the dimensions of the enclosing container. The pressure is now *so* low that the enclosed space is considered to be a vacuum and collisions with gas molecules are now practically non-existent. Since no molecules are present in a vacuum, no obstacle to electron movement exists if these are injected into the space.

In the gas-filled condition, we see that conduction is affected by pressure. If 2 electrodes, between which there is a P.D., are introduced into the gas-filled space, a directional motion of the charges can be expected. Because of the low pressure, the electrons have a long free path and can attain high velocities and gain motional energy on collision. The effect of the collisions will depend upon the velocity on impact with a gas molecule. The high velocity electrons may cause the molecules to merely bounce away from each

other. On the other hand the electron velocity on impact could be large enough to cause another electron to be jolted from 1 shell orbit of the neutral atom to another, thus exciting it and giving rise to the emission of light due to some of the collision energy having been absorbed. If the velocity on impact is very high the collision energy could cause an electron to be knocked out of a neutral gas atom thus ionising the molecule. The process results in the production of an additional free electron and an ionised molecule. The new atom is now available to join the collision action and could shock-ionise a further gas molecule. The effect is a cumulative one and, since there is a P.D. between the electrodes, a current would result, electrons making their way to the +ve plate or *anode* and ionised molecules to the −ve plate or *cathode*. Current will increase rapidly leading to a condition which could be disastrous unless stabilised by the inclusion of a resistor in the circuit. The above action forms the basis of discharge lamp operation and will be discussed further.

When considering the conditions for ionisation it is pointed out that if electrons could be made available, ionisation would continue, especially if electrodes at a potential were inserted into the evacuated or gas-filled container. Such electrons may be obtained by either *cold-emission* – tearing them away from the electrodes or *thermionic emission*, or the *photoelectric effect* where incident electromagnetic waves of sufficiently high frequency lead to electron emission. Thermionic emission is an important factor in existing Cathode Ray Oscilloscope (CRO) devices, although they are now becoming replaced by modern Liquid Crystal Displays (LCDs) and Plasma Screens. However, the stability of thermionic devices against ElectroMagnetic Pulse (EMP), often associated with nuclear devices, still makes them suitable for military maritime operations.

Cold electron emission

A vacuum, although a perfect insulator, nevertheless allows current flow if a potential high enough is applied across the electrodes. Electrons contained in the atoms of the metallic electrodes can be 'extracted' by the electric force due to the applied field or potential, provided it is strong enough to overcome the internal electrical bonds of the metal atoms. Movement of electrons constitutes a current, sometimes evident as a spark. Sparks may develop into an arc if a supply source is connected to the electrodes. If the discharge is maintained, sufficient to heat the electrodes, thermionic emission can provide a large supply of electrons and a discharge will continue. If the current resulting from the initial discharge is limited by the circuit resistance then a working arrangement can result in a source of illumination. If gas at a low pressure is introduced into the container then, once the initial electron emission is achieved by the high applied potential, ionisation of gas will occur and the electrode potential can be reduced to maintain the discharge

The cold-cathode discharge lamp

The basic principle concerning cold-cathode discharge lamp operation is use of a high voltage gradient across the gas to produce electron flow and the required collisions to cause ionisation. Electrons are initially extracted from the multiple cathodes by a large applied voltage between anode and cathode. To produce the initial ionisation, a 50% higher voltage than the running voltage is needed. When operating, a potential gradient of 1–1.3kV per metre is used, and since the total supply voltage for some lamps and signs can be in the region of 10kV, a mains transformer is used which is designed to have a high internal reactance so that the voltage settles down to a steady state value (or 'sits down' to the required running value), once the lamp is struck (figure 3.12).

The lamp, in its simple form, is used for advertising purposes and the discharge colour is determined by the gas used. For illumination purposes, mercury vapour is used together with a fluorescent coating on the inside of the tube. Long tubes (2.4m) with 3 or 4 in series are difficult to accommodate and the lamp has only limited applications for marine work. They are used for illumination and decoration on some of the larger passenger and cruise ships. A more familiar type of cold-cathode lamp is the sign lamp used for indicator purposes. This is arranged to work at mains voltage with a series resistance of about 10kΩ. Neon gas at high pressure is used and discharge is usually as a glow which surrounds the electrodes.
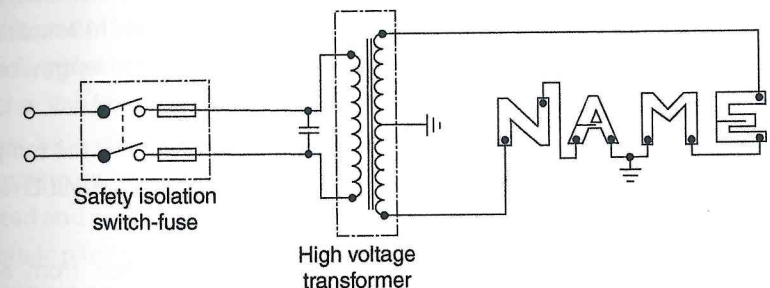
▲ Figure 3.12

Practice Examples

3.1. Ten thousand cubic millimetres of copper are (a) drawn into a wire 100 metres

and the resistance between opposite faces of the plate, if the resistance of the copper is $17\mu\Omega mm$ or 1.7×10^{-8} ohm-metres (2 significant figures).

3.2. A coil of copper wire has a resistance of 90Ω at 20°C and is connected to a 230V supply. By how much must the voltage be increased to keep the current constant, if the temperature of the coil rises to 60°C? Take the temperature coefficient of resistance of copper is 0.00428°C at 0°C (2 decimal places).

3.3. An electric kettle is fitted with a heater unit of 120Ω resistance. The efficiency is 84% and the voltage is 220V. How long will it take to heat 0.75 litre of water from 6°C to 100°C? Take the specific heat capacity of water as 4.2kJ/kg°C (nearest second).

3.4. Find the length of manganin wire required to make a 15.7Ω resistor, if the diameter is 0.315mm and the resistivity is $407\mu\Omega mm$ (nearest cm).

3.5. The cold resistance at 15°C of the field coil of a motor is 200Ω and the hot resistance is 240Ω. Determine the temperature rise assuming the temperature coefficient of resistance to be 0.0042°C at 0°C (1 decimal place).

3.6. A 2-core cable, each core of which is 300m long and of uniform cross-sectional area of $150mm^2$ is fed from one end at 240V. A load of 200A is taken off from the centre of the cable and a load of 100A from the far end. Calculate the voltage at each load. A single-core cable of similar material 880m in length and of uniform cross-sectional area of $50mm^2$ has a resistance of 0.219Ω (2 decimal places).

3.7. The resistance of a 100m length of copper conductor 1mm diameter, is 2.47Ω. Calculate the resistance of a cable 800m in length composed of 19 similar strands of conductor, but each 1.5mm diameter. Allow 5% increase in length for the 'lay' (twist) of each strand of the completed cable (3 decimal places).

3.8. A wire has a resistance of 10Ω at 0°C and 15Ω at 100°C. What is the temperature coefficient of the resistance of the material? At what temperature will its resistance be 30Ω (1 significant figure)?

3.9. A 200V, 200kW electric furnace must raise 500kg of brass from an initial temperature of 15°C to the melting point of 910°C. If overall efficiency is 0.8, calculate the operation time required. Specific heat capacity of brass is 0.39kJ/kg°C (nearest second).

3.10. A 230V electric water heater takes water at a mean temperature of 16°C and the mean temperature of the outlet water is 82°C. The cost at 2p per unit of the energy it consumes in a given period is £7.44. Determine the quantity of water used if the efficiency of the heater is 80%. Take the specific heat capacity of water as 4.2kJ/kg°C (3 significant figures).

4

ELECTROCHEMISTRY

I have been so electrically occupied of late that I feel as if hungry for a little chemistry.

Michael Faraday

The 3 main effects of current flow are those producing heat, chemical action and magnetism. In Chapter 3 the heating effect was discussed in some depth, while the magnetic effects will be covered in Chapter 5. This chapter will look at electrochemistry and studies which reveal the chemical action associated with current flow and the reversibility of this action. The electric cell was the principal source of electrical energy before the principles of electrodynamic induction were discovered and the electric generator developed – and is still important today.

The existence of static electricity, produced at a high potential and exhibiting itself as a stationary charge, had been known for centuries, and is associated with the lightning flash, the 'Van de Graaf' generator (1929), or friction effects, such as the attraction of paper by a piece of amber when the latter is rubbed. The early and sometimes accidental experiments of men like Galvani and Volta showed that electricity could be produced and controlled by various chemical means and led to the construction of the first 'voltaic piles' or batteries, consisting of several zinc and copper plates spaced with an absorbent material such as felt, soaked with acid or salt solution.

At the start of the nineteenth century the only practical way to produce electricity was by chemical means and, it is therefore unsurprising that, this branch of electrical engineering science was the first to be thoroughly investigated and developed. The *laws of electrolysis*, first propounded by Faraday in 1834, summarise the basics of a examining the relations between electrical and chemical action which is little changed today.

The conversion of electrical energy into chemical energy and its reverse action is an example of the principle of the conservation of energy. We will start with the conversion of electrical to chemical energy by the conduction of a current through a liquid.

Conduction in liquids depends on the presence of ions as current carriers. Pure distilled water is deionised and is a poor conductor but a few drops of sulphuric acid added to the water produces a conducting solution when subject to a P.D. This conduction is brought about by *electrolytic dissociation* – a process we will now examine.

Electrolytic dissociation

Consider common table salt (NaCl) in its dry crystalline form. A molecule of such a crystal consists of 1 atom of sodium (Na) and one atom of chlorine (Cl). In its normal state the sodium atom has only one electron in its valence shell instead of the possible 8. A chlorine atom has 7 electrons instead of the possible 8 in its outer shell and the combination (NaCl) is achieved by the one electron leaving the sodium atom and entering the chlorine atom. The chlorine atom thus absorbs the valence sodium electron, filling its outer shell. In this condition the chlorine atom becomes a –ve ion and the sodium atom a +ve ion. The salt molecule can be described as an *ionic* compound and is stable because of the attraction between the 2 atoms. The crystal is composed of the +ve and –ve ions arranged in regular patterns – the crystal lattice.

When the salt is dissolved in water, electrolytic dissociation occurs and some molecules break up allowing the chlorine and sodium atoms to separate. The chlorine atom however still keeps the extra electron taken from the sodium. The solution has now become an electrolyte with both chlorine ions and sodium ions being able to wander. The ions in the electrolyte are continuously recombining with other oppositely charged free ions while different molecules are breaking down elsewhere. The dissociation and recombination can be shown by:

$$NaCl \rightleftarrows Na^+ + Cl^-$$

Note. The representation of the electron distribution, i.e. chlorine has gained one electron hence the Cl⁻. Sodium on the other hand has become a +ve ion, hence Na⁺. It should be noted that the electron exchange number is one, i.e. the chemical valency number.

Electrolysis

The passage of current through a solution of an acid, alkali or salt produces a chemical change, explained by the theory of *dissociation*. The solution is called an *electrolyte* and

the process is termed electrolysis. The general theory is as follows. When an electrolyte is first made up, as would result from dissolving copper sulphate crystals in water, some molecules split into 2, independent of any external assistance, to form *ions*, carrying both +ve and –ve *charges*. Such ions are extremely mobile. If 2 plates, termed the *electrodes*, are immersed in the electrolyte and a P.D. applied across them, a current will pass through the solution. The +ve ions migrate, under the influence of the electric field due to the P.D., to the *cathode*, namely the electrode by which the current leaves the electrolyte. Such ions are called *cations*. The –ve ions, called *anions*, migrate to the *anode*, namely the electrode by means of which the current enters the solutions. Metal or hydrogen ions always carry +ve charges and travel with the current to appear at the cathode, whereas non-metallic ions travel in the opposite direction to the current and may appear at the anode or may engage in secondary reactions, some of which will be described shortly. If the electrodes are made from platinum, which is chemically inert, gas bubbles are produced on the electrodes. Consideration is now given to electrolysis with various liquids.

For the salt solution, ionic action is explained by the fact that Cl⁻ ions move to the anode and the Na⁺ ions move to the cathode and current flows. The Cl⁻ anions revert to their normal atomic structure by giving up their surplus electrons which are transferred round the external circuit. They are now neutral but in a very active state. Two such atoms combine to form a chlorine molecule and some of these rise to the surface as gas bubbles while others dissolve in the water. The Na⁺ cations meanwhile drift to the cathode, are neutralised on reaching it, and combine with the water to form sodium hydroxide and the hydrogen liberated will appear as bubbles at the cathode. The latter chemical action is shown by the equation:

$$2Na + 2H_2O = 2Na(OH) + H_2$$
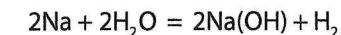
Consider next the case of dilute sulphuric acid (H_2SO_4). The dissociation and recombination can be shown by

$$H_2SO_4 \rightleftarrows H^+ + H^+ + SO_4^{--}$$
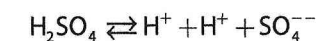
The hydrogen cations $2H^+$ move to the cathode and after giving up their charge are neutralised and rise to the surface as gas. The SO_4^{--} radicals are anions which move to the anode to give up their charge, but cannot exist in this form. They combine with hydrogen atoms of the water to reform acid molecules and liberate the oxygen as gas. Thus $SO_4 + H_2O = H_2SO_4 + O$. Note that the acid is not consumed but allows easy conduction by providing a good supply of ions.

If a copper sulphate ($CuSO_4$) solution is subjected to electrolysis with copper plates used as the electrodes, action occurs as shown below. When the electrolyte is made up, dissociation and recombination can be shown by

$$CuSO_4 \rightleftarrows Cu^{++} + SO_4^{--}$$

With the application of a potential across the electrodes, the cations (Cu^{++}) reach the cathode, give up their charges, are neutralised, and copper atoms are deposited on the copper electrode. The anions (SO_4^{--}), being radicals, cannot exist in this form and move to the anode and combine with the metal of this electrode to form copper sulphate which goes into solution. No gas is given off for this cell and copper is seen to be transferred from the anode to the cathode.

For the salt and acid cells, it is observed that once electrolysis proceeds, gas bubbles are released slowly and tend to blanket the electrodes, or said to cause *polarisation*. Under this condition, if the applied potential is removed and a sensitive voltmeter connected across the electrodes, a potential of some 1.5V will be indicated, caused, it is thought, by the slow progress of ions through the electrolyte. The voltmeter will also show a P.D. value between each platinum electrode and the electrolyte.

This polarising e.m.f. rapidly becomes ineffective if used to supply current to an external circuit. The cell appears to have an internal back e.m.f., confirmed by (1) Ohm's law not being followed and (2) electrolysis only starts satisfactorily if the applied voltage is about 2V. More is said about polarisation later but it is appropriate to point out that if tests are made on several different electrolytic cells, graphs can be drawn from the results, the ordinates being voltage applied across the electrodes (V) and current passing (I). The characteristics of a composite graph are seen to follow distinct types (figure 4.1). Characteristic 1 shows that Ohm's law is followed closely and is applicable to cells where no gas is given off. Characteristic 2 is followed for cells where gas is given off. It will be noted that little current flows until the applied voltage exceeds a critical value E_b – usually 1 to 2 volts and is linear after this. The explanation is also associated with polarisation.

A test condition can be investigated for the acid cell with different metals used for the electrodes. Consider zinc and copper with no external application of a P.D. Dissociation of the acid occurs as before, giving H^+ and SO_4^{--} ions. Chemical action is now involved in that sulphate ions readily combine with zinc atoms but not with copper atoms. Thus zinc dissolves to zinc sulphate and leaves behind 2 electrons, i.e. $Zn + SO_4^{--} \rightarrow Zn^{++} +$ 2 electrons. The zinc electrode then acquires a –ve potential or becomes the cathode for an external circuit. The H^+ ions on reaching the copper electrode acquire 2 electrons to leave this electrode with a +ve charge and thus is able to act as the anode for an external circuit.
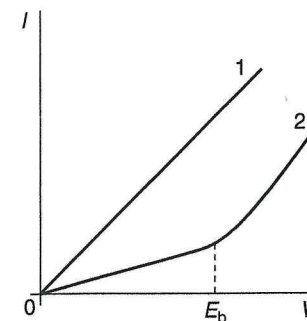
Here then we have the fundamental difference between an electrolytic and voltaic cell. For the electrolytic cell with electrodes of similar material and no applied e.m.f., chemical action is not possible as there is no apparent preference for a combination between the electrolyte and one electrode or the other. As a consequence, chemical action is only possible if an external voltage is applied.

For the latter, as a result of electrolytic dissociation and because electrodes of different metals are used, chemical action is possible if the circuit is completed. The chemical energy made available by the reaction is converted into electrical energy.

Electrolytic Cells

The whole arrangement consisting of electrodes and electrolyte, as described above, is often called an *electrolytic cell* to distinguish it from a *voltaic cell* described later. Electrolysis does not occur with solids or gases and is only possible for certain liquids. Some, like oils, are non-conductors, while others, like mercury, conduct without decomposition. The remaining liquids are electrolytes, which can be defined as liquids which decompose when current is passed through them. The electrolytic cell can be constructed to enable experiments and measurements to be made with great accuracy. In this form it is referred to as a *Voltameter*.

The water voltameter (sulphuric acid solution)

Figure 4.2 shows construction of the apparatus which is made of glass, with platinum electrode plates placed at A and C. The lead-in wires passed through rubber corks, are
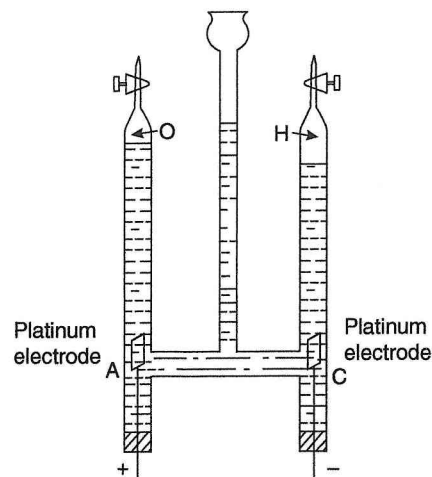
▲ Figure 4.2

not exposed to the solution to prevent corrosion. The voltameter is filled with acidified water and the platinum electrodes are connected to a battery of 2 volts or more. Current passes from the anode to the cathode and bubbles of gas are given off which rise into the graduated tubes.

If care is taken before passing current to fill both tubes with acidified water by opening the taps and then closing them after all the air is expelled, certain conclusions can be made from the experiment. After a period of time, the gas collected will be found to be hydrogen at the cathode and oxygen at the anode. The ratio of the volumes of H to O will be 2:1 and the amount of gas collected would be proportional to the current strength and the time it flows or more generally the quantity of charge passed.

In the acidified water there are sulphuric acid molecules which divide into 3 ions, 2 of hydrogen carrying +ve charges H^+, H^+ and one with −ve charge represented by (SO_4^{--}). Note the total +ve hydrogen charge equals the −ve sulphate or sulphion charge, but the ions migrate under the influence of the electric field. So the H^+ ions give up their charges at the cathode and are liberated as hydrogen gas. Sulphions proceed to the anode, but as they cannot exist in a free state they combine with 2 hydrogen H^+ ions or atoms from the water liberating oxygen as in the equation. Thus: $2SO_4 + 2H_2O = 2H_2SO_4 + O_2$. Oxygen rises from the anode and collects in the tube above it. The H_2SO_4 goes into solution and the electrolyte is decomposed so the water appears to be used up, but the acid content remains the same and the solution gets stronger; that is, its specific gravity rises.

The copper voltameter (copper sulphate solution)

Figure 4.3 shows the usual arrangement. A copper sulphate solution, made from crystals dissolved in pure water, is contained in a glass or glazed earthenware tank. Electrodes are made from pure copper sheet. The $CuSO_4$ molecule splits into 2 ions, Cu^{++} and SO_4^{--}. When a P.D. is applied across the electrodes and current is passed, copper ions migrate to the copper cathode to combine with it and give up their charges. The sulphions give up their charges at the anode and combine with copper from this electrode to form copper sulphate. Thus copper appears to be taken from one electrode and deposited on the other.

The chemical equations for the electrodes are:

(1) Cathode: $CuSO_4 = Cu + SO_4$.

(2) Anode: $Cu + SO_4 = CuSO_4$.

During electrolysis a small amount of gas may be noted at the plates – due to water decomposition in the solution as described for the water voltameter. Furthermore some complex action may occur in the electrolyte due to sulphions combining with hydrogen in the water to form H_2SO_4. Oxygen from the water is released to combine with anode copper to give copper oxide. This oxide then dissolves in the H_2SO_4 to give $CuSO_4$. Irrespective of the action the final result is a simple one, in that the loss in mass of the anode equals the gain in mass of the cathode.

Various forms of voltameter can be constructed for electrolysis research. Thus a silver voltameter may be used consisting of silver (Ag) plates and a silver nitrate ($AgNO_3$)
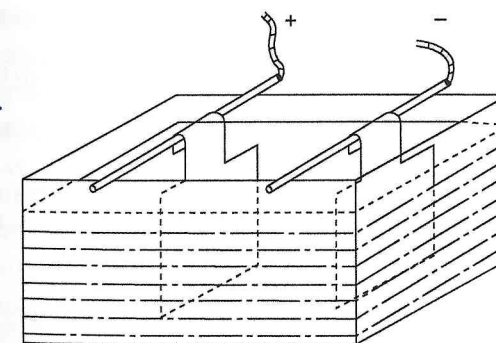
▲ Figure 4.3

solution. The examples described could be connected in series and the same quantity of electricity passed through all voltameters. If the electrodes were washed and carefully weighed before electrolysis and then washed and weighed again after electrolysis, certain conclusions would be reached which were first described by Faraday (1834) in his laws of electrolysis.

Quantitative laws of electrolysis (Faraday's laws)

(1) The mass of an element liberated from or deposited on an electrode is proportional to the quantity of electricity which has passed.

(2) The masses of elements liberated from or deposited on electrodes by a given quantity of electricity are proportional to their Chemical Equivalent Weight.

Consider the first law. It is found by experiment that the mass of any material deposited or liberated always depends on the quantity of electricity which has passed. Thus $m \propto Q$ coulombs or $m \propto It$. This proportion can be modified to:

$$m = zIt$$

where z is a constant depending on the substance deposited. z is termed the *electrochemical equivalent* of the element.

Electrochemical equivalent (E.C.E.)

The mass in grammes or kilogrammes liberated by one coulomb of electricity is called the E.C.E. of a substance. Thus, 10 amperes flowing through a copper voltameter for 1000 seconds will result in 10 000 coulombs having passed and 3.3g of copper will be deposited. Thus the E.C.E. of copper $= \dfrac{3.3}{10000} = 0.00033 \text{g/C}$. Similarly that for hydrogen would be 0.000 010 4, for oxygen 0.000 082 9 and for silver 0.001 118g/C.

With the use of SI units, it is better to think in terms of the kilogramme and the E.C.E. is defined as the mass (in kilogrammes) of a substance liberated by the passage of one coulomb. Thus the E.C.E. of copper would be 330×10^{-9}kg/C. Since the milligramme is also an accepted SI unit, the E.C.E. can be given as mg/C. Thus for copper it will be 0.33mg/C.

The first law of electrolysis leads to a method of stating the unit of current, which was considered accurate enough to allow an original definition for the International

Ampere. This was defined as, that constant current which deposits silver at the rate of 1118×10^{-9}kg per coulomb when passed through a solution of silver nitrate in water.

The formula deduced above allows the solution of problems associated with electrolysis and practical electroplating. The unit in which the E.C.E. is given should be noted.

Example 4.1. Find the time taken to deposit 11.4g of copper when a current of 12A is passed through the copper sulphate solution contained in a copper voltameter (nearest minute).

The E.C.E. of copper should be taken as 330×10^{-9}kg/C.

$$\text{Since } m = zIt \text{ then } t = \frac{m}{Iz}$$

$$\text{So } t = \frac{11.4 \times 10^{-3}}{12 \times 330 \times 10^{-9}}$$

$$t = 2880 \text{ seconds} = 48 \text{ minutes.}$$

Atomic weight, valency, chemical equivalent weight

The second law of electrolysis is deduced by examining the results of tests made with a number of different voltameters in series, having been subjected to the passage of the same quantity of electricity. The results of the experiment show that the mass of the substances deposited or liberated at the electrodes is proportional to the chemical equivalent of substances. If the atomic weight of a substance is known, its E.C.E. can be found provided the valency is known and the E.C.E. of hydrogen assumed. Thus if the chemical equivalents of hydrogen, oxygen, copper and silver were 1, 8, 31.8 and 107 respectively, the masses of H, O, Cu and Ag liberated by the same quantity of electricity would be in the same proportion and therefore the E.C.E. of a substance is the E.C.E. of hydrogen multiplied by the chemical equivalent of the substance. Thus taking the E.C.E. of hydrogen as 0.0104 mg/C, that of silver will be $0.0104 \times 107 = 1.118$ mg/C.

To conclude our deductions from the second law, it is worth defining the following terms.

ATOMIC WEIGHT. Atoms are extremely small and determination of their absolute masses present considerable difficulties. The mass of a hydrogen atom is about 1.67×10^{-24}g and it is customary, even in SI units, to practically refer to the relative

weights of the atoms of various substances in terms of the atom of hydrogen. Thus the term atomic weight is used and is the weight of an atom of a substance in relation to the mass of an atom of hydrogen. Thus the value for oxygen is 16, i.e. it has 16 times the weight of an atom of hydrogen.

VALENCY. This is described as the combining ratio of a substance, for example, oxygen is 2, whereas that of hydrogen is 1, so water is represented by the symbol H_2O. The valency of an element can also be defined as the number of atoms of hydrogen with which one atom of the element can combine. So the valency of a sulphate is 2 since, for example, in sulphuric acid H_2SO_4, 2 atoms of hydrogen are required to combine with the sulphate.

CHEMICAL EQUIVALENT WEIGHT (C.E.Wt). It is the ratio of atomic weight to valency, or

$$\text{Chemical Equivalent Weight} = \frac{\text{Atomic weight}}{\text{Valency}}.$$

It is the weight of a substance which will combine with one part by weight of hydrogen or 8 parts by weight of oxygen. So the C.E.Wt or *combining weight* of hydrogen is 1, that of oxygen is 8, copper 31.8, etc., as found in chemical tables.

From the second law we have:

$$\frac{\text{Mass of material X liberated}}{\text{Mass of hydrogen liberated}} = \frac{\text{C.E.Wt of material X}}{\text{C.E.Wt of hydrogen}}$$

$$\text{or } \frac{m_X}{m_H} = \frac{\text{C.E.Wt of X}}{\text{C.E.Wt of H}} \text{ also from the expression } m = zIt$$

$$\frac{z_X It}{z_H It} = \frac{\text{C.E.Wt of X}}{\text{C.E.Wt of H}} \text{ or } z_X = z_H \times \frac{\text{C.E.Wt of X}}{\text{C.E.Wt of H}}$$

But the chemical equivalent of hydrogen = 1

so $z_X = z_H$ (C.E.Wt of substance X)

$$\text{or } z_X = z_H \times \frac{\text{Atomic weight of substance X}}{\text{Valency of substance X}}$$

Example 4.2. How many amperes will deposit 2g of copper in 15 minutes, if the current is kept constant? Given the E.C.E. of hydrogen as 0.0104mg/C, the atomic weight of copper as 63.56 and the valency of copper as 2 (2 decimal places).

Then $z_{cu} = z_H \times \dfrac{63.56}{2} = 0.0104 \times 31.8$

whence $I = \dfrac{m}{zt} = \dfrac{2}{0.33 \times 10^{-3} \times 15 \times 60}$

or $I = 6.72A$.

Example 4.3. A voltameter consists of a solution of zinc sulphate and electrodes of zinc and carbon. If current is passed in at the carbon electrode, zinc is deposited on the zinc electrode and oxygen is given off at the carbon plate. If a current 3.5 amperes is passed for 1 hour, find the mass of zinc deposited and oxygen liberated from the solution. The E.C.E. of zinc can be taken as 338×10^{-9}kg/C, the atomic weight as 65.38 and the valency as 2. Take the atomic weight of oxygen as 16 and the valency as 2.

From relationships already deduced:

$$m_{zn} = z_{zn}It \quad \text{or} \quad m_{zn} = 338 \times 10^{-9} \times 3.5 \times 3600$$

$$\text{or zinc deposited} = 425.88 \times 10^{-5}$$

$$= 4.26 \times 10^{-3}\text{kg or } 4.26\text{g}$$

$$\text{Also } z_{zn} = z_H \left(\frac{\text{At wt of Zn}}{\text{Valency of Zn}} \right)$$

$$\text{and } z_0 = z_H \left(\frac{\text{At wt of O}}{\text{Valency of O}} \right)$$

$$\text{so } \frac{z_0}{z_{zn}} = \frac{\text{At wt of O}}{\text{Valency of O}} \times \frac{\text{Valency of Zn}}{\text{At wt of Zn}}$$

$$= \frac{\text{At wt of O}}{\text{At wt of Zn}} \times \frac{2}{2}$$

$$\text{Thus } z_0 = z_{zn} \frac{\text{At wt of O}}{\text{At wt of Zn}} = 338 \times 10^{-9} \times \frac{16}{65.38}$$

$$= \frac{338 \times 10^{-9}}{4.086}$$

$$\text{or } z_0 = 82.5 \times 10^{-9}\text{kg/C or } 82.5 \times 10^{-6}\text{g/C}$$

$$\text{and } m_0 = 82.5 \times 10^{-6} \times 3.5 \times 3600 \text{ as } m = zIt = 1.05\text{g.}$$

Back e.m.f. of electrolysis

The circuit laws established in Chapter 1 govern the conditions for most practical circuits. The loads of such circuits are mainly resistive, such as the coils of resistance wire in appliances like electric heaters and filament lamps. Such loads, termed *passive* loads, are recognised by the fact that they conform to Ohm's law. For other types of loads such as the electric motor, accumulator or storage battery when being charged, Ohm's law is not directly applicable and they represent *active* loads, of which some electrolytic cells are an example. The difference between pure ohmic resistance and that offered to the passage of current by such electrolytic cells is now considered.

Figure 4.4 shows a simple circuit for which the source of supply is a battery made up of 3 similar voltaic cells in series. Assume that the current through an electrolytic cell made up as shown is adjusted and maintained at 3 amperes by a variable resistor provided for this purpose. If the supply potential is reduced by removing one of the voltaic cells, it is assumed that, as the e.m.f. has been reduced to $\frac{2}{3}$ of the original value, the current will fall to $\frac{2}{3} \times 3 = 2$ amperes. In fact the new current strength will be well below this value, the value as expected from Ohm's law.

Experiment shows that an extra current controlling factor is present in a circuit involving an electrolytic cell and the result is explained by considering that a back *e.m.f.* is produced by the cell, so that the following equation represents the experimental conditions:

$$V = E_b + IR_i$$

Here V represents the voltage applied to the cell, E_b is the back e.m.f. of the cell, I the current causing electrolysis and R_i the internal resistance of the cell.

The magnitude of the back e.m.f., for any electrolytic cell, can be found by further experiment. Here the basic action is described, as noted for the simple water voltameter shown in the diagram (figure 4.5).

Electrodes are immersed in an electrolyte of sulphuric acid. When the switch is closed current passes from anode to cathode and both electrodes become coated with bubbles
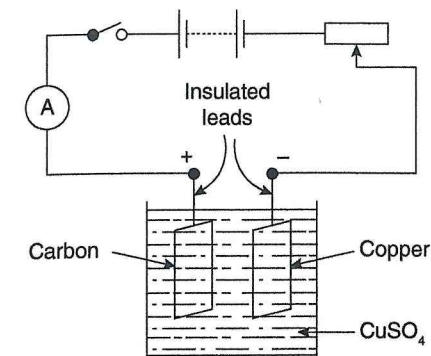
▲ Figure 4.4
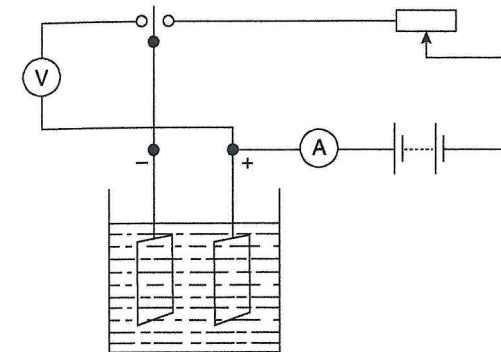
▲ Figure 4.5

second position so as to connect a sensitive voltmeter across the cell only. The main circuit current will have stopped, but the voltmeter will register a voltage across the plates, which gradually falls as the bubbles disperse, due to the flow of the small current through the voltmeter. The voltage or e.m.f. is due to the presence of the gas bubbles, and it will be noted that the cell polarity remains the same, namely that the current flows through the meter from anode to cathode in the external circuit so as to be in the opposite direction in the cell to the current flow which caused electrolysis. The value of the back e.m.f. is obviously important in that, if the applied voltage is less than this back e.m.f., electrolysis cannot take place. At start a small current flows but once polarisation begins, the back e.m.f. rises to equal the applied voltage and the current then ceases.

In the case of water the value of the back e.m.f. E_b can be calculated. It is known that 1g of water when formed by combustion of hydrogen in oxygen produces about 15.96kJ of heat. If we assume the energy required to separate H and O in 1g of water is identical, the electrical energy required will also be 15.96kJ.

Since both H and O are released by electrolysis it follows that the total mass of gas or water released by 1C will be equal to the E.C.E. of hydrogen + E.C.E. of oxygen = 0.000 010 4 + 0.000 082 9 or z_{HO} = 0.000 093 3g, i.e. 93.3×10^{-9}kg are released.

Thus 1C releases 0.000 093 3g of gas (or liberates this mass of water) and the electrical energy required to decompose this mass of water = $15\,960 \times 0.000\,093\,3 = 1.49$J.

If this electrical energy is produced by the work done against the back e.m.f. E_b, then the applied voltage will have a value V where $V = E_b$ and the energy produced by the passage of 1C = $V \times 1$J. Thus $V \times 1 = 1.49$ or $V = 1.49$ and hence $E_b = 1.49$V.

During electrolysis, decomposition of the liquid produces ions, which once dissociated, try to recombine and due to their slow progress through the electrolyte a back e.m.f. is produced. For water the back e.m.f. is about 1.5V and substituted for E_b in the formula: $V = E_b + IR_i$. Thus for current flow the applied voltage must be greater than the back e.m.f. by the voltage drop due to resistance of the electrolyte.

A back e.m.f. of appreciable value exists for electrolytic cells made up with electrodes of *dissimilar* materials, but if both electrodes are of the same material, as for the copper voltameter, then the back e.m.f. value is *so* small that it can generally be neglected. This is explained by the fact that no difficulty is experienced in enabling the dissociated ions to recombine. They can readily combine with the electrolyte as stated in the description of the copper voltameter, and the all important result is that the passage of current does not produce an overall chemical change. All that happens is that copper is transferred materially from anode to cathode. It should be noted that for the water voltameter, although the electrodes are of the same material, namely platinum, a back e.m.f. still appears when they are coated with H and O gas bubbles. The bubbles have the effect of insulating the electrodes and retarding the passage of ions. This condition results in a back e.m.f. of about 1.5V.

General observations show that when the products of electrolysis possess chemical energy, then the equivalent electrical energy must have been supplied through electrolysis and a back e.m.f. of appreciable value must exist, for example, in the electrolysis of water. Hydrogen and oxygen will recombine in an explosive manner to form water producing heat and light. The energy latent for this recombination was derived from the electrical energy put in during electrolysis and a back e.m.f. clearly must have been present. For an electrolytic cell, such as the copper voltameter, since the product of the process possesses no chemical energy it can be assumed that a cell using electrodes of the same material has negligible e.m.f.

RESISTANCE OF ELECTROLYTES. The resistance R_i of a liquid conductor is proportional to the length and inversely proportional to the cross-sectional area (the same as for a conductive wire). It also varies with the nature of the electrolyte and the concentration, however, the temperature coefficient is a negative one. Because of the back e.m.f. effect already discussed, resistance is difficult to measure. The value obtained by dividing the voltage drop across a cell, by the current flowing will give a resistance value for the electrolyte much greater than the true figure and is erroneous because of neglecting the back e.m.f. E_b.

POWER EXPENDED DURING ELECTROLYSIS. If the voltage equation for an electrolytic cell is $V = E_b + IR_i$ then for a current flow of I amperes the power equation becomes $VI = E_bI + I^2R_i$

Here VI represents the power applied to a cell, E_bI represents the power required to produce chemical dissociation and I^2R_i represents the heat energy produced in the cell generating a temperature rise.

Example 4.4. Find the voltage needed to pass a current of 4A through a copper voltameter with an internal resistance of 0.014Ω and a back e.m.f. of 0.25V. Find the power utilised to produce electrolysis and that wasted in heating the electrolyte. Find the overall % efficiency of the voltameter if used as a plating vat (2 significant figures).

Since $V = E_b + IR_i = 0.25 + (4 \times 0.014) = 0.306$V

Power utilised for electrolysis = $0.25 \times 4 = 1$W

Power wasted = $I^2R_i = 16 \times 0.014 = 0.224$W

Efficiency of the cell for electrolysis = Power used for electrolysis/Total power

$$= \frac{1}{(1 + 0.224)} = \frac{1}{1.224}$$

$$= 0.82 \text{ or } 82\%.$$

Primary and Secondary Cells

Many of the fundamentals of voltaic or *galvanic* action, as it was called originally, have been mentioned in this chapter, but we now consider in detail, the theory concerned with conversion of chemical energy into electrical energy, the production of an e.m.f. by chemical action. Generation of this e.m.f. is best studied by describing the action of a simple cell.

The simple voltaic cell

If a piece of commercial zinc is dropped into a glass jar containing dilute sulphuric acid, the zinc is seen to corrode and hydrogen gas bubbles given off. The jar is also found to warm and it is deduced that heat is given off by the chemical action. If then, a piece of pure zinc is similarly experimented with, no such effects occur, nor do they happen if a dissimilar piece of pure metal such as copper is substituted for the pure zinc.

The chemical action noticed for commercial zinc is considered due to 'local action' and explained by the presence of impurities in the zinc, the chief of which are iron and lead. A local closed circuit is made, between a particle of iron and the zinc, as both are in contact and the acid is common to both dissimilar metals. A small cell is considered to be formed and as current flow is possible, generation of an e.m.f. is thought to accompany the chemical action.

Consider 2 electrodes immersed in a solution of dilute sulphuric acid as shown by the diagram (figure 4.6). Electrodes should be plates of pure dissimilar metals, such as zinc and copper and placed to avoid touching each other. No action will take place for the arrangement described but if an ammeter and a resistance load are connected as illustrated, a current will flow when the switch is closed. The current through the external circuit will be from the copper +ve pole or anode to the zinc −ve pole or cathode, and hydrogen bubbles are given off from the copper plate while the zinc plate is slowly eaten away.

It is noted that the ammeter will show the current falling slowly until it ceases after a while. The copper plate will be covered completely with hydrogen bubbles and if these are wiped off with a glass rod, the current will be found to restart and the cycle of action
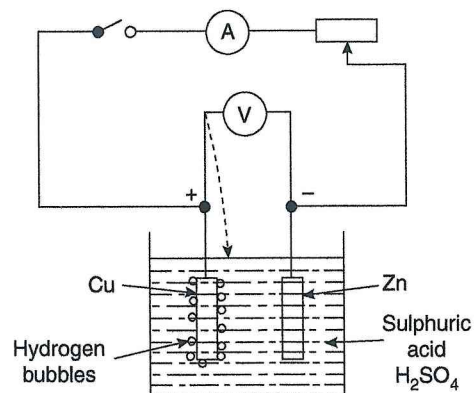
▲ Figure 4.6

repeat itself. Hydrogen bubbles appear to control the chemical action of the cell which is said to have 'polarised' when the current ceases.

Further simple experiments with different combinations of electrodes and electrolytes will produce different e.m.f.s, recorded by the voltmeter. When supplying current, the negative pole of each arrangement will corrode away and for every simple cell polarisation will occur. A more complete explanation of the observation made can now be developed.

THE CELL E.M.F. The voltmeter in figure 4.6 will show that an e.m.f. or voltage exists across the electrodes of a cell even when on O.C. When the circuit is completed, current flows because of this e.m.f. and chemical action is observed which continues to generate an e.m.f. and maintain current. The chemical action of a voltaic cell results in a definite e.m.f. which is the result of the action of both electrodes or plates with the electrolyte. If the voltmeter was connected on O.C. between the zinc and the solution, the former will be negative to the latter by some 0.63V. When connected between the electrolyte and the copper plate, the voltmeter will record 0.47V, with copper positive to the sulphuric acid. With the instrument connected across both plates a reading of 1.1V as expected. Production of a cell e.m.f. is explained by the electrolytic theory. When an electrolyte is made up, the molecules split into ions which are electrically charged and mobile. An electrolyte is thus an ionised solution and when a metal is immersed in it, some of the metal appears to enter the solution, so there is an immediate merging of surface ions of the metal with the ions of the electrolyte. Thus the barrier between the metal and electrolyte is not the surface of the former but along some layer of electrical potential equilibrium, which has caused ion exchange to cease. The action on immersing various metals into an electrolyte differs for the different metals. Thus for zinc in dilute sulphuric acid there is a greater tendency for +ve ions to pass to the solution than for +ve ions of the electrolyte to pass to the zinc. The zinc plate thus becomes deficient in +ve ions and becomes negative to the solution by a voltage of 0.63V. For copper in sulphuric acid, a different action takes place. This metal becomes positive to the solution because there is a greater tendency for the +ve charged hydrogen ions of the electrolyte to move to the copper than for the +ve ions of the metal plate to pass to the solution and the copper rises to a potential of 0.46V with respect to the electrolyte.

Exchange of ions results in a potential between the electrodes and solution which gradually opposes the exchange until it eventually ceases and equilibrium is established. Thus the arrangement of zinc−copper electrodes in sulphuric acid results in a P.D. of 1.1V between the electrodes. On O.C., we see that the initial +ve Zn ions migrating to the solution combine with the sulphions to form zinc sulphate, liberating +ve hydrogen ions which move to and accumulate on the copper plate. Thus the chemical action is

explained by the formula $Zn + H_2SO_4 = ZnSO_4 + 2H$. The potentials build up within the cell, quickly bring the ion migrations to an end and chemical action stops. If now the O.C. condition is changed to that of a closed circuit, by joining the copper electrode to the zinc through an external circuit, the chemical action is immediately noted to recommence. Current is seen to flow and the formation of zinc sulphate continues with liberation of hydrogen at the copper plate.

The O.C. e.m.f. of 1.1V, which is the result of the initial ion migration, can now cause a current, the magnitude of which is determined by the circuit resistance. A flow of current is a movement of electrons, passing from the zinc cathode to the copper anode. The electrons move round the external circuit from the zinc cathode to neutralise the +ve anode charges, thus making way for further migrations of +ve hydrogen ion charges in the cell. The initial cell action, as described for the O.C. condition, can now continue and the cell functions by maintained chemical action, provided polarisation is avoided. We can now conclude our study of cell action by saying that all metal electrodes produce an e.m.f. as a result of the ion exchange action with the electrolyte, and that they can be arranged in a table, in order of the value of their e.m.f.s.

THE ELECTROCHEMICAL SERIES. If any 2 elements shown in the table are used for a cell, the element *lowest* in the series is the +ve terminal, when considered with respect to the external circuit. The list comprises the usual elements which are mainly metals, but hydrogen and carbon are found to behave like metals and are included. To illustrate the use of the table, the ordinary dry cell as used for a hand-torch can be considered. This cell uses carbon for the +ve electrode and zinc for the −ve electrode.

Materials well spaced apart in table 4.1 are usually used for practical cells. For the torch battery the e.m.f. is about 1.5V per cell.

Table 4.1

Potassium	Lead
Sodium	Tin
Magnesium	Hydrogen
Aluminium	Copper
Manganese	Zinc
Cadmium	Iron
Nickel	Mercury
Silver	Platinum
Gold	Carbon

POLARISATION. For the electrolysis of water, it was noted that gas was liberated at the electrodes and this resulted in a decrease of current and ultimately in a change in cell action. Once polarisation (collection of gas on the electrodes) occurs, the electrodes have, in effect, changed – these now being hydrogen and oxygen. Voltaic action can now result to set up an e.m.f. operating in the reverse direction to the voltage applied to cause electrolysis introduced as the cell back e.m.f.

When the simple voltaic cell supplies current, polarisation will occur. The circuit current gradually falls, even though the chemical action of the cell appears to proceed. Close examination reveals that as the hydrogen bubbles make their way to the copper plate, not all are liberated here and rise to the surface. Some bubbles stick to the plate and this tendency increases until the whole plate is covered with bubbles so the cell becoming increasingly ineffective as a source of e.m.f. The layer of gas surrounding the +ve plate causes a polarising effect because (1) gas has a high resistance, so that any area of the plate covered with bubbles is almost insulated and cannot allow the passage of current. Thus the internal resistance of the cell rises as the gas layer increases and the circuit current falls as a direct result. (2) As hydrogen covers the copper plate, it begins to make its presence felt in that it effectively replaces the +ve copper electrode by a hydrogen electrode and thus reduces the e.m.f. of the cell. It will be seen from the table of the electromotive series that the spacing between zinc and hydrogen is smaller if compared with that for zinc and copper. The cell e.m.f. is thus much reduced giving the final result as described.

Once the cause of polarisation became known it was apparent that, in order to make the simple cell an effective source of electrical energy, a method of preventing collection of hydrogen bubbles was needed.

The simplest forms of depolarisers developed operate chemically, combining with liberated hydrogen to convert it into water, preventing the gas from reaching the +ve electrode and blanketing it. The methods by which this is accomplished will be seen when examples of primary cells are studied.

Batteries

These can be wet, dry or even gel type cells.

The *dry cell* (primary cell) is made of a carbon rod surrounded by manganese dioxide with an electrolyte of ammonium chloride. The cell is sealed and activated when connected to a circuit, such as a torch light. Different chemicals can be used, but the principle is the same. The usual voltage from a cell is 1.5V

The *wet cell* (secondary cell) is simply a container which has plates of different materials that are submerged together in an electrolyte. A chemical reaction takes place between the plates and positive particles will migrate through the electrolyte to the negative plate. A circuit is made between the plates and this will continue until the electrolyte has been expended. This type of battery is frequently used for motor boats.

The *gel cell* is a sealed unit very similar to the wet cell. The electrolyte is contained within the unit and it cannot be opened. The battery will only last a relatively limited lifetime, and upon discharge must be replaced. This type of battery is now fitted in many new cars.
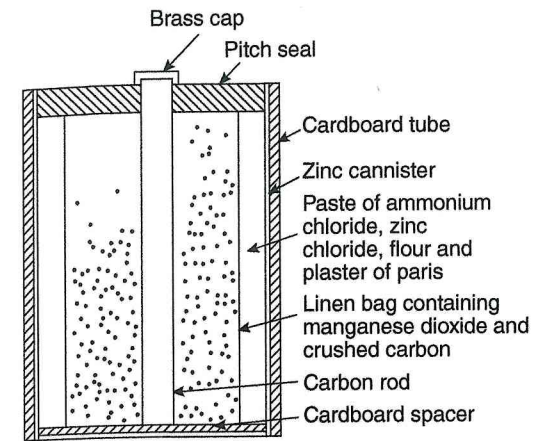
The primary cell

Under this heading are considered practical cells, which are suitable for providing a constant e.m.f. when operating under everyday conditions. They are, however, cells which obtain their electrical energy from chemical energy, the active material being used up in the process. They differ from secondary cells in that the latter utilise materials which are *not* consumed when the cells provide electrical energy. The secondary cell can be electrically 'charged' so that its electrodes are chemically converted into materials which enable the cell to provide an active e.m.f. for supplying electrical energy. In this condition the cell discharges and the electrode materials again change chemically, reverting back to those of the uncharged state. The whole cycle of charge and discharge can then be repeated.

Primary cells suffer from the 2 main disadvantages of the simple cell: (1) polarisation and (2) local action.

Polarisation is overcome by the use of a suitable chemical depolariser which is therefore an essential component of cell construction. Local action is minimised by using pure metal, such as zinc free from impurities like iron and lead. In its basic form the primary cell is a wet cell, which is not used to any extent today, although it was used for railway signalling in places where no electric mains was available. In the dry form the Leclanche cell is still common and attention should be paid to its construction and action.

THE LECLANCHE CELL (Dry type). One form of construction is illustrated (figure 4.7) which is a cross-sectional view of a typical practical cell.

The depolariser, manganese dioxide (MnO$_2$), is mixed with powdered carbon and packed round a central carbon rod. This assembly is placed in a linen bag which serves as the porous pot of the cell. The negative electrode is a pressed zinc cannister which

Brass cap
Pitch seal
Cardboard tube
Zinc cannister
Paste of ammonium chloride, zinc chloride, flour and plaster of paris
Linen bag containing manganese dioxide and crushed carbon
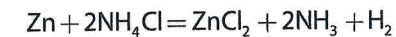Carbon rod
Cardboard spacer

▲ Figure 4.7

contains the linen bag assembly and the electrolyte, made up as a paste of ammonium chloride, zinc chloride, flour and plaster of Paris. One method of closing the cannister is to seal it with pitch.

The following chemical formula defines the chemical action:

Action at negative electrode:

$$Zn + 2NH_4Cl = ZnCl_2 + 2NH_3 + H_2$$

Action at positive electrode:

$$H_2 + 2MnO_2 = Mn_2O_3 + H_2O$$
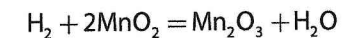
The form of cell as described is in most general service, but other forms have been developed for incorporating into the layered battery type, used for portable radio sets, calculators, etc. The reader should complement the information given here by referring to books specialising in the practical treatment of battery operated equipment.

The secondary cell (or accumulator)

Because of the importance of this cell as a means of storing electricity (it is sometimes called a storage cell), the reader is advised to consult other books giving more details of modern constructional methods, applications and maintenance requirements. Figure 4.8 shows only the basic construction and the description sets out only the elementary principles. The modern accumulator uses 'pasted' plates to allow maximum
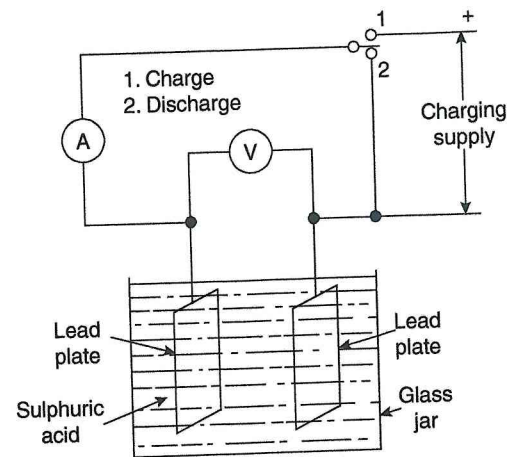
▲ Figure 4.8

use of available material and the process used in 'forming' the cell is too detailed for a book of basic theory. It is hoped, however, that the information given below will provide sufficient knowledge to enable the action of the lead–acid accumulator to be understood. The nickel–iron, or nickel–cadmium alkaline battery also functions on similar principles, although the plate materials and electrolyte differ. This type of cell is also important and should be thoroughly investigated.

The lead–acid cell

The simple accumulator consists of 2 lead (Pb) plates immersed in dilute sulphuric acid, the whole assembly being contained in a glass or moulded ebonite container. The cell has to be worked into a suitable condition before it can be used for storing electricity and the process is carried out by alternatively 'charging' and then 'discharging' the cell. If a D.C. supply is connected to the plates as in the diagram (figure 4.8, switch position 1) and the cell subjected to electrolysis by passing current through it, oxygen and hydrogen gases are given off at the electrodes. As for the water voltameter, the first stage of the reaction will be decomposition of the acid (H_2SO_4). A molecule of acid dissociates to produce hydrogen ions and sulphions (SO_4). The sulphions move to the +ve plate, reacting with the water to form sulphuric acid and oxygen. The latter attacks the +ve plate only to form lead dioxide (PbO_2), which causes the original lead electrode to become a dark brown colour. The hydrogen ions discharge at the cathode and liberated in a gaseous state. The first chemical action is thus at the +ve plate only but if the supply is switched off (switch-intermediate position), the cell is now found to have the properties of a voltaic cell and provides

If the cell is short-circuited (switch position 2), it behaves like a primary cell, passing a current for a short time during which period it discharges. The solution is electrolysed in the reversed direction and the original negative plate now acts as the anode with its lead ions reacting with the sulphions of the electrolyte to form lead sulphate. Thus at –ve plate $Pb + SO_4 = PbSO_4$ (lead sulphate). The hydrogen ions from the electrolysis during discharge, move to the original +ve plate, now the cathode. The hydrogen reduces the lead dioxide to lead oxide which in turn reacts with the acid to form lead sulphate. Thus at the +ve plate the chemical action is $PbO_2 + H_2 + H_2SO_4 = PbSO_4 + 2H_2O$. Both plates are converted into lead sulphate and will be in white colour.

If the charging cycle is repeated (switch – position 1), the direction of current flow in the electrolyte is again reversed and the $PbSO_4$ on the +ve plate becomes lead dioxide (PbO_2). This is a complex result of the electrolysis of the acid. Sulphions move to the +ve plate, react with the water to form H_2SO_4 and O_2. The latter attacks the +ve plate to form PbO_2 and more H_2SO_4. The chemical action at the +ve plate is $SO_4 + H_2O = H_2SO_4 + O$ and $O + H_2O + PbSO_4 = PbO_2 + H_2SO_4$. At the negative plate, lead is produced by the hydrogen ions liberated by the acid decomposition, moving to this electrode and reducing the lead sulphate to 'spongy lead'. The chemical action at the –ve plate is:

$$PbSO_4 + H_2 = H_2SO_4 + Pb$$
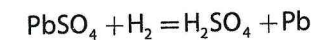
After a number of cycles of charging and discharging the plates become porous and the capacity of the cell is increased. When a cell is fully charged, chemical conversions are completed and hydrogen is freely given off, resulting in 'gassing' – the accepted term indicating a full charge.

The lead–acid accumulator in its practical form is provided with 'pasted' plates. Here the active material is applied to plates in the form of a paste, the backbone of the plate being a lead–antimony grid. One 'forming' charge converts the paste into lead dioxide on the +ve plate and spongy lead on the –ve plate. Irrespective of the method of production the charge and discharge action can be summarised by the following chemical equation.

Table 4.2

Charged			Discharged		
+ve Pole		–ve Pole	+ve Pole	–ve Pole	
Lead dioxide	Sulphuric acid	Lead	Lead sulphate	Water	Lead sulphate
PbO_2 +	$2H_2SO_4$ +	Pb =	$PbSO_4$ +	$2H_2O$ +	$PbSO_4$

It will be seen that during discharge water is formed, thus diluting and reducing the specific gravity of the electrolyte. During charge, acid is formed and the tests to check a fully charged cell include:

(1) S.G. of cell charged (1.20 to 1.27); discharged (1.17 to 1.18).
(2) Voltage on O.C., charged 2.2V per cell or higher.
(3) 'Gassing' on charge.
(4) Positive plate – rich dark brown colour. Negative plate – slate grey.

The alkaline cell (nickel–iron or nickel–cadmium)

Two variations of this cell are in common use, being the results of some 110 years developments and patents were taken out simultaneously by Edison in America and Jungner in Sweden. Both men devised a cell with reversible action which used iron or cadmium for one plate and nickel hydrate for the other. The construction of both types is basically the same. Active materials are enclosed in steel tubes or flat interlocking pockets perforated over the whole surface area with many minute holes. The tubes (Edison form) or pockets (Jungner form) are assembled into steel retaining frames to form the +ve plate. The –ve plate, for both forms of cell, consists of a steel frame into which are assembled the flat pockets. Groups of plates of the same polarity are bolted together to steel terminal pillars. Separators of sheet ebonite are used and the plate groups assembled in a steel container. When the plates expand after the initial 'forming' process, no internal movement is possible. The cell terminals for each plate group are brought through the lid in suitable insulated glands. Batteries of cells are built up in hardwood crates, the cell containers being at a potential. Other forms of enclosure in plastic cases have been developed in recent years leading to a lighter battery and dispensing with the need for insulated wooden crates.

Both forms of cell use the same electrolyte of dilute (21%) potassium hydroxide and have the same e.m.f. of 1.25V. The electrolyte takes no active part but functions merely as a conductor, transferring the hydroxyl (OH⁻) ions from one plate to the other when charging or discharging. The cell is inert on O.C. and the electrolyte S.G. (about 1.18) doesn't alter. The battery is often described by the tradename Nife (nickel–iron, Ni–Fe). It is a robust battery which is resistant to overcharging and short-circuiting and can have very long life. It is often used in backup situations where it can be continuously charged and can last for over 20 years. Due to its low specific energy, poor charge retention and high cost of manufacture, other types of rechargeable batteries have displaced the nickel–iron battery in most applications, one of the reasons the nickel–cadmium version is favoured and has been developed. A typical cell has an outer casing, a separator between electrodes, a positive electrode and a negative electrode and is usually cylindrical to maximise energy density (figure 4.9).

The chemical reactions in the alkaline cell are complex but as a guide the following equation shows the operation. This equation is given for the nickel–cadmium version. For the nickel–iron version it is similar except that Fe replaces Cd and $Fe(OH)_2$ the $Cd(OH)_2$. For a fully charged cell the nickel hydrate is at a high degree of oxidation and the –ve material is reduced to pure cadmium. On discharge the nickel triple-hydrate is reduced to a lower degree of oxidation (double hydrate) and the cadmium of the –ve plate converted to a hydroxide.
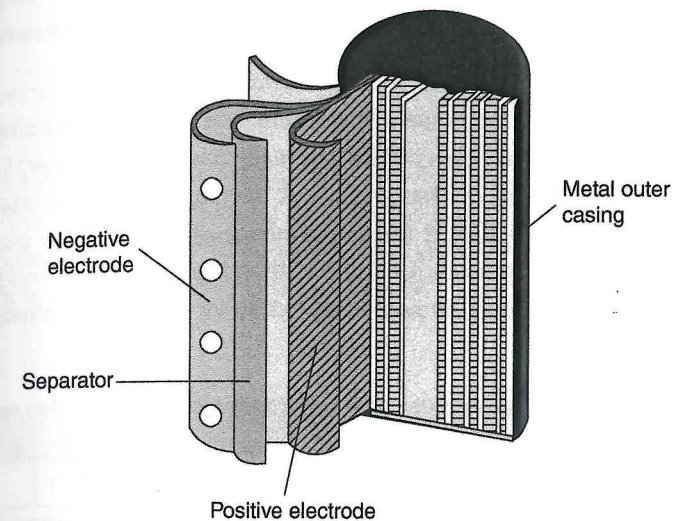
▲ Figure 4.9 *Nickel–cadmium cell: outer metal casing, separator, positive electrode and negative electrode*

Table 4.3

	.	+ve plate				–ve plate
		Nickel hydrate		Potassium hydroxide		Cadmium
CHARGED		$2Ni(OH)_3$	+	KOH	+	Cd
DISCHARGED	↕	Nickel hydroxide		Potassium hydroxide		Cadmium hydroxide
		$2Ni(OH)_2$	+	KOH	+	$Cd(OH)_2$

When the alkaline cell is charged the e.m.f. is about 1.5V, decreasing rapidly to 1.3V then more slowly to 1.0V on discharge. The average is 1.25V so for a given voltage the number of cells is 1.7 times the number of lead–acid cells required. The cell is extremely robust and can be 'short-circuited' without damage. Chemical action is completely reversible and no fumes are given off. Provided the plates are covered with electrolyte, cells will stand almost indefinitely in any state of charge without significant deterioration. The internal resistance of an alkaline cell is 10–50% more than that of the lead–acid type but this depends upon design factors. Conductivity is adjusted by mixing finely powdered carbon with the nickel hydrate in the pockets of the +ve plate while iron powder can be mixed with the cadmium powder in the −ve plate pockets. These additions prevent the active materials solidifying and the porous requirement of the overall plate construction being lost.

CAPACITY OF A CELL. This is the *ampere hour* figure it can yield on a single discharge, until the e.m.f. falls to about 1.8V per cell (for the lead–acid cell). Generally the capacity is based on a 10-hour rate of discharge, since it decreases as the rate of discharge increases. Research has shown that the performance of a cell can be improved by working the +ve plate at a higher current density than the negative. This is achieved by keeping the plates of equal area, providing an extra −ve plate, i.e. always making the outside plates *negative*.

EFFICIENCY OF A CELL. This may be expressed in terms of (1) the Ampere hour input and output, (2) the Watt hour input and output. Thus:

$$\text{Ampere hour efficiency} = \frac{\text{Ampere hours of discharge}}{\text{Ampere hours of charge}}$$

The ampere hour efficiency neglects the varying voltages during charge and discharge. Since this is important, we have an energy efficiency compared to a quantity efficiency, and

$$\text{Watt hour efficiency} = \frac{\text{Watt hours of discharge}}{\text{Watt hours of charge}}$$

$$= \frac{\text{Average discharge volts} \times \text{Amperes} \times \text{Hours}}{\text{Average charge volts} \times \text{Amperes} \times \text{Hours}}$$

Example 4.5. A battery is charged with a constant current of 16A for 11 hours after which time it is considered to be fully charged, its voltage per cell being recorded as

2.2V. Find its ampere hour efficiency if it is (1) discharged at a rate of 16A for 10 hours and (2) 28A for 4 hours. In either case discharge was discontinued when the voltage per cell fell to 1.8V (2 significant figures).

(1) Ampere hour input = 16 × 11 = 176

Ampere hour output = 160

$$\therefore \text{Efficiency} = \frac{160}{176} = 0.91 \text{ or } 91\%$$

(2) Ampere hour input = 16 × 11 = 176

Ampere hour output = 28 × 4 = 112

$$\therefore \text{Efficiency} = \frac{112}{176} = 0.63 \text{ or } 63\%.$$

Example 4.6. A 12V accumulator is charged by means of a constant current of 16A passed for 11 hours. The P.D. during charging varies as shown. The battery is then discharged at a constant current of 16A for 10 hours, the P.D. again varying as shown. Calculate the Watt hour efficiency of the battery (1 decimal place).

Table 4.4

	Reading No.	Time (hr)	Charge (V)	Discharge (V)
Start	1	–	10.8	12.6
	2	1	11.0	12.4
	3	2	11.5	12.2
	4	3	11.8	12.0
	5	4	12.0	11.8
	6	5	12.2	11.6
	7	6	12.4	11.4
	8	7	12.6	11.2
	9	8	12.8	11.0
	10	9	13.0	10.9
	11	10	13.1	10.8
	12	11	13.2	**127.9**
			146.4	