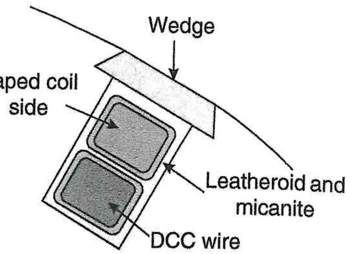
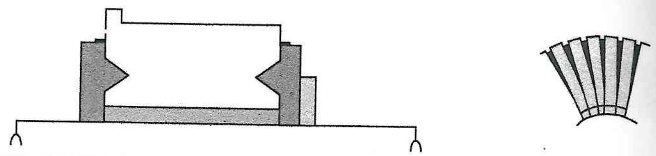


▲ Figure 12.5



▲ Figure 12.6

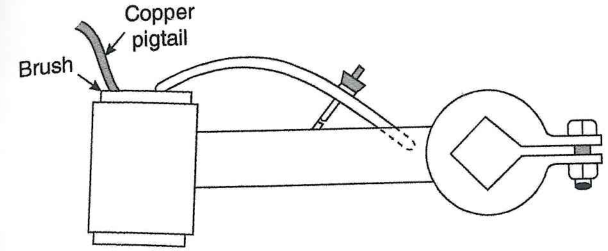
windings are enamelled or cotton-covered wire placed in and wound by hand. Semi-enclosed slots are used with fibre inserts closing the slots. Conductors are arranged into a closed winding and considered in detail after discussing machine construction.

SHAFT. This is made from forged mild steel and designed so it doesn't bend when at maximum speed.

COMMUTATOR. This consists of copper segments insulated from each other by mica. Segments are mounted on but insulated from a sleeve secured to the shaft and clamped by an end-ring which can be bolted or screwed as shown (figure 12.6). Insulated cone-shaped rings of micanite insulate segments from the steel clamping assembly. Armature windings are soldered to these segments.

BRUSHES. A brush is pressed onto a commutator by a pressure arm and connected to the holder by a braided copper wire 'pigtail' moulded into the brush. One or more brush-holders may be carried on an insulated spindle mounted on the brush rocker-ring itself clamped once the brush position is set. Modern D.C. machine brushes are made of moulded carbon or graphite. Figure 12.7 is a typical arrangement.

BEARINGS. For most industrial D.C. machines bearings are a ball or roller type. The advantages are (1) axial length is shorter than the journal type bearings and (2) after packing with grease, long periods of service are possible. For marine work journal bearings give quieter running and are preferred as they resist 'transmitted' vibrations. A steel shaft runs in a brass or cast-iron sleeve lined with metal. For small or medium size machines, bearings are carried in the end shields, but for large machines bearings are separate.



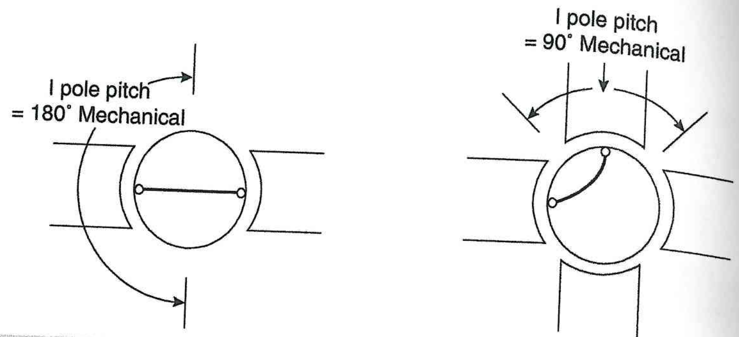
▲ Figure 12.7

D.C. Armature Winding Arrangements

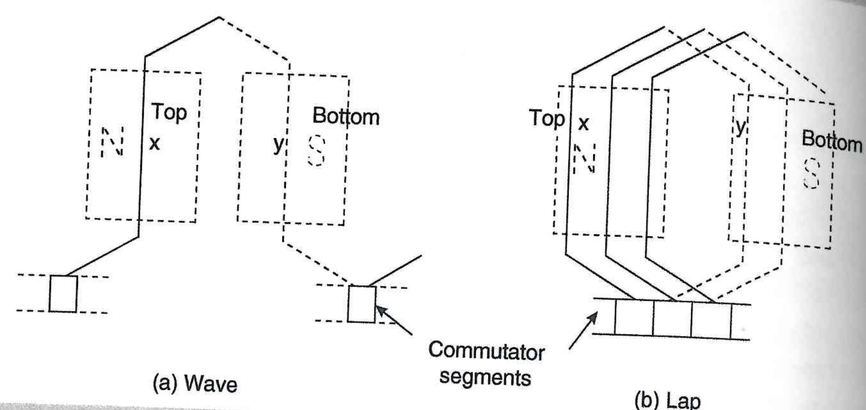
The simplest winding is built up from single-turn coils of span equal to 1 'pole pitch', i.e. 180° for a 2-pole machine. For a 4-pole machine the coil span is still 1 pole pitch, but is now 90° as shown (figure 12.8). It is unusual to make the span exactly equal to 1 pole pitch and many small machines have an odd number of slots. Each slot carries 2 coil sides, i.e. it contains more than 1 conductor. D.C. windings are usually of a 2-layer type, a coil side lying at the bottom of a slot and another at the top, but 4, 6 or even 8 coil sides may be contained in 1 slot as it may be impracticable to have many slots. There are 2 basic methods of connecting conductors on an armature after forming into either single or multi-turn coils and a complete winding falls into 2 distinct types namely: (1) a wave or (2) a lap winding.

(1) The WAVE or 2-circuit winding results in 2 parallel paths irrespective of the number of machine poles. Two sets of brushes only are needed but it is usual to fit as many sets of brushes as the machine has poles. Figure 12.9a shows the layout.

(2) The LAP or multi-circuit winding results in as many paths in parallel and as many sets of brushes as the machine has poles. Figure 12.9b shows the layout. In building up a winding it is vital to connect coil elements so the induced e.m.f.s in conductors add, in the same way as cells are connected in series so their e.m.f.s add to give the required battery voltage. Thus conductor *X* is in series with conductor *Y* which occupies relatively the same position as *X* but is under a pole of *reversed* polarity. A coil element formed by conductors *XY* is then connected in series with a similarly placed coil element under a pair of poles so the required voltage for a parallel armature path is achieved. For a wave winding connection are as shown. For a lap winding the same rule is followed, except that all coil elements under a pair of poles are connected in series *before* the winding connects to conductors.



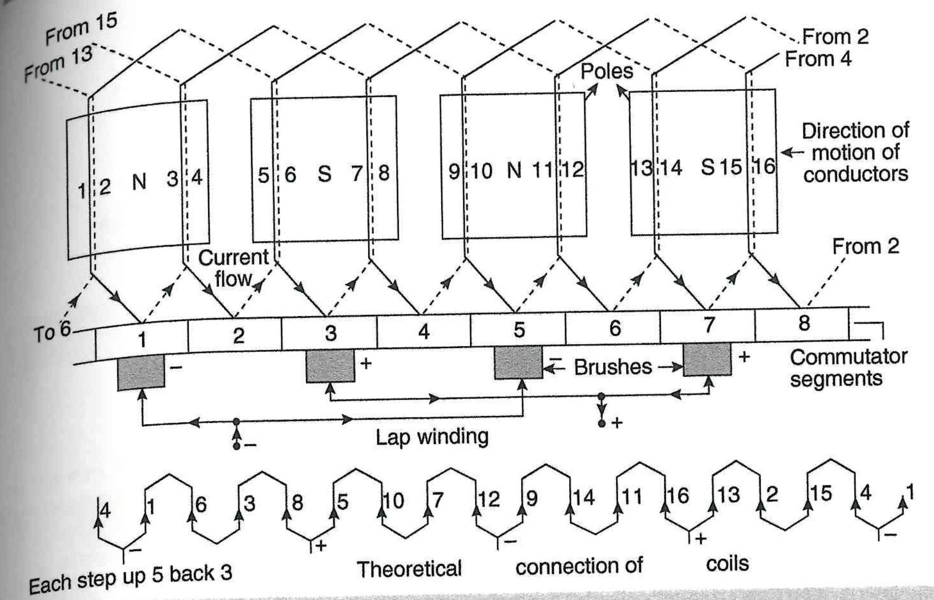
▲ Figure 12.8



▲ Figure 12.9

The following example illustrates both lap and wave windings. A small armature is designed to have 8 one-turn coils each having 2 conductors. There is 1 commutator segment to a coil, i.e. 8 commutator segments. If only 2 coil sides are accommodated in a slot there must be 8 armature slots. If a 4-pole system were used there would be 2 slots/pole, with a true pole pitch of 2, the pole pitch being the number of armature slots divided by the number of poles. As the coil sides are under the influence of the correct field poles, winding pitch must be as nearly as possible equal to the pole pitch. So the winding pitch will also be equal to 2 or a coil should embrace 2 teeth.

The LAP winding is considered first. For such a winding, connecting up of conductors is such that the winding progresses round the armature by being pitched alternatively forwards and backwards. If figure 12.10 is considered, it is seen that conductor No. 1 is connected to No. 6 spaced 2 teeth away (2 commutator segments). No. 6 is then connected to No. 3 and so on. The winding thus progresses by 1 slot until it is closed by all the slots having been occupied and conductor No. 15 is connected to No. 1 through No. 4.



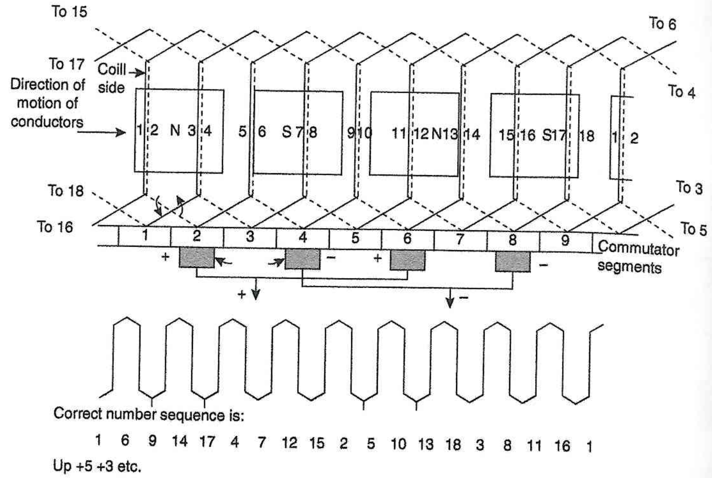
▲ Figure 12.10

If now a WAVE winding is needed a preliminary examination will show that this could not be achieved via the coil connections of figure 12.11. If the winding started at No. 1 proceeded to No. 6 and then on through Nos 9 and 14 it would close back onto conductor No. 1. It is obvious that an armature with 8 slots would not be suitable for such a wave winding and one of 7 or 9 slots must be considered. A 9-slot armature winding will give a winding pitch of length slightly less than the true pole pitch length and is suitable. Consider the diagram (figure 12.11). Conductor No. 1 is connected to No. 6 as before, then connected to Nos 9, 14, 17 and then to No. 4, i.e. the winding passes into the slot *next* beyond that which it started. The winding, doesn't close immediately and if the connecting-up proceeds as described, the winding will progress 4 times round the armature before the close is made at the starting slot by conductor No. 11 being joined to No. 1 through No. 16. This is a suitable winding but 9 coils will be used with 9 armature slots and 9 commutator segments.

Armature winding details are found elsewhere as machine design and armature winding is specialist work.

The D.C. Generator

D.C. armature and commutator theory shows that commercial D.C. is best obtained using an armature wound with several coils connected so all the coils except those



▲ Figure 12.11

short-circuited by brushes are in the circuit. The armature being a continuous closed winding splits itself electrically into several parallel paths. There are 2 key armature winding types: (1) a lap winding or (2) a wave winding. For a lap winding the number of parallel paths in an armature is always equal to the number of poles. For a wave winding the number of parallel paths is always 2, irrespective of the machine's number of poles.

The e.m.f. equation

Consider the diagram (figure 12.12) and the factors for the machine. A simple expression for the composite armature is deduced and it is vital to memorise it. A student must be able to prove it from first principles.

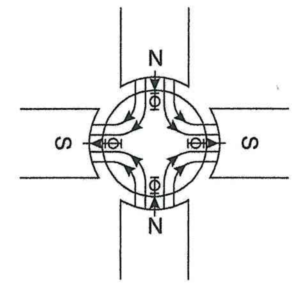
Let N = the machine speed in rev/min, P = the number of poles. Φ = the flux/pole (webers). Z = the number of armature conductors. A = the number of parallel paths of the armature winding.

In 1 second an armature revolves $\frac{N}{60}$ times so in 1 revolution, 1 conductor cuts a flux

$= P \times \Phi$ webers

$\therefore$ In 1 second, 1 conductor cuts $P\,\Phi\,\frac{N}{60}$ webers.

From Faraday's law the magnitude of the e.m.f. generated in volts is given by the flux cut/second



▲ Figure 12.12

so the e.m.f. generated in 1 conductor $= \frac{P\Phi N}{60}$ volts.

If the armature winding divides into A parallel paths the e.m.f. of a parallel path is also a machine's e.m.f.

In a parallel path there are $\frac{Z}{A}$ series conductors, so the e.m.f. of 1 parallel path =

the machine e.m.f. $= \frac{P\Phi N}{60} \times \frac{Z}{A}$ Thus $E = \frac{Z\Phi N}{60} \times \frac{P}{A}$ volts, where E = the generated voltage.

Example 12.1. The armature of a 4-pole, shunt generator is lap wound and generates 216 volts when run at 600 rev/min. The armature has 144 slots with 6 conductors/slot. If this armature is rewound and *wave connected*, determine the e.m.f. generated at the same speed and flux/pole.

From the e.m.f. equation e.m.f. $= 216 = \frac{(6\times144)\times\Phi\times600\times4}{60\times4}$ or $\Phi = \frac{216}{60\times144}$ webers

Note. This is a *Lap-wound* armature so $A = P = 4$.

For a *Wave-wound* armature $A = 2$

$\therefore E = \frac{6\times144}{60} \times \frac{216}{60\times144} \times \frac{600}{2} \times 4 = 216\times2 = 432V.$

Characteristics

These are graphs which show the behaviour of any machine type. For example, consider the e.m.f. equation. For a given machine, all the factors except Φ and *N* are constant. The equation can be written $E = k\Phi N$ where $k = \frac{ZP}{60A}$ Thus $E \alpha \Phi$ if N is kept constant and $E \alpha$ N, if Φ is kept constant.

If Φ and *N* both vary *E* will vary accordingly. Thus the voltage generated is controlled by varying the machine speed or flux as shown by deducing the 'no-load' characteristics.

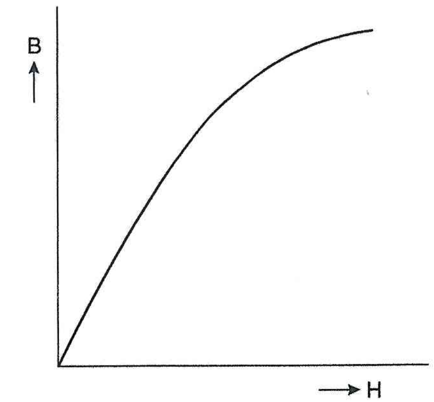
Associated Magnetic Circuit Effects

In Chapter 6 the iron or steel cored electromagnet was considered. As the magnetic circuit is an essential part of a D.C. machine, it is vital to consider 2 effects influencing generator characteristics.

The first effect is that of residual magnetism. Experiment shows that if an electromagnet's iron core is magnetised by passing a current through the energising coil, when the current is switched off and the magnetising m.m.f. removed, the magnetism or magnetic flux *will not* completely disappear, although in theory, it should not exist. It is important for readers to appreciate that this effect occurs.

The second factor is the *saturation effect* of iron when subject to a m.m.f. If a magnetic circuit uses iron as the medium for conveying flux then, as the m.m.f. increases, the flux increases up to a point when the straight-line relation between m.m.f. and flux Φ and consequent flux density *B* is no longer followed. Thus for an iron sample, if *B* is plotted against *H* (the magnetising force/metre), the '*B–H* curve' is obtained. The resulting graph is a straight line for *only* a short part of its length. It appears as shown (figure 12.13) indicating that iron *saturates*, i.e. no matter how much the m.m.f. increases, once the curve bends over and flattens out no increased flux Φ or consequent *B* value is produced, irrespective of the magnetising force's strength.

The flattening out or saturation effect is considered to be due to all the molecular magnets aligning with the magnetic field. The saturating effect is apparent when investigating the relationship of generated voltage *E* with flux Φ in a D.C. machine's magnetic circuit.



▲ Figure 12.13

The no-load characteristic

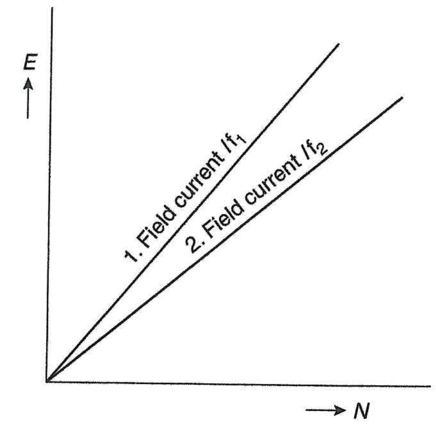
Theory shows that generated voltage depends on machine flux and speed and the no-load characteristics can be considered under the following headings.

VARIATION OF *E* WITH *N* (Flux Φ constant). A permanent-magnet generator is rarely used for practical applications, but the test can be made by controlling the current of separately energised field electromagnets. This current or *field* current I_f flowing through the field coils, creates an m.m.f. which results in flux in the air gaps. If this current is a constant value I_{f1}, the flux will be constant. Tests are made by varying the speed at which a machine is driven and noting the voltage generated. As flux Φ is constant and $E \propto N$, a straight-line graph as shown by (1) of the diagram (figure 12.14), results. If the field current is then adjusted to a smaller but constant value I_{f2}, and the test repeated, a straight-line graph such as (2) results. The deduction assumed, namely *E* varies directly with *N*, is proven.

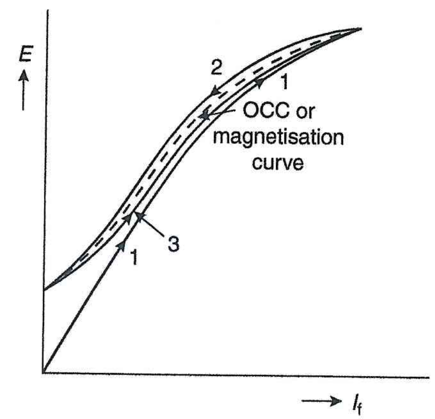
VARIATION OF *E* WITH Φ (Speed *N* constant). Flux variation is achieved by controlling the energising current I_f in the field coils or 'exciting current'. If no residual magnetism is present then, if I_f increases, m.m.f. increases and flux in the air gaps increases.

The generated e.m.f. increases accordingly and a *B–H* type of curve (1) is as shown (figure 12.15) if *E* is plotted to a base of I_f. *Note.* Φ cannot be readily measured but its effects are assessed by knowing the appropriate exciting current values.

Curve (1) at first increases as a straight line, flattening out to horizontal as the magnet saturates. When saturation occurs, if the field current is reduced, curve (2) results. This



▲ Figure 12.14



▲ Figure 12.15

curve lies slightly *above* the original curve (1) and for decreasing values of I_f, the values of E are above those obtained for the ascending curve (1). The cause of the difference between curves (1) and (2) is magnetic hysteresis (discussed in Chapter 6). When the field current is finally reduced to zero, some generated e.m.f. is present while the machine runs at constant speed N. The e.m.f. is due to residual magnetism, which is essential if a generator is to be self-exciting. The e.m.f. due to residual magnetism can only be removed by demagnetising the field system. If the value of I_f is increased again, curve (3) is followed which closes up on curve (1). The diagram overemphasises the difference between curves (1) and (2). In a modern machine this difference is not large and if a mean curve is drawn (dotted) this is the *magnetisation* or *open-circuit characteristic* (O.C.C.) curve which is important, plotted as generated voltage or e.m.f. to a base of field current. In theory relating to A.C. and D.C. generators and motors problems will refer to it before they are solved.

Types of D.C. Generator

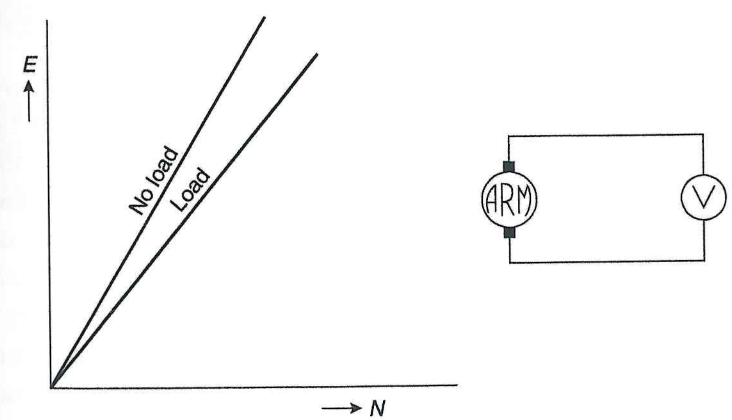
The machine is classified in different ways. As generator types change by varying the magnet system, i.e. either the magnetic material or connection of the field energising coils differ. In this book generators are described as follows: (1) the permanent-magnet type of generator, (2) the separately excited type of generator, (3) the self-exciting type of generator, further subdivided under the headings of: (3a) shunt-connected, (3b) series-connected and (3c) compound-connected.

(1) The permanent-magnet type of generator

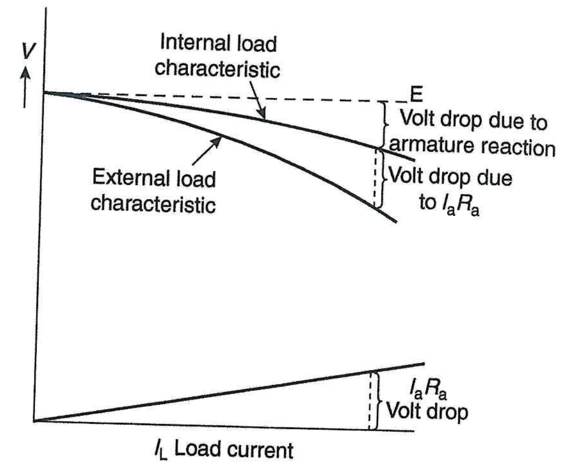
This type is not used much as it is hard to make large permanent magnets or to vary the magnetic field to control generator output. The most common uses are as electrical tachometers (speed indicators), hand-operated insulation testers and primary exciters for large alternators.

THE LOAD CHARACTERISTIC. Φ is constant so the load characteristic is almost identical to the no-load characteristic. A tachometer arrangement is shown (figure 12.16).

If a sensitive voltmeter is used, i.e. one requiring negligible current, generator output current will be small so the armature voltage drop (I_aR_a) will be negligible. R_a is the armature's ohmic resistance and I_a the armature current. The load terminal voltage V is nearly equal to the generated e.m.f. E and the voltmeter calibrated in revolutions per minute (rev/min).



▲ Figure 12.16



▲ Figure 12.17

(2) The separately excited type of generator

Knowing $E \propto N$ if Φ is constant, the no-load characteristic will be a straight line as discussed. However, if N is constant and Φ varied the characteristic will vary as the B–H curve and an O.C.C. will result. The 2 characteristic variations are shown (figures 12.14 and 12.15).

THE LOAD CHARACTERISTIC. This is obtained by setting the field current to give the normal rated voltage at the correct speed and by applying load in stages takes current between 0 to 25% overload. For a small generator, loading is applied by switching in banks of similar wattage lamps connected in parallel. If terminal voltage V is plotted versus load current I_L, the external load characteristic is obtained (figure 12.17).

If a machine is stopped and armature resistance R_a measured by the ammeter/voltmeter method with a separate low-voltage supply, the I_aR_a voltage-drop line can be plotted. If various I_aR_a voltage-drop values are added to the external characteristic the *internal load characteristic* is obtained by construction. The difference between this line and the horizontal line of the theoretical generated e.m.f. E shows the voltage drop due to any *armature reaction* effects, which were explained in Volume 7, but are described here briefly. It is the passage of current through an armature which sets up a magnetic field which interacts with the main field, weakening and distorting the latter. The magnitude of generated e.m.f. reduces and commutation is effected negatively.

Load characteristics are introduced to illustrate the effects responsible for a voltage drop inside the generator, when the machine is on load. In problems armature reaction

is seldom mentioned but the armature is usually credited with a resistance value greater than its ohmic value to allow for a total internal voltage drop. The voltage equation will be:

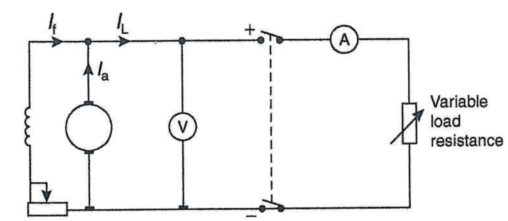
$$E = V + I_aR_a$$

The separately excited D.C. generator is used for applications such as machines supplying current to electroplating vats, with some 6–10kA needed at voltages of 6–12V with output controlled by varying a separately excited field.

(3a) The shunt-connected generator

Figure 12.18 shows the typical connections for this machine. Armature current is fed to both the load circuit and the parallel field circuit which although taking a small current in comparison with the load must be considered.

$$\text{Thus: } I_a = I_f + I_L$$



▲ Figure 12.18

A machine's shunt field is connected across the generator's terminals. Field coils form a high-resistance circuit wound with many turns of fine wire, i.e. the ampere-turns are produced by a small current value and many turns.

$$\text{As before: } E = V + I_aR_a$$

Example 12.2. A 4-pole, wave-wound generator delivers 40kW at 200V. Its armature has 181 turns and a resistance of 0.01Ω. The air-gap flux/pole is 0.02Wb. Calculate the machine's speed, neglecting the voltage drop at the brushes and taking the shunt-field resistance as 50Ω.

$$\text{Load current} = \frac{\text{kilowatts}}{\text{voltage}} \text{ or } I_L = \frac{40\,000}{200} = 200\text{A}$$

$$\text{Shunt}-\text{field current } I_f \; \frac{V}{R_f} = \frac{200}{50} = 4\text{A}$$

So armature current $I_a = 200 + 4 = 204\text{A}$

Armature voltage drop $= 204 \times 0.01 = 2.04\text{V}$

Generated voltage = terminal voltage + voltage drop in armature

$E = V + I_a R_a = 200 + 2.04 = 202.04\text{V}$ Thus $N = 840$ rev/min.

Note. The armature is assumed to be wound with single-turn coils, i.e. 2 conductors/turn.

THEORY OF SELF-EXCITATION. As shunt-connected generators use the principle of self-excitation, it is key to explaining the theory involved. If a field system has residual magnetism, armature rotation generates a small e.m.f. which causes a field current to produce more flux, which in turn causes more e.m.f. to giving a continuous positive feedback condition. Voltage rises and only flattens off when the voltage drop across the field equals the terminal voltage.

Note. Field current must be in the correct direction through coils to aid the original residual flux build-up.

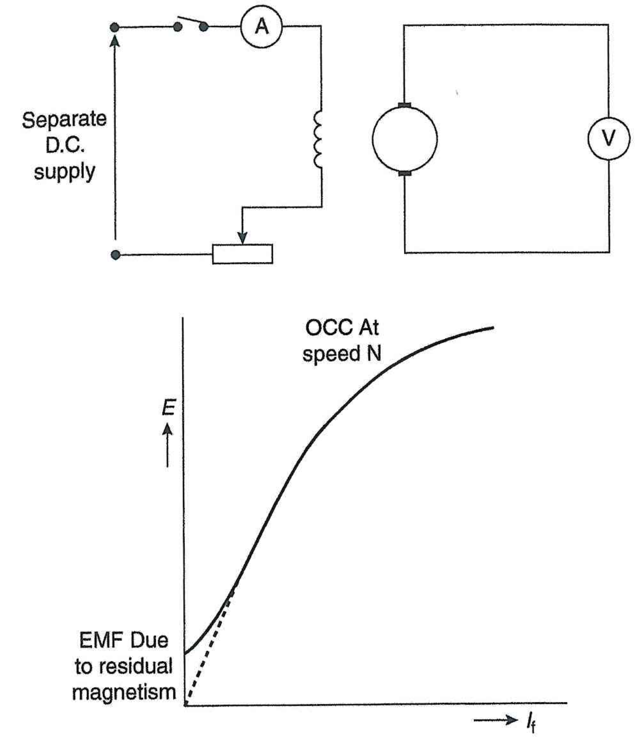
Summarising, the following are the conditions necessary for self-excitation:

1. There must be residual magnetism sufficient to generate a small e.m.f. when an armature rotates at the correct speed.
2. The shunt-field circuit must be continuous and connected so current flow causes flux to build up, assisting the original residual flux.
3. The shunt-field circuit resistance must be less than the *critical resistance* determined from the O.C.C. when the machine runs at a particular speed.

Critical resistance is helpful to understand the conditions for self-excitation to occur which is very important and also the subject of examination questions.

The magnetisation curve or O.C.C. applied to self-excitation, critical resistance

Figure 12.19 illustrates a circuit and its characteristic obtained with a separately excited generator. The initial part of the O.C.C. graph is complex, due to the effects of residual

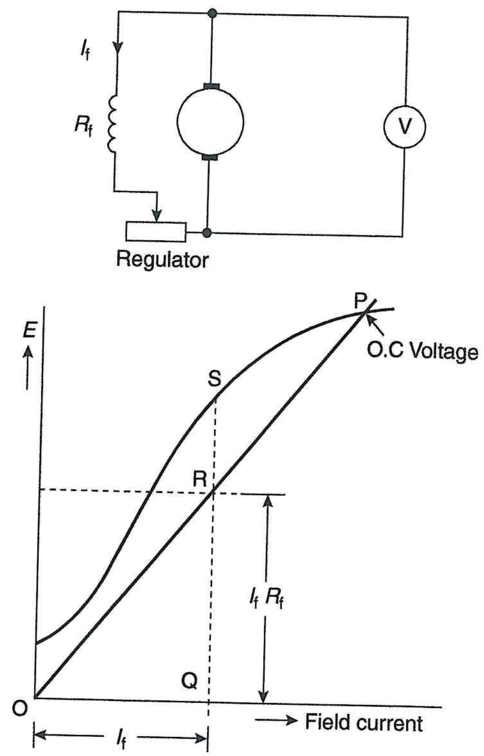


▲ Figure 12.19

magnetism. If the graph starts from zero it is usually straight, as the machine's magnetic circuit involves air gaps and saturation conditions which are reached gradually. Full saturation conditions are seldom attained. It is stressed that the O.C.C. depends on speed. When the field is *shunt-connected*, provided the conditions set out above are fulfilled, a generator will self-excite and an O.C. voltage value is attained where the voltage drop in the shunt field is equal to the generated terminal voltage.

This condition is illustrated (figure 12.20) and is best understood by considering the O.C.C. and field voltage-drop line. The magnetisation curve for any speed N is drawn from results obtained by separate excitation. Imagine a shunt field and regulator with a resistance of R_f ohms, assuming a current of value I_f amperes to flow, the field voltage drop will be $I_f R_f$ volts.

Plot this value (e.g. point R) and extend the straight line through R from zero to cut the O.C.C. at point P. At this intersection point (P), the voltage drop across the field equals the applied terminal voltage and conditions balance. Consider the I_f condition shown, where generated voltage SQ is greater than the field voltage drop RQ by SR volts. More current flows in the field circuit because of this voltage difference and both graphs rise until a point of intersection is reached.



▲ Figure 12.20

Example 12.3. A D.C. generator when separately excited and driven at 1000 rev/min gave the following test values on O.C.

Field current (A)	0	0.16	0.48	0.66	0.8	1.0	1.29
O.C. e.m.f. (V)	6.25	50	150	200	225	250	275

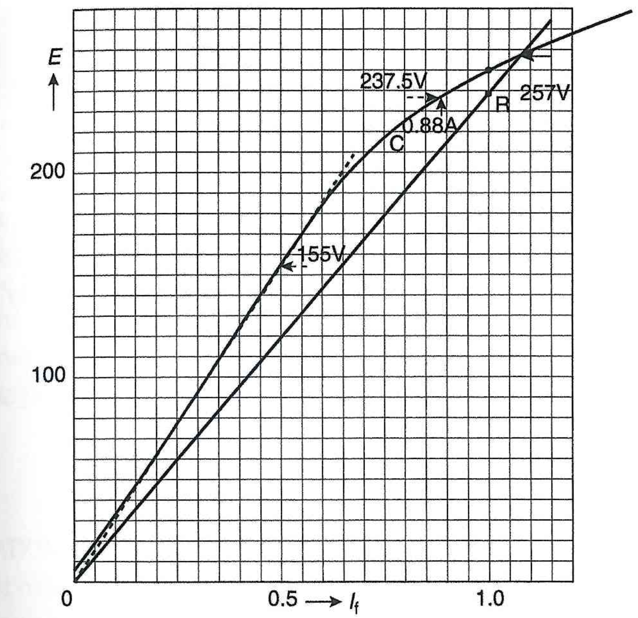
Field windings are then shunt-connected. Find (a) the voltage to which the machine will self-excite on O.C. when driven at 1000 rev/min and the shunt-field circuit resistance is 240Ω, (b) the value of the regulator resistance to be added or subtracted from the field circuit to allow the generator to self-excite to 237.5V and (c) the value of the critical resistance at this speed.

(a) Plot the O.C.C. as shown (figure 12.21). Using the graph, take any value of field current, viz. 1A. The voltage across the field circuit with 1A flowing will be 1 × 240 = 240V. Plot point (R) and draw the field voltage-drop line through the origin as shown. The O.C. voltage to which the machine self-excites is 257V.

(b) For the machine to excite to 237.5V, note this value on the O.C.C. and join it to zero to obtain the new field resistance voltage-drop line. Note the field current for 237.5V, this is 0.88A. From Ohm's law, the field-circuit resistance is $\frac{237.5}{0.88} = 269.9 = 270\Omega$.

$\therefore$ Resistance to be added = 270 − 240 = 30Ω.

(c) Neglecting the start of the graph (due to residual magnetism), draw a tangent through the origin. Read the voltage value on this tangent and the corresponding field current. For example: 155V and 0.5A. The critical resistance will be $\frac{155}{0.5}\text{ohms} = 310\Omega$.



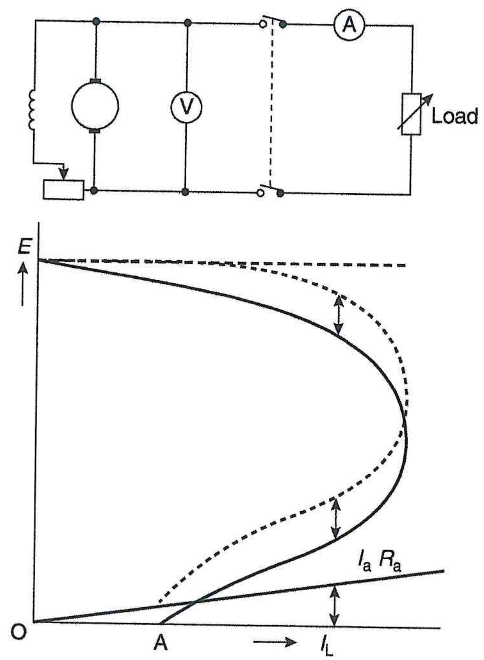
▲ Figure 12.21

CRITICAL RESISTANCE. The effect of altering the shunt-field resistance or regulator is seen by reference to the example and graph (figure 12.21). Reduce R_f and the slope of the field resistance voltage-drop line becomes less and the intersection point with the O.C.C. moves higher up, i.e. the generator O.C. voltage is raised. The opposite occurs with R_f increased. If R_f is increased until the field voltage-drop line lies outside the magnetisation curve, there will be no point of intersection and the generator will not self-excite. If R_f is reduced, the slope of the field voltage-drop line decreases until the line lies along or becomes tangential to the O.C.C. The resistance value deduced from the field voltage-drop line falls until it reaches the value given by the line tangential to the O.C.C. The resistance value for this condition is the *critical resistance*, and depends on speed. For any speed, if field resistance is less than the critical resistance, a machine will self-excite if the other conditions are satisfied.

THE LOAD CHARACTERISTIC. The test circuit graph is shown (figure 12.22). Armature resistance R_a is measured. The external load characteristic is plotted from test results and the internal characteristic drawn as described for a separately excited machine. Features of the load characteristics are (1) rapid fall-off of terminal voltage and (2) a 'bend-back' of the characteristic on itself.

(1) When the external circuit is connected to a load, there is a voltage drop in the armature. Terminal voltage falls, resulting in a decrease of field exciting current. This in turn causes the external characteristic to 'slump down' more than for the separately excited machine. The armature reaction effect is as for the separately excited machine, i.e. it is responsible for a voltage decrease, equivalent to an increased armature voltage drop, accounted for by giving the armature a R_a value *greater* than its ohmic resistance.

(2) As load resistance decreases, load current increases at first with a resulting terminal voltage fall, tending to slow an increase of load current. At first external load resistance decreases and consequent load current rise dominating and a rising current with



▲ Figure 12.22

falling terminal voltage is shown. At a certain current value the demagnetising effect of armature reaction, the armature-resistance voltage drops and the loss of field current due to reduced voltage, combine to produce a terminal voltage resulting in *less* load current even though load resistance is decreased and the curve bends back on itself! The armature may be short-circuited – a self-protecting effect. OA in figure 12.22 is caused by residual magnetism, but a sudden short-circuit may cause an excess armature reaction effect tending to cancel residual magnetism, thus demagnetising the machine which may then fail to self-excite when short-circuit is removed. The machine will need to be remagnetised before it can operate again. A shunt-connected machine can be used for most purposes where a simple generator is needed. Examples are battery chargers and small lighting-sets, the motor-car dynamo and some electrical systems.

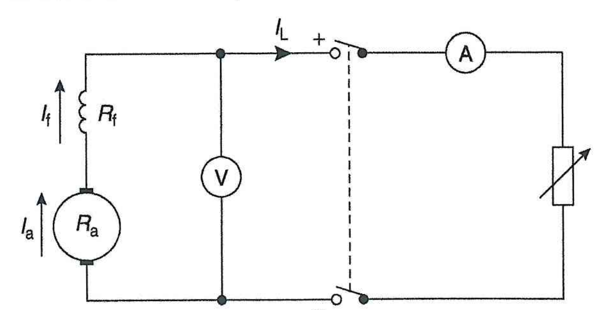
(3b) The series-connected generator

Figure 12.23 shows connections for this machine used in specialist work and in the compound generator.

The series field of this generator is designed to be connected in the main armature circuit to the load. Field coils are wound with a few turns of thick cable, i.e. the field ampere-turns are produced by a large current and a small number of turns. Thus $I_a = I_f = I_L$. Terminal voltage on load is V *and* the generated e.m.f. E is greater than V by the internal voltage drops in the armature and series field. Thus:

$$E = V + I_f R_f + I_a R_a = V + I_a(R_a + R_f)$$

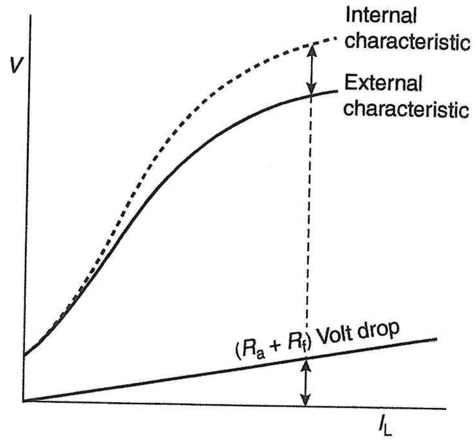
SELF-EXCITATION. The same theory and conditions apply as for the shunt-connected machine. It is noted that load resistance constitutes the field regulating resistance and so for any chosen speed there is a critical resistance value. If load resistance, i.e. the field-circuit resistance, is less than the critical value, the machine will self-excite. If the circuit resistance



▲ Figure 12.23

is above the critical value for that machine speed, self-excitation and voltage build-up will not occur.

THE LOAD CHARACTERISTIC. Consider the circuit shown (figure 12.24). A circuit switch is closed with load resistance at maximum which is reduced until the machine self-excites. Load current and terminal voltage settle at a fixed value but if the load is altered, new voltage and current values occur. This is repeated for decreasing and increasing load resistance, until a full external load characteristic is obtained (figure 12.24).



▲ Figure 12.24

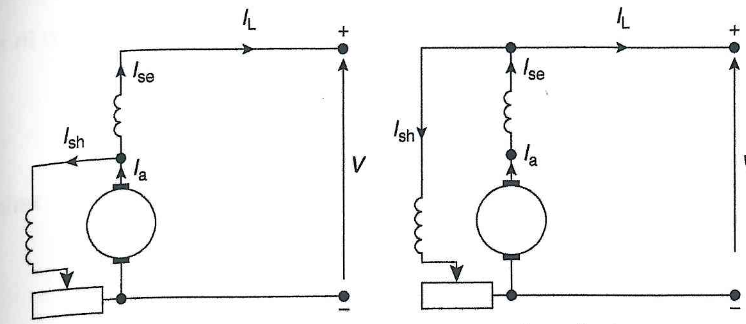
The machine is then shut down and R_a and R_f measured separately. The armature and series-field resistance voltage-drop lines are drawn and the internal load characteristic deduced. The effects of armature reaction may be investigated if an O.C.C. (obtained by separate excitation at the correct speed) was superimposed on the characteristics. The machine has the following disadvantages:

(1) It cannot self-excite until the load circuit is completed and its resistance value is less than the critical resistance.

(2) The voltage to which it self-excites depends on the load current and very little voltage control is possible.

(3) The load characteristic is a rising one and is unsuitable, in fact dangerous, and may result in load 'burn-out'.

A series generator is never used for normal generating purposes, but only certain applications. Machines of this type are specialist marine electrical systems only, for example, specific electric propulsion and winch controls.



▲ Figure 12.25

(3c) The compound-connected generator

This generator uses series and shunt fields, its characteristic is treated as made up of shunt and series-machine characteristics. The final characteristic depends on the relative strengths of each field. It is noted that the shunt field is the basic requirement; it is the shunt generator's performance which is improved on.

TYPES OF ELECTRICAL CONNECTION. Figure 12.25 shows how a machine is connected in either 'short' or 'long' shunt. There is no appreciable difference in the resulting generated voltage as is seen from the example.

Example 12.4. A 110V, compound generator has armature, shunt and series-field resistances of 0.06Ω, 25Ω and 0.04Ω respectively. A load consists of 200 lamps each rated 55W, 110V. Find the generated e.m.f. and armature current if the generator is connected (a) long shunt and (b) short shunt.

(a) LONG SHUNT. Load current $I_L = \dfrac{200 \times 55}{110} = 100\text{A}$

Shunt-field current $I_{sh} = \dfrac{110}{25} = 4.4\text{A}$

Series-field current I_{se} and armature current $I_a = 100 + 4.4 = 104.4\text{A}$

Generated voltage $E = V + I_a(0.06 + 0.04) = 110 + 104.4 \times (0.1)$ or $E = 120.44\text{V}$.

There are 2 fields with different current values, symbol I_f is not used for field current, but instead symbols I_{sh} and I_{se} are used in both figure 12.25 and this' example.

(b) SHORT SHUNT. Load current I_L (as before) = 100A = I_{se} Voltage drop in the series field = $I_{se} R_{se}$

$= 100 \times 0.04 = 4V$

Voltage applied to the shunt field = terminal voltage + voltage drop in series field = $110 + 4 = 114V$

Shunt-field current $I_{sh} = \frac{114}{25} = 4.56A$

Armature current $I_a = 100 + 4.56 = 104.56A$

$E = V +$ voltage drop in series field + voltage drop in armature

$E = 110 + 4 + (104.56 \times 0.06) = 120.27V.$

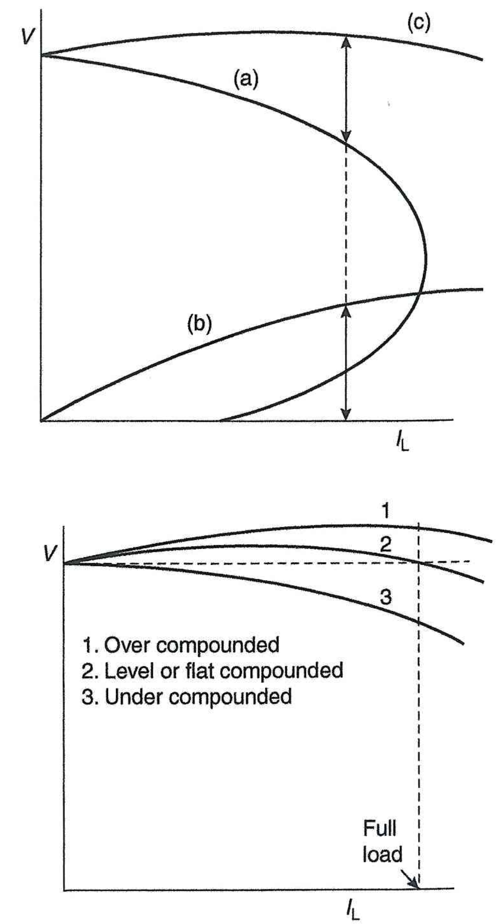
TYPES OF FIELD ARRANGEMENT. The series field is often connected so the flux produced adds to the shunt-field flux. For this common arrangement the machine is said to be *cumulatively* connected. All generators, used for supplying lighting and power for electrically driven auxiliary machinery aboard ship, have this connection. If the series field is connected to weaken the shunt field, the generator is *differentially* connected. This arrangement is used for specialist work such as certain types of welding generator.

THE LOAD CHARACTERISTIC. Figure 12.26a shows graphs due to directly loading a machine in the manner described. Curve (a) shows the characteristic if a shunt field only is used. Curve (b) is obtained with the series field only. Curve (c) results from use of both fields. Any point on this characteristic is obtained by adding voltages obtained from graphs (a) and (b), for any one value of load current.

Figure 12.26b shows how the load characteristics of a compound generator vary by altering the relative strength of the series field, so-called flat-compounding as required by most regulations. The curve is not quite flat and a rise in voltage between no load and full is called 'the hump'. It may be 6–7% for small generators, but in most cases is about 2–3%. Overcompounding compensates for the supply line voltage drop. This is shown by the example.

Example 12.5. A factory is sited some way from a generating-station and takes 100A at 200V. The supply cable resistance is 0.02Ω/core. Find the percentage compounding required for the generator.

Voltage drop in the line on full load $= 100 \times 2 \times 0.02 = 4V$



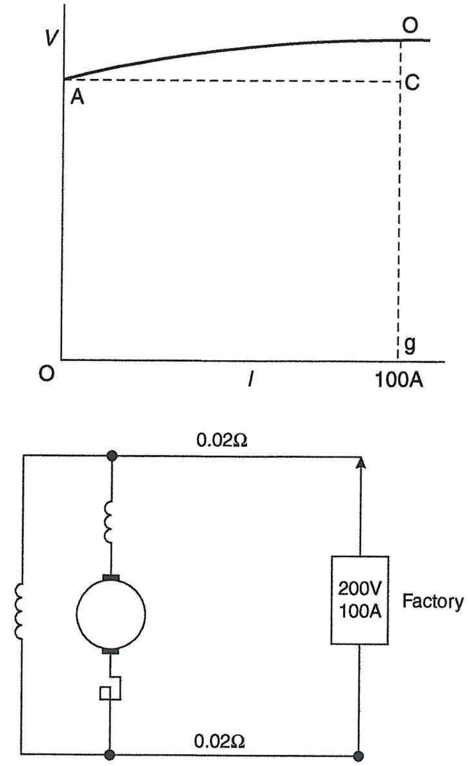
▲ Figure 12.26

To supply 200V to a factory, generated voltage must be 204V. A suitable overcompounded characteristic is shown (figure 12.27) and the compounding can be expressed as the rise on full load to the O.C. voltage.

Thus percentage compounding

$$= \frac{DC}{CB} = \frac{DC}{AO} = \frac{204 - 200}{200} = 4/200 = 0.02 \text{ or } = 0.02 \times 100 = 2\%$$

Thus a generator is required to be 2% overcompounded. Further work on the D.C. generator is covered in more advanced studies, but sufficient knowledge allows a study of the motor to be made. The following examples form a useful conclusion to this chapter.



▲ **Figure 12.27**

Example 12.6. A shunt generator is to be converted into a compound generator by addition of a series-field winding. From a test on the machine with shunt excitation a field current of 3A gives 440V on no load and 4A gives 440V at the full-load current of 200A. A shunt winding has 1600 turns/pole. Find the number of series turns required/pole.

Ampere-turns/pole required to give 440V on O.C.

$= 3 \times 1600 = 4800\text{At}$

Ampere-turns/pole required to give 200A at 440V on load

$= 4 \times 1600 = 6400\text{At}$

Full-load ampere-turns must be increased by $6400 - 4800 = 1600\text{At}$

But these 1600At/pole are obtained from the series field which passes 200A

Thus the required number of series turns/pole $= \dfrac{1600}{200} = 8.$

Example 12.7. A 4-pole, compound generator has a lap-wound armature and is connected in short shunt. The resistances of the armature and fields are 0.1Ω and 50Ω (shunt), 0.08Ω (series). The machine supplies a load consisting of sixty 100V, 40W lamps in parallel. Calculate the total armature current, the current/armature path and the generated e.m.f.

Since this is a lap-connected armature $A = P$

For 1 lamp, since $P = VI \therefore I = \dfrac{40}{100} = 0.4\text{A}$

The load current $I_L = 60 \times 0.4 = 24\text{A}$

Voltage drop in series field $= 24 \times 0.08 = 1.92\text{V}$

Voltage across shunt field $= 101.92\text{V}$

Shunt-field current $= \dfrac{101.92}{50} = 2.04\text{A}$

Armature current $= 24 + 2.04 = 26.04\text{A}$

Current per armature path $= \dfrac{26.04}{4} = 6.51\text{A}$

Generated voltage = terminal voltage + voltage drop in series field + voltage drop in armature

$= 100 + 1.92 + (26.04 \times 0.1) = 104.52\text{V}.$

Practice Examples

12.1. The armature of a 4-pole, shunt generator is lap wound and generates 216V when running at 600 rev/min. The armature has 144 slots with 6 conductors/slot. If the armature is rewound to be wave connected, find the e.m.f. generated at the same speed and flux/pole (3 significant figures).

12.2. A compound-wound, long-shunt D.C. generator has an output of 250A at 220V. The equivalent resistances of the armature, series and shunt windings are 0.025, 0.015 and 176Ω respectively. There is a 2V voltage drop across the brushes. Find the induced voltage (2 decimal places).

12.3. The curve of induced e.m.f. against excitation current for a separately excited generator when run on no load at 1200 rev/min is given by:

E.m.f. (V)	15	88	146	196	226	244	254
Excitation current (A)	0	0.4	0.8	1.2	1.6	2.0	2.4

Deduce the voltage to which a machine will self-excite if the shunt-field resistance is set at 90Ω and the machine run at 900 rev/min (2 decimal places).

12.4. A 220V, 4-pole, wave-wound, shunt generator has an armature resistance of 0.1Ω and a field resistance of 50Ω. Calculate the flux/pole, if the machine has 700 armature conductors, runs at 800 rev/min and is supplying a 38kW load (4 decimal places).

12.5. In a 250kW, 440/480V, overcompounded generator, the flux/pole required to generate 440V on no load is 0.055Wb at 620 rev/min. The resistances of the armature, interpoles and series field are 0.01, 0.005 and 0.005Ω respectively. Find the flux/pole required at full load, the speed now being 600 rev/min. Neglect the current taken by the shunt field (4 decimal places).

12.6. Estimate the series turns/pole needed for a 50kW compound generator to develop 500V on no load and 550V on full load. Assume a long-shunt connection and that the ampere-turns required per pole on no load are 7900 whereas the ampere-turns required per pole on full load are 11 200 (1 significant figure).

12.7. A 4-pole machine has a lap-wound armature with 90 slots each containing 6 conductors. If the machine runs at 1500 rev/min and the flux/pole is 0.03Wb, calculate from first principles the e.m.f. generated. If the machine is run as a shunt generator with the same field flux, the armature and field resistances being 1.0Ω and 200Ω respectively. Calculate output current when the armature current is 25A. Through a fall in speed the e.m.f. becomes 380V. Calculate load current in a 40Ω load (2 decimal places).

12.8. A D.C. generator gave the following O.C.C. when driven at 1000 rev/min.

Field current (A)	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6
Armature voltage (V)	32	58	78	93	104	113	120	125

If the machine is run as a shunt generator at 1000 rev/min, the shunt-field resistance being 100Ω, find (a) the O.C. voltage, (b) the critical value of the

shunt-field resistance and (c) the O.C. voltage if the speed was raised to 1100 rev/min, the field resistance being kept constant at 100Ω (all 3 significant figures).

12.9. Calculate the input power to drive a shunt generator having an output of 50kW at 230V, if under these conditions the bearing, friction, windage and core loss is 1.6kW and the total voltage drop at the brushes is 2V. The resistance of the armature is 0.034Ω and that of the field circuit 55Ω (4 significant figures).

12.10. A D.C. generator when separately excited and run at 200 rev/min gave the following test results:

Field current (A)	0	1	2	3	4	5	6	7	8	9
O.C. voltage (V)	10	38	61	78	93	106	115	123	130	135

The field is then shunt-connected and then run at 400 rev/min. Determine (a) the e.m.f. to which a machine will excite when field-circuit resistance is 36Ω (3 significant figures), (b) the critical field-circuit resistance value (2 significant figures), (c) the extra resistance needed in a shunt-field circuit to reduce e.m.f. to 220V (1 decimal place) and (d) the critical speed when field-circuit resistance is 36Ω (3 significant figures).

13

THE D.C. MOTOR

Every discovery opens a new field for investigation of facts, shows us the imperfection of our theories. It has justly been said, that the greater the circle of light, the greater the boundary of darkness by which it is surrounded.

Sir Humphry Davy

A D.C. machine will run as a motor if its field and armature are connected to a suitable supply. The 'motoring' action is based on the fundamental law described in Chapter 5, which stated that a force acts on a conductor if it lies in a magnetic field and carries current. Figure 13.1 shows one arrangement.

Direction of force

The 4 small diagrams (figure 13.2) show that to reverse the force direction and thus the direction the armature will rotate in, the conductor current must be reversed with respect to the magnetic flux.

Practically it must be remembered that if a motor runs in the 'wrong' direction when first connected, rotation reversal is obtained by reversing the supply leads to the armature circuit. A hand rule exists to help memorise motor action and is comparable with that shown in Chapter 12 for the generator.

LEFT-HAND RULE (Fleming's). Figure 13.3 shows this rule. The first and second fingers represent flux and current respectively, as for the right-hand rule. The direction of force on the conductor is represented by the thumb. *Note.* As for the right-hand rule, thumb, index finger and second finger *must* be placed at right angles to each other.

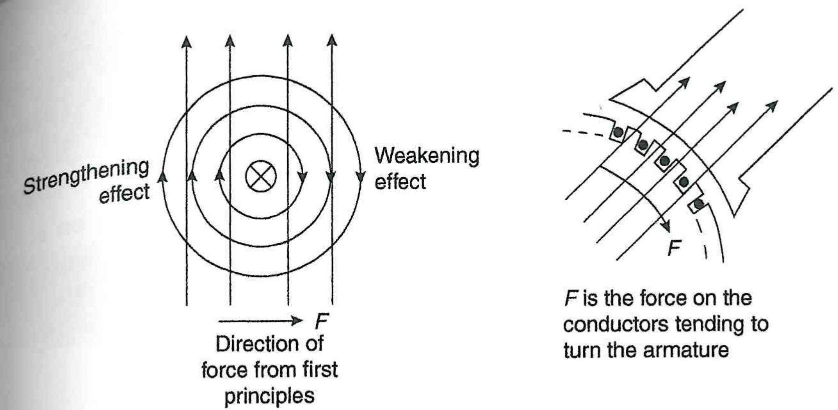


Figure 13.1

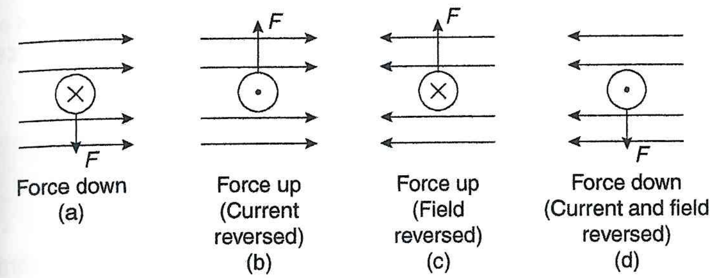


Figure 13.2

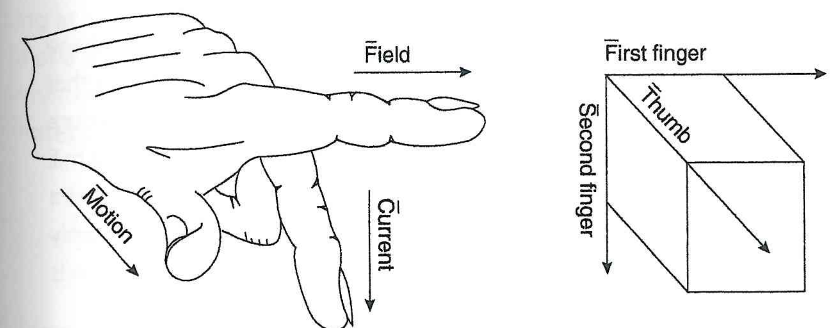


Figure 13.3

Magnitude of force

From the principles set out in Chapter 5, it was shown that the force acting on a conductor in a magnetic field is proportional to flux density, current and the active conductor length in the field. The law is summarised by the formula: $F = BI\ell$ newtons but the force *magnitude* also depends on the angle of the conductor to the field direction, which is *maximum* when they are at right angles.

Example 13.1. Calculate the force in newtons, on a conductor, 0.5m long, carrying a current of 500A at right angles to a magnetic field of uniform density 0.8T (3 significant figures).

$$\text{Since } F = BI\ell$$
$$\text{Then } F = 0.8 \times 500 \times 0.5 = 200 \text{ newtons.}$$

If the conductor is situated on an armature at a radius of *r* metres, the torque produced on the shaft is expressed as $F \times r$ newton metres.

Back e.m.f. of a motor

If a motor rotates due to the torque produced by the armature conductors, then these same conductors will cut the magnetic field. From Faraday's law an e.m.f. is induced, the magnitude of which is given by the generator expression developed in Chapter 12, namely:

$$E = \frac{Z\Phi N}{60} \times \frac{P}{A} \text{volts}$$

From first principles the induced e.m.f. direction is such as to *oppose* the applied voltage and a condition of balance results. Since, the rotation direction will be opposite to that for a generator operated under the same directions of flux and current in the armature conductors, the induced e.m.f. *opposes* current flow and is termed a 'back e.m.f.' This e.m.f. must be less than the terminal voltage *V* to permit motoring. Thus the armature starts as a passive load, but as it rotates, it accelerates until a balance condition is reached when supply voltage equals the armature voltage drop plus the generated back e.m.f. This condition is expressed by the voltage equation and the motor armature operates as an active load.

Voltage equation

$V = E_b + I_a R_a$. This equation explains the voltage conditions of

armature voltage drop caused by the armature current I_a passing through the armature resistance R_a. If a problem is encountered where a brush voltage drop is given this must be added. The equation is comparable with the generator terminal voltage equation $V = E - I_a R_a$. A thought about the difference in the equations summarises the basics of generator and motor action.

Current equation

$$\text{Since } V = E_b + I_a R_a \text{ then } I_a R_a = V - E_b$$

$$\text{and } I_a = \frac{V - E_b}{R_a}$$

This equation shows how motor current depends on the back e.m.f. generated. Starting conditions are illustrated.

$$\text{At start } E_b = 0 \therefore I_{as} = \frac{V}{R_a}$$

Usually R_a is small to minimise the armature-resistance voltage drop for working conditions, so I_a will be very large. For example, a 220V motor with a 0.4Ω armature resistance may take a full-load current of 52A, but if started without taking special precautions, the starting current I_{as} will be $I_{as} = \frac{220}{0.4} = 550\text{A}$. This large starting current may give rise to undesirable starting conditions. It could easily 'blow' a fuse, or accelerate too rapidly, resulting in mechanical or electrical damage through excess sparking at the commutator. It is for this reason that, the starting current I_{as} is limited by use of a 'starter'. The basic feature of a starter is a variable resistance inserted into the armature circuit at starting which is gradually reduced or cut out as the motor accelerates up to normal working speed. At the 'instant of starting':

$$I_{as} = \frac{V}{R_{as} + R_s} \text{ where } R_s \text{ is the full value of thestarting resistance.}$$

Speed equation

The equation is vital for understanding motor action. It is obtained by rearranging the

Since $V = E_b + I_a R_a$ Then $E_b = V - I_a R_a$.

But $E_b = \frac{Z\Phi N}{60} \times \frac{P}{A}$, E_b is the generated e.m.f. and magnitude determined from the generator formula.

Hence $\frac{Z\Phi N}{60} \times \frac{P}{A} = V - I_a R_a$

or $N = \frac{(V - I_a R_a)}{Z\Phi} \times \frac{60A}{P}$ rev/min

Example 13.2. Calculate the full-load speed of a motor operating from a 440V supply, given $R_a = 0.75\Omega$, full-load armature current = 55A, the flux/pole = 0.02Wb and it is a 4-pole machine with a simple wave-wound armature of 43 slots and 12 conductors per slot (4 significant figures).

Number of armature conductors $Z = 43 \times 12 = 516$ For a wave-wound armature $A = 2$.

$P = 4$ and $\Phi = 0.02Wb$ Then $N = \frac{440 - (55 \times 0.75)}{516 \times 0.02} \times \frac{60 \times 2}{4} = 1160$ rev/min

Speed controlling factors

The deductions below are derived from the speed equation and are of vital importance. Students should ensure they understand the implication of each deduction in detail.

Since $N = \frac{(V - I_a R_a)}{Z\Phi} \times \frac{60A}{P}$, it is clear that for a particular machine only certain variables affect the expression. Thus 60, A, Z and P are all constants and written as k.

Then we have $N = \frac{k(V - I_a R_a)}{\Phi}$ or $N = k\frac{E_b}{\Phi}$ since $E_b = V - I_a R_a$.

If the expression $N = \frac{k(V - I_a R_a)}{\Phi}$ is considered, for the purposes of approximation, the small voltage drop $I_a R_a$ can be neglected and we have $N = \frac{kV}{\Phi}$ or $N \propto \frac{V}{\Phi}$ (approx.).

Thus speed can be controlled by varying V or Φ

Variation of V gives *direct* speed control, while varying Φ gives *inverse* speed control. It should be remembered that the real relation is $N \propto \frac{E_b}{\Phi}$ but under working conditions E_b is not very different from that of V, the $I_a R_a$ voltage drop being small. The practical application of the deduction leads to the basic systems of motor speed control in that:

Variation of voltage across the armature terminals produces a direct variation of speed, i.e. raising the armature voltage increases speed and *vice versa*.

In contrast:

Variation of field flux produces an inverse variation of speed, i.e. lowering flux increases speed and *vice versa*. Thus the relationship $N \propto \frac{V}{\Phi}$ (approx.) determines the motor speed characteristic shape.

Example 13.3. The armature resistance of a 200V shunt motor is 0.4Ω. The no-load and armature current = 2A (a term for when a motor runs unloaded). When loaded and taking an armature current = 50A, motor speed = 1200 rev/min. Find the approximate no-load speed (4 significant figures).

On no-load. Back e.m.f. $E_{b0} = V - I_{a0} R_a$

$= 200 - (2 \times 0.4) = 199.2V$

On load. Back e.m.f. $= E_{b1} = V - I_{a1} R_a$

$= 200 - (50 \times 0.4) = 180V$

Also since $N = k\frac{E_b}{\Phi}$ Then $N_0 = \frac{kE_{b0}}{\Phi_0}$ and $N_1 = \frac{kE_{b1}}{\Phi_1}$

Since this is a shunt motor, the field is unaffected by loading the armature so $\Phi_1 = \Phi_0$.

$\therefore \frac{N_0}{N_1} = \frac{kE_{b0}}{\Phi_0} \Big/ \frac{kE_{b1}}{\Phi_1} = \frac{E_{b0}}{E_{b1}}$ since k, Φ_1 and Φ_0 cancel

$N_0 = N_1 \times \frac{E_{b0}}{E_{b1}} = 1200 \times \frac{199.2}{180}$ or $N_0 = 1328$ rev/min

Types of D.C. Motor

As for the generator, motor-field windings can be connected in shunt, series or a combination to give a compound arrangement. The motor is a machine which takes current from the supply and the fields are considered a load *added* to the armature circuit.

The shunt motor

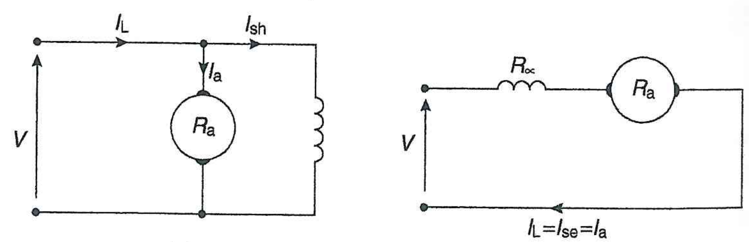
The arrangement is shown (figure 13.4). It is seen that $I_L = I_a + I_{sh}$. A supply voltage V is applied to both the armature and the field circuits but where the 'equivalent resistance' treatment for a parallel circuit *cannot* be supplied to find I_L because, although R_{sh} is a passive load, the armature is an active load when the machine runs. The shunt motor is essentially a constant-speed machine used for most applications.

$$\text{Here } I_{sh} = \frac{V}{R_{sh}} \quad \text{and} \quad I_a = I_L - I_{sh}$$

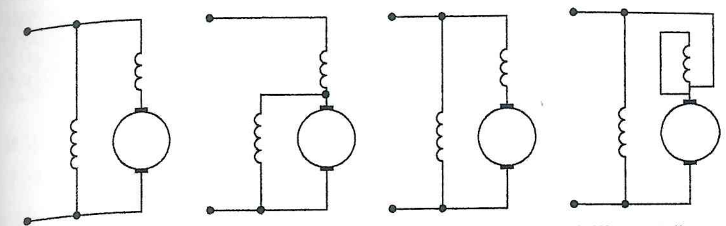
The series motor

The series motor arrangement is as shown (figure 13.4). Here $I_L = I_{se} = I_a$. The voltage equation is modified slightly as, if V is taken as the supply voltage, allowance is made for the voltage drop in the series field. The equation must be written as:

$$V = I_{se}R_{se} + I_aR_a + E_b = E_b + I_aR_a + I_aR_{se} \quad \text{or} \quad V = E_b + I_a(R_a + R_{se}).$$



▲ Figure 13.4



▲ Figure 13.5

The voltage applied to the armature is equal to *V minus* the voltage drop in the series field and a voltage drop at the brushes (if present). As machine current rises with increasing load, the voltage across the armature falls and speed changes accordingly. The ohmic value R_{se} is kept as small as possible for this machine type. This is a variable speed motor used mainly for traction, hoist, crane and winch work.

The compound motor

As shown by the diagram (figure 13.5), the motor, like the generator, can be connected as a long-shunt or short-shunt machine. The 2 fields can be connected to assist or oppose each other. If the resultant flux is strengthened by the arrangement, fields are said to be 'cumulatively' connected. If fields are however connected to *weaken* each other, the motor is 'differentially' connected, which is rarely used.

Most marine motors are cumulatively compounded machines. The relative strengths of the shunt and series fields are chosen by the performance required and is considered when characteristics are studied in detail.

Equations

The power equation

This equation shows the conversion from electrical to mechanical power and the causes of electrical loss. It is used for deducing the torque equation, and gives the student little difficulty provided he properly understands the voltage equation.

Since $V = E_b + I_a R_a$ and the armature is the cause by which electrical energy supplied is converted into mechanical energy, the following deduction is possible. Multiply the expression by I_a and study the result.

$$\text{Thus } V = E_b + I_a R_a \text{ becomes } VI_a = E_b I_a + I_a^2 R_a$$

VI_a is a measure of the power input to the armature circuit $I_a^2 R_a$ indicates a resistance loss and is the power lost, being converted into heat in the armature. It is known as a *copper loss* and is due to armature resistance. $E_b I_a$ is a measure of the armature power developed, as seen if the expression is rearranged:

$$\underset{\text{Input power}}{VI_a} - \underset{\text{Copper loss}}{I_a^2 R_a} = \underset{\text{Output power}}{E_b I_a}$$

Note. Output power $E_b I_a$ is in watts and is the mechanical power developed by the armature conductors and is *not* a true measure of the shaft output until the machine's mechanical losses, for example, those due to friction and windage, have been subtracted. When data concerning mechanical losses is not given, an estimate of shaft output power can still be obtained.

Example 13.4. A 4-pole motor has a wave-wound armature of 594 conductors. Armature current = 30A and the flux per pole is = 0.009Wb. Calculate the total power developed when running at 1400 rev/min. Estimate the shaft output power if mechanical losses absorb 10% of developed power.

For this machine $P = 4$, $A = 2$, $Z = 594$ and $\Phi = 0.009\text{Wb}$

$$E_b = \frac{Z\Phi N}{60} \times \frac{P}{A} = \frac{594 \times 0.009 \times 1400}{60} \times \frac{4}{2} = 249.48\text{V}$$

The power developed = $E_b I_a = 249.48 \times 30 = 7484.4\text{W}$

= 7.5kW (approx.)

As mechanical power loss = 10% of 7.5kW

= 0.75kW then shaft output power = 7.5 – 0.75 = 6.75kW.

The torque equation

This is an important expression, often developed from first principles for examination purposes. The method here involves the power and voltage equations and is considered to be the simplest. As the armature's electrical power output = $E_b I_a$ watts and the mechanical power developed is given by:

$$\frac{2\pi \times \text{speed (rev/min)} \times \text{torque (newton metres)}}{60}$$

We can write: $E_b I_a = \dfrac{2\pi NT}{60}$

$$\text{or } T = \frac{60}{2\pi N} \times E_b I_a = \frac{60}{2 \times 3.14} \times \frac{E_b I_a}{N}$$

Substituting for E_b in terms of machine data, we have:

$$T = \frac{60}{2 \times 3.14} \times \frac{Z\Phi N}{60} \times \frac{P}{A} \times I_a$$

$$\frac{60}{2 \times 3.14 \times 60} \times Z\Phi I_a \frac{P}{A} = 0.159 Z\Phi I_a \frac{P}{A} \text{ or } T = 0.159 Z\Phi I_a \frac{P}{A}\text{Nm.}$$

Torque controlling factors

For the torque equation, the factors which affect torque are determined. Thus for any machine 0.159, Z, P and A are all constants and can be written together as k, giving the expression $T = k\Phi I_a$ or $T \propto \Phi I_a$. The torque developed varies directly with either flux and/or the armature current and this fact is used for problems and determining machine characteristics. It is stressed that, for a shunt motor for different loading conditions Φ is basically constant with: $T \propto I_a$. For a series motor however, Φ is *not* constant and is often taken as proportional to I_a. Therefore, if $\Phi \propto I_a$ and $T \propto \Phi I_a$, we can write for a series motor $T \propto \Phi I_a^2$. This deduction is used in Example 13.5.

Example 13.5. A series motor running at a speed = 600 rev/min develops 3kW and takes a current of 40A. If the starting current is limited by means of a starter to 60A, find the starting torque. Neglect armature reaction effects and assume the magnetic circuit is unsaturated (1 decimal place).

As the magnetic circuit is unsaturated, it is assumed that $\Phi \propto I_{se} \propto I_a$, so $T \propto \Phi I_a$ or $T = kI_a^2$. There are 2 possible torque conditions.

$$\text{Thus: when running } T_1 = kI_{a1}^2.$$

$$\text{At starting } T_2 = kI_{as}^2 \text{ or } kI_{a2}^2.$$

When running at 600 rev/min, output = 3kW and T_1 is given by:

$$3000 = \frac{2\pi N T_1}{60} \quad \text{or} \quad T_1 = \frac{3000 \times 60}{2 \times \pi \times 600} = 47.8\text{Nm}.$$

$$\text{Also } \frac{T_2}{T_1} = \frac{kI_{a2}^2}{kI_{a1}^2} \quad \text{or} \quad T_2 = T_1\left(\frac{I_{a2}}{I_{a1}}\right)^2 = 47.8\left(\frac{60}{40}\right)^2$$

So the starting torque T_2 = 107.6Nm.

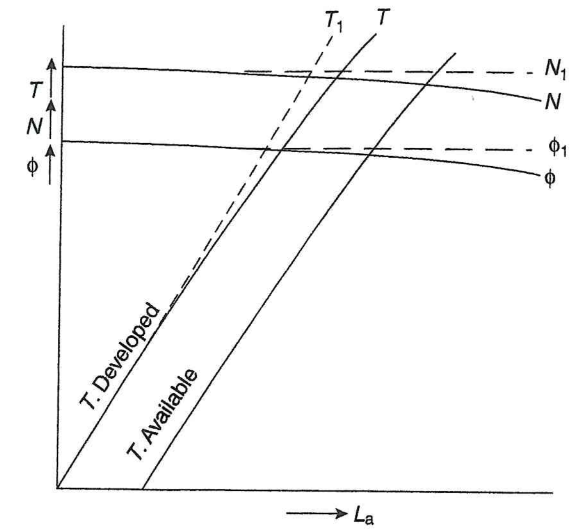
Motor Characteristics

The behaviour of shunt, series and compound motors is illustrated by their characteristics which are considered under (1) electrical load characteristics and (2) mechanical characteristic. Electrical characteristics show speed and torque in terms of armature current while the mechanical characteristic shows speed related to torque, assuming a constant applied terminal voltage. Electrical characteristics are important as they show machine performance when loaded. The mechanical characteristic shows the motor's suitability for a particular application. Characteristics are checked by making a load test on a motor, but its theoretical performance is reasoned from the 2 expressions: $N \propto \frac{V}{\Phi}$ (approx.) and $T \propto \Phi I_a$.

The shunt motor

Electrical characteristics

SPEED. If flux Φ is constant, assuming a constant applied voltage V, N is considered constant over the load range, as $N \propto V$ and V is constant. Speed is unaffected by



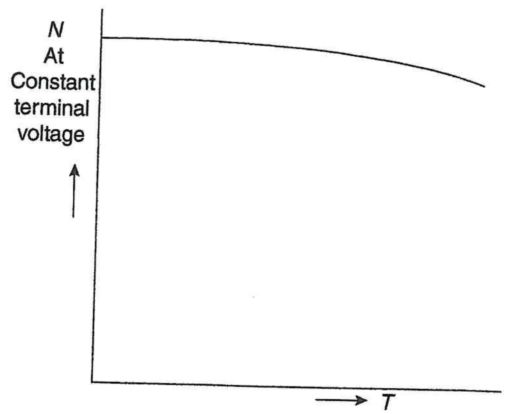
▲ Figure 13.6

I_a and the theoretical graph shown dotted in figure 13.6, as N_1 against I_a. This motor is considered to be a constant-speed machine although, in practice speed falls slightly with load, as shown. This is because the back e.m.f. reduces slightly (the speed drop from no load to full load is about 2% for large machines and 6% for small machines), as the armature voltage drop I_aR_a increases. Although field current I_{sh} and flux Φ are constant, the armature reaction effect causes the overall resulting flux Φ to drop slightly. As $N \propto \frac{E_b}{\Phi}$ it should be constant if both E_b and Φ variations are proportional. Weakening flux however means a rise in armature current due to the corresponding drop in E_b. The I_aR_a drop increases as a result, and so the speed lowering effect of a reduced E_b is greater than the speed raising effect of a falling Φ! The net result is speed falling slightly over the load range of I_a.

TORQUE. T varies as I_a giving a straight line through the origin, since Φ is assumed constant. In practice Φ is weakened by the armature reaction and T drops as a result, departing from a theoretical straight line T_1. Torque available at the shaft is everywhere lower due to rotational losses. Thus 2 torque characteristics are shown in figure 13.6.

Mechanical characteristic

As illustrated (figure 13.7), this is obtained by plotting N against T and is seen to drop slightly. Shunt motors are considered to be constant-speed machines and have about a 4% drop in speed from no load to full load. They are used for all constant-speed drives for example, machine tools, centrifugal pumps, purifiers, etc.



▲ Figure 13.7

The series motor

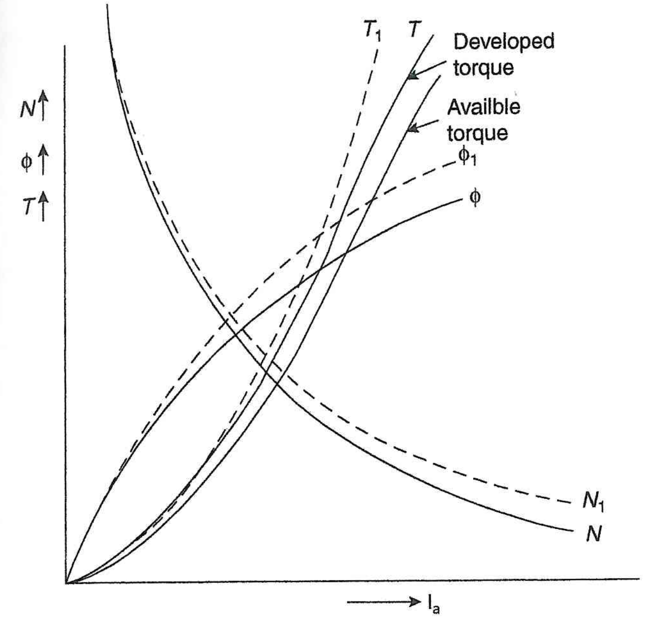
Electrical characteristics

SPEED. For this machine, the load current value is also that of the field current and, allowing for the armature reaction effects, it is seen (figure 13.8) that the useful flux Φ is slightly less than that given by the magnetisation curve Φ_1. As Φ increases with load and N varies as $\frac{1}{\Phi}$, the speed must drop and the curve conforms to an inverse variation, flattening out as saturation of Φ occurs. The no-load flux is small but speed can be excessive. For this reason a series motor should never run 'light' as it is liable to 'race' and may be damaged by a virtual centrifugal force. Like a shunt motor, N is lower than N_1 for the reasons described.

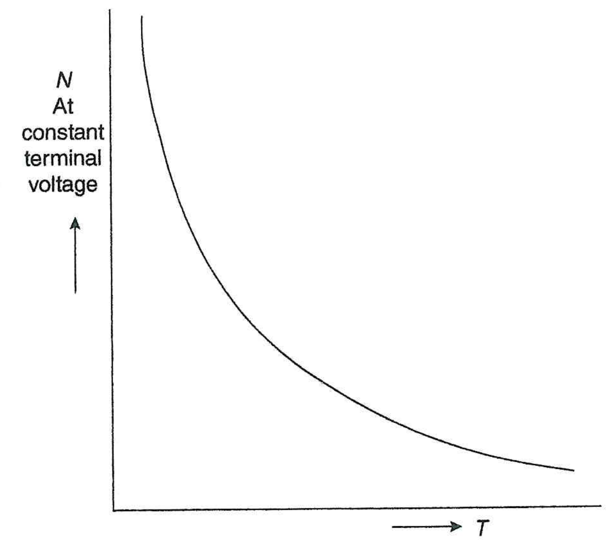
TORQUE. Field saturation is not usually achieved over the working load range and Φ is assumed proportional to I_a. As torque is proportional to $\Phi \times I_a$ we have from earlier that: $T_1 \propto I_a^2$ so the curve follows a parabola. On heavy loads, as Φ starts to saturate, $T \propto I_a$ so the graph tends to a straight line passing through the origin for low values of I_a, as shown (figure 13.8). As for the shunt motor, due to machine losses, torque available at the shaft is less than the developed torque. At start $T \propto I_a^2$ and so the starting torque is very high, being one advantage of this type of motor.

Mechanical characteristic

This is shown (figure 13.9) and given by plotting the N and T values, for the same



▲ Figure 13.8



▲ Figure 13.9

in shape to the speed-current curve (figure 13.8). Series motors are variable speed machines, giving low speed on heavy loads which are ideal for traction, winch, hoist and fan work. Their excellent starting torque is used to advantage when heavy masses

The compound motor

Field connections give either cumulative or differential flux result. The former is usual and the latter used only for exceptional motor duties. The shunt and series motor have such good characteristics, that compounding is only used as a means to minimise disadvantages which occur in simple machines. For example, the series motor tends to race on no load, which can be limited by providing a stabilising shunt field, and so the compounding for a machine may be arranged to give either a *strong shunt, weak series* effect, or a *strong series, weak shunt* field combination. The characteristics are considered for these arrangements.

CUMULATIVE CONNECTION OF FIELDS. Here the 2 fields assist each other to give a resultant strengthening flux. Machine characteristics depend on the relative field strengths.

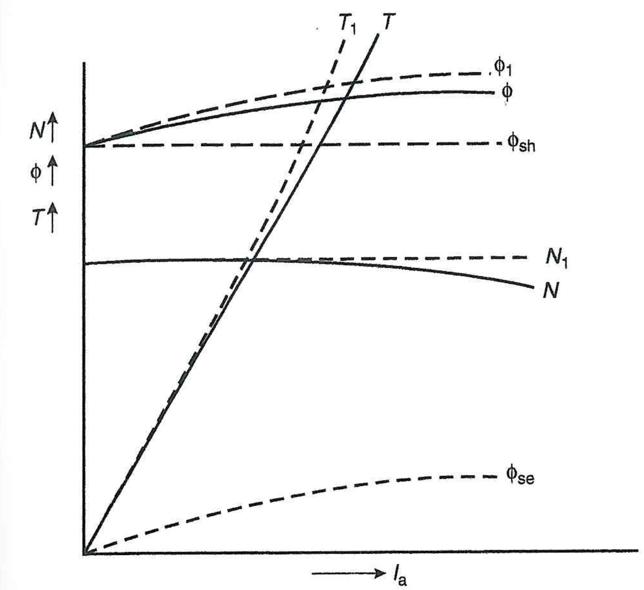
(1) *Strong shunt – weak series*. The characteristics of the shunt motor are so good, that in practice it is suitable for most motor drive duties. The provision of a weak series field doesn't materially alter the load characteristics, but the field gives an improved starting torque – as explained below.

Electrical characteristics

SPEED. Figure 13.10 shows the characteristic. As flux rises due to the series field, this will have a speed-lowering effect since $N \propto \frac{V}{\Phi}$ (approx.).

Speed tends to 'sit down' more than it would for the same machine *without* a series field. If the series field is weak, its effect is unobserved in the speed characteristic, differing little from the shunt motor. However, when a machine is coupled to a flywheel, a stronger series field can be used, so sudden load application momentarily slows down with a rise of I_a. Motor speed tends to 'sit down' and the required driving power is obtained from the flywheel. This arrangement enables a motor and the electrical system to be protected from undue shock and is used for motors driving specialised loads, such as the rolls in steelworks, as well as presses and hammers.

TORQUE. During starting, when voltage is applied to a shunt field, due to its self-inductance (a winding of thin wire and many turns), a back e.m.f. is induced which opposes the shunt field current. The shunt field current builds up slowly and the torque ($T \propto \Phi I_a$) is small in spite of a large armature current. A series field which passes the starting current I_{as} will produce a flux to strengthen the shunt flux, the net flux at starting



▲ Figure 13.10

used for starting with heavy loads, for example, compressors, centrifugal pumps, certain machine tools. Once the machine accelerates, the characteristic follows that of a shunt motor, and the armature reaction effect alters the theoretical characteristic from T_1 to T as shown (figure 13.10).

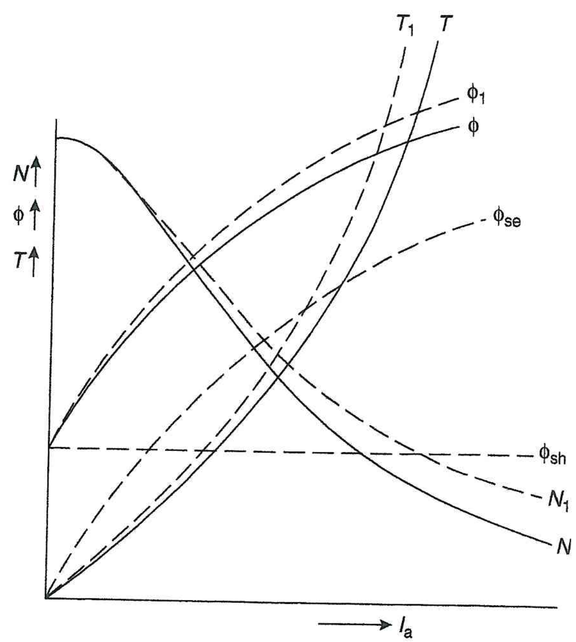
Mechanical characteristic

This characteristic is generally similar to that of a shunt motor.

(2) *Strong series – weak shunt*. Again the characteristic of the series motor is suitable for appropriate applications that its basic performance features are retained. Its main disadvantage, for example, a tendency to race on light load, must be removed and this is the purpose of the shunt field.

Electrical characteristics,

SPEED. It is seen from the diagram (figure 13.11) that although net flux varies, it follows the magnetisation curve, i.e. it never falls to zero as is the case for the series motor. In effect the shunt field dominates on light loads and the machine runs as a shunt motor at a fixed speed. Once load is applied, the series field asserts itself and the speed characteristic passes from that of a shunt machine to that of a series machine. The tendency for racing on no load is removed and this is the typical characteristic of a



▲ **Figure 13.11**

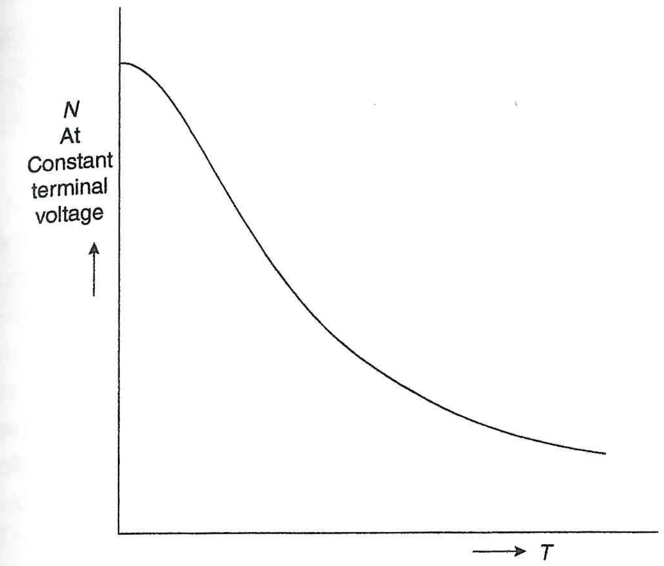
ship's D.C. winch. *Note*. The effect of armature reaction and voltage drop on E_b is seen at higher current values and speed N is lower than the theoretical value N_1.

TORQUE. Since a motor behaves like a shunt machine on light loads, the torque characteristic commences as a straight line through the origin but becomes parabolic as the series field increases. The armature reaction effect gives a slight reduction of net flux with a consequent falling off of torque T from the theoretical graph T_1 (shown dotted) figure 13.11.

Mechanical characteristic

The characteristic for this motor is shown (figure 13.12). It is similar to the electrical speed characteristic and can be deduced, as described for the series and shunt machines. The exact graph shape and position depends on the shunt and series fields' relative strengths.

DIFFERENTIAL CONNECTION OF FIELDS. This can help maintain a constant speed, for example, for an alternator drive. Increase of load results in an increase in the series flux and, as fields are opposed, resultant flux *decreases*. Speed, being inversely proportional to flux, increases to compensate for the fall due to application of a load.



▲ **Figure 13.12**

Machine speed and alternator frequency tend to remain constant but armature current increases appreciably to provide the required torque with reduced flux. A failing of the arrangement is the field cancelling effect at starting and that, due to the series field establishing itself quicker, a machine may start to run in reverse. Special arrangements must be made when starting a motor with a differentially connected field system.

Example 13.6. A 220V shunt motor runs on light load at a speed of 1250 rev/min and takes a current of 2.8A. On full load the current taken from the mains is 40A. Owing to armature reaction, the flux per pole is 4% less than the no-load value. Calculate the speed on full load if armature resistance = 0.29Ω and field resistance = 165Ω (4 significant figures.)

No load. Voltage across shunt field = 220V

Current through shunt field $= \dfrac{220}{165} = 1.332\text{A}$

Armature current = 2.8 − 1.33 = 1.47A

Voltage drop across armature $= I_{a0}R_a = 1.47 \times 0.29 = 0.426\text{V}$

and $E_{b0} = 220 - 0.426 = 219.574\text{V}$

Full load. Current through shunt field as before = 1.332A

Armature current = 40 − 1.332 = 38.67A Voltage drop across armature = $I_{a1}R_a$ = 38.67 × 0.29 = 11.23V and E_{b1} = 220 − 11.23 = 208.77V

Now since $E_b \propto \Phi N$ $\therefore E_b = k\Phi N$ and $\frac{E_{b0}}{E_{b1}} = \frac{\Phi_0 N_0}{\Phi_1 N_1}$ But $\Phi_1 = 0.96\Phi_0$

$$\therefore \frac{E_{b0}}{E_{b1}} = \frac{\Phi_0 N_0}{0.96\Phi_1 N_1} \text{ or } N_1 \frac{N_0 \times E_{b1}}{0.96 \times E_{b0}} \frac{1250 \times 208.77}{0.96 \times 219.57}$$

Thus N_1 = 1238 i.e. speed on full load = 1238 rev/min.

Example 13.7. A 220V series motor works with an unsaturated field taking a current of 100A and run at 800 rev/min. Calculate the speed the motor will run at when developing half the torque. Motor total resistance = 0.1Ω (4 significant figures).

$$\text{Here } T = kI_a^2 \quad \therefore \frac{T_1}{T_2} = \frac{I_{a1}^2}{I_{a2}^2} \text{ But } T_2 = 0.5T_1$$

$$\text{So } \frac{T_1}{0.5T_1} = \frac{100^2}{I_{a2}^2} \quad \text{or} \quad I_{a2}^2 = 100^2 \times 0.5 = 5000$$

$$\therefore I_{a2} = \sqrt{5000} = 70.7\text{A}$$

Also under the first condition $E_{b1} = V - I_{a1}(R_a + R_{se})$

$$= 220 - (100 \times 0.1) = 210\text{V}$$

Under the second condition $E_{b2} = V - I_{a2}(R_a + R_{se})$

$$= 220 - (70.7 \times 0.1) = 212.93\text{V}$$

$$\text{But } E_b = k\Phi N \text{ or } \frac{E_{b1}}{E_{b2}} = \frac{\Phi_1 N_1}{\Phi_2 N_2} \quad \text{also } \Phi \propto I_a$$

$$\text{so } \frac{E_{b1}}{E_{b2}} = \frac{I_{a1}N_1}{I_{a2}N_2} \text{ or } N_2 = \frac{I_{a1}N_1E_{b2}}{I_{a2}E_{b1}} = \frac{100 \times 800 \times 212.93}{70.7 \times 210}$$

so N_2 = 1147 rev/min

Motor Starters

The need for a starter to work with a motor was mentioned earlier, as at the instant of starting as the machine is *not* rotating, there is no back e.m.f. The current is thus limited by armature resistance alone, unless an arrangement is made to add further/extra resistance in the armature circuit. Thus for all but 'fractional output power' motors, with appreciable resistance, a resistor is inserted into the armature circuit and then removed in steps, as the motor accelerates up to its correct running speed. The arrangement is incorporated in a unit, called a 'motor starter' or more simply a 'starter' and consists of a tapped resistor and a switching device which enables resistance to be gradually reduced and finally eliminated altogether. A starter may incorporate other attachments as needed for safe motor operation which may include protective safeguards against the effects of a reduced working voltage or an overcurrent.

Although motor starters are studied elsewhere, it is appropriate to mention that the form of starter needed for a particular machine is mostly decided by the duty for which the motor is used. It may be a manually operated or automatic type; it may be designed for starting and stopping a motor and may require to be done only once a day. In contrast, the duty may require the motor to be started and stopped continually for long periods, with a winch or hoist. Such a starter is often referred to as a 'controller'. These observations show that a starter is an important item requiring attention, one which needs careful and routine maintenance and a thorough knowledge of its function from a theoretical and practical viewpoint.

Speed Control

As for the starter, so for the full treatment of speed control, further study has yet to be made. The basic methods whereby the speed of a D.C. motor can be controlled are discussed here, the reader is reminded of the basic deduction $N \propto \frac{E_b}{\Phi}$ or $N \propto \frac{V}{\Phi}$ (approx.). Varying the voltage applied to a motor armature and keeping flux constant varies the speed in direct proportion, and is termed 'voltage control'. Varying a machine's flux and keeping voltage constant will vary the speed in inverse proportions and is termed 'field control'.

FIELD CONTROL. This is the most common control type. When a motor is loaded, its i.e. keep it constant over the working range or to raise it above normal running speed. Field control is used because its addition into the field circuit is easily achieved, control is smooth and little energy wasted as heat.

It must be remembered that this type of control gives speed variation in an *upward* direction only. It is used for raising speed above normal and as flux is weakened, for the same driving torque, armature current rises. *Note.* $T \propto \Phi I_a$. Thus a motor may be of larger dimensions if speed variation is required and interpoles fitted to ensure good commutation across the working range.

VOLTAGE CONTROL. This is achieved in various ways for different kinds of D.C. motor but the key requirement is to reduce the voltage applied to the machine armature. Thus a large variable rheostat is connected in series with the armature or the latter supplied from a variable voltage supply. The method is used to lower speed and control is in a *downward* direction only.

A wide range in motor speed is obtained by combining field and voltage control and the methods of applying these are important enough to require further study. To meet the requirements of the duty for which a motor is required, the starter and speed controller may be incorporated into one unit. Since correct application and use of a motor is of vital importance to practical engineers, it is hoped the treatment given to the D.C. machine in Volume 7, will be seen as a valued addition.

Example 13.8. The armature of a motor has 660 conductors whose effective length is 410mm; of these, only 70% are simultaneously in the magnetic field. Flux density = 0.65T, the effective diameter of the armature = 300mm and each conductor carries a current = 80A. If armature speed = 800 rev/min calculate the output power developed (3 significant figures).

Force on one conductor is given by $F = BI\ell$ newtons

$\therefore F = 0.65 \times 80 \times 410 \times 10^{-3} \quad F = 21.32\text{N}$

Number of conductors in the field at any given instant = 0.7×660

$\therefore$ Total force = $21.32 \times 0.7 \times 660 = 9.85\text{kN}$

Torque = force × radius or $T = 9850 \times \frac{0.3}{2}$ newton metres

Thus $T = 9850 \times 0.15 = 1477.5\text{Nm}$

And power developed $= \frac{2 \times 3.14 \times 800 \times 9850 \times 0.15}{60} = 124\text{kW}$

Example 13.9. A shunt motor takes 180A. The supply voltage = 400V, the shunt-field

brushes, calculate (a) the motor back e.m.f. (2 decimal figures), (b) the output power developed (3 significant figures) and (c) the efficiency, neglecting all losses for which information is not given (1 decimal place).

Shunt-field current $= \frac{400}{200} = 2\text{A}$

Armature current = $180 - 2 = 178\text{A}$

Armature voltage drop = $178 \times 0.02 = 3.56\text{V}$

(a) Back e.m.f. = $400 - 3.56 - 2$ (voltage drop at brushes) = 394.44V

(b) Output power developed $= \frac{394.4 \times 178}{1000} = 70.2\text{kW}$

Efficiency $= \frac{\text{output}}{\text{input}} = \frac{394.4 \times 178}{400 \times 180}$

Efficiency or $\eta = 0.975$ or 97.5%

Example 13.10. A 4-pole D.C. motor with a lap winding is connected to a 200V supply mains. The armature carries 600 conductors and has a resistance = 0.3Ω. The shunt-field circuit resistance = 100Ω and the flux per pole = 0.02Wb. On no load armature current = 3A. If the normal full-load current in the armature is 50A, determine the drop in the motor speed from no load to full load. Neglect the effect of armature reaction (1 decimal place).

Back e.m.f. on no load $E_{b0} = 200 - I_{a0}R_a$

Shunt-field current $= \frac{200}{100} = 2\text{A} \quad I_{a0} = 3 - 2 = 1\text{A}$

$\therefore E_{b0} = 200 - (1 \times 0.3) = 200 - 0.3 = 199.7\text{V}$

No-load speed is given by N_0 where:

$$E_{b0} = \frac{Z\Phi_0 N_0}{60} \times \frac{P}{A}$$

or $199.7 = \frac{600 \times 0.02 \times N_0}{60} \times \frac{4}{4}$

Thus $199.7 = 0.2 \times N_0$

or $N_0 = \frac{199.7}{0.2} = 998.5\text{rev/min.}$

Back e.m.f. E_{b1} on full load is given by:

$E_{b1} = 200 - (48 \times 0.3)$

or $E_{b1} = 185.6V$

Since $E_{b1} = k\Phi_1 N_1$ and assuming a constant flux, then $\Phi_0 = \Phi_1$

or $\frac{E_{b0}}{E_{b1}} = \frac{k\Phi_1 N_1}{k\Phi_0 N_0}$ whence $N_1 = \frac{N_0 E_{b1}}{E_{b0}}$

Thus $N_1 = \frac{998.5 \times 185.6}{199.7}$

Full-load speed = 928 rev/min.

Example 13.11. Calculate the first resistance step of a starter for a 240V shunt motor with armature resistance = 0.5Ω, if the maximum current limit = 60A and the lower limit about 45A.

Let R_s = the total resistance of the series resistor put into the armature circuit. If is the armature current at start = I_{as} 60A and also $I_{as} = \frac{240}{R_a + R_s} = \frac{240}{0.5 + R_s}$

or $R_s + 0.5 = \frac{240}{60}$ giving $R_s = 4 - 0.5 = 3.5\Omega$

As the motor starts and accelerates up to speed, the starter handle is kept in position until the current falls to 45A. Thus the starting resistance is still in circuit, but a back e.m.f. is building to a final value given by E_{b1}.

Here $E_{b1} = 240 - 45(3.5 + 0.5) = 240 - 180 = 60V$

At this stage the handle is moved and a section of the starting resistor cut out. Let R_1 be the new value of the total starter resistance. Current rises to 60A but the back e.m.f. does not change until the motor speed changes. Thus at the instant of moving the handle:

$$240 = E_{b1} + I_a (R_a + R_1)$$

$$\text{or } 240 = 60 + 60\,(0.5 + R_1) \text{ so:}$$

$$0.5 + R_1 = \frac{240 - 60}{60} = 3 \text{ or } R_1 = 3 - 0.5 = 2.5\Omega.$$

Thus the resistance removed during the first movement of the handle after switching on, is:

$3.5 - 2.5 = 1\Omega$ The first resistance step is thus 1Ω.

Estimation of D.C. machine efficiency

The efficiency of a D.C. machine can be assessed by measuring its output power and comparing this with the input power. Thus in general:

$$\text{Efficiency} = \frac{\text{Power output}}{\text{Power input}}$$

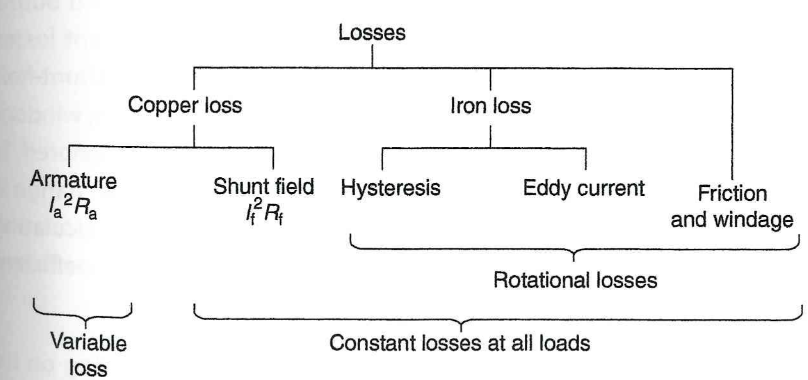
For small motors output is measured with a calibrated brake or a dynamometer and input power measured by electrical instrumentation. However, on large motors it is difficult to measure output power with any accuracy. Similarly mechanical input power to a generator is difficult to measure, and with large generators, the electrical output involves dissipation of much energy, usually as heat.

The Swinburne test was a technique devised to assess losses and estimate efficiency at any load from:

$$\eta = \frac{\text{Power input} - \text{losses}}{\text{Power input}} \text{ (for a motor)}$$

$$\eta = \frac{\text{Power input}}{\text{Power output} + \text{losses}} \text{ (for a generator)}$$

Although this test is rarely now considered in electrotechnology examinations, brief mention of the loss issues are discussed here. Possible losses are divided into 2 groups, those which vary with load and those which remain largely constant at all loads. These were introduced in Chapter 6 and summarised here:



▲ Figure 13.13

Rotational losses due to friction and windage are constant if the speed is constant. *Iron loss* is due to the core material (hysteresis) magnetic properties and eddy currents which are minimised by laminating the cores. Iron loss varies with load, but such variation has little effect on overall constant losses. Losses due to the windings resistance, or *Copper loss*, occurs in the armature and in field windings. Field copper loss is constant providing the supply voltage remains unaltered and the winding is shunt-connected. The only big losses which vary with load are due to the armature circuit resistance, the armature copper losses $I_a^2R_a$.

Hence for a motor:

$$\eta = \frac{\text{Power input} - \text{variable loss} - \text{constant loss}}{\text{Power input}}$$

$$\eta = \frac{VI - I_a^2R_a - I_f^2R_f - P_{WIF}}{VI} \times 100\%$$

For a generator:

$$\eta = \frac{\text{Power output}}{\text{Power output} + \text{variable loss} + \text{constant loss}}$$

$$\eta = \frac{VI}{VI + I_a^2R_a + I_f^2R_f + P_{WIF}} \times 100\%$$

To assess a machine's constant losses it is run as a motor on no load. As no output power is developed, all the input power overcomes the machine's constant losses. On a shunt-connected machine, this no-load input power supplies the shunt-field loss (VI_f or $I_f^2R_f$) and armature loss. This no-load armature loss comprises iron, windage and friction losses (P_{WIF}), and a small armature copper loss which is usually ignored. To make an accurate estimate of a machine's efficiency one must account for a change in copper loss due to resistance change as temperature rises. This involves calculating the new resistance value at the higher temperature using the temperature coefficient of resistance equation $R_T = R_0(1 + \alpha T)$, with rotational losses unaltered.

Example 13.12. A 250V D.C. shunt motor takes a current of 7A when running on no load. The armature and shunt field-circuit resistances are 0.15Ω and 125Ω respectively.

Find the machine efficiency when it (a) runs as a shunt motor taking a total current of 52A and (b) is driven as a shunt generator delivering 15kW at its output terminals (2 decimal places each).

On no load:

$$\text{Shunt-field current } I_f = \frac{V}{R_f} = \frac{250}{125} = 2\text{A}$$

$$\text{Armature current } I_{a_0} = I_L - I_f = 7 - 2 = 5\text{A}$$

No load armature power input = windage and friction losses + small $I_a^2R_a$ (often negligible losses).

$$\therefore P_0 = VI_{a_0} = 250 \times 5 = 1250\text{W}$$

$$\text{Field Copper loss } I_f^2R_f = VI_f = 250 \times 2$$

(a) At 52A load $I_a = 52 - 2 = 50\text{A}$ (I_f constant).

$$\text{Armature Copper loss} = I_a^2R_a = 50^2 \times 0.15 = 375\text{W}$$

$$\eta = \frac{VI - I_a^2R_a - I_f^2R_f - P_{WIF}}{VI} \times 100\ \%$$

$$\eta = \frac{(250 \times 52) - 375 - 500 - 1250}{(250 \times 52)} \times 100\% \quad \eta = 83.65\%$$

(b) Load current $I_L = \frac{15 \times 10^3}{250} = 60\text{A}$

$$\text{Shunt} - \text{field current } I_f = 2\text{A}$$

$$\text{Armature current } I_a = 60 + 2 = 62\text{A}$$

$$\text{Armature Cu Loss } I_a^2R_a = 622 \times 0.15 = 576.6\text{W}$$

$$\eta = \frac{VI}{VI + I_a^2R_a + I_f^2R_f + P_{WIF}} \times 100\%$$

$$\eta = \frac{(15 \times 10^3)}{(15 \times 10^3) + 576.6 + 500 + 1250} \times 100\% \quad \eta = 86.57\%$$

The efficiency at any load is estimated and a graph of efficiency against load current plotted, indicating the efficiency trend to give an accurate assessment of the load current where maximum efficiency occurs.

Practice Examples

13.1 A 110V series motor has a resistance of 0.120Ω. Determine its back e.m.f. when developing a shaft output of 7.5kW when efficiency is 85% (1 decimal place).

13.2 A 500V D.C. shunt motor has an input of 90kW when loaded. The armature and field resistances are 0.1Ω and 100Ω respectively. Calculate the back e.m.f. value (1 decimal place).

13.3 A 460V, D.C. motor takes an armature current of 10A at no load. At full load the armature current = 300A. If the resistance of the armature is 0.025Ω, what is the back e.m.f. value at no load and full load (1 decimal place)?

13.4 An armature winding of a D.C. motor has 240 conductors arranged in 4 parallel paths on an armature whose effective length and diameter are 400mm and 300mm respectively. Assuming average flux density in the air gap = 1.2T and the input to the armature = 40A, calculate (a) the force in newtons and torque in newton metres developed by one conductor, (b) the total torque developed by the complete winding, assuming all the conductors are effective and (c) the armature power output in watts, if the speed = 800 rev/min (all 1 decimal place).

13.5 A marine shunt motor is used for driving a 'fresh water' pump and found to take an armature current of 25A at 220V, when running on full load. Speed is measured to be 725 rev/min and the armature resistance = 0.2Ω. If field strength is reduced by 10% using a speed regulator and the torque is unchanged, determine the steady speed ultimately attained and the armature current, in rev/min.

13.6 A shunt generator delivers 50kW at 250V and runs at 400 rev/min. The armature and field resistances are 0.02Ω and 50Ω respectively. Calculate the machine's speed in rev/min when running as a shunt motor taking 50kW input at 250V. Allow 2V for brush-contact drop.

13.7 A 105V, 3kW D.C. shunt motor has a full-load efficiency of 82%. The armature and field resistances are 0.25Ω and 90Ω respectively. The full-load speed of the motor is 1000 rev/min. Neglecting armature reaction and brush drop, calculate the speed at which the motor will run at no load if the line current at no load is 3.5A. Calculate the resistance added to the armature circuit to reduce the speed to 800 rev/min, the torque remaining constant at full-load value (2 decimal places).

13.8 A shunt motor runs at 1000 rev/min when cold, taking 50A from a 230V supply. If the armature and field windings both increase in average temperature from 15°C to 60°C, as the motor warms up, determine the speed in rev/min when the motor is warm, given armature and field resistances are 0.2Ω and 200Ω at 15°C respectively. The total current drawn from the supply is constant. Neglect brush drop and armature reaction, and assume an unsaturated magnetic circuit, with resistivity temperature coefficient 0.40% from and at 15°C.

13.9 A 4-pole, shunt motor has a wave-wound armature of 294 conductors. The flux per pole = 0.025Wb and the armature resistance = 0.35Ω. Calculate (a) the speed of the armature in rev/min and (b) the torque developed when the armature takes a current of 200A from a 230V supply (1 decimal place).

13.10 A shunt motor runs at 600 rev/min from a 230V supply when taking a line current of 50A. Its armature and field resistances are 0.4Ω and 104.5Ω respectively. Neglecting armature reaction and with a 2V brush drop, calculate (a) the no-load speed in rev/min if the no-load line current is 5A, (b) the resistance placed in the armature circuit to reduce the speed to 500 rev/min when taking a line current = 50A (2 decimal places) and (c) the percentage reduction in the flux per pole so the speed = 750 rev/min, when taking an armature current of 30A with no added resistance in the armature circuit (1 decimal place).

SOLUTIONS TO PRACTICE EXAMPLES

Chapter 1

1.1. Let R be the equivalent resistance of the parallel arrangement.

$$\text{Then } \frac{1}{R} = \frac{1}{2} + \frac{1}{4} + \frac{1}{5} + \frac{1}{10} = \frac{10+5+4+2}{20}$$

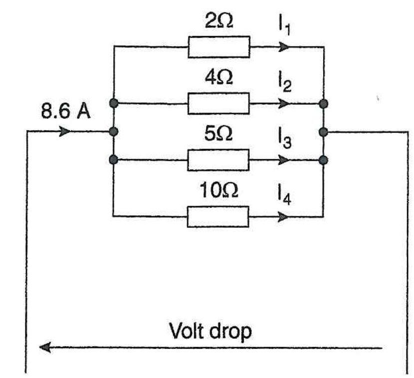
$$= \frac{21}{20} = \frac{2.1}{2} \text{ and } R = \frac{2}{2.1} = 0.952\Omega$$

Voltage drop across the arrangement = 8.6 × 0.952 = 8.19V

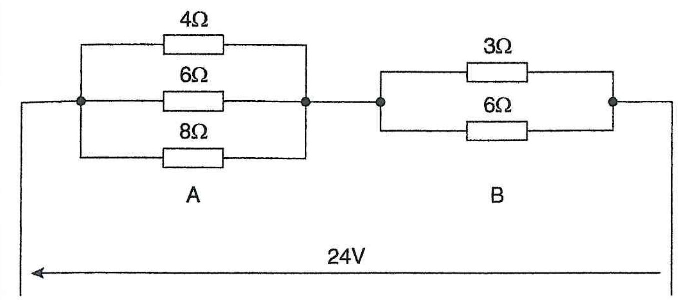
$$\text{Current } I_1 \text{ in } 2\Omega \text{ resistor} = \frac{8.19}{2} = 4.1\text{A}$$

$$\text{Current } I_2 \text{ in } 4\Omega \text{ resistor} = \frac{8.19}{4} = 2.1\text{A}$$

8.19



▲ Figure 1



▲ Figure 2

$$\text{Current } I_4 \text{ in } 10\Omega \text{ resistor} = \frac{8.19}{10} = 0.8\text{A}$$

Check. Total Current = 8.6A

1.2. For group A. Let R_A = the equivalent resistance, then:

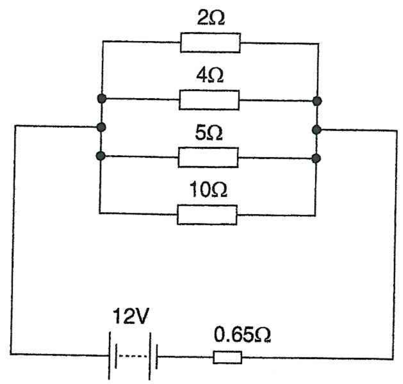
$$\frac{1}{R_A} = \frac{1}{4} + \frac{1}{6} + \frac{1}{8} = \frac{6+4+3}{24} \text{ or } R_A = \frac{24}{13} = 1.85\Omega$$

For group B. Let R_B = the equivalent resistance, then:

$$\frac{1}{R_B} = \frac{1}{3} + \frac{1}{6} = \frac{2+1}{6} = \frac{3}{6} \text{ or } R_B = 2\Omega$$

Total circuit resistance $R = R_A + R_B = 1.85 + 2 = 3.85\Omega$

Circuit current = $\frac{24}{\ldots}$ = 6.23A



▲ **Figure 3**

Voltage drop across group A = 1.85 × 6.23 = 11.53V

Voltage drop across group B = 2 × 6.23 = 12.46V

Check. Total voltage drop = 23.99 = 24.00V

Current in resistors – group A

$$\frac{11.53}{4} = 2.88\text{A}$$

$$\frac{11.53}{6} = 1.91\text{A}$$

$$\frac{11.53}{8} = 1.44\text{A}$$

Check I_{TOTAL} = 6.23A

Current in resistors – group B

$$\frac{12.46}{3} = 4.153\text{A}$$

$$\frac{12.46}{6} = 2.076\text{A}$$

Check I_{TOTAL} = 6.23A

1.3. From Q1.1 the equivalent resistance *R* of the load = 0.95Ω

The total resistance of the circuit = 0.95 + 0.65 = 1.6Ω

$I = V/R$

The circuit current $= \frac{12}{1.6} = 7.5\text{A}$

The terminal voltage = 7.5 × 0.95 = 7.1V

Current in 5Ω resistor $= \frac{7.125}{5} = 1.4\text{A}$

1.4. (a) Ammeter with shunt

Voltage drop across parallel arrangement for f.s.d. = $10 \times 15 \times 10^{-3}$ = 0.15V

Current to be carried by shunt

$$= 25 - (15 \times 10^{-3})$$

$$= 24.985\text{A}$$

Resistance of shunt $= \frac{0.15}{24.985}$

$$= 0.006\Omega$$

(b) Voltmeter with series resistance

Resistance of instrument circuit to drop 500V

$$= \frac{500}{15\times10^{-3}}$$

$$= 33\,333\Omega$$

∴ Series resistance to be added = 33 333 – 100

= 33 323Ω

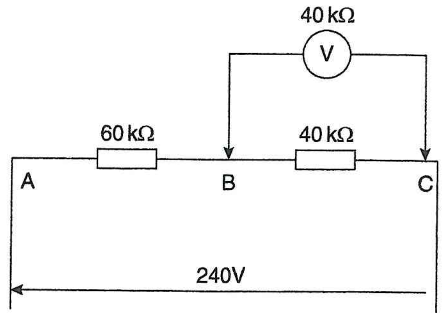
1.5. Parallel section BC has a resistance, given by:

$$\frac{1}{R} = \frac{1}{40} + \frac{1}{40} = \frac{2}{40} \text{ or } R = 20\text{k}\Omega$$

Total resistance of network = 60 + 20 = 80kΩ

Current taken by network $= \frac{240}{80000}$

$= 3 \times 10^{-3}$ A or 3mA



▲ **Figure 4**

Voltage drop across section BC

$= 3 \times 10^{-3} \times 20000$ volts = 60V = reading on voltmeter

1.6. Let E = e.m.f. of the battery and R_i its internal resistance.

then $E = 0.18\,(10 + R_i)$ … (i).

and $E = 0.08\,(25 + R_i)$ … (ii).

Equating (i) and (ii) $0.18\,(10 + R_i) = 0.08\,(25 + R_i)$

or $1.8 + 0.18\,R_i = 2 + 0.08\,R_i$

$\therefore (0.18 - 0.08)\,R_i = 2 - 1.8$

or $0.1\,R_i = 0.2$

or $R_i = 2\Omega$

Substituting in $E = 0.18\,(10 + 2)$

$= 0.18 \times 12$

$= 2.16$V

1.7. P.D. across 4Ω resistor = 4 × 1.5 = 6V

This is also the voltage drop across the other resistors in group A

$$\text{Current in } 2\Omega \text{ resistor} = \frac{6}{2} = 3.0\text{A}$$

$$\text{Current in } 6\Omega \text{ resistor} = \frac{6}{6} = 1.0\text{A}$$

$$\text{Current in } 8\Omega \text{ resistor} = \frac{6}{8} = 0.8\text{A}$$

$$\text{Current in } 4\Omega \text{ resistor} = \frac{6}{4} = 1.5\text{A}$$

The equivalent resistance R_B of parallel group B is obtained from:

$$\frac{1}{R_B} = \frac{1}{10} + \frac{1}{15} = \frac{3+2}{30} = \frac{5}{30} \text{ or } R_B = \frac{30}{5} = 6\Omega$$

So voltage drop across group B = 6 × 6.25 = 37.5V

$$\text{Current in } 10\Omega \text{ resistor} = \frac{37.5}{10} = 3.75\text{A}$$

$$\text{Current in } 15\Omega \text{ resistor} = \frac{37.5}{15} = 2.5\text{A}$$

Check. Total current = 6.25A

Voltage drop across group A = 6V

Supply Voltage = 6 + 37.5 = 43.5V

1.8. Generator O.C. voltage = 110V

Voltage drop in generator for 75A = 110 − 108.8 = 1.2V

$$\text{Internal resistance of generator} = \frac{1.2}{75} = 0.016\Omega$$

Voltage drop in cables = 108.8 − 105 = 3.8V

$$\text{Resistance of cables} = \frac{3.8}{75} = 0.0507\Omega$$

On 'short-circuit' the only limitation to current, is the resistance of the generator and the cables.

$$= 0.016 + 0.057 = 0.0667\Omega$$

$$\text{So short-circuit current} = \frac{110}{0.0667} = 1650\text{A}$$

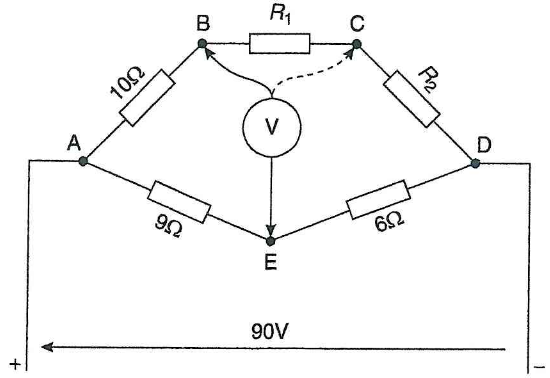
1.9. Meter voltage drop for f.s.d. = 1 × 0.12 volts = 120mV.

Since shunt voltage drop for 300A is 150mV, then the meter resistance must be increased by an external resistor of value 0.03Ω. Obtained from 1A × (0.03 + 0.12) Ω = 1 × 0.15 = 0.15V or 150mV.

Under this condition the actual current metered would be 301A. 300A will pass through the shunt and 1A through the ammeter.

Note. The resistor must be rated for this current, i.e. 1A. Thus $1^2 \times 0.03 = 0.03$W (see Chapter 2.)

1.10.



▲ **Figure 5**

P.D. across BC = 34 + 6 = 40V, since E is 6V above C and B in turn is 34V above E.

The P.D. across AE and ED is proportional to their resistance values.

$$\text{Thus P.D. across AE} = \frac{9}{15} \times 90 = 54\text{V with A +ve to E.}$$

$$\text{And P.D. across ED} = \frac{6}{15} \times 90 = 36\text{V with E +ve to D.}$$

Since A is positive to E by 54V and B is positive to E by 34V (voltmeter reading), then A is +ve to B by (54 – 34) = 20V.

Similarly B is above E by 34V and E is +ve to C by 6V then the P.D. across BC = 34 + 6 = 40V. E is above D by 36V and above C by 6V so C must be +ve to D by (36 – 6) = 30V.

$$\text{Thus the P.D.s across the resistors are } \left.\begin{aligned} AB &= 20\text{V} \\ BC &= 40\text{V} \\ CD &= 30\text{V} \end{aligned}\right\} \text{Total 90V}.$$

Also as the branches are *series* circuits, the ohmic values are proportional to the P.D.s.

$$\therefore \frac{\text{P.D. across AB}}{\text{P.D. across BC}} = \frac{10}{R_1} \text{ or } R_1 = \frac{10 \times 40}{20} = 20\Omega$$

$$\text{And, } \frac{\text{P.D. across CD}}{\text{P.D. across AB}} = \frac{R_2}{10} \text{ or } R_2 = \frac{10 \times 30}{20} = 15\Omega$$

$$\text{Current in branch ABCD} = \frac{90}{10 + 20 + 15} = \frac{90}{45} = 2\text{A}$$

$$\text{Current in branch AED} = \frac{90}{9 + 6} = \frac{90}{45} = 6\text{A}$$

Supply current = 2 + 6 = 8A

Chapter 2

2.1. Total mass to be lifted = (2 + 0.25) × 10^3kg

Force exerted = 2.25 × 10^3 × 9.81 newtons

Work done = 2.25 × 10^3 × 9.81 × 30 Nm

= 66.2175 × 10^4Nm or 662.175kJ

$$\text{Hoist output power} = \frac{\text{work done (joules)}}{\text{time (seconds)}}$$

$$= \frac{666175}{90} = 7.36\text{kW}$$

Power input = 200 × 50 = 11kW

$$\text{Efficiency} = \frac{7.36}{11} = 0.6689 \text{ or } 66.9\%$$

2.2. Since battery voltage of about 20V is needed, cells must be connected in a series–parallel arrangement. Ten cells in series will give 22V and this will be the e.m.f. of the battery irrespective of the number of identical parallel banks.

An arrangement of 10 cells in series with 3 such banks in parallel is a practical combination.

Battery e.m.f. = e.m.f. of 1 bank = 2.2 × 10 = 22V

Internal resistance of 1 bank = 0.3 × 10 = 30Ω

$$\text{Internal resistance of battery} = \frac{3}{3} = 1\Omega$$

$$\text{Resistance of 1 lamp} = \frac{V^2}{P} = \frac{20^2}{10} = 40\Omega$$

$$\text{or lamp current} = \frac{P}{V} = \frac{10}{20} = 0.5\text{A}$$

$$\text{and lamp resistance} = \frac{20}{0.5} = 40\Omega$$

$$\text{Resistance of 3 lamps in parallel} = \frac{40}{3} = 13.33\Omega$$

Total resistance of the complete circuit = 13.33 + 1 = 14.33Ω

$$\text{Circuit current} = \frac{22}{14.33} = 1.54\text{A}$$

$$\text{(b) Current taken by 1 lamp} = \frac{1.54}{3} = 0.513\text{A}$$

(a) Voltage drop in battery = 1.54 × 1 = 1.54V

Battery terminal voltage = 22 − 1.54 = 20.46V

(a) Power loss per cell = (current in 1 bank)^2 × resistance of a cell

$$= \left(\frac{1.54}{3}\right)^2 \times 0.3$$

$$= 0.5132 \times 0.3 = 0.079\text{W}$$

2.3. The equivalent head of water can be obtained thus: A pressure of 15 bars = 15 × 10^{5}Nm^{-2}. Specific weight of water is 10^3 × 9.81Nm^{-3}

$$\text{Then the head of water} = \frac{15 \times 10^5}{10^3 \times 9.81}$$

$$= 152.85\text{m}$$

Force required to lift 12 700 litres or 12.7 × 10^3 kg is 12.7 × 103 × 9.81 newtons

$$= 124\,587\text{N}$$

Work done per hour = 124 587 × 152.85 Nm = 19.064MJ

$$\text{Pump power output} = \frac{19.064 \times 10^6}{3600} = 5.296\text{kW}$$

Input to pump or output of motor

$$= \frac{5.296}{\ldots} = 6.465\text{kW}$$

$$\text{Input to motor} = \frac{6.465}{0.89} = 7.275\text{kW}$$

$$\text{Motor current} = \frac{7275}{220} = 33.1\text{A}$$

2.4. Battery e.m.f. = 4 × 2.2 = 8.8V

Terminal voltage of battery = voltage drop across resistor

= 5 × 1.4V = 7V

Voltage drop in battery = 8.8 − 7 = 1.8V

$$\text{Internal resistance of battery} = \frac{1.8}{1.4} = 1.29\Omega$$

$$\text{Internal resistance of 1 cell} = \frac{1.29}{4} = 0.32\Omega$$

For parallel working:

$$\text{Internal resistance of battery} = \frac{0.32}{4} = 0.08\Omega$$

E.m.f. of battery = e.m.f. of 1 cell = 2.2V

Total circuit resistance = 5 + 0.08 = 5.08Ω

$$\text{Circuit current} = \frac{2.2}{5.08} = 0.43\text{A}$$

2.5. Winch output

= 5 × 10^3 × 9.81 × 36.5 Nm per minute

= 4.905 × 36.5 × 104 joules per minute

$$= \frac{4.905 \times 36.5 \times 10^4}{60} \text{ joules per second}$$

$$= \frac{179.033}{6} \times 10^3 \text{watts} = 29.84\text{kW}$$

Since the winch is 75% efficient, the input must be $29.84 \times \frac{100}{75}$ kilowatts

$$= 29.84 \times \frac{4}{3} = 39.78\text{kW}$$

Input to winch = output of motor

∴ power rating of motor = 39.78kW

If the motor efficiency is taken as 85%,

The electrical input would be $\frac{39.78}{0.85} = 46.8\text{kW}$

Current taken from the mains $= \frac{46800}{220} = \frac{2340}{11} = 212.7\text{A}$

2.6. Lighting load = 100 × 100 = 10 000W and

200 × 60 = 12 000W

= 10 + 12 = 22kW

Heating load = 25kW

Miscellaneous loads = 30 × 220 = 6.6kW

Total load = 22 + 25 + 6.6 = 53.6kW

Generator output = 53.6kW

Generator input $= \frac{53.6}{0.85} = 63.1\text{kW}$

Now generator input = engine output.

So engine must develop 63.1kW

2.7. O.C. e.m.f. of battery = 4.3V

O.C. e.m.f./cell $= \frac{4.3}{3} = 1.43\text{V}$

Value of load resistor $= \frac{4.23}{0.4} = 10.575\Omega$

Voltage drop in battery = 4.3 – 4.23 = 0.07V

Internal resistance of battery $= \frac{0.07}{0.4} = 0.175\Omega$

Internal resistance of 1 cell $= \frac{0.175}{3} = 0.058\Omega$

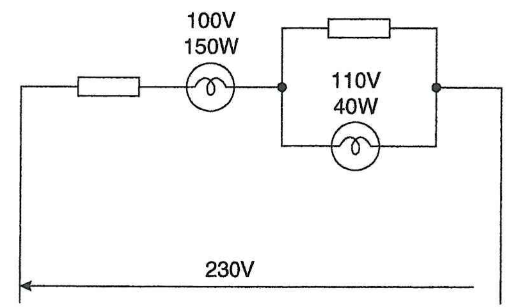
With a cell *reversed*, the e.m.f. of 2 cells cancel each other and the effective e.m.f. = that of 1 cell = 1.43V.

Let *I* be the current under this condition.

Then $1.43 = I(10.57 + 0.175) \quad \therefore \quad I = \frac{1.43}{10.75}\ 0.134\text{A}$

Note. For the solution, the internal resistance of a cell is assumed to be the same in both the forward and reverse direction in the absence of any further detailed information.

2.8.



▲ Figure 6

Current for 40W lamp $= \frac{40}{110} = 0.363\text{A}$

Resistance of 40W lamp $= \frac{110}{0.363} = 303\Omega$

Current for 150W lamp $= \frac{100}{1.5} = 1.5\text{A}$

Resistance of 150W lamp $= \frac{100}{1.5} = 66.66\Omega$

Parallel circuit has to carry 1.5A

∴ Current in shunt resistor = 1.5 – 0.363 = 1.137A

Voltage drop across shunt = 110V

Resistance of shunt $= \frac{110}{1.137} = 96.8\Omega$

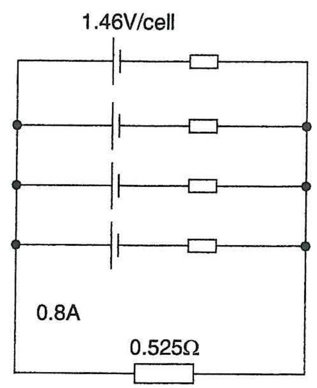
110V is dropped across the parallel circuit

100V is dropped across the series lamp

∴ 230 – 210 = 20V must be dropped across the series resistor which carries 1.5A

$\therefore$ Resistance value of series resistor $= \dfrac{20}{1.5} = 13.3\Omega$

2.9.



▲ **Figure 7**

E.m.f. of battery = e.m.f. of 1 cell for parallel working = 1.46V

Let R_i = the internal resistance of the battery

and R_c = the internal resistance of 1 cell.

The total resistance of the circuit = $0.525 + R_i$

Circuit voltage drop = $0.8\,(0.525 + R_i)$

= applied e.m.f. = 1.46V

$\therefore\ 1.46 = (0.8 \times 0.525) + 0.8\,R_i$

$= 0.42 + 0.8\,R_i$ or $0.8\,R_i = 1.46 - 0.42 = 1.04$

So $R_i = \dfrac{1.04}{0.8} = 1.3\Omega$

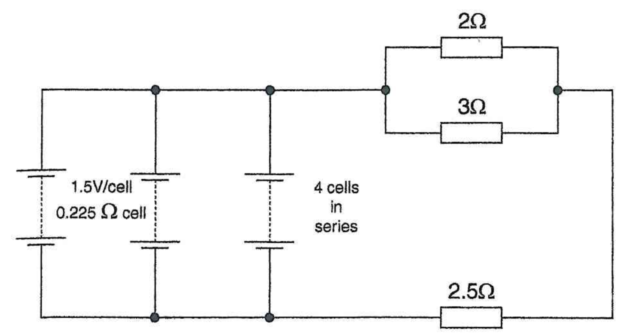
Now since the cells are in parallel, then

$$\frac{1}{R_i} = \frac{1}{R_c} + \frac{1}{R_c} + \frac{1}{R_c} + \frac{1}{R_c} = \frac{4}{R_c}$$

or $R_c = 4 \times R_i = 4 \times 1.3$

and internal resistance of 1 cell = 5.2Ω

2.10.



▲ **Figure 8**

Battery e.m.f. = e.m.f. of 1 bank = 4 × 1.5 = 6V

Battery resistance $= \dfrac{\text{resistance of 1 bank}}{3} = \dfrac{4 \times 0.225}{3} = 0.3\Omega$

Load resistance = 2.5 + R (resistance of parallel section)

Here $\dfrac{1}{R} = \dfrac{1}{3} + \dfrac{1}{2} = \dfrac{5}{6}$ or $R = \dfrac{6}{5} = 1.2\Omega$

Load resistance = 2.5 + 1.2 = 3.7Ω

Resistance of complete circuit = 3.7 + 0.3 = 4Ω

Circuit current $= \dfrac{6}{4} = 1.5$A

Voltage drop in battery = 0.3 × 1.5 = 0.45V

Battery terminal voltage = 6 − 0.45 = 5.55V

Power rating of 2.5Ω resistor = $I^2R = 1.5^2 \times 2.5 = 5.6$W

Current in 2Ω resistor $= \dfrac{\text{voltage drop}}{\text{resistance}} = \dfrac{\text{total current} \times R}{2}$

$$= \frac{1.5 \times 1.2}{2} = 0.9\text{A}$$

Power rating of 2Ω resistor = $0.9^2 \times 2 = 1.6$W

Current in 3Ω resistor = 1.5 – 0.9 = 0.6A

Power rating of 3Ω resistor = $0.6^2 \times 3 = 1.1$W

Energy conversion = energy in external resistors + energy in battery

= time (total wattage of external resistors + battery resistance power wastage)

$= t\,(5.625 + 1.62 + 1.08 + 1.5^2 \times 0.3)$

= 3600 (8.325 + 0.675) = 32400J

This could also be obtained thus:

Energy = e.m.f. × current × time

= 6 × 1.5 × 3600

= 32 400J or 32.4kJ

Chapter 3

3.1. (a) Volume = area × length or $A = \dfrac{V}{l} = \dfrac{10 \times 10^3}{100 + 10^3}$

$= 0.1\text{mm}^2$

$$\text{Then } R = \frac{el}{A} = \frac{17 \times 10^{-6} \times 100 \times 10^3}{10^{-1}} = 17\Omega$$

(b) Area of plate = 100 × 100 = 104mm²

$$\text{Thickness of plate} = \frac{10 \times 10^3}{10^4} = 1\text{mm}$$

$$\text{This is the length in the expression } R = \frac{\rho\ell}{A}$$

$$\therefore R = \frac{17 \times 10^{-6} \times 1}{10^4}\text{ ohms} = 1.7 \times 10^{-3}\mu\Omega$$

Alternatively using = 1.7×10^{-8} ohm-metres for (a) – as an example.

$$R = \frac{1.7 \times 10^{-8} \times 100}{0.1 \times 10^{-6}} = 17\Omega$$

3.2. Since $R_{20} = R_0\,(1 + \alpha 20)$ and $R_{60} = R_0\,(1 + \alpha 60)$

$$\text{Then } \frac{R_{60}}{R_{20}} = \frac{R_0(1 + \alpha 60)}{R_0(1 + \alpha 20)}$$

$$\text{and } R_{60} = \frac{R_{20}[1 + (60 \times 0.004\,28)]}{[1 + (20 \times 0.004\,28)]}$$

$$\text{or } R_{60} = \frac{90(1 + 0.2568)}{1 + 0.856} = \frac{90 \times 1.2568}{1.0856}\text{ ohms} = 104.4\Omega$$

$$\text{Current taken by coil at 20°C} = \frac{230}{90} = 2.56\text{A}$$

At 60°C to keep current constant, voltage must be 2.56 × 104.4 = 267.26V. So the voltage is raised by 267.26 – 230 = 37.26V

3.3. Assuming 1 litre of water to have a mass of 1 kg so the mass of 0.75 litre of water

= 0.75 × 1 = 0.75kg

Heat required = 0.75 × 4.2 × (100 – 6) = 296kJ

$$\text{The current taken by the heater is } \frac{220}{120} = 1.83\text{A}$$

and the power rating of the heater = 220 × 1.83 = 403.3W

Since the heater is 84% efficient, only 403.3 × 0.84 watts are available to heat the water.

$$\therefore \text{time of heating} = \frac{296 \times 10^3}{403.3 \times 0.84}\text{ seconds} = 873\text{s}$$

$$= \frac{873}{60}\text{minutes} = 14\text{min } 33\text{s}$$

3.4. Since $R = \dfrac{\rho\ell}{A}$ then $\ell = \dfrac{RA}{\rho}$

$$\ell = \frac{15.7 \times \pi \times 0.315^2}{407 \times 10^{-6} \times 4}\text{mm} = 3\text{cm}$$

3.5. Since $\frac{R_2}{R_1} = \frac{R_0(1+\alpha T_2)}{R_0(1+\alpha T_1)}$

$$\therefore R_2 = \frac{R_1(1+\alpha T_2)}{1+\alpha T_1}$$

Then $\frac{R_2}{R_1} \times (1+\alpha T_1) = 1+\alpha T_2$

So $1+\alpha T_2 = \frac{240}{200}[1+(0.0042 \times 15)]$

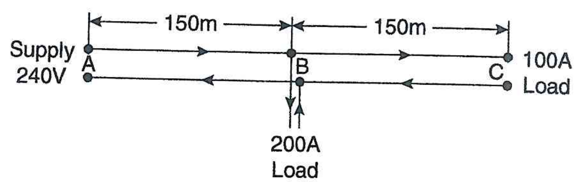
$= 1.2\,(1+0.063)$

and $\alpha T_2 = 1.2 + 0.0756 - 1 = 0.2756$

thus $T_2 = \frac{0.2756}{0.0042} = 65.6°C$

Temperature rise = 65.6 − 15 = 50.6°C

3.6.



▲ **Figure 9**

The resistance of a cable core 880m long and area 50mm² = 0.219Ω then the resistance of a cable core 880m long and area 150mm² = $\frac{0.219}{3} = 0.073\Omega$ and the resistance of a cable core 150m long and area 150mm² $= \frac{0.073 \times 150}{880} = 0.0124\Omega$

Current in length AB = 300A

Resistance of length AB = 2 × 0.0124 = 0.0248Ω

Voltage drop in length AB = 300 × 0.0248 = 7.44V

Voltage at 200A load = 240 − 7.44 = 232.56V

Voltage drop in section BC $= \frac{1}{3}$ that in AB as the current is $\frac{1}{3}$, all else being the same

$$\therefore \text{Voltage drop in BC} = \frac{7.44}{3} = 2.48\text{V}$$

Voltage at 100A load, i.e. at C = 232.56 − 2.48 = 230.08V

3.7. Resistance of 1mm diameter cable $R_1 = \frac{el_1}{A_1}$

or $2.47 = \frac{e \times 100 \times 10^3 \times 4}{\pi \times 1^2}$

Let R_2 = resistance of $800 \times \frac{105}{100}$ = 840m of 1.5mm diameter cable

Then $R_2 = \frac{e \times 840 \times 10^3 \times 4}{\pi \times 1.5^2}$

So $\frac{R_2}{R_1} = \frac{e \times 840 \times 10^3 \times 4}{\pi \times 1.5^2} \Big/ \frac{e \times 100 \times 10^3 \times 4}{\pi \times 1^2} = \frac{840 \times 1^2}{100 \times 1.5^2}$

or $R_2 = 2.47 \times 8.4 \times \left(\frac{1}{1.5}\right)^2 = 9.22\Omega$

Since there are 19 strands in parallel, the resistance of the complete cable, being inversely proportional to area, will be reduced by 19

$$\therefore \text{Resistance of cable} = \frac{9.22}{19} = 0.49\Omega$$

3.8. Since $R = R_0(1+\alpha T)$ or $15 = 10[1 + (\alpha \times 100)]$

$= 10 + (\alpha \times 1000)$ or $5 = 1000\alpha$ and $\alpha = 0.005$

or using the definition

$$\alpha = \frac{\text{increase of resistance per 0°C rise in temperature}}{\text{resistance at 0°C}}$$

$$= \frac{15-10}{100}\Big/10 = \frac{5}{1000} = 0.005$$

Also since

$R = R_0 (1 + \alpha T)$ then $30 = 10 (1 + 0.005T)$

and $30 = 10 + 0.05T$

or $20 = 0.05T$ and $T = \dfrac{20}{0.05}$ °C = 400°C

3.9. Heat required by brass = 500 × 0.39 × (910 − 15) kilojoules

= 5 × 39 × 895

or energy required by brass = 174.525kJ

$$\text{Energy taken from supply} = \frac{174.525 \times 10^3 \times 100}{80}$$

$= 21.816 \times 10^4$KJ

Time taken to expend this energy at the rate of 200kW

$$= \frac{21.816 \times 10^4}{200} \text{ seconds}$$

= 1091s

= 18.18min or 18 min 11s

3.10. Electrical energy used $= \dfrac{744}{2}$ = 372 units = 372kW h

= 372 × 3600KJ

Energy passed to heat water = 372 × 360 × 0.8 kilojoules

= 1. 071 × 360kJ

Heat energy received by water = 1.07×10^6 kilojoules

Temperature rise of water = 82 − 16 = 66

$$\text{So quantity of water} = \frac{1.07 \times 10^6}{66 \times 4.2} \text{ kilogrammes} = 3860\text{kg}$$

Assuming 1 litre has a mass of 1kg then the quantity of water used = 3860 litres

Chapter 4

4.1. Input to accumulator = 6 × 18 ampere hours

Output from accumulator = 3.5 × 28 ampere hours

$$\text{Ampere hour efficiency} = \frac{3.5 \times 28}{6 \times 18} = 0.907$$

= 90.7%

4.2. Mass of deposit, $m = zIt$

or $(19.34 - 14.52) \times 10^{-3} = 330 \times 10^{-9} \times I \times 50 \times 60$

$$\text{or } I = \frac{4.82 \times 10^{-3}}{50 \times 60 \times 330 \times 10^{-9}} \text{ amperes}$$

= 4.869A

Error in reading = 5.1 − 4.869

= 0.231A (high)

This is better expressed as a percentage thus:

$$= \frac{\text{Difference between false and true reading}}{\text{true reading}} \times 100$$

$$= \frac{5.1 - 4.869}{4.869} \times 100$$

= 4.75% (high)

4.3. E.m.f. of battery = 40 × 1.9 = 76V

or $E_b = 76$V

Internal battery resistance = 40 × 0.0025

= 0.1Ω

Total resistance of circuit = 1 + 0.1 = 1.1Ω = R

For charging $V = E_b + IR$

Thus $90 = 76 + (I \times 1.1)$

$$\text{or } I = \frac{90 - 76}{1.1} = \frac{14}{1.1} \text{ amperes}$$

= 12.72A

4.4. Area of nickel deposited = $\pi \times 100 \times 10^{-3} \times 150 \times 10^{-3} = 0.0471 m^2$

Volume of nickel deposited = $47.1 \times 10^{-3} \times 0.5 \times 10^{-3}$

$= 23.55 \times 10^{-6} m^3$

Mass of nickel deposited, $m = 23.55 \times 10^{-6} \times 8.6 \times 10^3$

$= 0.202\ 53 kg$

Now $m = zIt$ so $0.202\ 53 = 302 \times 10^{-9} \times I \times 8 \times 3600$

or $202.53 \times 10^{-3} = 30.2 \times 8 \times 36 \times 10^{-6} I$

$$\text{giving } I = \frac{202.53 \times 10^3}{30.2 \times 8 \times 36} = 23.3A$$

4.5. Discharge ampere hours = $6 \times 12 = 72$

Charge ampere hours = $6 \times 22 = 88$

$$\text{Ampere hour efficiency} = \frac{72}{88}$$

$= 0.82$ or 82%

Discharge watt hours per cell = $6 \times 12 \times 1.2 = 86.4$

Charge watt hours per cell = $4 \times 22 \times 1.5 = 132$

$$\text{Watt hour efficiency} = \frac{86.4}{132} = 0.65 \text{ or } 65\%$$

4.6. Battery voltage at start of charge = $80 \times 1.8 = 144V$

$= E_{b1}$

Battery voltage at end of charge = $80 \times 2.4 = 192V$

$= E_{b2}$

No battery resistance is given and is thus neglected.

Let R_1 = control resistance at start of charge

Then $V = E_{b1} + IR_1$ or $230 = 144 + 5 R_1$

Thus $5R_1 = 230 - 144 = 86$

$$R_1 = \frac{86}{5} 17.2\Omega \text{ (maximum value)}$$

Let R_2 = control resistance at end of charge

Then $230 = 192 + 5 R_2$ or $5 R_2 = 230 - 192 = 38$

$$R_2 = \frac{38}{5} = 7.6\Omega \text{(minimum value)}$$

$$\text{Charging time} = \frac{60}{5} = 12h \text{ (approx.)}$$

(Leave charging say 13h to allow for losses.)

4.7. Area of deposit = $2 \times 50 \times 150 = 15\ 000 mm^2$

Volume of deposit = $15 \times 10^3 \times 10^{-6} \times 0.05 \times 10^{-3}$

$= 0.75 \times 10^{-6} m^3$

Mass of deposit, $m = 0.75 \times 10^{-6} \times 8800 = 0.0066 kg$

$$\text{Now } m = zIt \text{ so } I = \frac{m}{zt}$$

$$\text{or } I = \frac{6.6 \times 10^{-3}}{330 \times 10^{-9} \times 30 \times 60} \quad \text{Thus } I = 11.1A$$

4.8. Discharge ampere hours = $4 \times 40 = 160$

Charge ampere hours = $8 \times 24 = 192$

$$\text{Ampere hour efficiency of battery} = \frac{160}{192} = 0.833 \text{ or } 83.3\%$$

Discharge watt hours = $4 \times 40 \times 1.93 \times 40$

Charge watt hours = $8 \times 24 \times 2.2 \times 40$

$$\text{Watt hour efficiency of battery} = \frac{4 \times 40 \times 1.93 \times 40}{8 \times 24 \times 2.2 \times 40}$$

$= 0.731$ or 73.1%

4.9. Voltage drop in leads = $10 \times 1 = 10V$

Voltage drop due to battery internal resistance = $10 \times 0.01 \times 30 = 3V$

At start of charge, if R_1 is the external resistance

Then $200 = (30 \times 1.85) + 10 + 3 + 10R_1$,

or $200 = 55.5 + 13 + 10R_1$,

$$\text{and } 200 - 68.5 = 10 R_1, \text{ thus } R_1 = \frac{131.5}{10} = 13.15\Omega$$

At end of charge, if R_2 is the external resistance

Then $200 = (30 \times 2.2) + 10 + 3 + 10R_2$

or $200 = 66 + 13 + 10R_2$

and $200 - 79 = 10R_2$ thus $R_2 = \frac{121}{10} = 12.1\Omega$

At start 13.15Ω are needed

At end 12.1Ω are needed

4.10. Here $m = zIt$ or $t = \frac{m}{zI} = \frac{4.2 \times 10^{-3}}{330 \times 10^{-9} \times 3.5}$ seconds

Thus $t = 3636\text{s} = 60.6$ min, i.e. 60 min 36s

From the second law of electrolysis or by proportion:

$$\frac{\text{Mass of hydrogen liberated}}{\text{Mass of copper liberated}} = \frac{\text{Chemical equivalent of hydrogen}}{\text{Chemical equivalent of copper}}$$

Thus mass of hydrogen $= \frac{1 \times 4.2}{31.8} = 0.1321\text{g}$

Chapter 5

5.1. $F = BI\ell$ newtons

$= 0.25 \times 100 \times 1 = 25$ newtons per metre length

5.2. M.m.f. $F = 4 \times 250 = 1000\text{At}$

(a) Magnetising force $H = \frac{F}{\ell}$ = m.m.f. per metre length

$$= \frac{1000}{500 \times 10^{-3}} = 2000\text{At/m}$$

(b) Flux density $B = \mu_0 \times H = 4 \times \pi \times 10^{-7} \times 2000$

$= 8 \times \pi \times 10^{-4}$ teslas

Cross-sectional area of ring $= 400 \times 10^{-6}\ \text{m}^2$

$\therefore$ Flux $\Phi = B \times A = 8 \times \pi \times 10^{-4} \times 400 \times 10^{-6}$

$= 1.0048 \times 10^{-6}$ webers or $1.005\mu\text{Wb}$

5.3. M.m.f. F produced $= 3200 \times 1$

$= 3200\text{At}$

The magnetising force H or m.m.f./m $= \frac{F}{\ell} = \frac{3200}{800 \times 10^{-3}}$

$= 4000\text{At/m}$

Also since $B = \mu_0 \times H$

$$B = 4 \times \pi \times 10^{-7} \times 4000$$

$$= 16 \times \pi \times 10^{-4} \text{ teslas}$$

Area of solenoid $= \frac{\pi \times d^2}{4} = \frac{\pi \times 20^2 \times 10^{-6}}{4} = \pi \times 10^{-4}$

So $\Phi = B \times A = 16 \times \pi^2 \times 10^{-8}$ Webers

5.4. Magnetising force H of a long, straight conductor

$$= \frac{I}{2\pi r} = \frac{2000}{2 \times \pi \times 0.8} \text{ ampere-turns/metre}$$

or H at conductor X, due to current in Y,

$$= \frac{1000}{0.8 \times \pi} \text{ ampere-turns/metre}$$

and B at conductor X due to current in Y $= \mu_0 \times H$

$$= \frac{4 \times \pi \times 10^{-7} \times 1000}{0.8 \times \pi} = \frac{10^{-3}}{2} \text{ teslas}$$

So $F = BI\ell$ newtons

$$= \frac{10^{-3}}{2} \times 2000 \times 1 = 1 \text{ newton/metre length}$$

5.5. Current to give f.s.d.

$$= \frac{50 \times 10^{-3}}{10} = 5 \times 10^{-3}\text{A}$$

Force exerted on 1 conductor

$$= Bl\ell = 0.1 \times 5 \times 10^{-3} \times 25 \times 10^{-3}$$

$$= 12.5 \times 10^{-6} \text{ newtons}$$

Force exerted on all conductors on both sides of the coil

$$= 100 \times 2 \times 12.5 \times 10^{-6}$$

$$= 2500 \times 10^{-6} \text{ newtons}$$

Torque exerted by coil

$$= \text{Force} \times \text{radius}$$

$$= 2500 \times 10^{-6} \frac{30}{2} \times 10^{-3}$$

$$= 37.5 \times 10^{-6} \text{ Nm}$$

Therefore the controlling torque of the spring

$$= 37.5 \times 10^{-6}\text{Nm}$$

$$\text{or} = 37.5\mu\text{Nm}$$

5.6. Flux density B in air gap $= \frac{0.05}{650 \times 10^{-6}}$

$$= \frac{5 \times 10^2}{6.5} \text{ teslas}$$

Also $B = \mu_0 \times H \quad \therefore H = \frac{B}{\mu_0} = \frac{5 \times 10^2}{6.5 \times 4 \times \pi \times 10^{-7}}$

$= 6.12 \times 10^7$ ampere-turns/metre

Air gap = 3mm = 3×10^{-3}m

$\therefore$ Required ampere-turns = $6.12 \times 10^7 \times 3 \times 10^{-3}$

= 183 600At

5.7. $F = Bl\ell$ newtons = $0.6 \times 150 \times 1 = 90$N/m

Assuming current flows away from the observer, then the force acts right to left to move the conductor horizontally.

5.8. $F = Bl\ell$ newtons = $0.5 \times 25 \times 400 \times 10^{-3} = 5$N

5.9. Force on 1 conductor = $0.6 \times 0.8 \times 250 \times 10^{-3}$

Force on 800 conductor = $0.6 \times 8 \times 250 \times 10^{-3} \times 8 \times 10^2$

$$= 9.6 \times 10^2 \text{ N}$$

Torque on armature = $9.6 \times 10^2 \times 100 \times 10^{-3} = 96$Nm

Power developed is given by $\frac{2\pi NT}{60}$ watts

$$= \frac{2 \times \pi \times 1000 \times 96}{60} = 10.05\text{kW}$$

5.10. Magnetising Force H of a long straight conductor

$$= \frac{I}{2\pi r} \text{ ampere-turns/metre}$$

$$= \frac{250}{2 \times \pi \times 25 \times 10^{-3}}$$

$$= \frac{10^4}{2 \times \pi}$$

Also $B = \mu_0 \times H$

or $B = 4 \times \pi \times 10^{-7} \times \frac{10^4}{2 \times \pi}$

$= 2 \times 10^{-3}$ teslas

Again $F = Bl\ell = 2 \times 10^{-3} \times 250 \times 1$

= 0.5 newtons

Mutual force per metre run = 0.5N

Chapter 6

6.1. (a) Total m.m.f., $F = 5 \times 500 = 2500\text{At}$

Mean circumference $= \pi d = \pi \times 300 \times 10^{-3}$

$= 0.942\text{m}$

So magnetising force, $H = \dfrac{F}{\ell} = \dfrac{2500}{0.942}$

$= 2654\text{At/m}$

(b) Since $\mu_0 = \dfrac{B}{H}$ then $B = \mu_0 H$

and $B = 4 \times \pi \times 10^{-7} \times 2654$

$= 0.0033\text{T} = 3.3\text{mT}$

(c) Total flux, $\Phi = BA$

$= 0.0033 \times 1000 \times 10^{-6}$ weber

$= 3.3 \times 10^{-6}\text{Wb}$ or $= 3.3\mu\text{Wb}$

6.2. $B = \dfrac{\Phi}{A} = \dfrac{500 \times 10^{-6}}{400 \times 10^{-6}} = 1.25\text{T}$

Also since $B = \mu H = \mu_r \mu_0 H$ then

$$H = \frac{1.25}{2500 \times 4 \times \pi \times 10^{-7}} = \frac{1.25}{\pi \times 10^{-3}} \text{ ampere-turns/metre}$$

Length of iron $= 250 \times 10^{-3}$ m

So total m.m.f., $F = \dfrac{250 \times 10^{-3} \times 1.25}{\pi \times 10^{3}}$

or $F = 99.7\text{At}$

Required ampere-turns = 99.7, say 100.

6.3. (a) $B = \dfrac{\Phi}{A}$ then $B = \dfrac{400 \times 10^{-6}}{500 \times 10^{-6}} = \dfrac{400}{500} = 0.8T$

Also, as H is given by $\dfrac{\text{total magnetomotive force}}{}$

Then $H = \dfrac{F}{\ell} = \dfrac{500}{1} = 500\text{At/m}$

Also, since $B = \mu H = \mu_o \mu_r H$ then

$$\mu_r = \frac{B}{\mu_o H} = \frac{0.8}{4 \times \pi \times 10^{-7} \times 500}$$

or relative permeability = 1275

(b) Reluctance $= \dfrac{\text{Length}}{\mu \times \text{Area}} = \dfrac{\ell}{\mu_o \mu_r \times A}$ ampere-turns/weber

$$= \frac{1}{4 \times \pi \times 10^{-1} \times 1275 \times 500 \times 10^{-6}} = 1.25\text{MA/Wb}$$

6.4. (a) H = ampere-turns permetre $= \dfrac{F}{\ell} = \dfrac{400 \times 2.5}{1.25}$

Thus $H = \dfrac{1000}{1.25} = 800\text{At/m}$

Also $B = \dfrac{\phi}{A} = \dfrac{0.000\ 75}{1500 \times 10^{-6}} = 0.5\text{T}$

Again $B = \mu\text{H}$ or $\mu = \dfrac{B}{H} = \dfrac{0.5 \times 1.25}{1000}$

Also $\mu = \mu_r \mu_o$

$\therefore \mu_r = \dfrac{\mu}{\mu_o} = \dfrac{0.625}{1000 \times 4\pi \times 10^{-7}}$

Thus relative permeability = 497.5

(b) Reluctance, $S = \dfrac{\ell}{\mu A} = \dfrac{1.25 \times 10^{-3}}{0.625 \times 1500 \times 10^{-6}}$ ampere-turns/weber = 1.33MA/Wb

(c) Since $F = H\ell = 800 \times 1.25 = 1000\text{At}$

6.5. Area of air gap $= 1200 \times 10^{-6}\ \text{m}^2$

$\therefore B_A = \dfrac{1.13 \times 13^{-3}}{12 \times 10^{-4}} = \dfrac{11.3}{12}$ teslas

Also since $B = \mu_o H$

Then H for air $= \dfrac{B}{\mu_0} = \dfrac{11.3}{12 \times 4\pi \times 10^{-7}}$

$= 75 \times 10^4$ ampere-turns/metre

The m.m.f. for the air gaps is given by: $75 \times 10^4 \times 2 \times 2 \times 10^{-3}$ ampere-turns = 3000At

The area of iron is the same as for the air gaps, and the B value of the iron is the same.

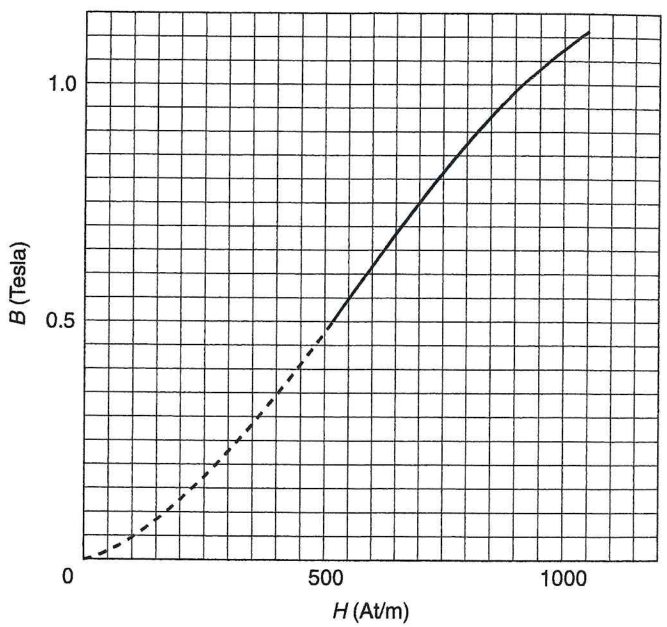
$$\text{or } B_1 = \frac{1.13 \times 10^{-3} \times 10^4}{12} = 0.942\text{T}$$

Using the graph of figure 10, for a flux density of 0.942T, the ampere-turns per metre length of the iron = 850.

Since length of iron path = 0.6m

$\therefore$ M.m.f. for iron = $0.6 \times 850 = 6 \times 85 = 510$At

Total m.m.f. required = 3000 + 510 or 3510At



▲ **Figure 10**

6.6. Circumference of flux path = $\pi \times 0.2 = 0.628$m

Length of air gap = $2 \times 10^{-3} = 0.002$m

Length of iron = 0.626m

This problem is best solved by trial and error thus:

Assume a flux density of 0.5T in the air gap and iron since these are of the same cross-sectional area. Then from the graph of figure 10:

M.m.f. for iron = $520 \times 0.626 = 326$At

$$\text{Since } B = \mu_0 H \;\therefore\; H = \frac{B}{\mu_0} = \frac{0.5}{4\pi \times 10^{-7}}$$

$$\frac{0.5 \times 2 \times 10^{-3}}{4\pi \times 10^{-7}} = 795\text{At}$$

Total m.m.f. would be (326 + 795) = 1121At. Thus too low a flux density has been assumed.

Again, assume a flux density 0.6T, then:

M.m.f. for iron = $585 \times 0.626 = 365.21$At

$$\text{M.m.f. for air} = \frac{0.6 \times 2 \times 10^{-3}}{4\pi \times 10^{-7}} \text{ or } \frac{6}{5} \text{ of that required for 0.5T}$$

$$= \frac{6}{5} \times 795 = 954\text{At}$$

Total m.m.f. would be 365.2 + 954 = 1319At – still too low. Assume a flux density of 0.7T. Then:

M.m.f. for iron = $660 \times 0.626 = 413.16$At

$$\text{M.m.f. for air} = \frac{7}{5} \times 795 = 1113\text{At}$$

Total m.m.f. would be 413 + 1113 = 1526At

Thus for an exciting ampere-turn value of $3 \times 500 = 1500$, the estimated flux density in the air gap would be a little less than 0.7T.

6.7. As the B value in the cores is 1.2T the At/m required will be 650, as seen from the graph of figure 11. The total m.m.f. for the cores will be:

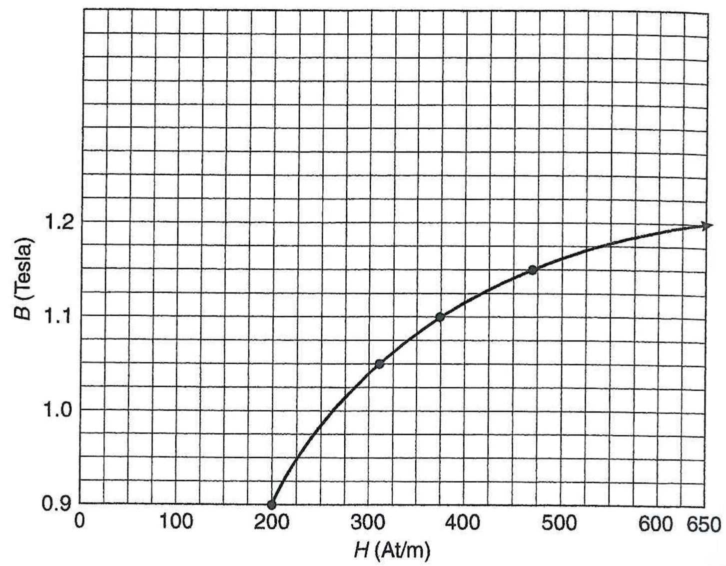
$$2 \times 160 \times 10^{-3} \times 650 = 208\text{At.}$$

In the yokes, the flux is the same as that for the cores and the flux density will therefore be different, as the areas are different.

Thus flux, $\Phi = 1.2 \times \frac{\pi}{4} \times 50^2 \times 10^{-6}$ weber

B in yokes $= 1.2 \times \frac{\pi}{4} \times \frac{25 \times 10^{-4}}{47 \times 47 \times 10^{-6}}$ teslas

$\therefore B = 1.066\text{T}$



▲ **Figure 11**

From the graph, the At/m for a density of 1.066T is 330 mean length of flux path in yokes $= (2 \times 130) + (2 \times 47) = 354\text{mm} = 0.354\text{m}$

Total m.m.f. for the yokes $= 330 \times 354 \times 10^{-3} = 116.8\text{At}$

Total m.m.f. for complete magnetic circuit = 208 + 116.8 = 324.8 say 325At

6.8. (a) Pull of magnet = 196.2N or 98.1N per contact face

Also Pull $= \frac{B^2 A}{2\mu_o} = \frac{B^2 A}{2 \times 4\pi \times 10^{-7}}$ newtons

$\therefore 98.1 = \frac{B^2 A}{8\pi \times 10^{-7}}$

Whence $B^2 = \frac{98.1 \times 8\pi \times 10^{-7} \times 4}{\pi \times 15 \times 15 \times 10^{-6}}$

$= 1.396$

and $B = \sqrt{1.396} = 1.185\text{T}$

From the graph of figure 11, for a B value of 1.185T, the H value = 560At/m.

Area of one contact face $= \frac{\pi}{4} \times 15^2 \times 10^{-6}\ \text{m}^2$

Flux, $\Phi = BA = 1.185 \times \frac{\pi}{4} \times 225 \times 10^{-6}$ weber

$= 2.09 \times 10^{-4}\,\text{Wb}$

Since B for the horse-shoe magnet = 1.185T

then, from figure 11, the H value = 560 At/m

Length of magnet path $= \pi \times \frac{115}{2} = 180.5\text{mm} = 0.1805\text{m}.$

Note. Mean circumference of ring

$= 2\pi \times$ (radius of ring)

$= 2\pi \times$ (50 + radius of rod)

$= 2\pi \times (50 + 7.5)$

$= \pi \times 115$ millimetres

Then m.m.f. for magnet $= 560 \times 0.1805 = 101.1$ At

B value for armature $= \frac{\text{Flux}}{\text{Area}} = \frac{2.09 \times 10^{-4}}{15 \times 10^{-6}} = 0.932\text{T}$

and H value = 215At/m (from figure 11)

Length of armature path = (115 + 15) = 130mm

$= 130 \times 10^{-3}$ m

So m.m.f. required $= 215 \times 130 \times 10^{-3} = 27.95$At

Total m.m.f. required = 101.1 + 27.95 = 129At

Current $= \frac{129}{480} = 0.268\text{A}$

(b) In the air gap $B = 1.15$T

$\therefore$ Flux, $\Phi = 1.15 \times \frac{\pi}{4} \times 15^2 \times 10^{-6} = 2.03 \times 10^{-4}$ weber

$$\text{Flux density in core} = \frac{2.03 \times 10^{-4}}{\frac{\pi \times 15^2}{4} \times 10^{-6}} = 1.15\text{T}$$

and from curve, H value = 470At/m

Length of core path = $\pi \times 57.5$ = 180.5mm = 0.1805m

M.m.f. for core = 0.1805 × 470 = 84.8At

$$\text{Flux density in armature} = \frac{2.03 \times 10^{-4}}{15^2 \times 10^{-6}} = 0.905\text{T}$$

From curve H value = 205At/m

Length of armature path = 130mm = 0.13m

M.m.f. for armature = 0.13 × 205 = 26.65At

Flux density in 1 air gap = 1.15T, but $B = \mu_o H$

$$\therefore H = \frac{B}{\mu_o} = \frac{1.15}{4\pi \times 10^7} = \frac{1.15 \times 10^7}{12.56} \text{ ampere-turns/metre}$$

M.m.f. for 2 air gaps

$$= \frac{2 \times 0.5 \times 10^{-3} \times 1.15 \times 10^7}{12.56}$$

$= 9.125 \times 10^2$ ampere-turns = 912.5At

Total m.m.f. for circuit = 84.8 + 26.65 + 912.5

= 1023.95 At

6.9. (a) $\text{Area of air gap} = \frac{\pi 100^2}{4} \times 10^{-6}$

$$= \frac{\pi \times 10^{-2}}{4} \text{ m}^2$$

$$\text{Volume of air gap} = \frac{\pi \times 10^{-2}}{4} \times 2.5 \times 10^{-3}$$

$$= \frac{\pi}{16} \times 10^{-4} \text{m}^3$$

$$\text{Flux density in gap} = \frac{0.004 \times 4}{\pi \times 100^2 \times 10^{-6}} = 0.508\text{T}$$

$$\text{Energy stored in joules} = \frac{B^2}{2\mu_o} \times \text{volume}$$

$$= \frac{0.508^2}{2 \times 4\pi \times 10^{-7}} \times \frac{\pi}{16} \times 10^{-4} = 2\text{J}$$

$$\text{(b) Pull (newtons)} = \frac{B^2 A}{2\mu_o} = \frac{0.508^2 \times \pi \times 10^{-2}}{2 \times 4\pi \times 10^{-7} \times 4} = 806\text{N}$$

6.10. Air gap. Useful flux = 0.05Wb/pole

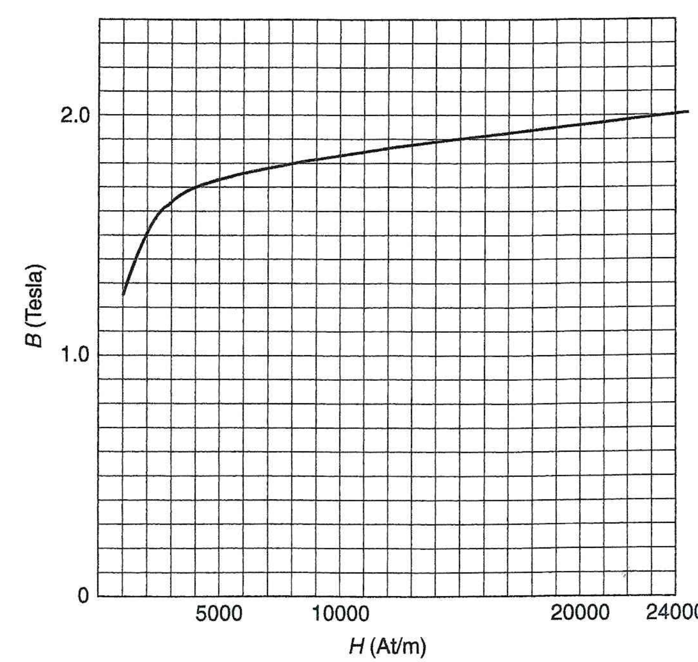
$$\text{Flux density in air gap} = \frac{0.05}{60000 \times 10^{-6}} = 0.833\text{T}$$

Also $B = \mu_o H$

$$\therefore H \text{ value for air} = \frac{B}{\mu_o} = \frac{0.833}{4\pi \times 10^{-7}}$$

$= 66.2 \times 10^4$ At/m

M.m.f. for air gap = $66.2 \times 10^4 \times 5 \times 10^{-3}$ = 3310At



▲ Figure 12

Pole

Total flux = 0.05 × 1.2 = 0.06Wb

Flux density in pole $= \dfrac{0.06}{40000 \times 10^{6}}$ or $B = 1.5$T

From the magnetic characteristic, plotted for figure 12, a flux density (B) of 1.5T gives an H value of 2000At/m.

Thus m.m.f. required for pole = $2000 \times 250 \times 10^{-3}$ = 500At

Teeth

Total flux = 0.05Wb (same as the gap)

Flux density in teeth $= \dfrac{0.05}{25000 \times 10^{-6}} = 2$T

From the characteristic, a flux density of 2T, gives an H value of 24000At/m.

Thus m.m.f. required for teeth = $24\,000 \times 45 \times 10^{-3}$

= 1080At

Total field coil m.m.f. = 3310 + 500 + 1080 = 4890At

Chapter 7

7.1. Dynamic Induction $E = B\ell v$

$$= 40 \times 10^{-6} \times 1.4 \times \frac{100 \times 10^{3}}{3600} \text{ volts} = 1.55 \times 10^{-3} \text{ volts}$$

= 0.001 55V or 1.55mV

7.2. In 1 revolution, the flux cut = $6 \times 64 \times 10^{-3}$ webers

The number of revolutions per second $= \dfrac{1000}{60} = \dfrac{100}{6}$

∴ Flux cut per second $= 6 \times 64 \times 10^{-3} \times \dfrac{10^{2}}{6}$

= generated voltage = 6.4V

The conductors in series per parallel path 468 ...

So voltage per parallel path = 6.4 × 78 = 499.2V

= the e.m.f. of the machine

Current per conductor = current per parallel path = 50A

Current in 6 parallel paths = 50 × 6 = 300A

So power developed $= \dfrac{499.2 \times 300}{1000} = 149.76$kW

Check $E = \dfrac{Z\Phi N}{60} \times \dfrac{P}{A}$

Note. This formula is introduced in Chapter 12. Its use is shown here.

Thus $E = \dfrac{468 \times 64 \times 10^{-3} \times 1000}{60} \times \dfrac{6}{6} = 499.2$V

7.3. Change of flux = $(30 - 2)\,10^{-3}$ weber = 28mWb

Induced e.m.f. = rate of change of flux-linkages

$$= \frac{2000 \times 28 \times 10^{-3}}{0.12} = 466.6\text{V}$$

7.4. Diameter of armature = 0.2m

Circumference = $\pi d = \pi \times 2 \times 10^{-1}$ = 0.628m

In 1 second the armature turns $\dfrac{500}{60}$ revolutions

∴ In 1 second a coil side travels $\dfrac{500}{60} \times 0.628$ metres

or v = 5.233m/s

So $E = B\ell v = 1.2 \times 2 \times 0.4 \times 5.233$ volts = 5.024V

7.5. Let E_1 Φ_1 and N_1 be the values under the original conditions and E_2 Φ_2 and N_2 be the values under the final condition. Here N_1 and N_2 are the speed conditions.

Also for a generator $E \propto \Phi N$ or $E = k\Phi N$

We know that $\Phi \propto B$ and $N \propto v$ thus the more general form $E \propto \Phi N$ (introduced later) can be used. k is a constant.

$$\frac{E_2}{E_1} = \frac{k\Phi_2 N_2}{k\Phi_1 N_1} \quad \text{or} \quad E_2 = \frac{E_1 \Phi_2 N_2}{\Phi_1 N_1}$$

and $E_2 = \dfrac{200 \times 19 \times 10^{-3} \times 1100}{20 \times 10^{-3} \times 1000}$ $E_2 = 209$V

7.6. Diameter of coil = 0.2m

Circumference of coil = $\pi \times 0.2 = 0.628$m

$$\text{Speed} = \frac{1200}{60} = 20\text{rev/s}$$

$\therefore$ velocity of coil side = $20 \times 0.628 = 12.56$m/s

Now $E = B\ell v$ volts

$$= 0.02 \times 0.3\ 0012.56 \text{ volts per conductor}$$

Total e.m.f. = $2 \times 3 \times 10^{-3} \times 12.56 \times 400 = 30.1$V

This is true if the conductors cut the field at right angles and so the maximum value of e.m.f. generated is 30.1V

Time for 1 revolution = $\frac{1}{20}$ seconds . In one revolution one cycle is generated, so the frequency of the generated e.m.f. = 20 cycles per second or, 20 Hertz

7.7. Average e.m.f. in volts = rate of change of flux-linkages

Change of flux = $(4 - 1.5)10^{-3} = 2.5 \times 10^{-3}$ webers

Time for change = 0.04s

So rate of change of flux-linkages = $1200 \times \frac{2.5 \times 10^{-3}}{4 \times 10^{-2}} = 75$V

7.8. Flux per pole = $0.09 \times 0.92 = 8.28 \times 10^{-2}$ webers

In one revolution a conductor cuts $4 \times 8.28 \times 10^{-2}$

$$= 33.12 \times 10^{-2} \text{ webers}$$

Also in 1 second the armature revolves $\frac{600}{60} = 10$ times

So by 1 conductor, the flux cut per second

$$= 33.12 \times 10^{-2} \times 10 = 3.312\text{Wb}$$

and the induced voltage per conductor = 3.312V

The number of armature conductors is $2 \times 210 = 420$

These are arranged in 4 parallel paths.

There are thus $\frac{420}{4} = 105$ conductors in series per parallel path

The e.m.f. of 1 parallel path = $105 \times 3.312 = 347.76$V

The generated e.m.f. = the e.m.f. of 1 parallel path = 347.76V

7.9. (a) Ampere-turns of solenoid = $400 \times 6 = 2400$At

The magnetising force H at the centre = ampere-turns/metre

$$= \frac{2400}{1.5} = 1600\text{At/m}$$

The flux density B at the centre of the solenoid and small coil

$$= \mu_0 H = 4 \times \pi \times 10^{-7} \times 1600 = 64 \times \pi \times 10^{-5}$$

Area of small coil = $\frac{\pi d^2}{4} = \frac{\pi \times (10 \times 10^{-3})^2}{4} = \frac{\pi \times 10^{-4}}{4}$ m^2

So the flux linked = $64 \times \pi \times 10^{-5} \times \frac{\pi \times 10^{-4}}{4}$ weber = 0.158μWb

(b) Average induced e.m.f. = rate of change of flux-linkages

$$= \frac{50 \times 16 \times \pi^2 \times 10^{-9}}{50 \times 10^{-3}} = 0.158\text{mV}$$

7.10. Coil A. Associated flux $\Phi = 18 \times 10^{-3}$ webers

Associated flux-linkages during reversal = turns × flux decrease to zero and then its build up to full value Φ in the *reversed* direction.

$$= 1000\,[0.018 - (-0.018)] = 1000(0.018 + 0.018)$$

$$= 1000 \times 2 \times 0.018 = 36 \text{ weber-turns}$$

Time of reversal = 0.1s

and the induced e.m.f. = rate of change of flux-linkages

$$= \frac{36}{0.1} = 360\text{V (average value)}$$

Coil B. Only 80% flux is associated and there are 500 turns.

∴ Associated flux-linkages during reversal

$$= 500 \times 0.8 \times 2 \times 0.018 = 14.4 \text{ weber-turns}$$

$$\text{Induced e.m.f.} = \frac{14.4}{0.1} = 144\text{V (average value)}$$

Alternatively:

Proportion of e.m.f. in Coil B to e.m.f. in Coil A

$$= 360 \times \frac{500}{1000} = 180\text{V, if full flux is associated}$$

For only 80% flux, e.m.f. is reduced in proportion

$$= 180 \times 0.8 = 144\text{V}$$

Chapter 8

8.1. For a series combination, the equivalent capacitance is given

By C, where $\frac{1}{C} = \frac{1}{0.02} + \frac{1}{0.04}$ (in microfarads)

So $C = \frac{0.04}{3} = 0.0133\mu\text{F}$ Also $Q = CV$

$\therefore Q = 0.0133 \times 10^{-6} \times 10^{2}$ coulombs

$$\text{Then } V_1 = \frac{1.33 \times 10^{-6}}{0.02 \times 10^{-6}} = 66.7\text{V}$$

$$\text{and } V_2 = \frac{1.33 \times 10^{-6}}{0.04 \times 10^{-6}} = 33.3\text{V}$$

The voltage drops are respectively 66.7V and 33.3V

8.2. The final 2 parallel 5μF capacitors are equivalent to 1 unit of 10μF.

The capacitance C of the branch, consists of: 20μF, 10μF and 20μF in series given by:

$$\frac{1}{C} = \frac{1}{20} + \frac{1}{10} + \frac{1}{20} \text{ or } C = 5\mu\text{F}$$

The series circuit is in parallel with a 5μF capacitor with an equivalent capacitance = 10μF. The final arrangement between A and B is equivalent to a 20μF, 10μF and 20μF capacitor in series. The equivalent capacitance is given by:

$$\frac{1}{C} = \frac{1}{20} + \frac{1}{10} + \frac{1}{20} \text{ or } C = \frac{20}{4} = 5\mu\text{F}$$

8.3. Since $Q = CV$. ∴ the quantity of electricity received *initially* is given by $Q = 1000 \times 10^{-6} \times 100 = 10^{5} \times 10^{-6}$

$$= 10^{-1} \text{ coulombs.}$$

Since the plates are separated by an insulated rod there is no loss of charge and hence Q remains the same.

Under the new condition since, as before, $Q = CV$

$$\text{Then } V = \frac{Q}{C} = \frac{10^{-1}}{300 \times 10^{-6}} = 333.3\text{V}$$

Hence the P.D. will increase by $333.3 - 100 = 233.3$V

8.4. The capacitor is made from 10 plates in parallel, making 1 assembly, interleaved with 9 plates in parallel forming the other plate assembly. There will be 18 mica separators or 18 electric fields and the total capacitance will be 18 times the capacitance between 1 pair of plates.

$$\text{Thus } C \text{ of 1 pair of plates} = \frac{\epsilon A}{d} = \frac{\epsilon_o \epsilon_r A}{d}$$

$$\text{or } C = \frac{8.85 \times 10^{-12} \times 7 \times 2580 \times 10^{-6}}{0.1 \times 10^{-3}}$$

$$= 1.6 \times 10^{-9} \text{ farads}$$

or with 18 units in parallel $C = 18 \times 1.6 \times 10^{-9}$ farads

$$= 0.0288\mu\text{F}$$

8.5. Since $Q = CV$, then $Q = 3 \times 10^{-4} \times 10^{-6} \times 10 \times 10^3$

$$= 3 \times 10^{-6} \text{ coulombs}$$

$$\therefore \text{ Flux density, } D = \frac{Q}{A} = \frac{3 \times 10^{-6}}{10000 \times 10^{-6}}$$

$$= 3 \times 10^{-4} \text{ coulomb per m}^2$$

$$\text{Also, permittivity, } \in = \frac{\text{electricity flux density}}{\text{electric force}} = \frac{D}{E}$$

$$\text{And electric force, } E = \frac{V}{d} = \frac{10 \times 10^3}{1 \times 10^{-3}} 10 \times 10^6 \text{ volts m}^{-1}$$

$$\text{Hence } \in = \frac{3 \times 10^{-4}}{10 \times 10} \text{ also } \in = \in_o \times \in_r$$

$$\text{And } \in_r = \frac{\in}{\in_o} = \frac{3 \times 10^{-4}}{10 \times 10^6 \times 8.85 \times 10^{-12}}$$

or $\in_r = 3.39$

8.6. $C = \dfrac{\in A}{d}$ $A = 6 \times 10^4 \times 10^{-6}\text{m}^2 = 6 \times 10^{-2} \text{ m}^2$

$$d = 3.5 \times 10^{-3} \text{ m}$$

and $\in = \in_o \times \in_r = 8.85 \times 10^{-12} \times 3$

$$\text{Hence } C = \frac{8.85 \times 10^{-12} \times 3 \times 6 \times 10^{-2}}{3.5 \times 10^{-3}} = 4.55 \times 10^{-10} \text{ F}$$

$$\text{Energy, } W = \frac{1}{2} CV^2 \text{ joules}$$

$$= \frac{1}{2} \times 4.55 \times 10^{-10} \times 300^2$$

$$= 20.475 \times 10^{-6} \text{ joules} = 20.48 \mu\text{J}$$

8.7. A 10-plate capacitor is made from two 5-plate assemblies interleaved with each other and separated by the dielectric. There are thus 9 electric fields or the final capacitance is 9 times that of 1-plate arrangement.

$$\text{Hence } C = \frac{\in A}{d} \text{ where } A = 1500 \times 10^{-6}\text{m}^2$$

$$d = 0.3 \times 10^{-3} \text{ metres}$$

$$\in = \in_o \times \in_r$$

$$C = \frac{8.85 \times 10^{-12} \times 4 \times 15 \times 10^{-4}}{3 \times 10^{-4}} = 1.77 \times 10^{-10} \text{ farads}$$

$$= 1.77 \times 10^{-4} \text{ microfarads}$$

$$\text{Total capacitance} = 9 \times 1.77 \times 10^{-4} = 0.0016\mu\text{F}$$

8.8. Let C = capacitance of the series arrangement,

$$\text{then } \frac{1}{C} = \frac{1}{20} + \frac{1}{30} = \frac{5}{60} \text{ or } C = 12\mu\text{F}$$

The charge stored is given by $Q = CV = 12 \times 10^{-6} \times 600$

$$= 72 \times 10^{-4} \text{ coulombs.}$$

$$\text{P.D. across } 20\mu\text{F capacitor A} = \frac{7.2 \times 10^{-3}}{20 \times 10^{-6}} = 360\text{V}$$

P.D. across 30μF capacitor B = 600 − 360 = 240V

If P.D. across B is 400V then P.D. across the parallel arrangement will be 200V. The equivalent capacitance must be 60μF (double) since the voltage is half that across B. C must be 30μF, being in parallel with A.

$$\text{Also the energy stored, } W = \frac{1}{2} CV^2$$

$$= \frac{1}{2} \times 40 \times 10^{-6} \times 200^2 \text{ joules. Thus } W = 0.8\text{J}$$

8.9. Since

$Q = CV$ and $Q = It$ then $It = CV$

or $I = C \dfrac{V}{t}$ where V = the voltage change then,

a. $I = 40 \times 10^{-6} \times \dfrac{100}{1 \times 10^{-3}}$ amperes $= 4\text{A}$

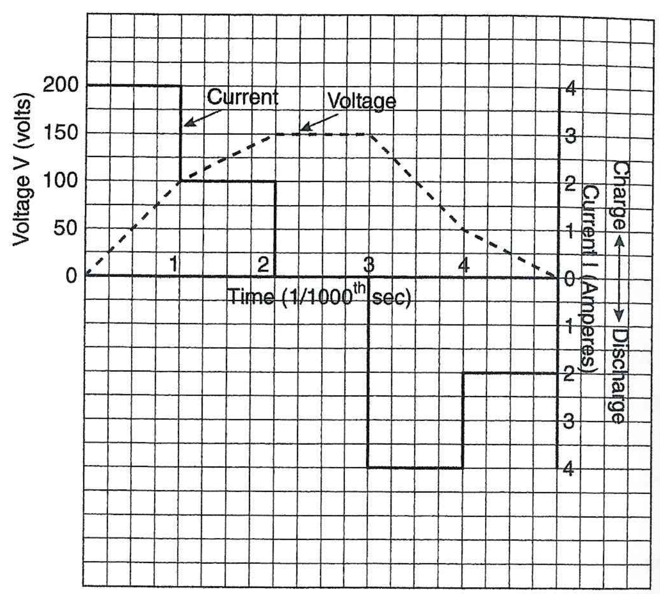
b. $I = 40 \times 10^{-6} \times \dfrac{50}{1 \times 10^{-3}}$ amperes $= 2A$

c. $I = 40 \times 10^{-6} \times \dfrac{0}{1 \times 10^{-3}}$ amperes $= 0A$

d. $I = 40 \times 10^{-6} \times \dfrac{100}{1 \times 10^{-3}}$ amperes $= 4A$

e. $I = 40 \times 10^{6} \times \dfrac{50}{1 \times 10^{-3}}$ amperes $= 2A$

Figure 13 shows the current and voltage conditions.



▲ **Figure 13**

8.10. The mean diameter of the insulation = 10 + 12 = 22mm

The dielectric area = mean circumference × length

$= \pi d \times 1000\ m^2$

$= \pi \times 22 \times 10^{-3} \times 10^3$

$= 69.1\ m^2$

Also, from the above,

$$D = \frac{CV}{A} = \frac{0.289 \times 10^6 \times 11 \times 10^3}{69.1} \text{ coulomb m}^{-2}$$

$$= 46 \times 10^{-6}\ C/m^2$$

Again $\in = \dfrac{D}{E} = \dfrac{46 \times 10^{-6}}{11 \times 10^5}$

Also $\in = \in_0 \times \in_r$

$$\therefore \in_r = \frac{46 \times 10^{-6}}{11 \times 10^{-5} \times 8.85 \times 10^{-12}}$$

$$\therefore \in_r = 4.73$$

Chapter 9

9.1. A scale of 10mm = 1A is used and I_1 is the reference phasor drawn horizontally. The diagram, drawn geometrically to scale, shows the solution with I the resultant current = 9.23A lagging I_1, by 6°.

If the above is checked mathematically

$I_H = 4 \cos 0 + 6 \cos 30 + 2 \cos 90$

$= (4 \times 1) + (6 \times 0.866) + (2 \times 0)$

$= 4 + 5.196 = 9.196A$

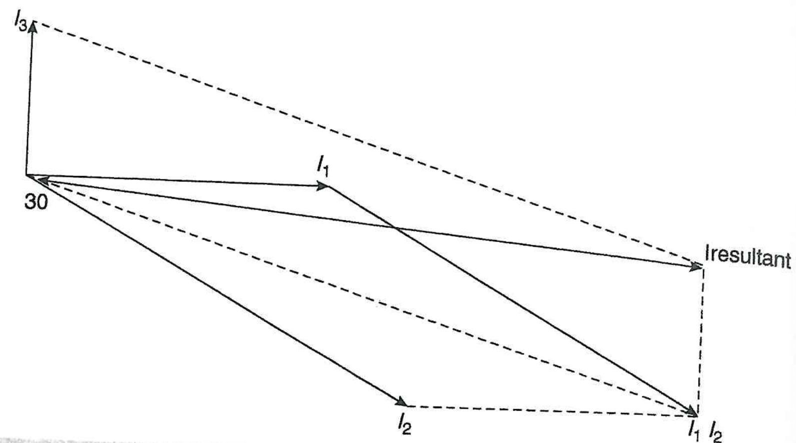
$I_V = 4 \sin 0 - 6 \sin 30 + 2 \sin 90$

$= (4 \times 0) - (6 \times 0.5) + (2 \times 1) = 0 - 3 + 2$

$= -1A$

$I = \sqrt{(9.196 \times 9.196 + 1)} = 9.24A$

$$\text{Cos } \theta = \frac{9.2}{9.24} = 0.995 \text{ and } \theta = 6° \text{ (approx.)}$$



▲ Figure 14

9.2. Since $v = V_m \sin(2\pi ft)$

$$\therefore \sin(2\pi ft) = \frac{200}{282.8} = 0.707$$

Now the angle whose sine is 0.707 is = 45°

$\therefore 2\pi ft = 45°$

$$\text{or } t = \frac{45}{22 \times 180 \times 25}$$

$= 0.005$ seconds = 5ms

(a) The first time is 5ms after zero value

(b) Time for 1 cycle $= \frac{1}{25} = 0.04\text{s}$

Time for $\frac{1}{2}$ cycle = 0.02s

The second time is 0.02 – 0.005 = 0.015s or 15ms after zero value

9.3. A simple phasor diagram illustrates the problem and its mathematical solution.

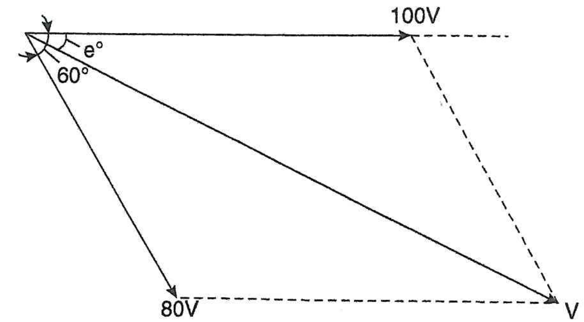
Horizontal component $V_H = 100 + 80 \cos 60$

$= 100 + 80 \times 0.5 = 140\text{V}$

Vertical component $V_v = 0 - 80 \sin 60$

$= 0 - 80 \times 0.866$

$= -69.28\text{V}$



▲ Figure 15

Resultant $V = \sqrt{140^2 + 69.28^2}$

$= 156.3\text{V}$

$$\cos\theta = \frac{140}{156.3}\ 0.8955. \quad \therefore \ \theta = 26°26'$$

Since maximum values were used for the phasor the resultant is a maximum which lags the 100V values by 26°26′

9.4. The waveform is shown in figure 16. Erecting mid-ordinates, measuring and squaring these gives the following columns.

$i_1 = 0.22 \quad i_1^2 = 0.05$

$i_2 = 0.60 \quad i_2^2 = 0.36$

$i_3 = 0.92 \quad i_3^2 = 0.85 \qquad$ Total of $i_2 = 19.17$

$i_4 = 1.25 \quad i_4^2 = 1.56 \qquad$ Average of $i^2 = \frac{19.17}{10}$

$i_5 = 1.55 \quad i_5^2 = 2.40 \qquad = 1.917$

$i_6 = 1.8 \quad i_6^2 = 3.24 \qquad \therefore$ r.m.s. value $= \sqrt{1.917}$

$i_7 = 1.97 \quad i_7^2 = 3.87 \qquad = 1.385\text{A}$

$i_8 = 1.92 \quad i_8^2 = 3.68$

$i_9 = 1.56 \quad i_9^2 = 2.44 \qquad$ Power dissipated obtained from

$i_{10} = 0.85 \quad i_{10}^2 = 0.72 \qquad I^2R = 1.917 \times 8 = 15.34\text{W}$