EXAMPLE 8.8

The three-phase induction motor of Example 8.6 is connected to a three-phase source at 480 V. Determine the following:

a. Slip and motor speed at which maximum torque occurs

b. Rotor current at this slip

c. Maximum torque

Solution From Example 8.6, the machine constants are given as follows:

$$R_1 = 0.100\ \Omega \qquad X_1 = 0.35\ \Omega$$
$$R_2 = 0.125\ \Omega \qquad X_2 = 0.40\ \Omega$$

The terminal voltage per phase is taken as reference phasor; thus,

$$\mathbf{V}_1 = (480/\sqrt{3})\,\underline{/0^\circ} = 277.1\,\underline{/0^\circ}\ \text{V} \quad \text{(line-to-neutral)}$$

The synchronous speed is

$$n_s = 120f/p = (120)(60)/4 = 1800\ \text{rpm}$$

a. By using Eq. 8.39, the slip at maximum torque is found as

$$s_{max} = \frac{0.125}{\sqrt{(0.10)^2 + (0.35 + 0.40)^2}} = 0.165$$

Therefore, the speed at maximum torque is

$$n_{max} = (1 - s_{max})n_s = (1 - 0.165)1800 = 1503\ \text{rpm}$$

b. The rotor current at s_{max} is given by

$$I_{2,max} = \frac{V_1}{\sqrt{(R_1 + R_2/s_{max})^2 + (X_1 + X_2)^2}}$$
$$= \frac{277.1}{\sqrt{(0.10 + 0.125/0.165)^2 + (0.35 + 0.40)^2}} = 243.2\ \text{A}$$

c. The maximum torque is found by using Eq. 8.40. Thus,

$$T_{e,max} = \frac{3(277.1)^2}{2(188.5)[0.10 + \sqrt{(0.10)^2 + (0.35 + 0.40)^2}]} = 713\ \text{N-m}$$

EXAMPLE 8.9

The three-phase, 25-hp, 440-V induction motor of Example 8.2 is operated at rated voltage and rated frequency. Determine the following:

a. Slip at maximum torque

b. Maximum torque

c. Value of slip when the rotor resistance is doubled

d. Value of maximum torque when rotor resistance is doubled

Solution

a. The approximate per-phase equivalent circuit is shown in Fig. 8.12. The rated terminal voltage per phase is

$$V_1 = 440/\sqrt{3} = 254.0\ \text{V} \quad \text{(line-to-neutral)}$$

The slip s_{max} at maximum torque is given by

$$s_{max} = \frac{R_2}{\sqrt{R_1^2 + (X_1 + X_2)^2}}$$
$$= \frac{0.35}{\sqrt{(0.50)^2 + (1.20 + 1.20)^2}} = 0.143$$

b. The rotor current $I_{2,max}$ at s_{max} is computed as

$$I_{2,max} = \frac{V_1}{\sqrt{(R_1 + R_2/s_{max})^2 + (X_1 + X_2)^2}}$$
$$= \frac{254.0}{\sqrt{(0.50 + 0.35/0.143)^2 + (1.20 + 1.20)^2}} = 66.8\ \text{A}$$

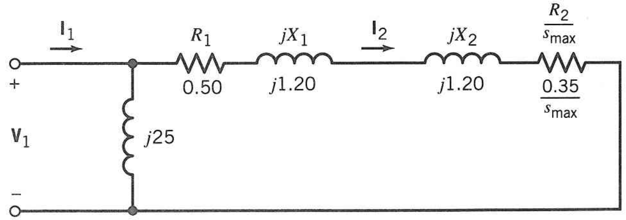

FIGURE 8.12 Approximate equivalent circuit for motor of Example 8.9.

Therefore, the maximum torque $T_{e,max}$ is calculated as

$$T_{e,max} = \frac{3I^2_{2,max}(R_2/s_{max})}{\omega_s}$$

$$= \frac{3(66.8)^2(0.35/0.143)}{188.5} = 174 \text{ N-m}$$

c. When the rotor resistance is doubled, the new value of slip s_{max} at maximum torque is given by

$$s_{max} = \frac{2(0.35)}{\sqrt{(0.50)^2 + (1.20 + 1.20)^2}} = 0.286$$

d. When the rotor resistance is doubled, the new value of the rotor current $I_{2,max}$ is found by using the value of s_{max} computed in part (c). Thus,

$$I_{2,max} = \frac{254.0}{\sqrt{(0.50 + 0.70/0.286)^2 + (1.20 + 1.20)^2}} = 66.8 \text{ A}$$

Therefore, the new value of maximum torque $T_{e,max}$ is found as

$$T_{e,max} = \frac{3I^2_{2,max}(R_2/s_{max})}{\omega_s}$$

$$= \frac{3(66.8)^2(0.70/0.286)}{188.5} = 174 \text{ N-m}$$

DRILL PROBLEMS

D8.13 A three-phase, 20-hp, 440-V, six-pole, 60-Hz, wye-connected induction motor has a starting torque of 98 N-m and a full-load torque of 72 N-m. Calculate (a) the starting torque when the applied voltage is reduced to 300 V and (b) the applied voltage in order to develop a starting torque equal to the full-load torque.

D8.14 For the motor in Problem D8.11, determine (a) the slip at pullout (maximum) torque and (b) the pullout torque.

D8.15 A three-phase, 10-hp, 230-V, four-pole, 60-Hz, wye-connected induction motor develops its full-load torque at a slip of 4.5% when operating at 230 V and 60 Hz. The per-phase equivalent circuit impedances of the motor are

$$R_1 = 0.35\ \Omega \qquad X_m = 15.0\ \Omega$$
$$X_1 = 0.50\ \Omega \qquad X_2 = 0.50\ \Omega$$

The mechanical and core losses are assumed to be negligible. Determine the following:

a. Rotor resistance R_2

b. Slip at maximum torque

c. Maximum torque

d. Rotor speed at maximum torque

e. The starting torque of this motor

8.5 SINGLE-PHASE INDUCTION MOTORS

One of the most common types of residential and commercial loads is the single-phase induction motor. These single-phase motors are rated at less than one horsepower (1 horsepower = 746 watts). Many different designs are available. In the home, these motors are most commonly used as fans and refrigerator compressors.

Compared to a three-phase induction motor, the fractional-horsepower motor is much simpler in construction but much more difficult to analyze. Much of the design of the single-phase motor has been done by building and testing prototype motors until the desired characteristics are obtained. Many different types of single-phase motors have been built in order to match the varying torque requirements of various appliances. Differing cost is another reason for the availability of numerous different types of motors. The chief difference between the various types of single-phase motors lies in the method for starting them. For more details of the types of single-phase induction motors, see, for example, Ref. 5.

In this section, the equivalent circuit of a single-phase induction motor is developed. This equivalent circuit is valid for all types of single-phase motors.

8.5.1 Equivalent Circuit and Performance Analysis

A single-phase motor receives its power from a 60-Hz, single-phase source of electric power. This source causes a current to flow in the stator winding. In Chapter 5, the mmf of such a current was analyzed. It was determined that this mmf can be resolved into two revolving mmfs, rotating in opposite directions. The rotor is rotating at a speed of n_r. The component mmf rotating in the same direction as the rotor is called the forward-revolving field, and the oppositely rotating mmf is called the backward-rotating field. Each rotating mmf induces a voltage in the rotor winding. Therefore, two equivalent circuits are built: one for the forward component mmf and one for the backward-rotating component field. Then, the two component fields are combined, and the two equivalent circuits are interconnected.

The forward-rotating component field rotates at synchronous speed n_s in the same direction as the rotor. Therefore, the slip s^+ of the rotor with respect to the forward-rotating field may be expressed as

$$s^+ = s = \frac{n_s - n_r}{n_s} = 1 - \frac{n_r}{n_s} \tag{8.42}$$

As shown in Chapter 5, the amplitude of the forward component mmf is one half of the stator mmf. Hence, one half of the stator current may be associated with the forward mmf. The equivalent circuit for this situation is similar to that of three-phase induction motor as shown in Fig. 8.1, with the modification that the core loss represented by R_c is omitted from the equivalent circuit of the single-phase machine. The core loss is treated separately and is usually lumped with the rotational losses. Thus, the equivalent circuit is as shown in Fig. 8.13. In the diagram, $\mathbf{V}_1^+$ is the stator voltage corresponding to the forward component.

Next, the backward component field is considered. The stator current corresponding to this field is one half of the stator current, that is, $\frac{1}{2}I_1$. Because the rotor and the backward fields are rotating in opposite directions, the slip s^- of the rotor with respect to the backward-rotating field is expressed as

$$s^- = \frac{n_s - (-n_r)}{n_s} = 1 + \frac{n_r}{n_s} \tag{8.43}$$

Substituting $s^+ = s$ from Eq. 8.42 into Eq. 8.43 yields

$$s^- = 2 - s^+ \tag{8.44}$$

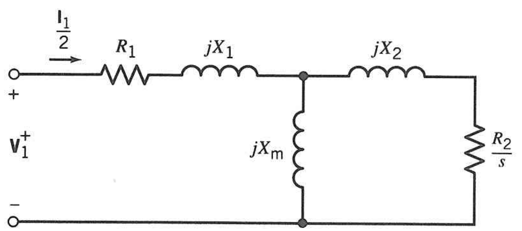

FIGURE 8.13 Equivalent circuit corresponding to forward mmf.

The equivalent circuit corresponding to the backward-revolving field is shown in Fig. 8.14.

The terminal voltage may be expressed in terms of its components:

$$\mathbf{V}_1 = \mathbf{V}_1^+ + \mathbf{V}_1^- \tag{8.45}$$

Since one half of the current $\mathbf{I}$ flowing in an impedance Z has the same performance effect as the current $\mathbf{I}$ flowing in the impedance $\frac{1}{2}Z$, the equivalent circuits shown in Figs. 8.13 and 8.14 may be combined to form the overall equivalent circuit shown in Fig. 8.15. In this equivalent circuit, the resistances $\frac{1}{2}R_1$ and $\frac{1}{2}R_1$ are combined into R_1, and $\frac{1}{2}X_1$ and $\frac{1}{2}X_1$ are likewise combined into X_1. This equivalent circuit can be used to analyze the performance of a single-phase induction motor.

EXAMPLE 8.10

A single-phase, $\frac{1}{4}$-hp, 110-V, four-pole, 60-Hz induction motor has the following parameters:

$$R_1 = 2.0\ \Omega \qquad R_2 = 4.0\ \Omega$$
$$X_1 = 2.5\ \Omega \qquad X_2 = 2.5\ \Omega \qquad X_m = 50.0\ \Omega$$

The core loss is 25 W, and the mechanical (friction and windage) losses are 15 W. The motor operates at rated voltage and rated frequency at a slip of 5%. Determine the following:

- **a.** Motor speed
- **b.** Input current
- **c.** Output power
- **d.** Efficiency of the motor
- **e.** Output torque

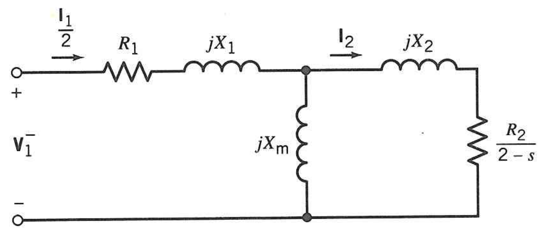

FIGURE 8.14 Equivalent circuit corresponding to backward mmf.

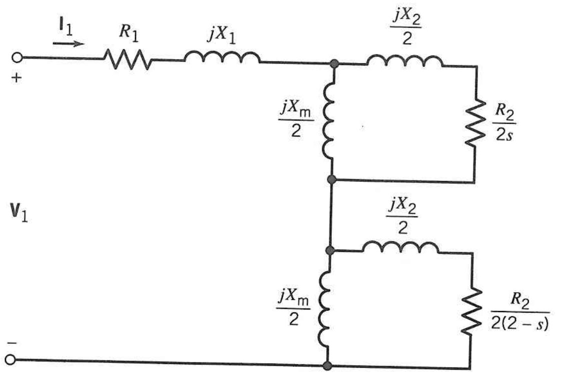

FIGURE 8.15 Single-phase induction motor equivalent circuit.

Solution

a. The motor speed is given by

$$n_r = (1 - s)n_s = (1.0 - 0.05)1800 = 1710 \text{ rpm}$$
$$\omega_r = 2\pi(1710)/60 = 179.1 \text{ rad/s}$$

b. The equivalent circuit for the single-phase motor is shown in Fig. 8.16. The applied voltage $\mathbf{V}_1$ is taken as reference phasor; thus,

$$\mathbf{V}_1 = 110\underline{/0^\circ}$$

The input impedance is found as follows:

$$\begin{aligned} Z_{in} &= (2.0 + j2.5) + [j25 \| (40 + j1.25)] \\ &\quad + [j25 \| (1.0256 + j1.25)] \\ &= 13.85 + j21.56 = 25.62\underline{/57.3^\circ} \end{aligned}$$

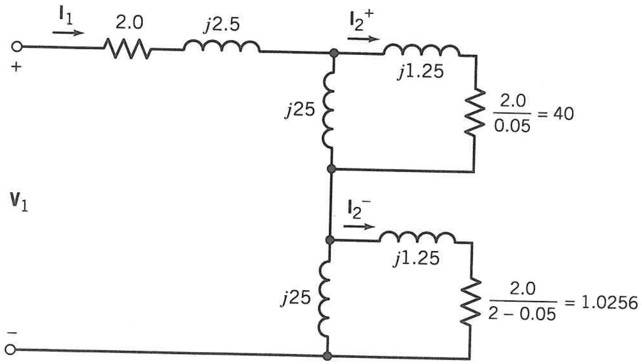

FIGURE 8.16 Equivalent circuit for the single-phase ...

Therefore, the input current is found as follows:

$$\mathbf{I}_1 = \frac{110\underline{/0^\circ}}{Z_{in}} = \frac{110\underline{/0^\circ}}{25.62\underline{/57.3^\circ}} = 4.29\underline{/-57.3^\circ}$$

The components of the rotor current are computed as follows:

$$\mathbf{I}_2^+ = \mathbf{I}_1[j25/(j25 + 40 + j1.25)] = 2.24\underline{/-0.6^\circ}$$
$$\mathbf{I}_2^- = \mathbf{I}_1[j25/(j25 + 1.0256 + j1.25)] = 4.08\underline{/-55.1^\circ}$$

c. The input power factor is

$$\text{PF} = \cos 57.3^\circ = 0.54 \text{ lagging}$$

Thus, the power input to the motor is given by

$$P_{in} = V_1 I_1 \text{PF} = (110)(4.29)(0.54) = 254.8 \text{ W}$$

The losses of the motor include the following:

$$\begin{aligned} \text{Stator copper loss} &= R_1 I_1^2 = (2.0)(4.29)^2 = 36.8 \text{ W} \\ \text{Rotor copper loss} &= (R_2/2)(I_2^+)^2 + (R_2/2)(I_2^-)^2 \\ &= (2.0)(2.24)^2 + (2.0)(4.08)^2 = 43.3 \text{ W} \\ \text{Core loss} &= 25.0 \text{ W} \\ \text{Mechanical losses} &= 15.0 \text{ W} \end{aligned}$$

The sum of the above losses amounts to

$$\Sigma(\text{losses}) = 36.8 + 43.3 + 25.0 + 15.0 = 120.1 \text{ W}$$

Therefore, the power output of the motor is given by

$$P_{out} = P_{in} - \Sigma(\text{losses}) = 254.8 - 120.1 = 134.7 \text{ W}$$

d. The efficiency of the motor is computed as

$$\eta = P_{out}/P_{in} = (134.7/254.8)100\% = 53\%$$

e. The output torque of the motor is given by

$$T_{out} = P_{out}/\omega_r = 134.7/179.1 = 0.752 \text{ N-m}$$

DRILL PROBLEMS

D8.16 A single-phase, 60-Hz induction motor has six poles and runs at 1020 rpm. Determine (a) the slip with respect to the forward-rotating field and (b) the slip with respect to the backward-rotating field.

D8.17 A single-phase, 110-V, four-pole, 60-Hz induction motor is operating at 1710 rpm. The parameters of the motor equivalent circuit are $R_1 = R_2 = 8\ \Omega$, $X_1 = X_2 = 15\ \Omega$, and $X_m = 120\ \Omega$. Determine the following:

a. Line current

b. Power factor

c. Developed torque

8.5.2 Starting Single-Phase Induction Motors

Single-phase induction motors are classified according to the method used to produce their starting torque. These starting techniques differ in cost and in the amount of starting torque produced. There are three major starting techniques:

a. Split-phase windings

b. Capacitor type

c. Shaded pole

Split-Phase Motors A split-phase motor has two stator windings, a main stator winding (M) and an auxiliary starting winding (A), with their axes displaced 90 electrical degrees in space. The auxiliary winding is designed to be switched out of the circuit at some set speed by a centrifugal switch. This winding has a higher R/X ratio than the main winding, so its current leads the main winding current. The higher R/X ratio is accomplished by using smaller wire for the auxiliary winding. A schematic diagram of a split-phase winding induction motor is shown in Fig. 8.17.

The current in the auxiliary winding always peaks before the current in the main winding, and therefore the magnetic field of the auxiliary winding peaks before the magnetic field from the main winding. The direction of rotation of the motor is determined by whether the space angle of the magnetic field from the auxiliary winding is 90° ahead or 90° behind the angle of the main winding. That angle can be changed from 90° ahead to 90° behind just by switching connections on the auxiliary winding while the main winding connection is unchanged; then the direction of rotation of the motor is reversed.

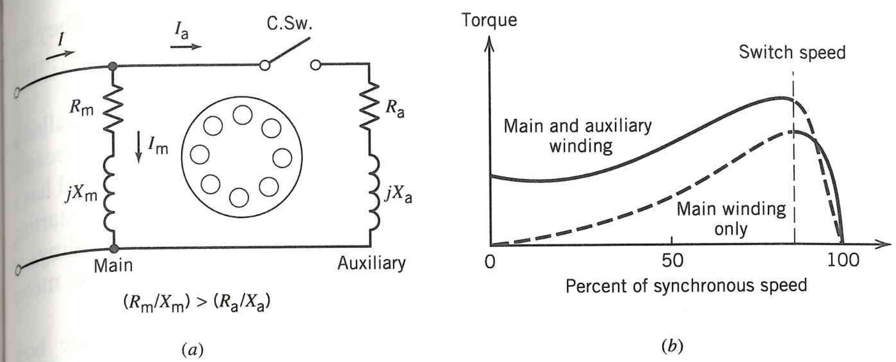

FIGURE 8.17 Split-phase induction motor: (*a*) connection; (*b*) torque-speed characteristic.

Split-phase motors have a moderate starting torque with a fairly low starting current. They are used for applications that do not require high starting torques such as fans, blowers, and centrifugal pumps. They are available for sizes in the fractional-hp range and are quite inexpensive.

Capacitor-Start Motors A capacitor is placed in series with the auxiliary winding of the motor. By proper selection of capacitor size, the mmf of the starting current in the auxiliary winding can be adjusted to be equal to the mmf of the current in the main winding and the phase angle of the current in the auxiliary winding can be made to be 90° leading the current in the main winding. Since the two windings are physically separated, the 90° phase difference in current will yield a single rotating stator magnetic field, and the motor will behave as though it were starting from a three-phase supply. In this case, the starting torque of the motor can be more than 300% of its rated value. A schematic diagram of a capacitor-start induction motor is shown in Fig. 8.18.

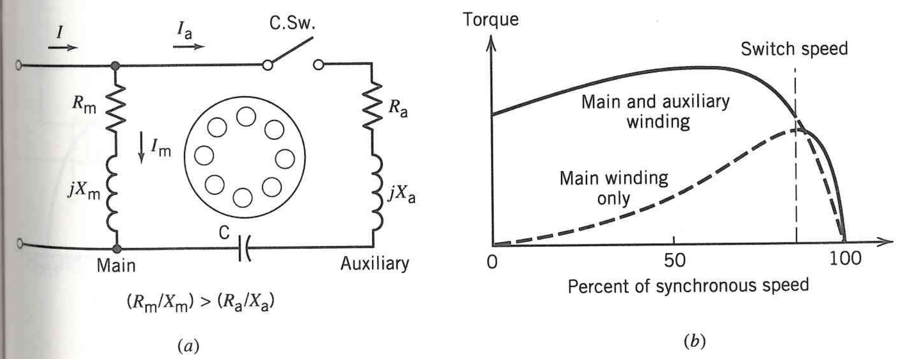

FIGURE 8.18 Capacitor-start induction motor: (*a*) connection; (*b*) torque-speed characteristic.

Capacitor-start motors are more expensive than split-phase motors. They are used in applications in which a high starting torque is absolutely required, such as in compressors, air conditioners, and pumps.

If the capacitor is left permanently in the motor circuit, the motor is called a permanent split-capacitor or capacitor-start-and-run motor. It is simpler because the starting switch is not needed. At normal loads, it is more efficient and has a higher power factor and a smoother torque characteristic. It has a lower starting torque because the capacitor is sized to balance the currents in the windings at normal load. A schematic diagram of a capacitor-start-and-run induction motor is shown in Fig. 8.19.

If the largest starting torque and the best running conditions are both needed, two capacitors are used with the auxiliary winding. This motor is called a capacitor-start–capacitor-run, or two-value capacitor, induction motor. The larger capacitor ensures that the currents in the two windings are balanced during starting, yielding very high torques. When the motor gets up to speed, the centrifugal switch opens and only the smaller capacitor, which is just large enough to balance currents at normal loads, remains connected. Thus, the motor operates at high power factor and high torque. The permanent capacitor is 10% to 20% of the starting capacitor. A capacitor-start–capacitor-run induction motor is shown in Fig. 8.20.

The direction of rotation of any capacitor-type single-phase induction motor may be reversed by switching the connections of its auxiliary winding.

Shaded-Pole Motors A shaded-pole induction motor has only one (main) winding. It has salient poles; one portion of each pole is surrounded by a short-circuited coil called a shading coil. The varying flux in the poles produced by the main winding induces a voltage and a current (in the shading coil) that opposes the original change in flux. This opposition retards the flux changes

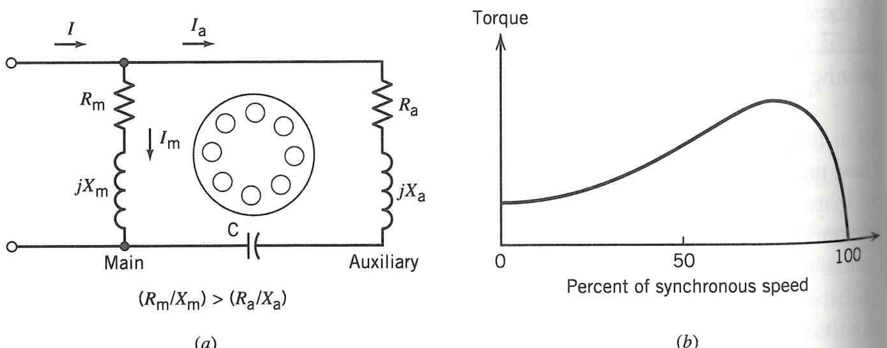

FIGURE 8.19 Capacitor-start-and-run induction motor: (*a*) connection; (*b*) torque-speed characteristic.

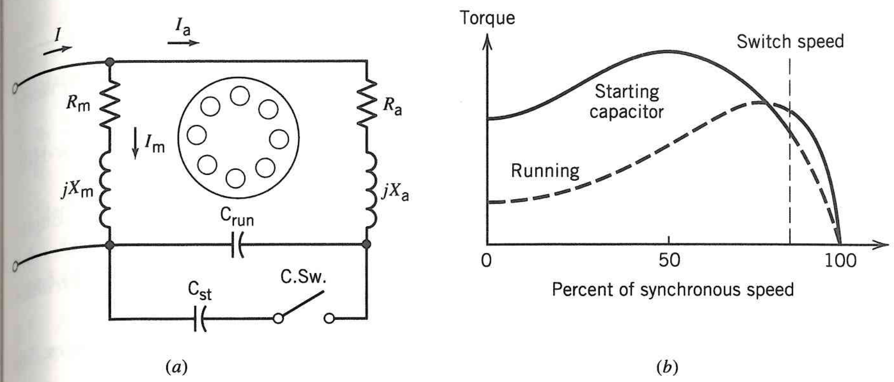

FIGURE 8.20 Capacitor-start–capacitor-run induction motor: (*a*) connection; (*b*) torque-speed characteristic.

under the shaded portion and produces an imbalance between the stator fields. The net torque and net rotation are in the direction from the unshaded to the shaded portion of the pole face.

Shaded-pole motors produce the least starting torque. They are less efficient and have much higher slip. They are also the cheapest design available. The direction of rotation cannot be reversed. A schematic diagram of a shaded-pole induction motor is shown in Fig. 8.21.

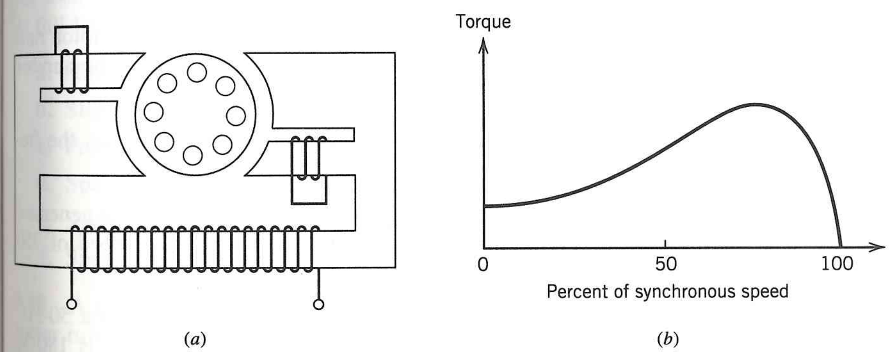

FIGURE 8.21 Shaded-pole induction motor: (*a*) connection; (*b*) torque-speed characteristic.

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PROBLEMS

8.1 A three-phase, 100-hp, 480-V, 60-Hz, squirrel-cage-rotor induction motor runs with no load on the shaft and is observed to run at 1194 rpm. Determine (a) the number of poles and (b) the frequency of the rotor currents at no load.

8.2 At full load, the motor of Problem 8.1 slows down to 1146 rpm. Find the frequency of the rotor currents.

8.3 A synchronous generator has four poles and is running at 1500 rpm. The generator supplies a six-pole induction motor, which is used to drive a load at a speed of 750 rpm. Determine the frequency of the rotor current of the motor.

8.4 A three-phase, six-pole induction motor is operating at 960 rpm from a 50-Hz, 230-V supply. The voltage induced in the rotor when the motor is blocked is 180 V. Determine

a. Slip speed
b. Rotor frequency
c. Rotor induced voltage at 960 rpm

8.5 A three-phase, 100-hp induction motor operates at rated load and a speed of 855 rpm when it is connected to a 440-V, 60-Hz source. The slip at this load is 0.05. Determine

a. Synchronous speed
b. Number of stator poles
c. Rotor frequency

8.6 A three-phase, 60-Hz, six-pole, wye-connected induction motor has a full-load speed of 1140 rpm. Calculate the (a) full-load slip and (b) slip rpm.

8.7 A three-phase, 15-hp, 208-V, 60-Hz, wye-connected induction motor runs at 1728 rpm when it delivers rated output power. Determine the following:

a. Number of poles of the machine
b. Slip at full load
c. Frequency of the rotor currents
d. Speed of the rotor field with respect to the stator
e. Speed of the rotor field with respect to the stator rotating field

8.8 A three-phase, 480-V, four-pole, 50-Hz induction motor operates at a slip of 3.5%. Determine the following:

a. Speed of the stator and rotor magnetic fields
b. Speed of the rotor
c. Slip speed of the rotor
d. Rotor frequency

8.9 A three-phase induction motor is supplied with power from a 60-Hz source. At full load the motor speed is 1728 rpm, and at no load the speed is nearly 1800 rpm. At full-load conditions, determine the following:

a. Number of poles
b. Slip
c. Frequency of the rotor voltages
d. Speed of the rotor field with respect to the rotor
e. Speed of the rotor field with respect to the stator
f. Speed of the rotor field with respect to the stator field

8.10 A three-phase induction motor is connected to a three-phase 60-Hz source. The motor runs at almost 1200 rpm at no load and at 1140 rpm at full load. Determine the following:

a. Number of poles
b. Slip at full load
c. Frequency of the rotor voltages

d. Speed of the rotor field with respect to the rotor

e. Speed of the rotor field with respect to the stator

8.11 A three-phase, 220-V, four-pole, 60-Hz induction motor has the following parameters referred to the stator.

$$R_1 = 0.3\ \Omega \qquad R_2 = 0.2\ \Omega$$
$$X_1 = 0.5\ \Omega \qquad X_2 = 0.5\ \Omega \qquad X_m = 15\ \Omega$$

The total rotational loss is 500 W. For a slip of 5%, calculate:

a. The motor speed

b. The input current

c. The input power factor

d. The shaft torque

e. The efficiency of the motor

8.12 A three-phase, 440-V, four-pole, 60-Hz, wye-connected induction motor takes a stator current of 50 A at 0.8 power factor while operating at a slip of 5%. The stator copper loss is 2500 W, and the total rotational losses are 3200 W. Calculate the efficiency of the motor.

8.13 A three-phase, 440-V, four-pole, 60-Hz induction motor produces 100 hp at the shaft at 1728 rpm. Determine the efficiency of the motor if rotational losses are 3200 W and stator copper losses are 2700 W.

8.14 A three-phase, 440-V, six-pole, 60-Hz, wye-connected induction motor takes 30 kVA at 0.8 power factor, and it runs at a slip of 3.5%. The stator copper losses are 500 W, and the rotational losses are 350 W. Compute the following:

a. Rotor copper losses

b. Shaft output torque and hp

c. Efficiency of the motor

8.15 A three-phase, 60-Hz, six-pole, wye-connected induction motor is rated at 20 hp and 440 V. The motor operates at rated conditions and a slip of 5%. The friction and windage losses are 250 W, and the core losses are 225 W. Find the following:

a. Shaft speed

b. Output power

c. Load torque

d. Induced torque

8.16 A three-phase, 440-V, 60-Hz, six-pole induction motor supplies an output power of 36 kW when operating at 1158 rpm. The combined rotational loss is 1.5 kW, the stator copper loss is 1.2 kW, and the input power factor is 0.866. Determine the following:

a. Copper losses of the rotor circuit

b. Stator current

c. Total input power

8.17 The following test results are obtained for a three-phase, 100-hp, 440-V, eight-pole, wye-connected induction machine.

	No Load (at 60 Hz)	Blocked Rotor (at 15 Hz)	DC
Voltage (V)	440	80	8
Current (A)	38	110	50
Power (W)	3800	4800	

Determine the parameters of the equivalent circuit.

8.18 A three-phase, 20-hp, 440-V, 60-Hz, wye-connected induction motor, operating at rated conditions, draws a line current of 18 A. Data from blocked-rotor and no-load tests at rated frequency and from a DC test are as follows:

	No Load	Blocked Rotor	DC
Voltage (V)	440	62	25
Current (A)	9	24	34
Power (W)	3750	1800	

Determine R_1, R_2, X_1, X_2, X_m, and $P_{\text{rotational}}$.

8.19 A three-phase induction motor is subjected to no-load and blocked-rotor tests, and the following data are obtained.

	No-Load Test (at 60 Hz)	Blocked-Rotor Test (at 15 Hz)
Voltage (V)	440	150
Current (A)	10	40
Power (W)	4500	6100

The stator resistance between any two leads is 1.2 Ω. Determine the parameters of the equivalent circuit of the motor.

8.20 A three-phase, 15-hp, 440-V, 60-Hz, wye-connected induction motor draws a line current of 16 A when operating at rated load conditions. No-load and blocked-rotor tests at rated frequency and a DC test provide the following data:

	No Load	Blocked Rotor	DC
Voltage (V)	440	35	6
Current (A)	6	16	12
Power (W)	1200	750	

Determine the parameters of the equivalent circuit of the motor.

8.21 A three-phase, 100-hp, 2400-V, 6-pole, 60-Hz induction motor is tested, and the following data are obtained:

	No-Load Test (at 60 Hz)	Blocked-Rotor Test (at 15 Hz)
Voltage (V)	2400	420
Current (A)	6	35
Power (W)	4200	13,500

The stator resistance is measured at 1.75 ohms per phase.

a. Find the parameters of the equivalent circuit.

b. At a slip of 0.25, calculate the torque and power output and the efficiency of this motor.

8.22 A three-phase, 25-hp, 230-V, 60-Hz, four-pole, induction motor operates at rated load. The motor has a rotor copper loss of 350 W and friction and windage loss of 275 W. Determine

a. Mechanical power developed

b. Air-gap power

c. Shaft speed

d. Shaft torque

8.23 A three-phase, 60-Hz, six-pole, wye-connected induction motor is rated at 100 hp and 440 V. The parameters of the equivalent circuit are

$$R_1 = 0.12\ \Omega \qquad R_2 = 0.10\ \Omega$$
$$X_1 = 0.25\ \Omega \qquad X_2 = 0.20\ \Omega \qquad X_m = 15\ \Omega$$

The friction and windage and core losses are 1.4 kW and 1.2 kW, respectively. For a slip of 3.5%, determine the following:

a. Line current and power factor

b. Air-gap power

c. Power converted from electrical to mechanical form

8.24 A three-phase, 25-hp, 440-V, 60-Hz, six-pole induction motor is operating at a slip of 4%. The core loss and the friction and windage losses at this load are 250 W and 125 W, respectively. The motor is wye connected, and the motor parameters in Ω/phase are

$$R_1 = 0.35 \qquad R_2 = 0.40$$
$$X_1 = 1.25 \qquad X_2 = 1.50 \qquad X_m = 25$$

Determine the following:

a. Line current and power factor

b. Real and reactive power input

c. Air-gap power

d. Mechanical power and torque developed

e. Shaft horsepower and torque

f. Efficiency of the motor

8.25 A six-pole, 60-Hz induction motor runs at 1020 rpm. The input to the rotor circuit is 4 kW. Calculate the rotor copper loss.

8.26 An induction motor delivers 50 kW to a load connected to its shaft. At this load condition, the efficiency of the motor is 88% and stator copper loss = rotor copper loss = core losses = friction and windage losses. Determine the slip.

8.27 A three-phase, 15-hp, 208-V, 60-Hz, six-pole, wye-connected induction motor has the following parameters per phase:

$$R_1 = 0.15\ \Omega \qquad R_2 = 0.10\ \Omega$$
$$X_1 = 0.50\ \Omega \qquad X_2 = 0.50\ \Omega \qquad X_m = 20\ \Omega$$

The friction and windage losses and the hysteresis and eddy-current losses are 350 W and 200 W, respectively. For a slip of 3.5%, find the following:

a. Line current and power factor

b. Horsepower output

c. Starting torque

8.28 A three-phase, four-pole, 60-Hz, three-phase induction machine is rated at 10 hp, 208 V, and 1755 rpm. The parameters of the equivalent circuit of the motor are as follows:

$$R_1 = 0.15\ \Omega \qquad R_2 = 0.15\ \Omega$$
$$X_1 = 0.40\ \Omega \qquad X_2 = 0.25\ \Omega \qquad X_m = 30\ \Omega$$

The combined rotational (friction and windage plus hysteresis and eddy-current) losses amount to 500 W. The motor operates at rated speed when connected to a 208-V and 60-Hz source. Calculate

a. Line current and power factor

b. Output torque

c. Efficiency of the motor

d. Starting current and torque

8.29 A three-phase, six-pole, 60-Hz, wye-connected induction motor delivers 20 kW at the shaft at a slip of 4.5%. The motor has total rotational losses of 1500 W. Calculate the following:

a. Rotor input

b. Output torque

c. Developed torque

8.30 A three-phase, 440-V, four-pole, 60-Hz induction motor takes 120 A of stator current at starting, and the motor draws 20 A while running at full load. The starting torque is 1.8 times the torque at full load at rated voltage. It is desired that the starting torque be equal to the full-load torque. Determine (a) the applied voltage and (b) the corresponding line current.

8.31 A three-phase, 440-V, wye-connected induction motor has a stator impedance of $1.0 + j1.6\ \Omega$ per phase. The rotor impedance referred to the stator is $0.8 + j1.4\ \Omega$ per phase. Determine the maximum electromagnetic power developed by the motor.

8.32 For the motor in Problem 8.23, determine the (a) slip at maximum (pullout) torque and (b) pullout torque.

8.33 A three-phase, 10-hp, 230-V, four-pole, 60-Hz, wye-connected, induction motor operates at rated voltage and rated frequency, and it develops full-load torque at a slip of 3.5%. The mechanical and core losses can be neglected. The impedance parameters of the motor in Ω/phase are as follows:

$$R_1 = 0.25 \qquad X_1 = 0.35 \qquad X_2 = 0.45 \qquad X_m = 40$$

Determine the (a) maximum torque and the slip at maximum torque and (b) starting torque.

8.34 A three-phase, 20-hp, 480-V, 60-Hz, six-pole, wound-rotor induction motor operates at rated conditions, and the motor runs at 1164 rpm with the rotor rheostat shorted. The motor parameters in Ω/phase are

$$R_1 = 0.30 \qquad R_2 = 0.40$$
$$X_1 = 1.50 \qquad X_2 = 2.00 \qquad X_m = 300$$

Determine the following:

- **a.** Slip at which maximum torque occurs
- **b.** Maximum torque
- **c.** Resistance to be inserted in the rotor circuit to operate the motor at rated torque and a speed of 1074 rpm

8.35 A three-phase, 25-hp, 440-V, 60-Hz, 1750 rpm, wound-rotor induction motor has the following equivalent circuit parameters:

$$R_1 = 0.20\ \Omega \qquad X_1 = 1.0\ \Omega$$
$$R_2 = 0.15\ \Omega \qquad X_2 = 0.8\ \Omega \qquad X_m = 30\ \Omega$$

The motor is connected to a three-phase, 440-V, 60-Hz supply.

- **a.** Calculate the starting torque.
- **b.** Determine the resistance of the rheostat in the rotor circuit such that the maximum torque occurs at starting.

8.36 A single-phase, $\frac{1}{4}$-hp, 110-V, 1725-rpm, 60-Hz, four-pole, capacitor-start induction motor has the following equivalent circuit parameters for the main winding.

$$R_1 = 2.5\ \Omega \qquad R_2 = 4.0\ \Omega$$
$$X_1 = 3.5\ \Omega \qquad X_2 = 3.5\ \Omega \qquad X_m = 50\ \Omega$$

The core loss at 110 V is 25 W and the friction and windage loss is 20 W. The motor is connected to a 110-V, 60-Hz supply, and it runs at a slip of 5%. Determine the following:

- **a.** Input current and power factor
- **b.** Input power
- **c.** Developed torque
- **d.** Efficiency of the motor

8.37 A single-phase, $\frac{1}{2}$-hp, 110-V, four-pole, 60-Hz induction motor has the following constants in ohms:

$$R_1 = 1.8 \qquad X_1 = 2.5$$
$$R_2 = 3.5 \qquad X_2 = 2.5 \qquad X_m = 60$$

The motor core loss is 35 W, and the friction and windage loss is 15 W. The motor operates at a slip of 0.05. Determine the (a) input power and (b) mechanical power output.

Nine

Transmission Lines

9.1 INTRODUCTION

An electric power system is a collection of equipment and devices whose common objectives are the production, conversion, transmission, distribution, and consumption of electric energy. The major components of an electric power system are generators, which convert mechanical energy into electricity; transformers, which change the voltage or current levels of an electric supply; power transmission lines, which are used to transfer power from one location to another; and a variety of auxiliary control and regulating equipment intended to vary the system characteristics.

Electric energy is produced in large quantities at various electric power plants by converting different forms of energy—fossil fuels, nuclear energy, water power, and so forth. Electric energy is transformed by the use of transformers to different voltage levels most suitable for transmission, distribution, and consumption.

Electric power is transmitted using overhead or cable lines to consumers at varied distances from its source. Electric energy is utilized by various conversion devices and mechanisms, such as electric motors, ovens, and electric lighting.

The need for power transmission lines arises from the fact that bulk electric power generation is done at large electric power plants remote from consumers. However, consumers require rather small amounts of energy, and they are scattered over wide areas. Thus, the transmission of electric energy over a distance offers a number of advantages, such as:

1. Use of remote energy sources
2. Reduction of the total power reserve of generators

3. Utilization of the time difference between various time zones when the peak demands are not coincident
4. Improved reliability of electric power supply

Power transmission lines are subdivided into overhead and cable lines. Overhead lines are made up of metal conductors suspended on insulators from a tower or post by suitable clamps. The tower or post may be made of metal, wood, or reinforced concrete depending on the purpose of the line, the operating voltage, economic considerations, and so on. Most modern overhead lines are constructed using *aluminum cable steel reinforced (ACSR)* conductors consisting of a central steel core on which aluminum wires are wound. The steel core increases the mechanical strength of the line, and the aluminum wires have good electrical properties. Multiwire conductors are preferred over single-wire conductors of equivalent cross-sectional area because of their improved flexibility and reduced skin effect.

The conductors of a transmission line may be configured in various ways. A few examples are shown in Fig. 9.1. The towers or poles carrying six transmission wires are called double-circuit lines. As described in Chapter 2, steel ground wires are sometimes placed atop the towers or poles for protection against direct lightning strikes.

In cable lines, the conductors are insulated from one another and are enclosed in protective sheaths. The practice is to lay the underground cables directly in the soil, or in a bed of sand, or within special cable ducts. At present, AC overhead transmission lines are favored because of their lower first costs and ease and simplicity of repair and maintenance. However, there is a tendency

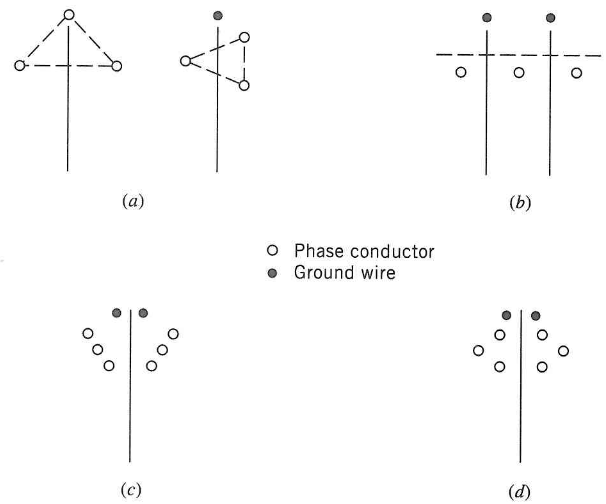


FIGURE 9.1 Transmission line configurations: (*a*) delta type; (*b*) horizontal; (*c*) reverse herring-bone; (*d*) barrel type.

toward more extensive utilization of cable lines, especially in new constructions in urban areas.

The maximum power that a transmission line can transmit, within some constraints, is called its carrying capacity. For overhead AC power transmission lines, the maximum power that can be transmitted may be approximated as being directly proportional to the square of the voltage and inversely proportional to the length of the transmission line. Similarly, the construction costs are also approximately proportional to voltage. Hence, to increase the power levels transmitted over long distances, the usual practice of electric utilities is to raise the transmission voltage.

In this chapter, the basic parameters of the conductors that are used in transmission lines are discussed. The various formulas for these parameters are presented, and the proper use of tables of conductor characteristics for obtaining values of the parameters is described. Models of transmission lines are then presented for use in power system studies.

9.2 RESISTANCE

The DC resistance of a conductor at a specified temperature T is given by

$$R_{\mathrm{DC},T} = \frac{\rho l}{A} \tag{9.1}$$

where

l = length of the conductor

A = cross-sectional area of the conductor

ρ = resistivity of the conductor

Two sets of units are generally used: the English system and the Système International (SI).

In the English system, the length l is given in feet, the cross-sectional area A is expressed in *circular mils (CM)*, and the resistivity ρ is specified in ohm-circular mil per foot (Ω-CM/ft). Note that one inch is equivalent to 1000 mils, and the cross-sectional area measured in circular mils is obtained by squaring the diameter expressed in mils. In other words, one circular mil is the area of a circle having a diameter of 1 mil.

In the SI system of units, the length is measured in meters, the area is given in square meters, and resistivity is expressed in ohm-meter (Ω-m).

Resistivity depends on the conductor material. Annealed copper is the international standard for measuring resistivity. At a temperature of 20°C, the resistivity of annealed copper is given as 10.37 Ω-CM/ft or 1.72×10^{-8} Ω-m.

When the DC resistance of a conductor at a certain temperature is known, the DC resistance at any other temperature may be found as follows:

$$R_{T2} = \left(\frac{M + T_2}{M + T_1}\right) R_{T1} \tag{9.2}$$

where

R_{T1} = DC resistance at temperature T_1 in °C

R_{T2} = DC resistance at temperature T_2 in °C

M = temperature constant in °C

The constant M is the temperature at which the extrapolated resistance of a particular conductor becomes zero. The temperature constants of certain materials, as well as their resistivities and percent conductivities, are given in Table 9.1.

The resistance of nonmagnetic conductors varies not only with temperature but also with frequency because of the *skin effect*. Electric current distribution in the conductor is not uniform. As frequency increases, current tends to flow nearer the outer surface of the conductor. This decreases the effective cross-sectional area of the conductor, thus increasing the resistance.

The resistances at 25, 50, and 60 Hz, as well as the DC resistance, are given in the tables of electrical characteristics of conductors in Appendix A. For other frequencies, the AC resistance is found as follows:

$$R_{\mathrm{AC}} = K R_{\mathrm{DC}} \ \Omega/\mathrm{mi} \tag{9.3}$$

where R_{DC} is the DC resistance in Ω/mi, and K is found from Table 5 in Appendix A as a function of the variable X, which is defined as follows:

$$X = 0.0636\sqrt{\frac{\mu_r f}{R_{\mathrm{DC}}}} \tag{9.4}$$

where the relative permeability μ_r is 1.0 for nonmagnetic materials.

Table 9.1 Resistivity and Temperature Constants of Different Materials

Material	Conductivity (%)	Resistivity Ω-m	Resistivity Ω-CM/ft	Temperature Constant (°C)
Annealed copper	100.0	1.72×10^{-8}	10.37	234.5
Hard-drawn copper	97.3	1.77×10^{-8}	10.66	241.5
Aluminum	61.0	2.83×10^{-8}	17.00	228.1
Iron	17.2	10.00×10^{-8}	60.00	180.0
Silver	108.0	1.59×10^{-8}	9.60	243.0

EXAMPLE 9.1

Determine the 60-Hz AC resistance in Ω/mi of a 1-inch-diameter, 97.3% conductivity, hard-drawn copper conductor at 75°C.

Solution The resistivity of 100% conductivity copper at 20°C is

$$\rho_{100} = 10.37\ \Omega\text{-CM/ft}$$

Thus, the resistivity of 97.3% conductivity copper at 20°C is

$$\rho_{97.3} = 10.37/0.973 = 10.66\ \Omega\text{-CM/ft}$$

Hence, the resistance at 20°C is

$$R_{\mathrm{DC},20} = \frac{\rho l}{A} = \frac{(10.66)(5280)}{(1000)^2} = 0.0563\ \Omega/\mathrm{mi}$$

Therefore, the DC resistance at 75°C is given by

$$R_{\mathrm{DC},75} = \left(\frac{M + T_{75}}{M + T_{20}}\right) R_{\mathrm{DC},20} = \left(\frac{241.5 + 75}{241.5 + 20}\right) 0.0563 = 0.0681\ \Omega/\mathrm{mi}$$

The variable X is computed as follows:

$$X = 0.0636\sqrt{\frac{\mu_r f}{R_{\mathrm{DC},75}}}$$

$$= 0.0636\sqrt{\frac{(1.0)(60)}{0.0681}} = 1.8878 \cong 1.9$$

From Table 5 in Appendix A, for $X = 1.9$, read off the value of K as 1.0644. Therefore, the AC resistance is

$$R_{\mathrm{AC}} = K R_{\mathrm{DC}}$$

$$= (1.0644)(0.0681) = 0.0725\ \Omega/\mathrm{mi}$$

DRILL PROBLEMS

D9.1 One thousand circular mils is designated by the abbreviation MCM. Derive an equation to convert from area expressed in MCM to area expressed in m^2.

D9.2 Determine the cross-sectional area in m^2 of a 795-MCM conductor. Find the AC resistance in Ω/km of a 795-MCM ACSR conductor at 50°C.

9.3 INDUCTANCE AND INDUCTIVE REACTANCE

The series inductance of a transmission line consists of two components: its internal inductance, which is due to magnetic flux inside the conductor, and its external inductance, which is due to magnetic flux outside the conductor. The internal inductance is considered first.

9.3.1 Internal Inductance

Consider the conductor of radius r carrying the current I shown in Fig. 9.2. The magnetic field intensity at a distance x from the center of the conductor is denoted by H_x.

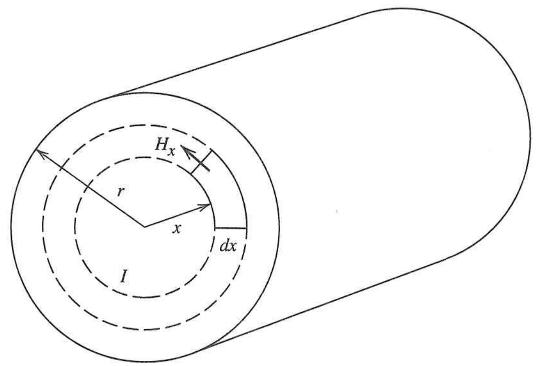
The mmf $(H_x l_{\mathrm{cir}})$ around the closed circular path indicated by broken lines in Fig. 9.2 is equal to the current I_x enclosed by the path. The length of the path is $l_{\mathrm{circle}} = 2\pi x$; thus,

$$H_x 2\pi x = I_x$$

$$= \left(\frac{\pi x^2}{\pi r^2}\right) I \tag{9.5}$$

Therefore,

$$H_x = \frac{x}{2\pi r^2} I \tag{9.6}$$



The flux density B_x is given by

$$B_x = \mu H_x = \frac{\mu x}{2\pi r^2} I \tag{9.7}$$

The differential magnetic flux is equal to the product of the magnetic flux density and the cross-sectional area $dA = (dx)$(axial length). Thus, the flux per unit length is

$$d\phi = \frac{\mu x}{2\pi r^2} I\, dx \tag{9.8}$$

The flux linkages $d\lambda$ are equal to the flux times the fraction of current linked:

$$d\lambda = \left(\frac{\pi x^2}{\pi r^2}\right) d\phi = \frac{\mu x^3}{2\pi r^4} I\, dx \tag{9.9}$$

Integrating Eq. 9.9 over the full radius of the conductor yields the total flux linkages inside the conductor.

$$\lambda_{\text{internal}} = \int_0^r \frac{\mu I}{2\pi r^4} x^3\, dx = \frac{\mu I}{2\pi r^4}\left(\frac{x^4}{4}\right)\Bigg|_0^r = \frac{\mu I}{8\pi} \tag{9.10}$$

For a relative permeability $\mu_r = \mu/\mu_0 = 1.0$, the permeability μ is equal to the permeability of free space $\mu_0 = 4\pi \times 10^{-7}$. Thus,

$$\lambda_{\text{internal}} = \tfrac{1}{2} I \times 10^{-7} \qquad \text{Wb-t/m} \tag{9.11}$$

Therefore, since $L = \lambda/I$, the internal inductance is found as

$$L_{\text{internal}} = \tfrac{1}{2} \times 10^{-7} \qquad \text{H/m} \tag{9.12}$$

9.3.2 External Inductance

Consider the external region surrounding the conductor carrying current I shown in Fig. 9.3. At the tubular element located at a distance x from the conductor center, the magnetic field intensity H_x is related to the current I as follows:

$$H_x 2\pi x = I \tag{9.13}$$

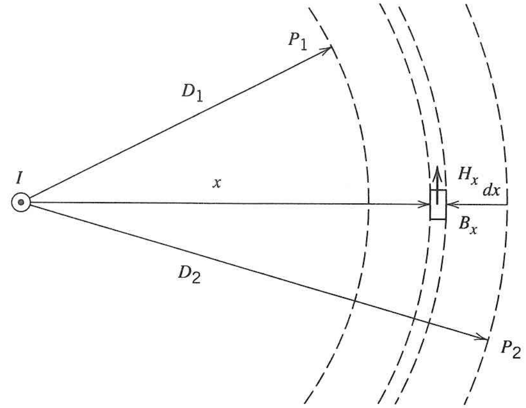


FIGURE 9.3 Surrounding region of a current-carrying conductor.

Solving for the magnetic field intensity from Eq. 9.13 yields

$$H_x = \frac{I}{2\pi x} \tag{9.14}$$

The magnetic flux density B_x in the element is expressed as

$$B_x = \mu H_x = \frac{\mu I}{2\pi x} \tag{9.15}$$

The magnetic flux per unit length of the tubular element of thickness dx is given by

$$d\phi = \frac{\mu I}{2\pi x} dx \tag{9.16}$$

Since the flux links the full current carried by the conductor,

$$d\lambda = d\phi = \frac{\mu I}{2\pi x} dx \tag{9.17}$$

Therefore, the total flux linkages between P_1 and P_2 are obtained by integrating Eq. 9.17 from $x = D_1$ to $x = D_2$.

$$\lambda_{12} = \int_{D_1}^{D_2} \frac{\mu I}{2\pi x} dx = \frac{\mu I}{2\pi}(\ln x|_{D_1}^{D_2} = \frac{\mu I}{2\pi} \ln\left(\frac{D_2}{D_1}\right) \tag{9.18}$$

For a relative permeability of 1, the permeability μ is equal to the permeability of free space $\mu_0 = 4\pi \times 10^{-7}$. Thus,

$$\lambda_{12} = 2 \times 10^{-7} I \ln\left(\frac{D_2}{D_1}\right) \quad \text{Wb-t/m} \tag{9.19}$$

Therefore, the inductance due only to the flux between P_1 and P_2 is

$$L_{12} = \frac{\lambda_{12}}{I} = 2 \times 10^{-7} \ln\left(\frac{D_2}{D_1}\right) \quad \text{H/m} \tag{9.20}$$

9.3.3 Inductance of a Single-Phase Line

Consider the two-wire transmission line with a separation distance D between conductors shown in Fig. 9.4. A current I flows toward the plane of the paper in conductor 1 and returns through conductor 2.

A line of flux produced by current in conductor 1 at a distance equal to or greater than $(D + r_2)$ from the center of conductor 1 does not link the circuit and cannot induce a voltage in the circuit. Stated differently, such a line of flux links a net current of zero.

Therefore, the inductance of the two-wire circuit due to current flowing in conductor 1 is given by

$$L_1 = L_{1,\text{internal}} + L_{1,\text{external}} \tag{9.21}$$

where

$$L_{1,\text{internal}} = \tfrac{1}{2} \times 10^{-7} \quad \text{H/m} \tag{9.22}$$

$$L_{1,\text{external}} = 2 \times 10^{-7} \ln\left(\frac{D}{r_1}\right) \quad \text{H/m} \tag{9.23}$$

Substituting Eqs. 9.22 and 9.23 into Eq. 9.21 and simplifying yields

$$L_1 = 2 \times 10^{-7} \ln\left(\frac{D}{r_1 e^{-1/4}}\right) \quad \text{H/m} \tag{9.24}$$

or

$$L_1 = 2 \times 10^{-7} \ln\left(\frac{D}{r'_1}\right) \quad \text{H/m} \tag{9.25}$$

where $r'_1 = r_1 e^{-1/4} = 0.7788 r_1$ is the *equivalent radius* of conductor 1.

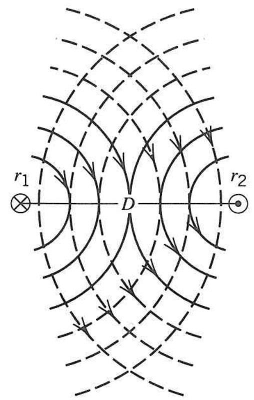


FIGURE 9.4 A two-wire transmission line.

Similarly, the inductance of the circuit due to current in conductor 2 is

$$L_2 = 2 \times 10^{-7} \ln\left(\frac{D}{r'_2}\right) \quad \text{H/m} \tag{9.26}$$

where $r'_2 = r_2 e^{-1/4} = 0.7788 r_2$ is the equivalent radius of conductor 2. Thus, the total inductance for the complete circuit is given by

$$L = L_1 + L_2 = 2 \times 10^{-7}\left[\ln\left(\frac{D}{r'_1}\right) + \ln\left(\frac{D}{r'_2}\right)\right]$$

$$= 4 \times 10^{-7} \ln\left(\frac{D}{\sqrt{r'_1 r'_2}}\right) \quad \text{H/m} \tag{9.27}$$

If $r'_1 = r'_2 = r'$, the total inductance reduces to

$$L = 4 \times 10^{-7} \ln\left(\frac{D}{r'}\right) \quad \text{H/m} \tag{9.28}$$

9.3.4 Inductance of a Three-Phase Circuit

Consider a three-phase transmission line whose conductors are not equilaterally spaced. This condition would result in different flux linkages and inductances in each phase. Since the unbalances are small, they may be neglected. Thus, the transmission line is assumed to be transposed, and each conductor occupies the original positions of the other conductors over equal distances. Figure 9.5 shows one transposition cycle of a completely transposed three-phase circuit.

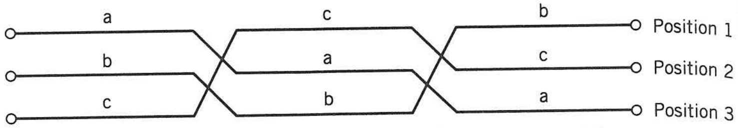


FIGURE 9.5 A three-phase completely transposed line.

For a completely transposed transmission line, the average inductance of each conductor over a complete cycle is the same. To find this average, a point F that is remote from the line is chosen as reference as shown in Fig. 9.6.

The flux linking phase a between conductor a and point F due only to the current in phase a is given by

$$\lambda_{\mathrm{aF,a}} = 2 \times 10^{-7} I_{\mathrm{a}} \ln\left(\frac{D_{\mathrm{aF}}}{r'_{\mathrm{a}}}\right) \tag{9.29}$$

The flux linking phase a between conductor a and point F due only to the current in phase b is

$$\lambda_{\mathrm{aF,b}} = 2 \times 10^{-7} I_{\mathrm{b}} \ln\left(\frac{D_{\mathrm{bF}}}{D_{12}}\right) \tag{9.30}$$

The flux linking phase a between conductor a and point F due only to the current in phase c is

$$\lambda_{\mathrm{aF,c}} = 2 \times 10^{-7} I_{\mathrm{c}} \ln\left(\frac{D_{\mathrm{cF}}}{D_{31}}\right) \tag{9.31}$$

The total flux linking phase a between conductor a out to point F is the sum of the flux linkages due to the three phase currents. Thus, by adding Eqs. 9.29, 9.30, and 9.31 and grouping similar terms, Eq. 9.32 is obtained.

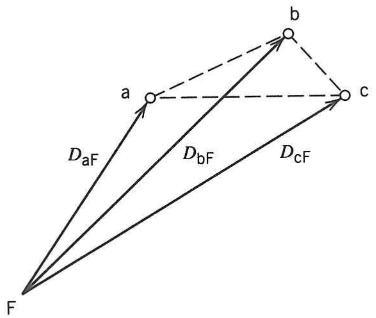


FIGURE 9.6 A three-phase unsymmetrically spaced line.

$$\lambda_{\mathrm{aF}} = 2 \times 10^{-7}\left[I_{\mathrm{a}} \ln\left(\frac{1}{r'_{\mathrm{a}}}\right) + I_{\mathrm{b}} \ln\left(\frac{1}{D_{12}}\right) + I_{\mathrm{c}} \ln\left(\frac{1}{D_{31}}\right)\right] + 2 \times 10^{-7}[I_{\mathrm{a}} \ln D_{\mathrm{aF}} + I_{\mathrm{b}} \ln D_{\mathrm{bF}} + I_{\mathrm{c}} \ln D_{\mathrm{cF}}] \tag{9.32}$$

If the point F is moved farther and farther away from the line, in the limit as F goes to infinity the three distances D_{aF}, D_{bF}, and D_{cF} become equal to each other:

$$D_{\mathrm{aF}} = D_{\mathrm{bF}} = D_{\mathrm{cF}} = D \tag{9.33}$$

Thus, Eq. 9.32 can be written as

$$\lambda_{\mathrm{aF}} = 2 \times 10^{-7}\left[I_{\mathrm{a}} \ln\left(\frac{1}{r'_{\mathrm{a}}}\right) + I_{\mathrm{b}} \ln\left(\frac{1}{D_{12}}\right) + I_{\mathrm{c}} \ln\left(\frac{1}{D_{31}}\right)\right] + 2 \times 10^{-7}(I_{\mathrm{a}} + I_{\mathrm{b}} + I_{\mathrm{c}}) \ln D \tag{9.34}$$

If I_{a}, I_{b}, and I_{c} form a balanced set of three-phase currents, then their sum is equal to zero:

$$I_{\mathrm{a}} + I_{\mathrm{b}} + I_{\mathrm{c}} = 0 \tag{9.35}$$

Therefore, the total flux linkage for phase a in the first transposition position is

$$\lambda_{\mathrm{a1}} = 2 \times 10^{-7}\left[I_{\mathrm{a}} \ln\left(\frac{1}{r'_{\mathrm{a}}}\right) + I_{\mathrm{b}} \ln\left(\frac{1}{D_{12}}\right) + I_{\mathrm{c}} \ln\left(\frac{1}{D_{31}}\right)\right] \tag{9.36}$$

Similarly, the total flux linkage for phase a in the second transposition position is given by

$$\lambda_{\mathrm{a2}} = 2 \times 10^{-7}\left[I_{\mathrm{a}} \ln\left(\frac{1}{r'_{\mathrm{a}}}\right) + I_{\mathrm{b}} \ln\left(\frac{1}{D_{23}}\right) + I_{\mathrm{c}} \ln\left(\frac{1}{D_{12}}\right)\right] \tag{9.37}$$

In the same manner, the total flux linkage for phase a in the third transposition position is given by

$$\lambda_{\mathrm{a3}} = 2 \times 10^{-7}\left[I_{\mathrm{a}} \ln\left(\frac{1}{r'_{\mathrm{a}}}\right) + I_{\mathrm{b}} \ln\left(\frac{1}{D_{31}}\right) + I_{\mathrm{c}} \ln\left(\frac{1}{D_{23}}\right)\right] \tag{9.38}$$

To find the flux linkage for phase a over one complete transposition cycle, Eqs. 9.36, 9.37, and 9.38 are added together and divided by three; upon simplification, this yields

$$\lambda_a = \frac{\lambda_{a1} + \lambda_{a2} + \lambda_{a3}}{3}$$
$$= \frac{2 \times 10^{-7}}{3}\left[3I_a \ln\left(\frac{1}{r'_a}\right) + (I_b + I_c) \ln\left(\frac{1}{D_{12}D_{23}D_{31}}\right)\right] \quad (9.39)$$

According to Eq. 9.35, the sum of the phase currents is equal to zero, or $(I_b + I_c) = -I_a$; hence,

$$\lambda_a = 2 \times 10^{-7} I_a \left[\ln\left(\frac{1}{r'_a}\right) - \ln\left(\frac{1}{D_{12}D_{23}D_{31}}\right)^{1/3}\right] \quad (9.40)$$

Therefore, the inductance for phase a of the completely transposed three-phase line is given by

$$L_a = 2 \times 10^{-7} \ln \frac{\sqrt[3]{D_{12}D_{23}D_{31}}}{r'_a} \quad \text{H/m} \quad (9.41)$$

Calculations for phases b and c yield similar results. Thus,

$$L_b = 2 \times 10^{-7} \ln \frac{\sqrt[3]{D_{12}D_{23}D_{31}}}{r'_b} \quad \text{H/m} \quad (9.42)$$

$$L_c = 2 \times 10^{-7} \ln \frac{\sqrt[3]{D_{12}D_{23}D_{31}}}{r'_c} \quad \text{H/m} \quad (9.43)$$

If all three phase conductors have the same equivalent radius $r' = re^{-1/4}$, then the inductance per phase is given by

$$L_a = L_b = L_c = 2 \times 10^{-7} \ln \frac{\sqrt[3]{D_{12}D_{23}D_{31}}}{r'}$$
$$= 2 \times 10^{-7} \ln \frac{\text{GMD}}{r'} \quad \text{H/m} \quad (9.44)$$

where GMD $= (D_{12}D_{23}D_{31})^{1/3}$ is the *geometric mean distance*.

For a three-phase, equilaterally spaced transmission line, the phase conductors have equal separation distances between them; that is, $D_{12} = D_{23} = D_{31} = D$. Therefore, the inductance per phase is given by

$$L_a = 2 \times 10^{-7} \ln\left(\frac{D}{r'}\right) \quad \text{H/m} \quad (9.45)$$

9.3.5 Stranded Conductors and Bundled Conductors

Stranded conductors are composed of two or more elements or strands of wire that are electrically in parallel. It is assumed that all the strands are identical and that they share the total current equally. For an n-strand conductor, the equivalent radius of the stranded conductor is referred to as its *geometric mean radius (GMR)*, and it is derived from the equivalent radius of a strand and the distances from the center of a strand to the centers of the rest of the strands. Thus,

$$\text{GMR} = \sqrt[N]{(r'D_{12}D_{13}\cdots D_{1n})(D_{21}r'D_{23}\cdots D_{2n})\cdots(D_{n1}D_{n2}\cdots r')} \quad (9.46)$$

where

$N = n^2$

$r' = re^{-1/4} = 0.7788r$ = equivalent radius of each strand

D_{km} = distance from the center of strand k to the center of strand m

Therefore, in terms of the GMR of the phase conductors, the inductance per phase given by Eq. 9.44 reduces to

$$L_a = L_b = L_c = 2 \times 10^{-7} \ln \frac{\text{GMD}}{\text{GMR}} \quad \text{H/m} \quad (9.47)$$

Bundled conductors consist of two or more conductors that belong to the same phase and are close together in comparison with the separation distances between the phases. The bundle consists of two, three, or four conductors. The three-conductor bundle usually has the conductors at the vertices of an equilateral triangle, and the four-conductor bundle usually has its conductors at the corners of a square.

Bundling of the conductors of a transmission line has several advantages. The increased number of conductors in each phase reduces the effects of corona for the very high transmission voltages. Bundling also results in reduced resistance and reduced reactance. The GMR for a bundled conductor is calculated in the same way as that for a stranded conductor. For this case, each conductor of the bundle is treated as a strand in the multistrand conductor.

The equivalent GMR_b of the bundled conductor is related to the conductor GMR_c as follows:

For a two-conductor bundle

$$\text{GMR}_b = \sqrt[4]{d^2\,\text{GMR}_c^2} = \sqrt{d\,\text{GMR}_c} \quad (9.48)$$

For a three-conductor bundle

$$\text{GMR}_b = \sqrt[9]{d^3 d^3\,\text{GMR}_c^3} = \sqrt[3]{d^2\,\text{GMR}_c} \quad (9.49)$$

For a four-conductor bundle

$$\text{GMR}_b = \sqrt[16]{d^4 d^4 (\sqrt{2}d)^4\,\text{GMR}_c^4} = 1.09\sqrt[4]{d^3\,\text{GMR}_c} \qquad (9.50)$$

9.3.6 Inductive Reactance of a Single-Phase Line

Inductive reactance, rather than inductance, is usually desired. For a single-phase, two-conductor distribution line with a separation distance D, the inductive reactance of one conductor is derived from Eq. 9.25 as follows:

$$X_L = 2\pi f L = 2\pi f \times 2 \times 10^{-7} \ln\left(\frac{D}{r'}\right)$$

$$= 4\pi f \times 10^{-7} \ln\left(\frac{D}{r'}\right) \quad \Omega/\text{m} \qquad (9.51)$$

$$X_L = 2.022 \times 10^{-3} f \ln\left(\frac{D}{r'}\right)$$

$$= 0.2794\left(\frac{f}{60}\right)\log\left(\frac{D}{r'}\right) \quad \Omega/\text{mi} \qquad (9.52)$$

$$X_L = 1.256 \times 10^{-3} f \ln\left(\frac{D}{r'}\right)$$

$$= 0.1736\left(\frac{f}{60}\right)\log\left(\frac{D}{r'}\right) \quad \Omega/\text{km} \qquad (9.53)$$

EXAMPLE 9.2

Determine the 60-Hz impedance in Ω/mi of the single-phase distribution line employing the 1-inch-diameter copper conductor of Example 9.1 with a separation distance of 4 feet.

Solution From Example 9.1:

$$R_a = 0.0725\ \Omega/\text{mi}$$

$$r' = re^{-1/4} = (0.5/12)e^{-1/4} = 0.0324\ \text{ft}$$

By Eq. 9.52, the inductive reactance of one conductor is

$$X_L = 2.022 \times 10^{-3} f \ln(D/r')$$

Therefore, the impedance of the single-phase line is

$$Z = 2(R_a + jX_L)$$

$$= 2(0.0725 + j0.584) = 0.145 + j1.168 = 1.177\angle 82.9^\circ\ \Omega/\text{mi}$$

DRILL PROBLEMS

D9.3 A single-phase, 60-Hz transmission line consists of two 1/0 copper conductors, hard drawn, 97.3% conductivity whose centers are 5 m apart. Find the inductive reactance in Ω/mi.

D9.4 A single-phase, 60-Hz transmission line consists of two 636,000 CM (or 636 MCM), 26/7, ACSR conductors. The total inductive reactance per phase is 1.65 Ω/mi. Find the distance between the two conductor centers.

D9.5 A single-phase, 60-Hz transmission line consists of two bundles, each arranged vertically. One bundle contains two conductors, each with a radius of 2.5 cm, that are 10 cm apart. The other bundle consists of three conductors, each of radius 1.5 cm, with a separation distance between adjacent conductors of 5 cm. The horizontal distance between the bundles is 8 m.

a. Find the geometric mean radius for each bundle.

b. Compute the geometric mean distance.

c. Determine the total inductance of the transmission line in mH/km and in mH/mi.

d. Find the total inductive reactance of the transmission line in Ω/mi.

9.3.7 Inductive Reactance of a Three-Phase Circuit

Consider the three-phase circuit consisting of three identical conductors. If the three conductors are symmetrically spaced, no transposition is necessary to maintain balanced conditions. The inductance was derived in Section 9.3.4, and the inductance formula is given by Eq. 9.45 for a separation distance D between conductors. Therefore, the inductive reactance per phase is found as follows:

$$X_L = 2\pi f L_a = 2\pi f \times 2 \times 10^{-7} \ln\left(\frac{D}{\text{GMR}}\right)$$

$$= 4\pi f \times 10^{-7} \ln\left(\frac{D}{\quad}\right) \quad \Omega/\text{m} \qquad (9.54)$$

$$X_L = 2.022 \times 10^{-3} f \ln\left(\frac{D}{\text{GMR}}\right)$$

$$= 0.2794\left(\frac{f}{60}\right)\log\left(\frac{D}{\text{GMR}}\right) \quad \Omega/\text{mi} \tag{9.55}$$

$$X_L = 1.256 \times 10^{-3} f \ln\left(\frac{D}{\text{GMR}}\right)$$

$$= 0.1736\left(\frac{f}{60}\right)\log\left(\frac{D}{\text{GMR}}\right) \quad \Omega/\text{km} \tag{9.56}$$

When the conductors are unsymmetrically spaced, the voltage drop for each conductor is different. For this case, the conductors are transposed; that is, the positions of the conductors are exchanged at regular intervals along the line so that each conductor occupies the original position of every other conductor over an equal distance. The average inductance of a transposed line was derived in Section 9.3.4, and the inductance formula is given by Eq. 9.44. Therefore, the inductive reactance per phase for the transposed three-phase circuit is found as follows:

$$X_L = 2\pi f L_a = 2\pi f \times 2 \times 10^{-7} \ln\left(\frac{\text{GMD}}{\text{GMR}}\right)$$

$$= 4\pi f \times 10^{-7} \ln\left(\frac{\text{GMD}}{\text{GMR}}\right) \quad \Omega/\text{m} \tag{9.57}$$

$$X_L = 2.022 \times 10^{-3} f \ln\left(\frac{\text{GMD}}{\text{GMR}}\right)$$

$$= 0.2794\left(\frac{f}{60}\right)\log\left(\frac{\text{GMD}}{\text{GMR}}\right) \quad \Omega/\text{mi} \tag{9.58}$$

$$X_L = 1.256 \times 10^{-3} f \ln\left(\frac{\text{GMD}}{\text{GMR}}\right)$$

$$= 0.1736\left(\frac{f}{60}\right)\log\left(\frac{\text{GMD}}{\text{GMR}}\right) \quad \Omega/\text{km} \tag{9.59}$$

9.3.8 The Use of Tables to Find Inductive Reactance

The inductive reactance of one conductor of a single-phase distribution line is given by Eq. 9.52, which may be rewritten as

$$X_L = 0.2794\left(\frac{f}{60}\right)\log\left(\frac{1}{r'}\right)$$

$$+ 0.2794\left(\frac{f}{60}\right)\log D \quad \Omega/\text{mi} \tag{9.60}$$

If both r' and D are given in feet, the first term on the right-hand side is denoted as x_a and is called *inductive reactance at 1-ft spacing* because it is equal to the inductive reactance of one conductor of a two-conductor line 1 foot apart. The second term on the right-hand side is called the *inductive reactance spacing factor* x_d. Thus, for the single-phase circuit, the total inductive reactance per conductor is given by

$$X_L = x_a + x_d \quad \Omega/\text{mi} \tag{9.61}$$

The values of x_a at 25, 50, and 60 Hz and the values of GMR at 60 Hz are given in the tables of conductor characteristics (Tables 1–4) in Appendix A. The values of x_d for various spacings are given in Table 6, also in Appendix A.

The inductive reactance of a three-phase transmission line with an equivalent spacing factor GMD is given by Eq. 9.58, which may be rewritten as

$$X_L = 0.2794\left(\frac{f}{60}\right)\log\left(\frac{1}{\text{GMR}}\right)$$

$$+ 0.2794\left(\frac{f}{60}\right)\log \text{GMD} \quad \Omega/\text{mi} \tag{9.62}$$

For the case of an equilaterally spaced, three-phase circuit, Eq. 9.62 equally applies with the GMD represented by the distance between conductors, that is, GMD $= D$.

If both GMD and GMR are expressed in feet, Eq. 9.62 can also be expressed as follows:

$$X_L = x_a + x_d \quad \Omega/\text{mi} \tag{9.63}$$

where x_a is the per-phase inductive reactance at 1-foot spacing and x_d is the per-phase inductive reactance spacing factor. As mentioned previously, the values of x_a and x_d may be found from the tables in Appendix A.

A simple procedure that can be used to find the inductive reactance of a three-phase circuit is as follows:

1. Read x_a from the tables of conductor characteristics.
2. (a) Evaluate GMD and read x_d from Table 6 using the GMD value as spacing factor, or (b) find the average of the three inductive reactance spacing factors corresponding to the distances between phase conductors.

EXAMPLE 9.3

Find the impedance of 2.5 miles of a single-phase ungrounded circuit consisting of 3 No. 5, 40% conductivity copperweld (CW) conductors that are spaced 6 feet apart and are expected to operate at 75% of current-carrying capacity.

Solution From Table 4-B of Appendix A, for 3 No. 5, 40% conductivity, copperweld (CW) conductors:

$$r_a = 1.772\ \Omega/\text{mi}$$
$$x_a = 0.617\ \Omega/\text{mi}$$

From Table 6, at 6-feet spacing:

$$x_d = 0.217\ \Omega/\text{mi}$$

Therefore, the impedance per unit length is

$$\begin{aligned} z &= r_a + j(x_a + x_d) \\ &= 1.772 + j(0.617 + 0.217) = 1.772 + j0.834\ \Omega/\text{mi} \end{aligned}$$

Hence, the total impedance for the two 2.5-mi conductors is

$$Z = (2)(2.5)(1.772 + j0.834) = 8.860 + j4.170\ \Omega$$

EXAMPLE 9.4

Determine the impedance of 80 miles of a 220-kV, three-phase circuit consisting of 795,000 CM, 26/7 ACSR conductors that are arranged in flat spacing with 20 feet between conductors.

Solution From Table 2-A in Appendix A, for 795 MCM ACSR,

$$r_a = 0.1288\ \Omega/\text{mi}$$
$$x_a = 0.3990\ \Omega/\text{mi}$$

From Table 6,

$$x_{d,20} = 0.3635\ \Omega/\text{mi}$$
$$x_{d,20} = 0.3635\ \Omega/\text{mi}$$
$$x_{d,40} = 0.4476\ \Omega/\text{mi}$$

Alternatively,

$$\text{GMD} = \sqrt[3]{(20)(20)(40)} = 25.2\ \text{ft}$$

From Table 6,

$$x_{d,26} = 0.3953\ \Omega/\text{mi}$$
$$x_{d,25} = 0.3906\ \Omega/\text{mi}$$

Interpolating,

$$x_d = 0.3906 + (0.2)(0.3953 - 0.3906) = 0.3915\ \Omega/\text{mi}$$

Thus,

$$z = 0.1288 + j(0.3990 + 0.3915) = 0.1288 + j0.7905\ \Omega/\text{mi}$$

Therefore, the total impedance for 80 miles of line is given by

$$Z = 80z = 80(0.1288 + j0.7905) = 10.30 + j63.24\ \Omega$$

DRILL PROBLEMS

D9.6 A three-phase, 60-Hz transmission line has solid cylindrical copper conductors arranged in the form of an equilateral triangle with 2 m spacing between conductors. Each conductor has a diameter of 2.5 cm. Calculate the inductance in H/m and the inductive reactance in Ω/km.

D9.7 A three-phase, 60-Hz transmission line has its conductors arranged in a triangular formation so that two of the distances between conductors are 8 m and the third distance is 10 m. The conductors are 795-MCM 26/7 ACSR. Determine the inductance and inductive reactance per phase per mile.

D9.8 A three-phase transmission line consists of bundled conductors that are placed at the corners of an equilateral triangle with a separation distance of 10 m between the centers of each bundle. Each bundle contains three conductors, each with a radius of 1 cm, with an equilateral spacing between conductors of 25 cm. Determine the (a) inductance per phase in mH/km and in mH/mi and (b) inductive reactance per phase in Ω/km and in Ω/mi.

9.4 CAPACITANCE AND CAPACITIVE REACTANCE

When a voltage is applied to a pair of conducting plates (or cylinders) separated by a nonconducting dielectric medium, charge of equal magnitude but opposite sign accumulates on the plates (or cylinders). The charge deposited is proportional to the applied voltage, the proportionality constant being the capacitance C.

Similarly, for a transmission line the potential difference between the conductors causes them to be charged just like a capacitor. The effective capacitance of the transmission line is dependent on the size and separation distance of the conductors.

In AC power systems, the transmission line is energized by a time-varying voltage. This alternating voltage also causes the charges on the conductors to vary with time. The alternately increasing and decreasing charges on the conductors give rise to the charging current. The charging current affects the power transmitted and the operating power factor, as well as the voltage drop along the transmission line.

Consider a long, straight, cylindrical conductor having a uniform charge throughout its length. According to *Gauss's law,* the electric flux density at a point P is equal to the flux leaving the conductor per unit length divided by the area of the surface in a unit length:

$$D = \frac{q}{2\pi x} \tag{9.64}$$

where

D = electric flux density, coulombs/m^2

q = charge on the conductor, coulombs/m

x = distance in meters from conductor to point P

The electric field intensity at point P is equal to the electric flux density divided by the permittivity ϵ; thus,

$$E = \frac{D}{\epsilon} = \frac{q}{2\pi\epsilon x} \tag{9.65}$$

The permittivity may be expressed in terms of the relative permittivity and the permittivity of free space; that is,

$$\epsilon = \epsilon_r \epsilon_0 \tag{9.66}$$

where

ϵ_r = relative permittivity (dielectric constant) = 1 for air

ϵ_0 = permittivity of free space

Next, consider a point P_1 located at a distance D_1 from the center of a conductor and another point P_2 at a distance D_2 from the center. The instantaneous potential difference, or voltage drop, from point P_1 to P_2 is found by integrating the electric field intensity over a radial path between the two equipotential surfaces passing through the two points as shown in Fig. 9.7.

Thus, the voltage drop V_{12} is found as follows:

$$V_{12} = \int_{D_1}^{D_2} E\,dx = \int_{D_1}^{D_2} \frac{q}{2\pi\epsilon x}\,dx = \frac{q}{2\pi\epsilon}\ln\left(\frac{D_2}{D_1}\right) \tag{9.67}$$

9.4.1 Capacitance of a Single-Phase Line

The voltage V_{ab} between the two conductors of the transmission line shown in Fig. 9.4, which has a separation distance D, is found as follows.

The voltage drop due to the charge q_a on conductor a is given by

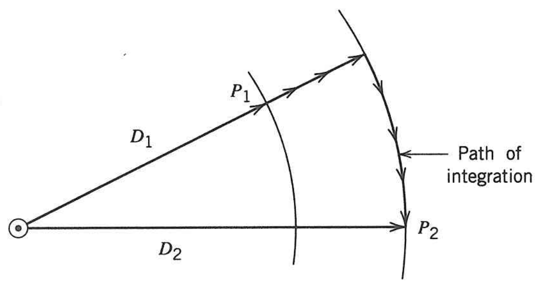
$$V_{ab,a} = \frac{q_a}{2\pi\epsilon}\ln\left(\frac{D}{r_a}\right) \tag{9.68}$$

Similarly, the voltage drop due to the charge q_b on conductor b is given by

$$V_{ba,b} = \frac{q_b}{2\pi\epsilon}\ln\left(\frac{D}{r_b}\right) \tag{9.69}$$

or

$$V_{ab,b} = -\frac{q_b}{2\pi\epsilon}\ln\left(\frac{D}{r_b}\right) \tag{9.70}$$



By the principle of superposition,

$$\begin{aligned} V_{ab} &= V_{ab,a} + V_{ab,b} \\ &= \frac{q_a}{2\pi\epsilon} \ln\left(\frac{D}{r_a}\right) - \frac{q_b}{2\pi\epsilon} \ln\left(\frac{D}{r_b}\right) \end{aligned} \tag{9.71}$$

Since $q_a = -q_b$,

$$\begin{aligned} V_{ab} &= \frac{q_a}{2\pi\epsilon}\left[\ln\left(\frac{D}{r_a}\right) + \ln\left(\frac{D}{r_b}\right)\right] \\ &= \frac{q_a}{2\pi\epsilon} \ln\left(\frac{D^2}{r_a r_b}\right) \end{aligned} \tag{9.72}$$

If $r_a = r_b = r$, then

$$V_{ab} = \frac{q_a}{\pi\epsilon} \ln\left(\frac{D}{r}\right) \tag{9.73}$$

Therefore, the capacitance between the conductors is given by

$$C_{ab} = \frac{q_a}{V_{ab}} = \frac{\pi\epsilon}{\ln(D/r)} \quad \text{F/m} \tag{9.74}$$

The potential difference between each conductor and ground is half of the potential difference between the two conductors. Thus, the capacitance to ground, or capacitance to neutral, is twice the capacitance from line to line. Therefore,

$$C_n = C_{an} = C_{bn} = \frac{2\pi\epsilon}{\ln(D/r)} \quad \text{F/m} \tag{9.75}$$

9.4.2 Capacitance of a Three-Phase Circuit

If the three-phase circuit is unsymmetrically spaced, it is assumed that it is completely transposed as shown in Fig. 9.5. Thus, the voltage to neutral in any transposition cycle will be the same. The voltage to neutral is derived in a manner similar to the derivation of Eq. 9.75. For the first transposition section,

$$V_{ab1} = \frac{1}{2\pi\epsilon}\left[q_a \ln\left(\frac{D_{12}}{r}\right) + q_b \ln\left(\frac{r}{D_{12}}\right) + q_c \ln\left(\frac{D_{23}}{D_{31}}\right)\right] \tag{9.76}$$

For the second transposition section,

$$V_{ab2} = \frac{1}{2\pi\epsilon}\left[q_a \ln\left(\frac{D_{23}}{r}\right) + q_b \ln\left(\frac{r}{D_{23}}\right) + q_c \ln\left(\frac{D_{31}}{D_{12}}\right)\right] \tag{9.77}$$

For the third transposition section,

$$V_{ab3} = \frac{1}{2\pi\epsilon}\left[q_a \ln\left(\frac{D_{31}}{r}\right) + q_b \ln\left(\frac{r}{D_{31}}\right) + q_c \ln\left(\frac{D_{12}}{D_{23}}\right)\right] \tag{9.78}$$

Thus, the average voltage across conductors a and b is

$$\begin{aligned} V_{ab} = \frac{1}{3}\frac{1}{2\pi\epsilon}\Bigg[& q_a \ln\left(\frac{D_{12}D_{23}D_{31}}{r^3}\right) + q_b \ln\left(\frac{r^3}{D_{12}D_{23}D_{31}}\right) \\ & + q_c \ln\left(\frac{D_{12}D_{23}D_{31}}{D_{12}D_{23}D_{31}}\right)\Bigg] \end{aligned} \tag{9.79}$$

Simplifying,

$$\begin{aligned} V_{ab} &= \frac{1}{2\pi\epsilon}\left[q_a \ln\left(\frac{\sqrt[3]{D_{12}D_{23}D_{31}}}{r}\right) - q_b \ln\left(\frac{\sqrt[3]{D_{12}D_{23}D_{31}}}{r}\right)\right] \\ &= \frac{1}{2\pi\epsilon}\left[q_a \ln\left(\frac{\text{GMD}}{r}\right) - q_b \ln\left(\frac{\text{GMD}}{r}\right)\right] \end{aligned} \tag{9.80}$$

Similarly, the average value of the voltage across conductors a and c is given by

$$V_{ac} = \frac{1}{2\pi\epsilon}\left[q_a \ln\left(\frac{\text{GMD}}{r}\right) - q_c \ln\left(\frac{\text{GMD}}{r}\right)\right] \tag{9.81}$$

For a balanced wye system, assuming a positive, or abc, phase sequence, the phase voltage may be calculated from the average values of the line-to-line voltages as follows:

$$V_{an} = \frac{V_{ab} + V_{ac}}{3} \tag{9.82}$$

Substituting Eqs. 9.80 and 9.81 into Eq. 9.82 yields

$$V_{an} = \frac{1}{3}\frac{1}{2\pi\epsilon}\left[2q_a \ln\left(\frac{\text{GMD}}{r}\right) - q_b \ln\left(\frac{\text{GMD}}{r}\right) - q_c \ln\left(\frac{\text{GMD}}{r}\right)\right] \tag{9.83}$$

For a balanced system,

$$q_a = -(q_b + q_c) \qquad (9.84)$$

Hence,

$$V_{an} = \frac{1}{2\pi\epsilon} q_a \ln\left(\frac{\text{GMD}}{r}\right) \qquad (9.85)$$

Therefore, the capacitance per phase is

$$C_n = \frac{q_a}{V_{an}} = \frac{2\pi\epsilon}{\ln(\text{GMD}/r)} \quad \text{F/m} \qquad (9.86)$$

9.4.3 Capacitive Reactance

For a single-phase line, the capacitance to neutral C_n is given by Eq. 9.75. When ϵ_0 is substituted for the permittivity ϵ, the capacitive reactance from one conductor to neutral is given by the following:

$$X_C = \frac{1}{2\pi f C_n} = \frac{2.862}{f} \times 10^9 \ln\left(\frac{D}{r}\right) \quad \Omega\text{-m} \qquad (9.87)$$

$$X_C = \frac{1.779}{f} \times 10^6 \ln\left(\frac{D}{r}\right) \quad \Omega\text{-mi} \qquad (9.88)$$

It should be noted that the capacitance to neutral C_n given by Eq. 9.75 is in units of farads per unit length. Thus, the total capacitance to neutral is obtained by multiplying C_n by the total length of the line. Therefore, to find the total capacitive reactance to neutral in ohms for the entire length of the line, the expressions given by Eqs. 9.87 and 9.88 must be divided by the length of the line.

For a three-phase, completely transposed transmission line with equivalent spacing factor GMD, the capacitance to neutral is given by Eq. 9.86. The capacitive reactance from one conductor to neutral, with ϵ_0 substituted for permittivity ϵ, is given by the following:

$$X_C = \frac{1}{2\pi f C_n} = \frac{2.862}{f} \times 10^9 \ln\left(\frac{\text{GMD}}{r}\right) \quad \Omega\text{-m} \qquad (9.89)$$

$$X_C = \frac{1.779}{f} \times 10^6 \ln\left(\frac{\text{GMD}}{r}\right) \quad \Omega\text{-mi} \qquad (9.90)$$

Again, it may be noted that the capacitive reactance X_C is given in units of Ω-m or Ω-mi. Therefore, to obtain the total capacitive reactance in Ω, the

expressions given by Eqs. 9.89 and 9.90 must be divided by the total length of the line.

9.4.4 The Use of Tables to Find Capacitive Reactance

The capacitive reactance from one conductor to neutral of a single-phase distribution line is given by Eq. 9.88 in Ω-mi. This may be rewritten as follows:

$$X_C = \frac{1.779}{f} \times 10^6 \ln\left(\frac{1}{r}\right) + \frac{1.779}{f} \times 10^6 \ln D \quad \Omega\text{-mi} \qquad (9.91)$$

If both D and r are expressed in feet, the first term on the right-hand side is called *capacitive reactance at 1-ft spacing* x'_a and the second term is called *capacitive reactance spacing factor* x'_d. Therefore, the capacitive reactance is given by

$$X_C = x'_a + x'_d \qquad (9.92)$$

The values of x'_a may be found from Tables 1–4 in Appendix A. The values of x'_d are given in Table 8 as a function of spacing.

For a three-phase transmission line with an equivalent spacing factor GMD, the capacitive reactance from one conductor to neutral is given by Eq. 9.90 in Ω-mi. This may be rewritten as follows:

$$X_C = \frac{1.779}{f} \times 10^6 \ln\left(\frac{1}{r}\right) + \frac{1.779}{f} \times 10^6 \ln \text{GMD} \qquad (9.93)$$

If both GMD and r are given in feet, Eq. 9.93 can also be expressed as follows:

$$X_C = x'_a + x'_d \qquad (9.94)$$

where x'_a is the per-phase shunt capacitive reactance at 1-foot spacing and x'_d is the per-phase shunt capacitive reactance spacing factor. The values of x'_a may be found from Tables 1–4 and the values of x'_d from Table 8 in Appendix A.

EXAMPLE 9.5

Determine the capacitive reactance of the three-phase circuit of Example 9.4. The transmission line is 80 miles long, and the conductors are of 795-MCM, 26/7 ACSR that are arranged in flat spacing with 20 feet between conductors.

Solution From Table 2-A in Appendix A:

$$x'_a = 0.0912 \text{ M}\Omega\text{-mi}$$

From Table 8:

$$x'_{d,20} = 0.0889 \text{ M}\Omega\text{-mi}$$

$$x'_{d,40} = 0.1094 \text{ M}\Omega\text{-mi}$$

$$x'_d = \tfrac{1}{3}(0.0889 + 0.0889 + 0.1094) = 0.0957 \text{ M}\Omega\text{-mi}$$

Therefore, the capacitive reactance is

$$x_C = x'_a + x'_d = 0.0912 + 0.0957 = 0.1869 \text{ M}\Omega\text{-mi}$$

For the 80-mile long transmission line,

$$X_C = x_C/80 = 186{,}900/80 = 2336 \ \Omega$$

DRILL PROBLEMS

D9.9 A single-phase, 60-Hz transmission line consists of identical conductors each having a radius of 2.5 cm. The separation distance between the conductors is 5 m. Determine (a) the capacitance of the line in μF/km and in μF/mi and (b) the capacitive reactance in Ω-mi.

D9.10 Calculate the capacitance to neutral in F/m and the admittance per phase in S/km for the three-phase transmission line described in Drill Problem D9.6.

D9.11 A three-phase, 60-Hz, transmission line has a total length of 120 miles. The conductors are 715.5-MCM 26/7 ACSR and are arranged in a triangular formation so that two of the distances between conductors are 20 ft and the third distance is 30 ft. Determine (a) the capacitance to neutral in μF/mi and the capacitive reactance to neutral in Ω-mi and (b) the total capacitance to neutral and the total capacitive reactance.

D9.12 A three-phase, 60-Hz, bundled transmission line consists of three 900-MCM ACSR conductors per bundle arranged with equilateral spacing of 40 cm between conductors. The spacing between bundle centers is 8, 8, and 12 m. Calculate the capacitive reactance to neutral in Ω-km.

9.5 TRANSMISSION LINE MODELS

Transmission lines are classified as short lines, medium-length lines, or long lines. A transmission line is considered a short line if it is less than 50 miles (or 80 km). Medium-length lines are those with lengths roughly between 50 miles and 150 miles (or 240 km). Transmission lines extending more than 150 miles are considered long lines and are usually represented in terms of their distributed parameters. For some applications, however, lines as long as 200 miles (or 320 km) can still be approximated using equivalent lumped parameters. In the following sections, various models are described for the transmission line corresponding to the three classifications.

9.5.1 The Short Transmission Line

The equivalent circuit of a short transmission line consists of a series combination of a resistance and an inductive reactance. It is shown in Fig. 9.8. This equivalent circuit is applicable for transmission lines up to 50 miles in length.

Since there are no shunt branches, the current will be the same at the sending and receiving ends:

$$\mathbf{I}_S = \mathbf{I}_R \tag{9.95}$$

The voltage at the sending end is given by

$$\mathbf{V}_S = \mathbf{V}_R + \mathbf{I}_R Z \tag{9.96}$$

The voltage regulation of a transmission line is defined as the increase in receiving-end voltage as the load is reduced from full load to no load with the sending-end voltage held constant. It is expressed in percent of the full-load voltage; thus,

$$\text{Voltage regulation} = \frac{V_{R,nl} - V_{R,fl}}{V_{R,fl}} 100\% \tag{9.97}$$

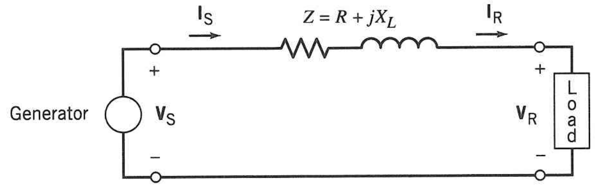


FIGURE 9.8 Equivalent circuit for a short transmission line.

EXAMPLE 9.6

A 60-Hz, three-phase transmission line is 40 miles long. It has a total series impedance of $35 + j140\ \Omega$ and negligible shunt admittance. It delivers 40 MW at 220 kV and 0.9 power factor lagging.

a. Find the voltage, current, and power factor at the sending end of the line.

b. Compute the voltage regulation and efficiency of the line.

Solution For a load of 40 MW, at 220 kV and 0.9 power factor lagging, the receiving-end current is given by

$$\mathbf{I}_R = \frac{40{,}000}{\sqrt{3}(220)(0.9)} \angle -\cos^{-1} 0.9 = 116.6 \angle -25.8^\circ$$

The sending-end voltage is found as follows:

$$\begin{aligned}\mathbf{V}_S &= \mathbf{V}_R + Z\mathbf{I}_R \\ &= (220/\sqrt{3}) \times 10^3 \angle 0^\circ + (35 + j140)(116.6 \angle -25.8^\circ) \\ &= 127{,}000 \angle 0^\circ + (144.3 \angle 76^\circ)(116.6 \angle -25.8^\circ) \\ &= 138.4 \angle 5.4^\circ \text{ kV} \quad \text{(line-to-neutral)} \\ &= 239.7 \angle 35.4^\circ \text{ kV} \quad \text{(line-to-line)}\end{aligned}$$

Since there is no shunt branch, the sending-end current is the same as the receiving-end current. Thus,

$$\mathbf{I}_S = \mathbf{I}_R = 116.6 \angle -25.8^\circ$$

The sending-end power factor is

$$\text{PF}_S = \cos[5.4^\circ - (-25.8^\circ)] = 0.86 \text{ lagging}$$

The percent voltage regulation is computed as

$$\begin{aligned}\text{Voltage regulation} &= \frac{V_{R,\text{nl}} - V_{R,\text{fl}}}{V_{R,\text{fl}}} 100\% \\ &= \frac{239.7 - 220}{220} 100\% = 8.95\%\end{aligned}$$

The sending-end real power is

$$\begin{aligned}P_S &= 3V_S I_S \text{PF}_S = 3(138.4 \times 10^3)(116.6)(0.86) \\ &= 41.6 \times 10^6 \text{ W} = 41.6 \text{ MW}\end{aligned}$$

Therefore, the efficiency of the line is found as follows:

$$\text{Efficiency} = (P_R/P_S)100\% = (40/41.6)100\% = 96.2\%$$

9.5.2 The Medium-Length Line

The shunt admittance, often a pure capacitance, is included in the equivalent circuit of a medium-length line. The total shunt capacitive admittance of the line is divided into two equal parts and placed at the sending and receiving ends of the line forming a *nominal* π circuit. The equivalent circuit is shown in Fig. 9.9 and is used to represent transmission lines that are between 50 to 150 miles long.

The sending-end voltage and current may be expressed in terms of the receiving-end voltage and current by using the ABCD transmission parameters as follows:

$$\mathbf{V}_S = A\mathbf{V}_R + B\mathbf{I}_R \tag{9.98}$$

$$\mathbf{I}_S = C\mathbf{V}_R + D\mathbf{I}_R \tag{9.99}$$

where

$A = D = ZY/2 + 1$

$B = Z$

$C = Y(ZY/4 + 1)$

$Z =$ total series impedance of the line

$Y =$ total shunt admittance of the line

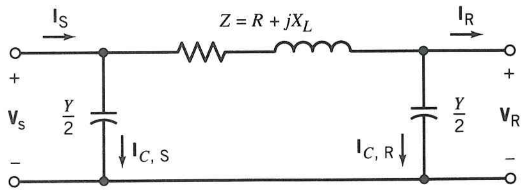


FIGURE 9.9 Equivalent circuit for a medium-length line.

At no load, the receiving-end voltage is $1/A$ times the sending-end voltage; thus, the voltage regulation is expressed as follows:

$$\text{Voltage regulation} = \frac{V_S/A - V_{R,fl}}{V_{R,fl}}100\% \tag{9.100}$$

EXAMPLE 9.7

A 60-Hz, three-phase transmission line is 125 miles long. It has a total series impedance of $35 + j140\ \Omega$ and a shunt admittance of $930 \times 10^{-6}\angle 90°$ S. It delivers 40 MW at 220 kV and 0.9 power factor lagging.

a. Determine the voltage, current, and power factor at the sending end of the line.

b. Find the voltage regulation and efficiency of the line.

Solution For a load of 40 MW, at 220 kV and 0.9 power factor lagging, the receiving-end current is given by

$$\mathbf{I}_R = \frac{40{,}000}{\sqrt{3}(220)(0.9)}\angle -\cos^{-1} 0.9 = 116.6\angle -25.8° \text{ A}$$

For the nominal π equivalent circuit, the ABCD parameters are computed as follows:

$$\begin{aligned} A = D &= ZY/2 + 1 \\ &= (35 + j140)(930 \times 10^{-6}\angle 90°)/2 + 1 \\ &= 0.935 + j0.0163 = 0.935\angle 1° \\ B = Z &= 35 + j140 = 144.3\angle 76°\ \Omega \\ C &= Y(ZY/4 + 1) \\ &= (930 \times 10^{-6}\angle 90°)[(35 + j140)(930 \times 10^{-6}\angle 90°)/4 + 1] \\ &= (-7.57 + j899.7) \times 10^{-6} = 900 \times 10^{-6}\angle 90.5° \text{ S} \end{aligned}$$

Thus, the sending-end voltage and current are given by

$$\begin{aligned} \mathbf{V}_S &= A\mathbf{V}_R + B\mathbf{I}_R \\ &= (0.935\angle 1°)(127{,}000\angle 0°) + (144.3\angle 76°)(116.6\angle -25.8°) \\ &= 130.4\angle 6.6° \text{ kV} \quad \text{(line-to-neutral)} \\ &= 225.8\angle 36.6° \text{ kV} \quad \text{(line-to-line)} \end{aligned}$$

$$\begin{aligned} \mathbf{I}_S &= C\mathbf{V}_R + D\mathbf{I}_R \\ &= (900 \times 10^{-6}\angle 90.5°)(127{,}000\angle 0°) + (0.935\angle 1°)(116.6\angle -25.8°) \\ &= 97.97 + j68.57 = 119.6\angle 35° \text{ A} \end{aligned}$$

The sending-end power factor is

$$\text{PF}_S = \cos(6.6° - 35.0°) = 0.88 \text{ leading}$$

At no load, the receiving-end voltage is $1/A$ times the sending-end voltage; thus, the voltage regulation is computed as follows:

$$\begin{aligned} \text{Voltage regulation} &= \frac{V_S/A - V_{R,fl}}{V_{R,fl}}100\% \\ &= \frac{225.8/0.935 - 220}{220}100\% = 9.77\% \end{aligned}$$

The sending-end real power is

$$\begin{aligned} P_S = 3V_S I_S \text{PF}_S &= 3(130.4 \times 10^3)(119.6)(0.88) \\ &= 41.17 \times 10^6 \text{ W} = 41.17 \text{ MW} \end{aligned}$$

Therefore, the efficiency of the line is found as follows:

$$\text{Efficiency} = (P_R/P_S)100\% = (40/41.17)100\% = 97.2\%$$

EXAMPLE 9.8

Solve Example 9.7 using per unit representation. Choose a power base of 50 MVA and a voltage base of 220 kV.

Solution

a. For the chosen $S_{base} = 50$ MVA and $V_{base} = 220$ kV, the current base is computed as follows:

$$I_{base} = \frac{S_{base}}{\sqrt{3}V_{base}} = \frac{50{,}000}{220\sqrt{3}} = 131.2 \text{ A}$$

The impedance base is calculated as

$$Z_{base} = \frac{(V_{base})^2}{S_{base}} = \frac{(220)^2}{50} = 968\ \Omega$$

Correspondingly, the admittance base is given by

$$Y_{base} = 1/Z_{base} = 1/968 = 1033 \times 10^{-6} \text{ S}$$

The series impedance is converted to per unit as follows.

$$Z = (35 + j140)/968 = 0.036 + j0.145 \text{ pu} = 0.149\underline{/76^\circ} \text{ pu}$$

The shunt admittance in per unit is found as follows:

$$Y = (j930 \times 10^{-6})/(1033 \times 10^{-6}) = j0.90 \text{ pu}$$

The ABCD parameters are computed as follows:

$$\begin{aligned} A = D &= ZY/2 + 1 \\ &= (0.149\underline{/76^\circ})(j0.90)/2 + 1 = 0.935\underline{/1^\circ} \\ B &= Z = 0.149\underline{/76^\circ} \text{ pu} \\ C &= Y(ZY/4 + 1) \\ &= j0.90[(0.149\underline{/76^\circ})(j0.90)/4 + 1] = 0.871\underline{/90.5^\circ} \text{ pu} \end{aligned}$$

The receiving-end voltage is taken as reference phasor. Thus,

$$\mathbf{V}_R = 1.0\underline{/0^\circ} \text{ pu}$$

The line delivers a per-unit power of

$$\mathbf{S}_R = \frac{40}{(0.9)(50)}\underline{/\cos^{-1} 0.9} = 0.889\underline{/25.8^\circ} \text{ pu}$$

Thus, the receiving-end current is given by

$$\mathbf{I}_R = \frac{(0.889\underline{/25.8^\circ})^*}{(1.0\underline{/0^\circ})^*} = 0.889\underline{/-25.8^\circ} \text{ pu}$$

The sending-end voltage and current are computed as follows:

$$\begin{aligned} \mathbf{V}_S &= A\mathbf{V}_R + B\mathbf{I}_R \\ &= (0.935\underline{/1^\circ})(1.0\underline{/0^\circ}) + (0.149\underline{/76^\circ})(0.889\underline{/-25.8^\circ}) \\ &= 1.0264\underline{/6.6^\circ} \text{ pu} = 225.8 \text{ kV} \end{aligned}$$

$$\begin{aligned} \mathbf{I}_S &= C\mathbf{V}_R + D\mathbf{I}_R \\ &= (0.871\underline{/90.5^\circ})(1.0\underline{/0^\circ}) + (0.935\underline{/1^\circ})(0.889\underline{/-25.8^\circ}) \\ &= 0.9114\underline{/35^\circ} \text{ pu} = 119.6 \text{ A} \end{aligned}$$

The sending end power factor is

$$\text{PF}_S = \cos(6.6^\circ - 35^\circ) = 0.88 \text{ leading}$$

b. At no load, the receiving-end voltage is $1/A$ times the sending-end voltage; thus, the voltage regulation is given by

$$\begin{aligned} \text{Voltage regulation} &= \frac{V_S/A - V_R}{V_R} 100\% \\ &= \frac{1.0264/0.935 - 1.0}{1.0} 100\% = 9.77\% \end{aligned}$$

The receiving-end real power is

$$P_R = 40/50 = 0.80 \text{ pu}$$

The sending-end real power is found as follows:

$$P_S = V_S I_S \text{PF}_S = (1.0264)(0.9114)(0.88) = 0.823 \text{ pu} = 41.16 \text{ MW}$$

The efficiency of the transmission line is given by

$$\text{Efficiency} = (P_R/P_S)100\% = (0.80/0.823)100\% = 97.2\%$$

9.5.3 The Long Transmission Line

In the analysis of long transmission lines of more than 150 miles, the lumped-parameter models that were used for short and medium-length lines may no longer provide the desired accuracy. It is then necessary to consider that the parameters are distributed uniformly throughout the length of the line.

Consider a point on the transmission line at a distance x from the receiving end of the line. The line-to-neutral voltage and the current flowing toward the receiving end are expressed, in terms of the distributed impedance and admittance parameters of the line, by differential equations of the form

$$\frac{dV}{dx} = zI \qquad (9.101)$$

$$\frac{dI}{dx} = yV \tag{9.102}$$

where

z = series impedance per unit length

y = shunt admittance per unit length

If Eq. 9.101 is differentiated and the expression for dI/dx is substituted into Eq. 9.102, the following is obtained.

$$\frac{d^2V}{dx^2} = yzV \tag{9.103}$$

Similarly, Eq. 9.102 can be differentiated and the expression for dV/dx substituted into Eq. 9.101 to obtain

$$\frac{d^2I}{dx^2} = yzI \tag{9.104}$$

The solutions of Eqs. 9.103 and 9.104 have been derived in several of the references listed at the end of the chapter; for example, see Ref. 11. Thus, the solutions are presented here without derivation. At a distance x from the receiving end of the line, the voltage and current are given by

$$\mathbf{V}_x = \tfrac{1}{2}(\mathbf{V}_R + \mathbf{I}_R Z_0)e^{\gamma x} + \tfrac{1}{2}(\mathbf{V}_R - \mathbf{I}_R Z_0)e^{-\gamma x} \tag{9.105}$$

$$\mathbf{I}_x = \tfrac{1}{2}\left(\frac{\mathbf{V}_R}{Z_0} + \mathbf{I}_R\right)e^{\gamma x} - \tfrac{1}{2}\left(\frac{\mathbf{V}_R}{Z_0} - \mathbf{I}_R\right)e^{-\gamma x} \tag{9.106}$$

respectively, where

$Z_0 = \sqrt{z/y}$ = characteristic or surge impedance

$\gamma = \sqrt{zy}$ = propagation constant

To obtain the equations relating the voltage and current at the sending end to the voltage and current at the receiving end, the distance is set equal to the length of the line; that is, $x = l$. Therefore,

$$\begin{aligned}\mathbf{V}_S &= \frac{1}{2}(e^{\gamma l} + e^{-\gamma l})\mathbf{V}_R + \frac{1}{2}Z_0(e^{\gamma l} - e^{-\gamma l})\mathbf{I}_R \\ &= (\cosh \gamma l)\mathbf{V}_R + (Z_0 \sinh \gamma l)\mathbf{I}_R \end{aligned} \tag{9.107}$$

$$\begin{aligned}\mathbf{I}_S &= \frac{1}{2}\frac{1}{Z_0}(e^{\gamma l} - e^{-\gamma l})\mathbf{V}_R + \frac{1}{2}(e^{\gamma l} + e^{-\gamma l})\mathbf{I}_R \\ &= \frac{1}{Z_0}(\sinh \gamma l)\mathbf{V}_R + (\cosh \gamma l)\mathbf{I}_R \end{aligned} \tag{9.108}$$

The transmission parameters of the long line are

$$A = D = \cosh \gamma l = \frac{1}{2}(e^{\gamma l} + e^{-\gamma l}) \tag{9.109}$$

$$B = Z_0 \sinh \gamma l = \frac{1}{2}Z_0(e^{\gamma l} - e^{-\gamma l}) \tag{9.110}$$

$$C = \frac{1}{Z_0}\sinh \gamma l = \frac{1}{2}\frac{1}{Z_0}(e^{\gamma l} - e^{-\gamma l}) \tag{9.111}$$

The equivalent circuit of the long transmission line is found by comparing its voltage and current equations with those for a nominal π circuit. Thus,

$$Z' = Z_0 \sinh \gamma l = Z\frac{\sinh \gamma l}{\gamma l} \tag{9.112}$$

$$\begin{aligned}\frac{Y'}{2} &= \frac{1}{Z_0}\frac{\cosh \gamma l - 1}{\sinh \gamma l} \\ &= \frac{Y}{2}\frac{\tanh(\gamma l/2)}{\gamma l/2}\end{aligned} \tag{9.113}$$

where

$Z = zl$ = total series impedance

$Y = yl$ = total shunt admittance

The equivalent circuit is shown in Fig. 9.10.

EXAMPLE 9.9

A 60-Hz, three-phase transmission line is 175 miles long. It has a total series impedance of $35 + j140\ \Omega$ and a shunt admittance of $930 \times 10^{-6}\angle 90°$ S. It delivers 40 MW at 220 kV and 0.9 power factor lagging.

a. Find the voltage, current, and power factor at the sending end of the line.

b. Determine the voltage regulation and efficiency of the line.

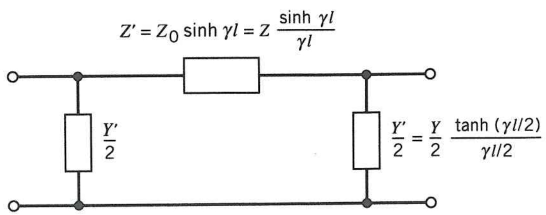


FIGURE 9.10 Equivalent circuit for a long transmission line.

Solution For a load of 40 MW, at 220 kV and 0.9 power factor lagging, the receiving-end current is given by

$$\mathbf{I}_R = \frac{40{,}000}{\sqrt{3}(220)(0.9)} \angle -\cos^{-1} 0.9 = 116.6\angle -25.8^\circ$$

For the long-line model, the transmission parameters are computed as follows:

$$Z_0 = \sqrt{\frac{35 + j140}{930 \times 10^{-6}\angle 90^\circ}} = 393.9\angle -7^\circ$$

$$\gamma l = \sqrt{(35 + j140)(930 \times 10^{-6}\angle 90^\circ)}$$
$$= 0.3663\angle 83^\circ = 0.0446 + j0.3636$$

$$e^{\gamma l} = e^{(0.0446 + j0.3636)} = 1.0456\angle 20.8^\circ$$

$$e^{-\gamma l} = e^{(-0.0446 - j0.3636)} = 0.9563\angle -20.8^\circ$$

$$A = D = \cosh \gamma l = \tfrac{1}{2}(e^{\gamma l} + e^{-\gamma l})$$
$$= \tfrac{1}{2}(1.0456\angle 20.8^\circ + 0.9563\angle -20.8^\circ) = 0.936\angle 1^\circ$$

$$B = Z_0 \sinh \gamma l = \tfrac{1}{2} Z_0 (e^{\gamma l} - e^{-\gamma l})$$
$$= \tfrac{1}{2}(393.9\angle -7^\circ)(1.0456\angle 20.8^\circ - 0.9563\angle -20.8^\circ) = 141.2\angle 76^\circ$$

$$C = (1/Z_0)\sinh \gamma l = \tfrac{1}{2}(1/Z_0)(e^{\gamma l} - e^{-\gamma l})$$
$$= \tfrac{1}{2}(1/393.9\angle -7^\circ)(1.0456\angle 20.8^\circ - 0.9563\angle -20.8^\circ)$$
$$= 910 \times 10^{-6}\angle 90^\circ$$

Thus, the sending-end voltage and current are given by

$$\mathbf{V}_s = (\cosh \gamma l)\mathbf{V}_R + (Z_0 \sinh \gamma l)\mathbf{I}_R$$
$$= (0.936\angle 1^\circ)(127{,}000\angle 0^\circ)$$
$$+ (141.2\angle 76^\circ)(116.6\angle -25.8^\circ)$$
$$= 130.1\angle 7^\circ \text{ kV} \quad \text{(line-to-neutral)}$$
$$= 225.4\angle 37^\circ \text{ kV} \quad \text{(line-to-line)}$$

$$\mathbf{I}_s = (1/Z_0)(\sinh \gamma l)\mathbf{V}_R + (\cosh \gamma l)\mathbf{I}_R$$
$$= (910 \times 10^{-6}\angle 90^\circ)(127{,}000\angle 0^\circ)$$
$$+ (0.936\angle 1^\circ)(116.6\angle -25.8^\circ)$$
$$= 120.6\angle 35.3^\circ \text{ A}$$

The sending-end power factor is

$$\text{PF}_S = \cos(7.0^\circ - 35.3^\circ) = 0.88 \text{ leading}$$

The voltage regulation is computed as follows:

$$\text{Voltage regulation} = \frac{V_S/A - V_{R,\text{fl}}}{V_{R,\text{fl}}} 100\%$$
$$= \frac{225.4/0.936 - 220}{220} 100\% = 9.46\%$$

The sending-end real power is

$$P_S = 3V_S I_S \text{PF}_S = 3(130.1 \times 10^3)(120.6)(0.88)$$
$$= 41.42 \times 10^6 \text{ W} = 41.42 \text{ MW}$$

Therefore, the efficiency of the line is found as follows:

$$\text{Efficiency} = (P_R/P_S)100\% = (40/41.42)100\% = 96.6\%$$

DRILL PROBLEMS

D9.13 Solve Example 9.6 using per unit representation. Choose a power base of 100 MVA and a voltage base of 220 kV.

D9.14 A three-phase transmission line is 45 mi long, and it delivers 50 MVA at 0.866 power factor lagging to a load connected to its receiving end at 115 kV. The transmission line is composed of 795-MCM, 26/7 ACSR conductors with flat horizontal spacing of 4 m between adjacent conductors. Determine the sending-end voltage, current, and power.

D9.15 A three-phase, 60-Hz transmission line delivers 25 MW at 69 kV and 0.8 power factor lagging to a load center substation at its receiving end. The transmission line consists of 336-MCM, 26/7 ACSR conductors arranged horizontally with a spacing of 5 m between adjacent conductors. The line is 75 km long. Determine the voltage, current, power, and power factor at the sending end of the line.

D9.16 A three-phase, 60-Hz, completely transposed transmission line has a length of 100 km and has a series impedance per phase of $0.25 + j0.85$ Ω/mi and a shunt admittance of 5.0×10^{-5} S/mi. The transmission line delivers 150 MW at 0.85 lagging power factor to a load connected to its receiving end. The line-to-line voltage at the receiving end is 138 kV. Determine the following:

a. Sending-end voltage and current
b. Sending-end real and reactive power
c. Total power dissipated in the transmission line

D9.17 A three-phase, 60-Hz transmission line has the following parameters: $R = 0.25\ \Omega/\text{mi}$, $X = 1.0\ \Omega/\text{mi}$, and $Y = 5.0\ \mu\text{S/mi}$. The line is 200 mi long, and it delivers 150 MW at 220 kV and unity power factor to a substation at its receiving end.

a. Calculate the voltage, current, power factor, and real power at the sending end.
b. Calculate the voltage regulation of the line.

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PROBLEMS

9.1 Find the AC resistance in Ω/km of a $\frac{3}{4}$-inch-diameter conductor made of 97.3% conductivity hard-drawn copper at 60 Hz and 75°C.

9.2 Find the AC resistance in Ω/mi of a 500-MCM conductor made of 97.3% conductivity hard-drawn copper at 50 Hz and 60°C.

9.3 A single-phase, 60-Hz transmission line has solid cylindrical copper conductors 1.5 cm in diameter. The conductors are arranged in a horizontal configuration with 1.25 m spacing between adjacent conductors. Calculate the following:

a. Inductance in mH/km due to internal flux linkages only
b. Inductance in mH/km due to both internal and external flux linkages
c. The total inductance of the line in mH/km.

9.4 A single-phase, 60-Hz transmission line consists of solid round aluminum wires each having a diameter of 1.5 cm. The conductor spacing is 3 m. Determine the inductance of the line in mH/mi.

9.5 Find the inductive reactance in Ω/km of a single-phase, 60-Hz transmission line consisting of 500-MCM ACSR conductors with a separation distance of 4 m.

9.6 A single-phase, 60-Hz transmission line consists of two bundles each containing two conductors arranged horizontally. Each conductor of a bundle has a radius of 2.0 cm, and they are 8 cm apart. The distance between the nearest conductors of each bundle is 6 m.

a. Find the geometric mean radius of each bundle.
b. Calculate the geometric mean distance.
c. Determine the total inductive reactance in Ω/km and in Ω/mi of the transmission line.

9.7 A single-phase, 60-Hz transmission line consists of two bundled conductors. Each bundle consists of three 500-MCM ACSR conductors placed at the corners of an equilateral triangle standing on one of its sides with 5 cm separation between conductor centers. The distance between the nearest conductors of each bundle is 8 m.

a. Compute the total inductance of the transmission line in mH/km and mH/mi.
b. Find the total inductive reactance in Ω/km and in Ω/mi.

9.8 A three-phase line has equilateral spacing between phase conductors with a separation distance of 4 m. Each conductor has a radius of 1.5 cm. Determine (a) the inductance per phase in mH/mi and (b) the inductive reactance per phase in Ω/km and in Ω/mi.

9.9 A three-phase transmission line has its conductors arranged horizontally and spaced such that $D_{13} = 2D_{12} = 2D_{23}$. The conductors are completely transposed. Determine the spacing between adjacent conductors in order to obtain an equivalent spacing of 5 m.

9.10 A three-phase, 230-kV, 60-Hz, completely transposed transmission line has one ACSR 1113-MCM conductor per phase and flat horizontal spacing with 5 m between adjacent conductors. Find the inductance in H/m and the inductive reactance in Ω/km.

9.11 A three-phase, 60-Hz transmission line has flat horizontal spacing. The conductors are ACSR, each having a GMR of 0.0278 ft, with a separation distance of 35 ft between adjacent conductors. Determine the inductive reactance per phase in ohms per mile. What is the name of this conductor?

9.12 A three-phase, 60-Hz, bundled transmission line consists of three 900-MCM ACSR conductors per bundle arranged with an equilateral spacing of 40 cm between conductors. The spacing between bundle centers is 8, 8, and 12 m. Calculate the inductive reactance in ohms per kilometer.

9.13 A three-phase, bundled, 345-kV, 60-Hz, completely transposed transmission line has flat horizontal spacing with 10 m between centers of adjacent bundles. Each bundle consists of three 954-MCM ACSR conductors per bundle, with equilateral spacing of 40 cm between conductors in the bundle. Calculate the inductive reactance in Ω/km.

9.14 A three-phase, 60-Hz transmission line has a flat horizontal spacing of 10 m between adjacent conductors. Each phase conductor is a 1113-MCM ACSR. Compare the inductive reactance in Ω/mi/phase of this transmission line with that of another transmission line using a bundle of two ACSR 26/7 conductors in each phase having the same total cross-sectional area of aluminum as the single-conductor line and 10 m spacing between bundle centers. The spacing between conductors in the bundle is 25 cm.

9.15 A single-phase, 60-Hz power line is supported on a horizontal crossarm, and its conductors are 2.5 m apart and carrying a current of 100 A. A telephone line is also supported on a horizontal crossarm 1.5 m directly below the power line, and the spacing between the conductors is 1.0 m. Determine (a) the mutual inductance between the power and telephone circuits and (b) the induced voltage per mile in the telephone line.

9.16 The single-phase power line and the telephone line described in Problem 9.15 are placed in the same horizontal plane. The distance between the nearest conductors of the two lines is 10 m. Determine (a) the mutual inductance between the power and telephone circuits and (b) the induced voltage per mile in the telephone line.

9.17 A three-phase, 60-Hz, power line is supported on a horizontal crossarm, and the separation distance between adjacent conductors is 2 m. The current flowing in the conductors is 100 A. A telephone line is supported on a horizontal crossarm that is placed 1.5 m below the power line, and the telephone wires are 1 m apart. Find the expression for the mutual inductance between the power line and the telephone line. Calculate the induced voltage per kilometer in the telephone line.

9.18 A three-phase power line and a telephone line are supported on the same tower. The three-phase line is arranged horizontally with a separation distance of 2.5 m between adjacent conductors. The telephone conductors are arranged vertically 0.5 m apart, and the top conductor is located 1 m directly below the middle conductor of the three-phase line. Determine the expression for the mutual inductance between the power line and the telephone line.

9.19 A single-phase transmission line consists of two conductors each having a diameter $\frac{1}{4}$ in. The conductors are 10 ft apart and are placed 40 ft above ground. Calculate the capacitance to neutral in farads per meter.

9.20 Calculate the capacitance to neutral in F/m and the admittance to neutral in S/km for the single-phase transmission line described in Problem 9.3.

9.21 A single-phase, 60-Hz transmission line consists of two bundles. Each bundle contains two identical conductors of 1 cm radius and placed 10 cm apart. The separation distance between the bundle centers is 5 m. Compute the capacitance and the capacitive reactance of 50 mi of this transmission line.

9.22 A three-phase, 60-Hz transmission line is 100 mi long and has flat horizontal spacing. Each phase conductor has an outside diameter of 3 cm, and the separation distance between adjacent conductors is 10 m. Calculate the capacitive reactance to neutral in Ω-m and the total capacitive reactance of the line in ohms.

9.23 Calculate the capacitance per phase in F/m and the admittance per phase in S/km for the three-phase transmission line described in Problem 9.10. Also calculate the total charging current taken by the transmission line capacitance when it is 125 km long and is operated at 345 kV.

9.24 A three-phase, 60-Hz transmission line is completely transposed. The separation distances between conductors are 7, 7, and 10 m. Each conductor has a diameter of 5 cm. The line-to-line voltage is 138 kV, and the length of the transmission line is 100 km. Determine the following:

a. Capacitance per phase

b. Total capacitive reactance per phase

c. Charging current taken by the total capacitance of the transmission line

9.25 A three-phase, 60-Hz transmission line has an equivalent equilateral spacing of 5 ft, and its capacitive reactance to neutral is given as 196.1 kΩ-mi. What would be the value of the capacitive reactance to neutral in kΩ-mi at 1-ft spacing and 25 Hz?

9.26 Calculate the capacitance per phase in F/m and the admittance per phase in S/km for the three-phase transmission line described in Problem 9.13. Also calculate the total reactive power in MVAR/km supplied by the line capacitance when it is operated at 345 kV.

9.27 A three-phase, 60-Hz transmission line has a flat horizontal spacing of 10 m between adjacent conductors. Each phase conductor is a 1113-MCM ACSR. Compare the capacitive reactance in Ω-mi/phase of this transmission line with that of another transmission line using a bundle of two ACSR 26/7 conductors in each phase having the same total cross-sectional area of aluminum as the single-conductor line and 10 m spacing between bundle centers. The spacing between conductors in the bundle is 25 cm.

9.28 A three-phase, 60-Hz transmission line is composed of 300-MCM 26/7 ACSR conductors that are equilaterally spaced with 1.5 m between conductor centers. The line is 50 km long, and it delivers 2.5 MW at 13.8 kV to a load connected to its receiving end. Find the sending-end voltage, current, real power, and reactive power for the following conditions.

a. 80% power factor lagging

b. Unity power factor

c. 90% power factor leading

9.29 A three-phase, 34.5-kV, 60-Hz, 40-km transmission line has a series impedance $z = 0.20 + j0.50\ \Omega/\text{km}$. The load at the receiving end absorbs 10 MVA at 33 kV. Calculate the following:

- **a.** ABCD parameters
- **b.** Sending-end voltage at a power factor of 0.9 lagging
- **c.** Sending-end voltage at a power factor of 0.9 leading

9.30 A three-phase, 60-Hz, completely transposed transmission line has the following total parameters:

$$Z_{\text{series}} = 10 + j50\ \Omega$$
$$Y_{\text{shunt}} = j30 \times 10^{-5}\ \text{S}$$

The transmission line is 80 mi long, and the line-to-line voltage at the receiving end is 230 kV. The load connected to the receiving end of the line may be represented by the load impedance $Z_L = 150\angle 36.9°\ \Omega$.

- **a.** Determine the current and the line-to-line voltage at the sending end.
- **b.** Find the voltage regulation.
- **c.** Calculate the real and reactive power at the sending end.
- **d.** Find the efficiency of the transmission line.

9.31 A three-phase, 230-kV, 60-Hz, 200-km transmission line has a series impedance $z = 0.10 + j0.35\ \Omega/\text{km}$ and a shunt admittance $y = j5.0 \times 10^{-6}$ S/km. The line delivers 250 MW at 220 kV and 0.95 power factor lagging to a substation connected to its receiving end. Using the nominal π circuit, calculate:

- **a.** The ABCD parameters
- **b.** The sending-end voltage and current
- **c.** The percent voltage regulation

9.32 A three-phase transmission line is 120 mi long. It has a total series impedance of $25 + j110\ \Omega$ and a total shunt admittance of $j5 \times 10^{-4}$ S. It delivers 175 MW at 220 kV and 0.9 power factor lagging to a load connected to its receiving end. Use per-unit representation and find

- **a.** The voltage, current, power factor, and real power at the sending end
- **b.** The voltage regulation

9.33 A three-phase, 60-Hz, 350-km transmission line has the following parameters: $R = 0.10\ \Omega/\text{km}$, $X_L = 0.4\ \Omega/\text{km}$, and $X_C = 350\ \text{k}\Omega\text{-km}$. The line delivers 150 MW at 230 kV and unity power factor to its receiving end. Determine the sending-end voltage, current, real power, and power factor. Use per-unit representation.

9.34 A three-phase, 60-Hz, 300-mi transmission line has a total series impedance $Z = 40 + j175\ \Omega$ and a total shunt admittance $Y = j10^{-3}$ S. The line delivers 300 MW at 220 kV and 0.9 power factor lagging.

- **a.** Determine the sending-end voltage, current, power factor, and real power.

- **b.** Repeat part (a) when the transmission line delivers 300 MW at 220 kV and unity power factor.

9.35 A three-phase, 500-kV, 60-Hz, 300-km transmission line has a series impedance $z = 0.04 + j0.50\ \Omega/\text{km}$ and a shunt admittance $y = j3.5 \times 10^{-6}$ S/km. The line delivers 1000 MW at 480 kV and unity power factor to its receiving end. Calculate the following:

- **a.** Characteristic impedance Z_0
- **b.** ABCD parameters
- **c.** Sending-end voltage and current
- **d.** Percent voltage regulation

9.36 Determine the equivalent π circuit for the transmission line of Problem 9.35 and compare it with the nominal π circuit.

Ten

Power Flow Solutions

10.1 INTRODUCTION

The planning, design, and operation of electric power systems require continuing and comprehensive analysis in order to determine system performance and to evaluate alternative system expansion plans. Because of the increasing cost of system additions and modifications, as well as soaring fuel costs, it is imperative that utilities consider a range of design options. In-depth analysis is necessary to determine the effectiveness of each alternative in alleviating operating problems and supplying load demands during normal and abnormal operating conditions, for peak and off-peak loadings, and for both present and future power systems.

As the size of the network grows, the determination of such system variables as voltage levels and power flows in the transmission lines, transformers, and generators becomes more and more difficult. A large volume of network data must be collected and handled accurately. This is aggravated by the dynamic nature of the system as it is reconfigured to meet changing system conditions. Thus, as the electric system grows in size and the number of its interconnections increases, planning for future expansion becomes increasingly complex.

The availability of fast and large computers has somewhat eased the work load of the power system engineer. Routine calculations can now be accomplished more efficiently and more extensively. Advances in device and system modeling, as well as developments in computational techniques, have greatly enhanced the analysis and planning tasks.

The high costs of system operations and system expansions and modifications have paved the way for the use of computers in power systems. To assist the engineer in his or her role as system operator and planner, the engineer is provided with digital computers and highly developed computer programs.

10.1.1 Scope of Power System Analysis

The performance of the power system is routinely analyzed under normal steady-state operating conditions, as well as under abnormal conditions in the presence of sudden, large disturbances. The principal power system problems usually considered include power flow analysis, fault analysis, transient stability analysis, and system protection.

Power flow calculations provide the power flows and voltages for a particular steady-state operating condition of the system. Power flow analysis is performed for small or large power systems, for high-voltage and low-voltage systems, and for existing and future systems. The power flow problem formulation and the popularly accepted solution methods such as Gauss–Seidel, Newton–Raphson, and fast-decoupled power flow methods are presented in this chapter.

The analysis of symmetrical and unsymmetrical faults, the description and applications of protection systems, and transient stability analysis are discussed in Chapter 11.

It is customary in power system analysis to use per-unit representation. The circuit parameters are expressed in per unit, and all calculations are performed in per unit. The power base is usually chosen to be the rating of one of the major pieces of equipment or is set as a company policy. The power base is the same for all parts of the power system. The voltage bases are normally chosen as the nominal voltages in the various parts of the system or are selected to be the voltage ratings of the transformer windings in order to maintain a unity per-unit turns ratio of the transformer. The current and impedance bases are computed on the basis of the previously selected values of power base and voltage bases.

10.1.2 One-Line Diagrams

A typical power system involves the interconnection between many different components, including generators, transformers, transmission lines, loads, circuit breakers, and switches. These interconnections are commonly referred to as *nodes* or *buses*.

A power system that operates under balanced conditions may be studied using per-phase analysis. Thus, it is sufficient to represent the three-phase power system by a single-phase power system in a simplified diagram commonly referred to as a *one-line, or single-line, diagram*. A one-line diagram of a sample power system is shown in Fig. 10.1.

The complexity of representation in the one-line diagram depends mainly on the type of study to be undertaken. For example, information about circuit breakers and relays may be important and have to be present in a system protection study; however, they may be omitted completely in a power flow analysis. The various real and reactive power demands at the different buses are

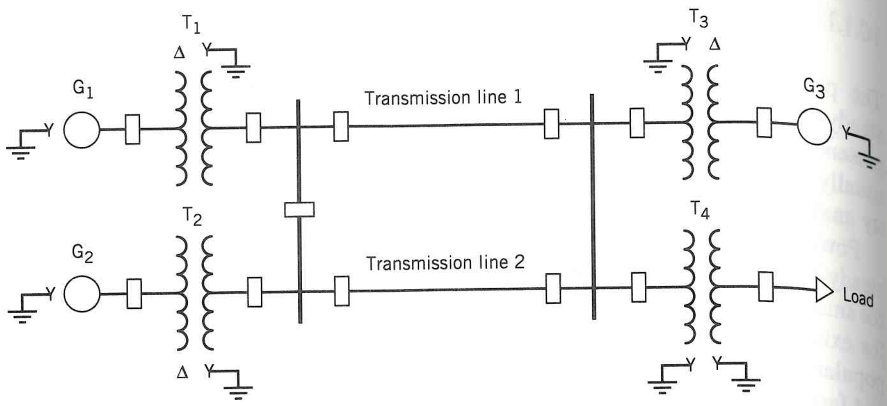

FIGURE 10.1 One-line diagram of a sample power system.

needed in power flow studies but may sometimes be neglected in short-circuit analysis.

The one-line diagram is used to build the equivalent circuit or impedance diagram of the power system. The models to be used for representing the generators, electrical loads, transmission lines, and transformers are selected and are included in the impedance diagram. The models are chosen depending on the analysis to be performed.

10.1.3 Power System Modeling

The equivalent circuit for the synchronous generator was derived in Chapter 7. The circuit model for the cylindrical-rotor machine consists of the per-phase generated voltage E_a and the per-phase armature resistance R_a and synchronous reactance X_s in series. Thus, the synchronous generator is represented by the equivalent circuit shown in Fig. 10.2.

The equivalent circuit of a transmission line was derived in Chapter 9 and is shown in Fig. 10.3. A nominal π circuit is chosen over a nominal T circuit in order not to introduce an additional node at the midpoint of the line. The series branch represents the series impedance per phase, and the shunt branch

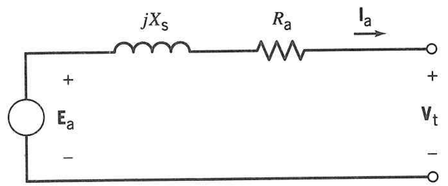

FIGURE 10.2 Equivalent circuit of a synchronous generator.

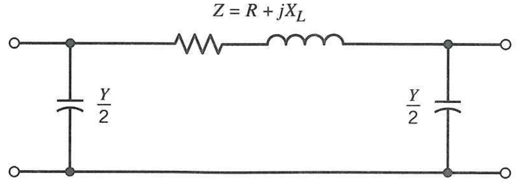

FIGURE 10.3 Equivalent circuit of a transmission line.

represents one half of the total capacitance of the line. For short transmission lines, the shunt branches are neglected.

The equivalent circuit of the three-phase transformer was derived in Chapter 4. The series impedance is given in terms of the per-phase equivalent resistance and leakage reactance of the transformer windings. The shunt magnetizing branch is generally neglected except when the transformer efficiency is being considered; then the core loss is included. Thus, the equivalent circuit is as shown in Fig. 10.4.

For the sample power system whose one-line diagram is shown in Fig. 10.1, the impedance diagram is derived by representing the various system components by their respective equivalent circuits. When all the resistances are neglected, the impedance diagram reduces to the reactance diagram shown in Fig. 10.5. From the figure, it may be seen that all shunt elements have also been assumed to be negligible. Thus, this model may be useful in short-circuit analysis, which will be discussed in Section 11.2.

The models and equivalent circuits described in this section are valid for studying balanced system conditions. Thus, they are useful for power flow analysis, as well as for balanced three-phase faults and transient stability analysis. They are also needed for unbalanced or unsymmetrical fault analysis, together with other additional models which are described later in Section 11.2.3.

10.2 POWER FLOW ANALYSIS

Power flow analysis is the solution for the static operating condition of a power system. It is the most frequently performed of routine power network calculations in digital computers. Power flow analysis is used in power system

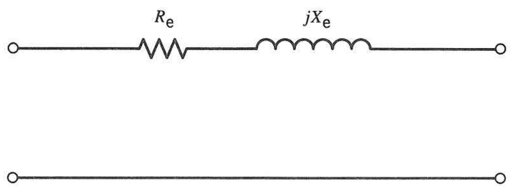

FIGURE 10.4 Equivalent circuit of a three-phase transformer.

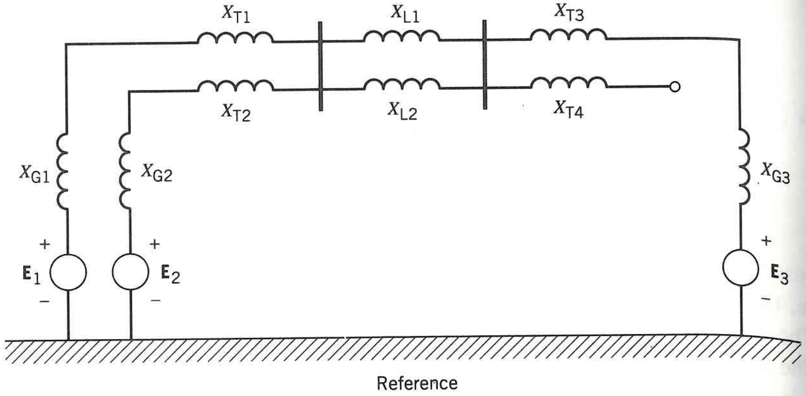

FIGURE 10.5 Impedance diagram for power system of Fig. 10.1.

planning, operational planning, and operations/control. It is also employed in multiple assessments, stability analysis, and system optimization.

Power flow programs compute the voltage magnitudes and phase angles at each bus of the network under steady-state operating conditions. These programs also compute the power flows and power losses for all equipment, including transformers and transmission lines. Thus, overloaded transformers and transmission lines are identified, and remedial measures can be designed and implemented.

Present-day computers have sufficiently large storage and high speeds. These enable the power system engineer to simulate and analyze the many different base cases, summer and winter peak conditions, and various contingency cases encountered in system operation and planning.

10.2.1 Power Flow Concepts

A power system comprising three buses is shown in Fig. 10.6. A generator is connected to each of the first two buses, and an electrical load is connected to

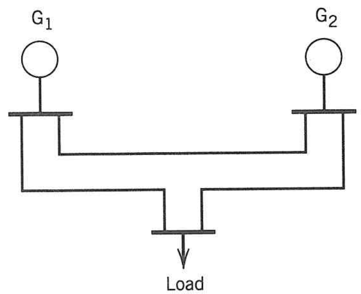

FIGURE 10.6 A three-bus power system.

the third bus. The real power and reactive power demands are known for the load bus, bus 3, and the generator voltages at buses 1 and 2 are also specified. The three transmission lines interconnecting the buses contain both resistance and reactance; thus, electric current flowing through these lines results in electrical losses.

The two generators must jointly supply the total load requirements and the power losses in the transmission lines. While doing so, the generators are constrained to operate within their power generation capabilities, and the power they deliver has to be at the desired voltage at the load. In addition, there should be no overloading of equipment, including transmission lines and transformers. Furthermore, there should be no bus voltages either above or below specified operating limits. In case of an equipment overload or voltage-limit violation, the generation schedules have to be adjusted, power flows in the transmission lines have to be rerouted, or capacitor banks have to be switched in order to bring the system conditions back to normal.

The requirements of power system operations described above are satisfied to a certain extent by undertaking power flow studies to determine appropriate measures and proper procedures. These analytical studies are also incorporated in power system planning and engineering design. These ensure that future expansions and modifications in the power system will redound to the ultimate objectives of reliability, economy, and quality of service.

10.2.2 Node-Voltage Equations

A three-bus power system consisting of two generators and one load interconnected by transmission lines is shown in Fig. 10.6. A per-phase impedance diagram that can be used in a power flow study for this three-bus system is shown in Fig. 10.7. This impedance diagram is basically an electrical network containing four nodes. The phasor voltages at nodes 1, 2, and 3 may be expressed with respect to the fourth node, which is taken as reference or ground.

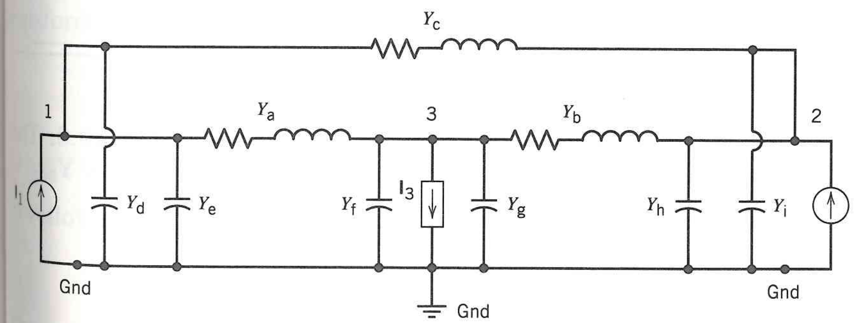

FIGURE 10.7 Impedance diagram for three-bus power system.

Kirchhoff's current law is applied to each node of the network of Fig. 10.7 successively, with the sum of currents flowing into the node set equal to the sum of currents flowing away from the node. Thus, the node-voltage equations for the three-bus system of Fig. 10.6 are obtained as follows:

$$\mathbf{I}_1 = (\mathbf{V}_1 - \mathbf{V}_2)Y_c + (\mathbf{V}_1 - \mathbf{V}_3)Y_a + \mathbf{V}_1(Y_d + Y_e) \qquad (10.1)$$
$$\mathbf{I}_2 = (\mathbf{V}_2 - \mathbf{V}_3)Y_b + (\mathbf{V}_2 - \mathbf{V}_1)Y_c + \mathbf{V}_2(Y_h + Y_i) \qquad (10.2)$$
$$-\mathbf{I}_3 = (\mathbf{V}_3 - \mathbf{V}_1)Y_a + (\mathbf{V}_3 - \mathbf{V}_2)Y_b + \mathbf{V}_3(Y_f + Y_g) \qquad (10.3)$$

Rearranging these equations yields

$$\mathbf{I}_1 = (Y_a + Y_c + Y_d + Y_e)\mathbf{V}_1 - Y_c\mathbf{V}_2 - Y_a\mathbf{V}_3 \qquad (10.4)$$
$$\mathbf{I}_2 = -Y_c\mathbf{V}_1 + (Y_b + Y_c + Y_h + Y_i)\mathbf{V}_2 - Y_b\mathbf{V}_3 \qquad (10.5)$$
$$-\mathbf{I}_3 = -Y_a\mathbf{V}_1 - Y_b\mathbf{V}_2 + (Y_a + Y_b + Y_f + Y_g)\mathbf{V}_3 \qquad (10.6)$$

These three equations may be written in standard matrix form as

$$\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \\ -\mathbf{I}_3 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \\ \mathbf{V}_3 \end{bmatrix} \qquad (10.7)$$

The vector on the left-hand side is called the bus injection-current vector $\mathbf{I}_{bus}$. On the right-hand side are the bus voltage vector $\mathbf{V}_{bus}$ and the *bus admittance matrix* $\mathbf{Y}_{bus}$, which relates the bus voltages to the injection currents.

The bus admittance matrix is square, sparse, and symmetrical when there are no phase shifters and no mutual coupling. A diagonal element Y_{kk} is called the self-admittance of bus k and is found by summing the primitive admittances of all lines and transformers connected to bus k, plus the admittances of shunt connections from bus k to reference. The off-diagonal element Y_{km} is the mutual admittance between bus k and bus m, and it is equal to the negative of the admittance of the line or transformer between buses k and m; thus, it is zero if there is no connection from bus k to bus m.

EXAMPLE 10.1

The one-line diagram of a four-bus power system is shown in Fig. 10.8. The line impedances are given in per unit. Find the bus admittance matrix $\mathbf{Y}_{bus}$.

Solution The elements of the bus admittance matrix are computed as follows:

$$Y_{12} = Y_{21} = -1.0/(0.05 + j0.15) = -2 + j6$$
$$Y_{13} = Y_{31} = -1.0/(0.10 + j0.30) = -1 + j3$$

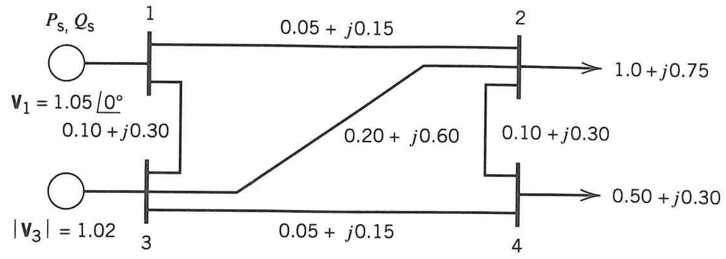

FIGURE 10.8 Four-bus power system of Example 10.1.

$$Y_{11} = (2 - j6) + (1 - j3) = 3 - j9$$
$$Y_{23} = Y_{32} = -1.0/(0.20 + j0.60) = -0.5 + j1.5$$
$$Y_{24} = Y_{42} = -1.0/(0.10 + j0.30) = -1 + j3$$
$$Y_{22} = (2 - j6) + (0.5 - j1.5) + (1 - j3) = 3.5 - j10.5$$
$$Y_{34} = Y_{43} = -1.0/(0.05 + j0.15) = -2 + j6$$
$$Y_{33} = (1 - j3) + (0.5 - j1.5) + (2 - j6) = 3.5 - j10.5$$
$$Y_{44} = (1 - j3) + (2 - j6) = 3 - j9$$

Hence, the bus admittance matrix is assembled as follows:

$$\mathbf{Y}_{bus} = \begin{bmatrix} 3.0 - j9.0 & -2.0 + j6.0 & -1.0 + j3.0 & 0.0 + j0.0 \\ -2.0 + j6.0 & 3.5 - j10.5 & -0.5 + j1.5 & -1.0 + j3.0 \\ -1.0 + j3.0 & -0.5 + j1.5 & 3.5 - j10.5 & -2.0 + j6.0 \\ 0.0 + j0.0 & -1.0 + j3.0 & -2.0 + j6.0 & 3.0 - j9.0 \end{bmatrix}$$

DRILL PROBLEMS

D10.1 The power system shown in Fig. 10.1 is operating at steady state. The reactance of each transmission line is $X = 20\ \Omega$. The generators and transformers are rated as follows:

G1: 20 MVA, 12 kV, $X = 1.20$ pu
G2: 60 MVA, 13.8 kV, $X = 1.40$ pu
G3: 50 MVA, 13.2 kV, $X = 1.40$ pu
T1: 25 MVA, 12/69 kV, $X = 0.08$ pu
T2: 75 MVA, 13.8/69 kV, $X = 0.16$ pu
T3: 60 MVA, 69/13.2 kV, $X = 0.14$ pu
T4: 75 MVA, 69/13.8 kV, $X = 0.16$ pu

Choose a power base of 100 MVA and a voltage base of 12 kV in the circuit

D10.2 In the one-line diagram of the four-bus power system shown in Fig. 10.8, the per-unit line impedances are given as follows:

$$\begin{aligned} \text{Line 1–2}: \quad & Z_{12} = 0.20 + j0.60 \\ \text{Line 1–3}: \quad & Z_{13} = 0.10 + j0.30 \\ \text{Line 2–3}: \quad & Z_{23} = 0.30 + j0.90 \\ \text{Line 2–4}: \quad & Z_{24} = 0.10 + j0.30 \\ \text{Line 3–4}: \quad & Z_{34} = 0.20 + j0.60 \end{aligned}$$

Find the bus admittance matrix $\mathbf{Y}_{\text{bus}}$.

10.2.3 Classification of Buses

The power system model used in power flow analysis shows the interconnections between generating plants and substations, substations and transmission lines, distribution lines and loads, and so forth. Three major types of nodes or buses are identified in the power network. These are listed ahead, together with their corresponding descriptive equations:

1. *Load bus, or P-Q bus k.* At a load bus, the net real and reactive power demands are specified, or scheduled; that is,

$$\begin{aligned} \mathbf{S}_k^{\text{sp}} &= P_k^{\text{sp}} + jQ_k^{\text{sp}} \\ &= (P_{\text{G}k} - P_{\text{D}k}) + j(Q_{\text{G}k} - Q_{\text{D}k}) \end{aligned} \tag{10.8}$$

where

$P_k^{\text{sp}}, Q_k^{\text{sp}}$ = specified real and reactive power at bus k

$P_{\text{G}k}, Q_{\text{G}k}$ = real and reactive power generation at bus k

$P_{\text{D}k}, Q_{\text{D}k}$ = real and reactive power demand at bus k

A physical load may or may not be present at bus k; in the latter case, $P_{\text{D}k}$ and $Q_{\text{D}k}$ are simply set equal to zero. When a generator is present at a P-Q bus, its real and reactive power generations are given and are fixed.

2. *Generator bus, or P-V bus m.* The generators are connected to P-V buses. At these buses, the real power generation and the voltage magnitude are specified; thus,

$$P_m^{\text{sp}} = P_{\text{G}m} - P_{\text{D}m} \tag{10.9}$$

$$|\mathbf{V}_m| = V_m^{\text{sp}} \tag{10.10}$$

where V_m^{sp} is the specified voltage magnitude at bus m. The voltage magnitude at a P-V bus is fixed at its specified, or controlled, value provided the reactive power generation is able to support the voltage; that is, the reactive power generated is within operating limits. Otherwise, the voltage is allowed to seek its proper level, and the bus is converted into a P-Q bus by specifying the reactive power at this bus.

3. *System slack, or swing bus s.* Because the system losses are not known precisely before the power flow solution, it is not possible to specify the real power injected at every bus. Hence, the real power of one of the generator buses is allowed to swing, and it supplies the slack between the scheduled real power generations and the sum of all loads and system losses. Therefore, the swing bus voltage magnitude is specified and its voltage phase angle is usually chosen as the system reference and set equal to zero. Thus,

$$|\mathbf{V}_s| = V_s^{\text{sp}} \tag{10.11}$$

$$\delta_s = 0° \tag{10.12}$$

where V_s and δ_s are the swing bus voltage magnitude and angle, respectively.

10.2.4 The Gauss–Seidel Method

In Section 10.2.2, the power system is represented as an electrical network that is characterized by node equations. Thus at bus k, the phasor current $\mathbf{I}_k$ injected into the bus is given by

$$Y_{k1}\mathbf{V}_1 + Y_{k2}\mathbf{V}_2 + Y_{k3}\mathbf{V}_3 + \cdots + Y_{kN}\mathbf{V}_N = \mathbf{I}_k \tag{10.13}$$

where

$\mathbf{V}_m$ = phasor voltage at bus m

Y_{km} = element of the bus admittance matrix

N = total number of buses in the power system

The specified, or scheduled, complex power injection at bus k may be expressed in terms of the current injected into the bus and the bus voltage phasor as follows:

$$\mathbf{S}_k^{\text{sp}} = P_k^{\text{sp}} + jQ_k^{\text{sp}} = \mathbf{V}_k\mathbf{I}_k^* \tag{10.14}$$

where

$(\,)^*$ = the complex conjugate

P_k^{sp} = specified real power injected into bus k

Solving Eq. 10.14 for $\mathbf{I}_k$ yields

$$\mathbf{I}_k = \frac{P_k^{sp} - jQ_k^{sp}}{\mathbf{V}_k^*} \tag{10.15}$$

Substituting Eq. 10.15 into Eq. 10.13,

$$Y_{k1}\mathbf{V}_1 + Y_{k2}\mathbf{V}_2 + \cdots + Y_{km}\mathbf{V}_m + \cdots + Y_{kN}\mathbf{V}_N = \frac{P_k^{sp} - jQ_k^{sp}}{\mathbf{V}_k^*} \tag{10.16}$$

Transposing all terms, except for the term $\mathbf{V}_k Y_{kk}$, from the left-hand side of Eq. 10.16, and dividing by Y_{kk} yields the expression for the phasor voltage $\mathbf{V}_k$ at bus k.

$$\mathbf{V}_k = \frac{1}{Y_{kk}}\left[\frac{P_k^{sp} - jQ_k^{sp}}{\mathbf{V}_k^*} - (Y_{k1}\mathbf{V}_1 + Y_{k2}\mathbf{V}_2 + \cdots + Y_{km}\mathbf{V}_m + \cdots + Y_{kN}\mathbf{V}_N)\right] \quad \text{for } m \neq k \tag{10.17}$$

The swing bus voltage is taken as the reference phasor, and its voltage magnitude is specified and phase angle set equal to zero. Equation 10.17 is used to write a system of $(N-1)$ simultaneous algebraic equations relating the phasor voltage at the individual buses with the corresponding power injections at the bus and phasor voltages at all the buses. These equations are coupled through the elements of the bus admittance matrix. This system of equations may be solved by an iterative technique such as the Gauss–Seidel method.

The *Gauss–Seidel method* is an iterative technique for solving a system of nonlinear algebraic equations such as that given in Eq. 10.17 for the unknown bus phasor voltages. With this procedure, the phasor voltage at a bus is found by using the latest computed values of the phasor voltages at the other buses. Thus, Eq. (10.17) may be rewritten as follows to demonstrate the procedure.

$$\mathbf{V}_k^{(i+1)} = \frac{1}{Y_{kk}}\left[\frac{P_k^{sp} - jQ_k^{sp}}{\mathbf{V}_k^{(i)*}} - \sum_{\substack{m=1\\ m\neq k}}^{N} Y_{km}\mathbf{V}_m^{(\beta)}\right]$$
$$\beta = i \quad \text{for } m > k$$
$$\beta = i+1 \quad \text{for } m < k \tag{10.18}$$

where i represents the iteration count.

In the solution of Eq. 10.18, the most recently computed values of $\mathbf{V}_k$ are used. The solution procedure bypasses the swing bus because the phasor voltage of the swing bus is already known. For a P-Q (or load) bus, the procedure is applied directly to obtain an improved estimate of the phasor voltage inasmuch

In a P-V (or generator) bus, the bus voltage V_k is specified; hence, Q_k^{sp} is not directly available. Therefore, an estimate Q_k^{calc} is computed by using the current estimates of the phasor voltages as follows:

$$Q_k^{calc} = -\text{Im}\,[\mathbf{V}_k^*(Y_{k1}\mathbf{V}_1 + Y_{k2}\mathbf{V}_2 + \cdots + Y_{km}\mathbf{V}_m + \cdots + Y_{kN}\mathbf{V}_N)] \tag{10.19}$$

In Eq. 10.19, Im[ ] indicates the imaginary part of the expression in the brackets. The computed value of the reactive power Q_k^{calc} is used to replace Q_k^{sp} in Eq. 10.18, and V_k is recalculated. Then the magnitude of V_k is reset to its specified value V_k^{sp}, but the new value of its phase angle is retained.

The Gauss–Seidel method has the attractions of simplicity, comparatively good performance, and nonstorage of previous values. It has a very reliable convergence characteristic, but the rate of convergence is quite slow. The rate of convergence is improved somewhat with the use of an accelerating factor α. Therefore, at the $(i+1)$th iteration, the phasor voltage is modified as follows:

$$[\mathbf{V}_k^{(i+1)}]_{mod} = \mathbf{V}_k^{(i)} + \alpha\,\Delta\mathbf{V}_k^{(i+1)} \tag{10.20}$$

In the second term on the right-hand side of Eq. 10.20, $\Delta\mathbf{V}_k^{(i+1)}$ is defined as

$$\Delta\mathbf{V}_k^{(i+1)} = \mathbf{V}_k^{(i+1)} - \mathbf{V}_k^{(i)} \tag{10.21}$$

When α is left at its default value of 1.0, the phasor voltage is not modified. The best value of acceleration factor to use is usually different for different systems. A value of 1.6 for α is generally considered a good choice.

The iterative process is said to have converged when the process no longer yields any improvement on the solution. At this point, the phasor voltages at all buses have been found and may be used to derive other information about the steady-state operating characteristics of the power system. The real and reactive power generation of the slack bus and the reactive power generations of the other generators may be found by using Eqs. 10.16 and 10.8. The real and reactive powers flowing through any transformer or transmission line, with series impedance Z and shunt admittance Y, connected between buses k and m may be found as follows:

$$\mathbf{S}_{km} = P_{km} + jQ_{km} = \mathbf{V}_k\left(\frac{\mathbf{V}_k - \mathbf{V}_m}{Z} + \frac{Y}{2}\mathbf{V}_k\right)^* \tag{10.22}$$

The real power and reactive power losses of any transformer or transmission line are derived from the line power flows computed by using Eq. 10.22.

$$\mathbf{S}_{km}^{loss} = \mathbf{S}_{km} + \mathbf{S}_{mk}$$
$$P_{km}^{loss} + jQ_{km}^{loss} = (P_{km} + P_{mk}) + j(Q_{km} + Q_{mk}) \tag{10.23}$$

By adding the losses in all transformers and transmission lines, the total system

EXAMPLE 10.2

With the bus data given for the power system shown in Fig. 10.8, determine the value of the voltage $\mathbf{V}_2$ at bus 2 that is produced by the first iteration of the Gauss–Seidel method. Bus number 1 is taken as swing bus.

Solution The specified real and reactive powers at bus 2 are

$$P_2^{sp} = -1.00 \text{ pu}$$
$$Q_2^{sp} = -0.75 \text{ pu}$$

The initial values of the bus voltages are

$$\mathbf{V}_1 = 1.05\angle 0^\circ \text{ pu} \qquad \mathbf{V}_2 = 1.00\angle 0^\circ \text{ pu}$$
$$\mathbf{V}_3 = 1.02\angle 0^\circ \text{ pu} \qquad \mathbf{V}_4 = 1.00\angle 0^\circ \text{ pu}$$

The voltage at bus 2, for the first iteration, is calculated as follows:

$$\mathbf{V}_2 = \frac{1}{Y_{22}}\left[\frac{P_2^{sp} - jQ_2^{sp}}{(\mathbf{V}_2)^*} - Y_{21}\mathbf{V}_1 - Y_{23}\mathbf{V}_3 - Y_{24}\mathbf{V}_4\right]$$

Substituting the initial values of the bus voltages and the elements of the admittance matrix,

$$\mathbf{V}_2 = \frac{1}{3.5 - j10.5}\left[\frac{-1.00 + j0.75}{(1.00\angle 0^\circ)^*} - (-2.0 + j6.0)(1.05\angle 0^\circ) - (-0.5 + j1.5)(1.02\angle 0^\circ - (-1.0 + j3.0)(1.00\angle 0^\circ)\right]$$

Simplifying,

$$\mathbf{V}_2 = 0.94\angle -3.9^\circ \text{ pu}$$

DRILL PROBLEMS

D10.3 A two-bus power system has a bus admittance matrix given by

$$\mathbf{Y}_{bus} = \begin{bmatrix} 2.0\angle -75^\circ & 1.5\angle 105^\circ \\ 1.5\angle 105^\circ & 2.0\angle -75^\circ \end{bmatrix} \text{ pu}$$

The power demand on bus 1 is $\mathbf{S}_{D1} = (1.2 + j0.9)$ pu and the power demand on bus 2 is $\mathbf{S}_{D2} = (0.4 - j0.3)$ pu. Bus 1 is selected as swing bus and its voltage is specified as $\mathbf{V}_1 = 1.05\angle 0^\circ$. Use the Gauss–Seidel iterative technique to find the per-unit voltage at bus 2.

D10.4 Calculate the per-unit complex power supplied by the swing generator of the power system described in Problem D10.3.

D10.5 A two-bus power system has a generator connected to bus 1. Bus 2 is connected to bus 1 through a transmission line with a series impedance $Z = (0.02 + j0.10)$ pu and a shunt admittance $Y = j0.20$ pu. The load demands on the buses are

$$\mathbf{S}_{L1} = (0.4 + j0.1) \text{ pu}$$
$$\mathbf{S}_{L2} = (0.8 + j0.2) \text{ pu}$$

Bus 1 is chosen as the slack or swing bus, and its phasor voltage is set to $V_1 = 1.0\angle 0^\circ$.

a. Form the bus admittance matrix $\mathbf{Y}_{bus}$ with the elements expressed in rectangular form.

b. Reformulate $\mathbf{Y}_{bus}$ with the matrix elements in polar form.

c. Write the expressions for P_1 and Q_1 and P_2 and Q_2 in terms of the bus voltage magnitudes and angles.

10.2.5 The Newton–Raphson Method

The specified, or scheduled, complex power injection into a bus k of the power system may be expressed as

$$P_k^{sp} + jQ_k^{sp} = \mathbf{V}_k\mathbf{I}_k^* \qquad (10.24)$$

Substituting the expression for the bus current injection given by Eq. 10.13 into Eq. 10.24 yields

$$P_k^{sp} + jQ_k^{sp} = \mathbf{V}_k \sum_{m=1}^{N} Y_{km}^*\mathbf{V}_m^* = \sum_{m=1}^{N} \mathbf{V}_k Y_{km}^*\mathbf{V}_m^* \qquad (10.25)$$

The bus voltages and the elements of the bus admittance matrix may be expressed in terms of their magnitudes and phase angles as

$$\mathbf{V}_k = V_k\angle\delta_k$$
$$\mathbf{V}_m = V_m\angle\delta_m \qquad (10.26)$$

Substituting the relations of Eq. 10.26 into Eq. 10.25 yields

$$P_k^{\text{sp}} + jQ_k^{\text{sp}} = \sum_{m=1}^{N} V_k V_m Y_{km} \angle \delta_k - \delta_m - \theta_{km} \qquad (10.27)$$

The real and imaginary parts of Eq. 10.27 are the specified real power and specified reactive power, respectively. Thus,

$$P_k^{\text{sp}} = P_{\text{G}k} - P_{\text{D}k} = \sum_{m=1}^{N} V_k V_m Y_{km} \cos(\delta_k - \delta_m - \theta_{km}) \qquad (10.28)$$

$$Q_k^{\text{sp}} = Q_{\text{G}k} - Q_{\text{D}k} = \sum_{m=1}^{N} V_k V_m Y_{km} \sin(\delta_k - \delta_m - \theta_{km}) \qquad (10.29)$$

Equations 10.28 and 10.29 are used to write a set of nonlinear algebraic equations for the various buses of the power system, and the resulting equations are referred to as *power flow equations*.

It is sometimes more convenient to express the bus admittance elements in terms of their conductance and susceptance components, that is,

$$Y_{km} = G_{km} + jB_{km} \qquad (10.30)$$

Thus, Eqs. 10.28 and 10.29 may be expressed alternatively as follows:

$$P_k^{\text{sp}} = V_k \sum_{m=1}^{N} V_m[G_{km} \cos(\delta_k - \delta_m) + B_{km} \sin(\delta_k - \delta_m)] \qquad (10.31)$$

$$Q_k^{\text{sp}} = V_k \sum_{m=1}^{N} V_m[G_{km} \sin(\delta_k - \delta_m) - B_{km} \cos(\delta_k - \delta_m)] \qquad (10.32)$$

To find the solution of the power flow problem, Eqs. 10.31 and 10.32 are used to write as many equations as necessary to compute the unknown bus voltage magnitudes and phase angles. For each bus k, except the swing bus, an equation for the bus real power (10.31) is written, because the bus real power has been specified or scheduled. For these buses, the voltage angles are unknown except for the swing bus angle, which is used as reference and set equal to zero. An equation for the reactive power (10.32) is written for each P-Q, or load, bus, because the bus reactive power has been specified and the voltage magnitude at this bus is to be computed. No reactive power equations are written for P-V, or generator, buses because the reactive power generations are not known; however, the bus voltage magnitudes are already known and are controlled (maintained) at their specified values as far as physically

The resulting system of power flow equations to be solved consists of nonlinear algebraic equations involving the products of voltage magnitudes and trigonometric functions of the phase angles. These equations are linearized using a Taylor series expansion, and an iterative procedure is introduced. At every iteration, the system of linearized equations is solved to find improvements to the current estimate of the solution. An example of such a procedure is the Newton–Raphson method, also called Newton's method.

The generalized *Newton–Raphson method* is an iterative technique for solving a set of simultaneous nonlinear equations involving the same number of unknown variables. Consider the vector equation **F** of the vector of unknown variables **X**.

$$\mathbf{F}(\mathbf{X}) = \mathbf{0} \qquad (10.33)$$

At every iteration, Eq. 10.33 is approximated by the first two terms of a Taylor series expansion. The linearized problem is used to solve for corrections, or improvements, to the current estimate of the vector of unknowns. The elements of the *Jacobian matrix J* are the first-order partial derivatives, which are evaluated at the current estimate of the solution. The Jacobian matrix is employed in the Newton–Raphson method as shown in the following.

$$\mathbf{J}\,\Delta\mathbf{X} = \left[\frac{\partial \mathbf{F}}{\partial \mathbf{X}}\right] \Delta\mathbf{X} = -\mathbf{F}(\mathbf{X}) \qquad (10.34)$$

Equation 10.34 is solved for the correction vector $\Delta\mathbf{X}$ by using any known method of solving a system of linear equations. The state vector is then updated as follows.

$$\mathbf{X}^{(\text{new})} = \mathbf{X}^{(\text{old})} + \Delta\mathbf{X} \qquad (10.35)$$

The process is repeated until the solution converges to within a specified tolerance.

In power flow calculations, the set of nonlinear power flow equations given in Eqs. 10.31 and 10.32 may be rewritten as Eqs. 10.36 and 10.37, respectively, which represent the real power and reactive power mismatches at bus k. These mismatches give the difference between the injected power at the bus and the sum of the powers flowing through the various lines connected to the other buses.

$$\Delta P_k = V_k \sum_{m=1}^{N} V_m[G_{km} \cos(\delta_k - \delta_m) + B_{km} \sin(\delta_k - \delta_m)] - P_k^{\text{sp}} = 0 \qquad (10.36)$$

$$\Delta Q_k = V_k \sum_{m=1}^{N} V_m[G_{km} \sin(\delta_k - \delta_m) - B_{km} \cos(\delta_k - \delta_m)] - Q_k^{\text{sp}} = 0 \qquad (10.37)$$

Equations 10.36 and 10.37 are linearized, and the resulting equations to be solved at each iteration are given ahead. In these equations, the real power mismatches and reactive power mismatches at the buses are expressed as linear functions of the incremental voltage angles and incremental voltage magnitudes.

$$\begin{bmatrix} \mathbf{H} & \mathbf{N} \\ \mathbf{M} & \mathbf{L} \end{bmatrix} \begin{bmatrix} \boldsymbol{\Delta\delta} \\ \mathbf{\Delta V} \end{bmatrix} = - \begin{bmatrix} \mathbf{\Delta P} \\ \mathbf{\Delta Q} \end{bmatrix} \tag{10.38}$$

The submatrices **H**, **N**, **M**, and **L** are found by taking the derivatives of Eqs. 10.36 and 10.37 with respect to the voltage angles and magnitudes. These expressions are given in Eqs. 10.39 to 10.42.

$$H_{km} = \frac{\partial P_k}{\partial \delta_m} = V_k V_m [G_{km} \sin(\delta_k - \delta_m) - B_{km} \cos(\delta_k - \delta_m)]$$

$$H_{kk} = \frac{\partial P_k}{\partial \delta_k} = -Q_k - B_{kk} V_k^2 \tag{10.39}$$

$$N_{km} = \frac{\partial P_k}{\partial V_m} = V_k [G_{km} \cos(\delta_k - \delta_m) + B_{km} \sin(\delta_k - \delta_m)]$$

$$N_{kk} = \frac{\partial P_k}{\partial V_k} = \frac{1}{V_k}(P_k + G_{kk} V_k^2) \tag{10.40}$$

$$M_{km} = \frac{\partial Q_k}{\partial \delta_m} = V_k V_m [-G_{km} \cos(\delta_k - \delta_m) - B_{km} \sin(\delta_k - \delta_m)]$$

$$M_{kk} = \frac{\partial Q_k}{\partial \delta_k} = P_k - G_{kk} V_k^2 \tag{10.41}$$

$$L_{km} = \frac{\partial Q_k}{\partial V_m} = V_k [G_{km} \sin(\delta_k - \delta_m) - B_{km} \cos(\delta_k - \delta_m)]$$

$$L_{kk} = \frac{\partial Q_k}{\partial V_k} = \frac{1}{V_k}(Q_k - B_{kk} V_k^2) \tag{10.42}$$

Because the swing bus voltage is taken as the reference phasor (i.e., $\mathbf{V}_s = 1.0\angle 0°$), real and reactive power mismatches are not included in Eq. 10.38. For P-V buses, reactive power mismatches and incremental voltage magnitudes are not included because the voltage magnitudes are set to the specified values. Also, if bus k is not directly connected to bus m, then the kmth element of submatrices **H**, **N**, **M**, and **L** are zero. The Jacobian matrix **J** has sparsity characteristics similar to those of the bus admittance matrix $\mathbf{Y}_{\text{bus}}$.

Equation 10.38 is solved directly for the incremental voltage angles and incremental voltage magnitudes that are used as corrections to the current estimate of the solution. Any known solution technique for a system of linear algebraic equations may be used to solve Eq. 10.38. These computed

corrections are used to determine new estimates of the voltage phase angles and voltage magnitudes as follows:

$$\boldsymbol{\delta}^{\text{new}} = \boldsymbol{\delta}^{\text{old}} + \boldsymbol{\Delta\delta}^{\text{new}} \tag{10.43}$$

$$\mathbf{V}^{\text{new}} = \mathbf{V}^{\text{old}} + \mathbf{\Delta V}^{\text{new}} \tag{10.44}$$

The iterative process is said to have converged when the bus real power mismatch vector $\mathbf{\Delta P}$ and reactive power mismatch vector $\mathbf{\Delta Q}$, which are computed by using Eqs. 10.36 and 10.37, have been reduced to within a specified tolerance. At this point, the voltage magnitudes and phase angles at all buses have been found, and they may be used to calculate the real and reactive powers flowing through any transmission line or transformer, the real and reactive power generation of the slack bus, and the reactive power generations of the other generators. In addition, the real and reactive power losses of any transmission line or transformer, as well as of the total system, may be found by using the computed line power flows.

The Newton–Raphson method converges most rapidly of any of the power flow solution techniques, especially when the solution point is close. It has a quadratic convergence characteristic. It is reliable and has minimal sensitivity to factors that cause poor convergence, such as the choice of swing bus and the presence of series capacitors in the network, which could cause problems in the Gauss–Seidel method.

EXAMPLE 10.3

For the power system shown in Fig. 10.9, the bus admittance matrix $\mathbf{Y}_{\text{bus}}$ is given by

$$\mathbf{Y}_{\text{bus}} = \begin{bmatrix} 3.0 - j9.0 & -2.0 + j6.0 & -1.0 + j3.0 \\ -2.0 + j6.0 & 2.5 - j7.5 & -0.5 + j1.5 \\ -1.0 + j3.0 & -0.5 + j1.5 & 1.5 - j4.5 \end{bmatrix}$$

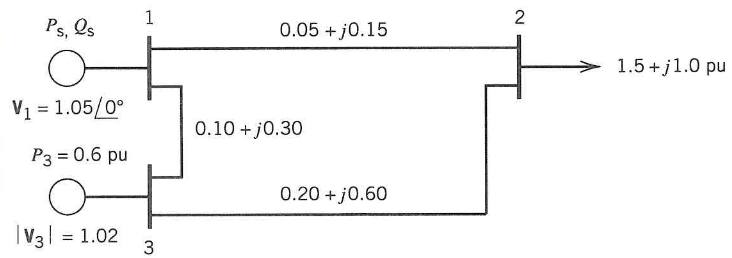

FIGURE 10.9 Power system of Example 10.3.

The per-unit bus voltages and power injections are also given in the figure.

a. For each bus k, specify the bus type, and determine which of the variables V_k, δ_k, P_k, and Q_k are input data and which are unknowns.

b. Assume an initial estimate of $\mathbf{V}_2 = 1.0\angle 0°$, and $\delta_3 = 0°$, and calculate the bus real and reactive power mismatches to be used in the first iteration of the Newton–Raphson power flow method.

c. Set up the linearized system of equations that are solved at each iteration of the Newton-Raphson power flow method.

Solution

a. The bus types and the classification of variables at each bus are shown in Table 10.1.

b. Using the initial estimates $\mathbf{V}_2 = 1.00\angle 0°$ and $\delta_3 = 0.0°$, the real and reactive bus power mismatches are calculated using Eqs. 10.36 and 10.37 with $\delta_{km} = (\delta_k - \delta_m)$ as follows:

$$\begin{aligned}
\Delta P_2 &= V_2[V_1(G_{21}\cos\delta_{21} + B_{21}\sin\delta_{21}) + V_2G_{22} \\
&\quad + V_3(G_{23}\cos\delta_{23} + B_{23}\sin\delta_{23})] - P_2^{sp} \\
&= 1.00\ [1.05(-2.0\cos 0 + 6.0\sin 0) + 1.00(2.5) \\
&\quad + 1.02(-0.5\cos 0 + 1.5\sin 0)] - (-1.5) \\
&= 1.39 \text{ pu} \\
\Delta P_3 &= V_3[V_1(G_{31}\cos\delta_{31} + B_{31}\sin\delta_{31}) \\
&\quad + V_2(G_{32}\cos\delta_{32} + B_{32}\sin\delta_{32}) + V_3G_{33}] - P_3^{sp} \\
&= 1.02\ [1.05(-1.0\cos 0 + 3.0\sin 0) \\
&\quad + 1.00(-0.5\cos 0 + 1.5\sin 0) + 1.02(1.5)] - 0.6 \\
&= -0.62 \text{ pu} \\
\Delta Q_2 &= V_2[V_1(G_{21}\sin\delta_{21} - B_{21}\cos\delta_{21}) - V_2B_{22} \\
&\quad + V_3(G_{23}\sin\delta_{23} - B_{23}\cos\delta_{23})] - Q_2^{sp} \\
&= 1.00\ [1.05(-2.0\sin 0 - 6.0\cos 0) - 1.00(-7.5) \\
&\quad + 1.02(-0.5\sin 0 - 1.5\cos 0)] - (-1.00) \\
&= 0.67 \text{ pu}
\end{aligned}$$

Table 10.1 Bus Type and Classification of Variables for Power System of Example 10.3

Bus Number	Bus Type	Input Data	Unknown
1	Swing	V_1, δ_1	P_1, Q_1
2	Load	P_2, Q_2	V_2, δ_2
3	Generator	P_3, V_3	δ_3, Q_3

c. The real and reactive power mismatches calculated above are used in the linearized system of equations to find corrections, or improvements, in the values of the voltage magnitudes and angles as shown in the following equation.

$$\begin{bmatrix} \dfrac{\partial P_2}{\partial \delta_2} & \dfrac{\partial P_2}{\partial \delta_3} & \dfrac{\partial P_2}{\partial V_2} \\ \dfrac{\partial P_3}{\partial \delta_2} & \dfrac{\partial P_3}{\partial \delta_3} & \dfrac{\partial P_3}{\partial V_2} \\ \dfrac{\partial Q_2}{\partial \delta_2} & \dfrac{\partial Q_2}{\partial \delta_3} & \dfrac{\partial Q_2}{\partial V_2} \end{bmatrix} \begin{bmatrix} \Delta\delta_2 \\ \Delta\delta_3 \\ \Delta V_2 \end{bmatrix} = - \begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \Delta Q_3 \end{bmatrix}$$

10.2.6 The Fast-Decoupled Power Flow Method

In the Newton–Raphson method, described in the previous section, the Jacobian matrix is formed and evaluated at the current estimate of the solution at every iteration. This large matrix is used to solve for the corrections or improvements to the current estimate of the solution. Thus at each iteration, the Newton–Raphson method performs either a matrix inversion or a Gaussian elimination or a triangular factorization of the large Jacobian matrix. Therefore, the Newton–Raphson method is more complicated and requires more computations per iteration and more storage space than the Gauss–Seidel technique. To alleviate some of the shortcomings of the Newton–Raphson method, the fast-decoupled method has been developed and is discussed in this section.

Any practical transmission system operating in steady state exhibits the characteristic of strong interdependence between real powers and bus voltage angles and between reactive powers and voltage magnitudes. Correspondingly, the coupling between P and V, as well as between Q and δ, is relatively weak. Based on the physical property of the problem, the *fast-decoupled method* solves the power flow problem by "decoupling," that is, solving separately, the P-δ and Q-V subproblems.

The first step in the fast-decoupled approach is to neglect the submatrices **M** and **N** in Eq. 10.38, giving two separate matrix equations:

$$\mathbf{H}\Delta\boldsymbol{\delta} = -\Delta\mathbf{P} \qquad (10.45)$$

$$\mathbf{L}\Delta\mathbf{V} = -\Delta\mathbf{Q} \qquad (10.46)$$

where the elements of the matrices **H** and **L** are given by Eqs. 10.39 and 10.42 and are rewritten here as follows:

$$H_{km} = V_kV_m[G_{km}\sin(\delta_k - \delta_m) - B_{km}\cos(\delta_k - \delta_m)] \qquad (10.47)$$

$$H_{kk} = -Q_k - B_{kk}V_k^2$$

$$L_{km} = V_k[G_{km}\sin(\delta_k - \delta_m) - B_{km}\cos(\delta_k - \delta_m)]$$
$$L_{kk} = \frac{1}{V_k}(Q_k - B_{kk}V_k^2) \tag{10.48}$$

For practical power systems, the phase angles of the bus voltages are seen to be generally small and close to the values of neighboring buses. In addition, the transformer and line reactances are normally greater than the corresponding resistances. Hence, the following assumptions are further made.

$$\cos(\delta_k - \delta_m) \cong 1 \qquad G_{km} \ll B_{km}$$
$$\sin(\delta_k - \delta_m) \cong 0 \qquad Q_k \ll B_{kk}V_k^2 \tag{10.49}$$

Thus, good approximations to Eqs. 10.45 and 10.46 are given by

$$\sum_{m=1}^{N} V_k(-B'_{km})V_m\,\Delta\delta_m = -\Delta P_k \tag{10.50}$$

$$\sum_{m=1}^{N} V_k(-B''_{km})\,\Delta V_m = -\Delta Q_k \tag{10.51}$$

Equations 10.50 and 10.51 may be simplified by moving the V_k terms to the right-hand side and setting the V_m terms in Eq. 10.50 to unity. The final fast-decoupled power flow equations may thus be expressed as two sets of linear algebraic equations with constant coefficients as follows:

$$\mathbf{B}'\Delta\boldsymbol{\delta} = \left[\frac{\Delta P_k}{V_k}\right] \tag{10.52}$$

$$\mathbf{B}''\Delta\mathbf{V} = \left[\frac{\Delta Q_k}{V_k}\right] \tag{10.53}$$

Both $\mathbf{B}'$ and $\mathbf{B}''$ are real, sparse, and constant matrices. $\mathbf{B}''$ is symmetrical, and $\mathbf{B}'$ is also symmetrical if the effects of phase shifters are neglected.

The elements of the matrix $\mathbf{B}'$ may be simplified by neglecting all the resistances and omitting the elements that affect predominantly the Q-V problem, such as shunt susceptances and transformer off-nominal taps. When the resistances are neglected, $\mathbf{B}'$ is formed from the reactance network by using the same procedure used in forming the bus admittance matrix $\mathbf{Y}_{\text{bus}}$. The matrix $\mathbf{B}''$ is the imaginary part of $\mathbf{Y}_{\text{bus}}$. The elements of $\mathbf{B}''$ may be simplified by omitting phase shifters.

In the fast-decoupled power flow method, the bus real power mismatches $\Delta\mathbf{P}$ are evaluated using Eq. 10.36 at the current estimate of the voltage magnitudes and phase angles. Equation 10.52 is solved for the improvements $\Delta\boldsymbol{\delta}$. The improvements $\Delta\boldsymbol{\delta}$ are then used to update the voltage phase angles using Eq. 10.43.

Using the newly computed values of the phase angles and the current estimates of the voltage magnitudes, the reactive power mismatches $\Delta\mathbf{Q}$ are calculated by using Eq. 10.37. Equation 10.53 is solved for the improvements $\Delta\mathbf{V}$. The improvements $\Delta\mathbf{V}$ are then used to update the voltage magnitudes using Eq. 10.44.

It is seen that the phase angles and voltage magnitudes are alternately updated while using the latest solution estimates in the calculation of the power mismatches. Convergence is achieved when both the real and reactive power mismatches are within specified tolerances. It may be noted that the mismatches are computed using the exact expressions, without approximations or decoupling; thus, the method converges to the true and exact solution.

The fast-decoupled power flow Eqs. 10.52 and 10.53 may be solved by any known method of solving a system of linear equations. The matrices $\mathbf{B}'$ and $\mathbf{B}''$ in these equations are constant matrices, which have the same size as the Jacobian submatrices $\mathbf{H}$ and $\mathbf{L}$, respectively, and have the same sparsity characteristics as the bus admittance matrix $\mathbf{Y}_{\text{bus}}$. If matrix inversion is used, both matrices $\mathbf{B}'$ and $\mathbf{B}''$ need to be inverted only once at the beginning of the solution process. If triangular factorization is employed, the triangular factors are calculated only once at the beginning of the iterative process.

The fast-decoupled power flow method converges reliably. The speed per iteration is approximately five times faster than in the Newton–Raphson method, and the storage requirements for both $\mathbf{B}'$ and $\mathbf{B}''$ matrices are also less.

EXAMPLE 10.4

For the four-bus power system shown in Fig. 10.8, form the $\mathbf{B}'$ and $\mathbf{B}''$ matrices that are used in the fast-decoupled power flow solution method.

Solution The $\mathbf{B}'$ matrix is formed using the same procedure as in building the bus admittance matrix with the resistances of the lines neglected. The reactance network, derived from Fig. 10.8, is shown in Fig. 10.10, and it is used to obtain the $\mathbf{B}'$ matrix.

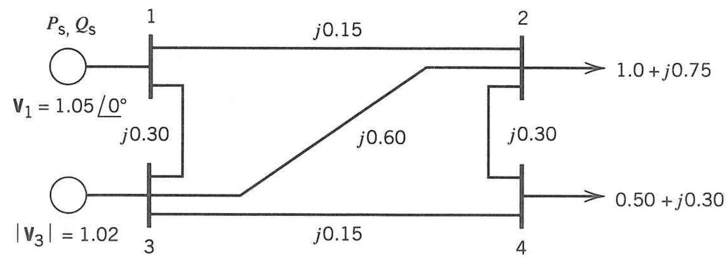

FIGURE 10.10 Reactance network for Example 10.4.

The elements of $\mathbf{B}'$ are found as follows:

$$jB_{12} = jB_{21} = -1.0/j0.15 = j6.667$$
$$jB_{13} = jB_{31} = -1.0/j0.30 = j3.333$$
$$jB_{14} = jB_{41} = 0$$
$$jB_{11} = -j6.667 - j3.333 = -j10$$
$$jB_{23} = jB_{32} = -1.0/j0.60 = j1.667$$
$$jB_{24} = jB_{42} = -1.0/j0.30 = j3.333$$
$$jB_{22} = -j6.667 - j1.667 - j3.333 = -j11.667$$
$$jB_{34} = jB_{43} = -1.0/j0.15 = j6.667$$
$$jB_{33} = -j3.333 - j1.667 - j6.667 = -j11.667$$
$$jB_{44} = -j3.333 - j6.667 = -j10$$

Thus, the $\mathbf{B}'$ matrix is given by

$$\mathbf{B}' = \begin{bmatrix} -10.000 & 6.667 & 3.333 & 0.000 \\ 6.667 & -11.667 & 1.667 & 3.333 \\ 3.333 & 1.667 & -11.667 & 6.667 \\ 0.000 & 3.333 & 6.667 & -10.000 \end{bmatrix}$$

The bus admittance matrix $\mathbf{Y}_{\text{bus}}$ was found in Example 10.1 and is repeated as follows:

$$\mathbf{Y}_{\text{bus}} = \begin{bmatrix} 3.0 - j9.0 & -2.0 + j6.0 & -1.0 + j3.0 & 0.0 + j0.0 \\ -2.0 + j6.0 & 3.5 - j10.5 & -0.5 + j1.5 & -1.0 + j3.0 \\ -1.0 + j3.0 & -0.5 + j1.5 & 3.5 - j10.5 & -2.0 + j6.0 \\ 0.0 + j0.0 & -1.0 + j3.0 & -2.0 + j6.0 & 3.0 - j9.0 \end{bmatrix}$$

The $\mathbf{B}''$ matrix is the imaginary component of bus admittance matrix $\mathbf{Y}_{\text{bus}}$. Thus,

$$\mathbf{B}'' = \begin{bmatrix} -9.0 & 6.0 & 3.0 & 0.0 \\ 6.0 & -10.5 & 1.5 & 3.0 \\ 3.0 & 1.5 & -10.5 & 6.0 \\ 0.0 & 3.0 & 6.0 & -9.0 \end{bmatrix}$$

10.3 AN APPLICATION OF POWER FLOW

To demonstrate the application of power flow for analyzing the performance of a power system, a 15-bus test system is considered. The one-line diagram of

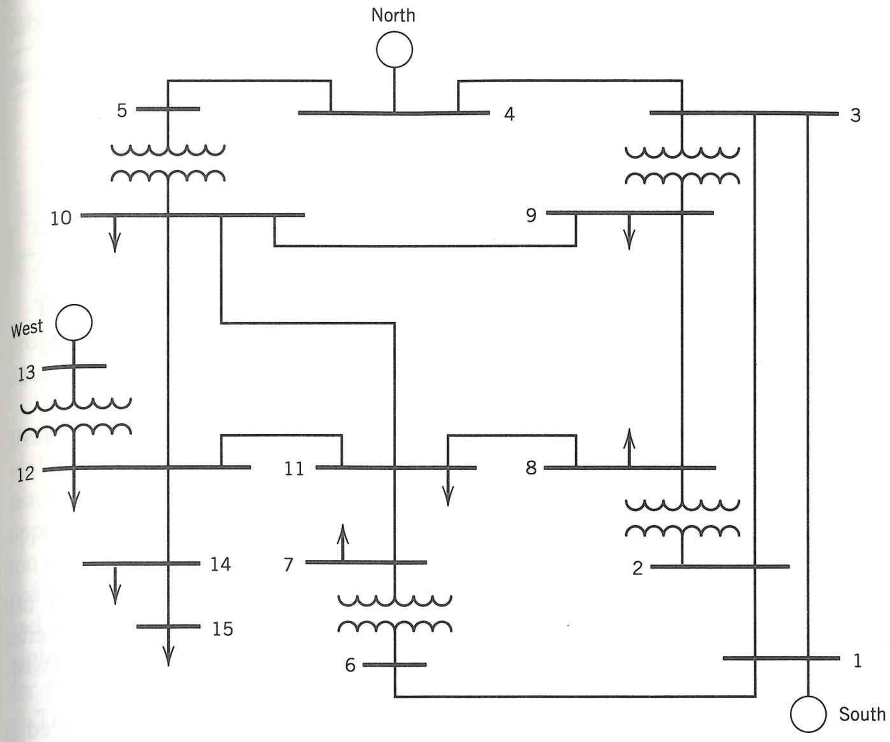

FIGURE 10.11 One-line diagram of test system.

The system power base used is 100 MVA, and the voltage bases are selected to match the nominal voltages in various parts of the system. The transformer data are provided in Table 10.2. Each of the transformers is fixed-tap, which is set at 100%, and the percent resistances and reactances given are based on the device ratings.

The system load data are given in Table 10.3, and the system generator data are shown in Table 10.4. The generators at buses 1 and 4 represent in-feeds from the neighboring power system that is charged with the bulk-power generation and transmission in the region. The South generator is connected to bus 1, which is chosen as slack bus.

Table 10.2 Transformer Data

Bus_{From}	Bus_{To}	Name	S_{Rated} (MVA)	V_{Rated} (kV)	% R	% X
2	8	Janice	200	230/115	0.5	6
3	9	Excel	200	230/115	0.5	6
5	10	Sharp	200	230/115	0.5	6
6	7	Kate	100	230/115	0.2	5
12	13	Center	150	115/13.8	0.5	5

Table 10.3 System Load Data

Bus	Name	P_{Load} (MW)	Q_{Load} (MVAR)
1	South	0	0
2	East	0	0
3	Main	0	0
4	North	0	0
5	Rich	0	0
6	Market	0	0
7	Ash	25	10
8	Cedar	40	20
9	Maple	18	10
10	Walnut	130	40
11	Oak	160	50
12	Center	100	30
13	West	0	0
14	Woods	50	20
15	Green	24	10

The transmission line parameters are calculated and converted to per unit based on the chosen bases. The transformer impedances are also converted to per unit on system bases. The load data and generator data are, likewise, converted to system bases.

The per-unit transformer data and transmission line data are presented in Tables 10.5 and 10.6, respectively. The system input bus data are given in Table 10.7. A "flat voltage" start is assumed; that is, the voltage phase angle at each bus is set equal to zero, while the voltage magnitude at each load bus is set equal to 1.0 per unit. The voltage at each generator bus is held at the specified value provided the generator reactive power stays within specified minimum and maximum limits.

The test system was analyzed using a noncommercial educational software package. This package includes programs employing either of the following power flow solution methods: Gauss–Seidel, Newton–Raphson, fast-decoupled; combined Gauss–Seidel/Newton–Raphson, or combined Gauss–Seidel/fast-decoupled. All programs are written in FORTRAN. Each solution method converges to the same solution.

Table 10.4 Generator Data

Bus	Name	S_{Rated} (MVA)	V_{Rated} (kV)	P_G (MW)	Q_G^{MIN} (MVAR)	Q_G^{MAX} (MVAR)
1	South	400	230	—	—	—
4	North	200	230	170	−50	110
13	West	150	13.8	120	0	90

Table 10.5 Transformer Input Data in Per Unit

Bus_{From}	Bus_{To}	Name	S_{Rated}	R (pu)	X (pu)
2	8	Janice	200	0.0025	0.0300
3	9	Excel	200	0.0025	0.0300
5	10	Sharp	200	0.0025	0.0300
6	7	Kate	100	0.0020	0.0500
12	13	Center	150	0.0033	0.0333

The Gauss–Seidel power flow method was selected, and the corresponding program was run. A converged solution was reached in 15 iterations. The bus data from the converged solution are given in Table 10.8.

Results of Simulation The program flagged the West generator as having violated its reactive generation limit. As can be seen from Table 10.8, the reactive power generated by the West generator is 1.18 per unit, exceeding its upper limit of 0.9 per unit. This means that the specified voltage at bus 13 is too high, and the generator cannot maintain it at this level. To find the correct operating voltage of the generator, the P-V bus 13 is converted to a P-Q bus and the reactive power demand at this bus is set equal to the negative of the violated limit Q_G^{max}, or −0.90 per unit. The program was run again to solve for the voltage magnitude at this bus. It was found to be 1.00 per unit.

The program also automatically checks all bus voltages and flags those that are less than 0.95 per unit or higher than 1.05 per unit. The voltages at load buses 14 and 15 are 0.935 and 0.922 per unit, respectively, which are both

Table 10.6 Transmission Line Input Data in Per Unit

Bus_{From}	Bus_{To}	R (pu)	X (pu)	$-jY$ (pu)	S_{Rated}
1	2	0.0100	0.0528	0.1119	3.585
1	3	0.0233	0.1232	0.2611	3.585
1	6	0.0083	0.0440	0.0933	3.585
2	3	0.0133	0.0704	0.1492	3.585
3	4	0.0100	0.0528	0.1119	3.585
4	5	0.0066	0.0352	0.0746	3.585
7	11	0.0277	0.1518	0.0271	2.012
8	9	0.0332	0.1822	0.0325	2.012
8	11	0.0222	0.1214	0.0217	2.012
9	10	0.0277	0.1518	0.0271	2.012
10	11	0.0277	0.1518	0.0271	2.012
10	12	0.0222	0.1214	0.0217	2.012
11	12	0.0166	0.0911	0.0163	2.012
12	14	0.0222	0.1214	0.0217	2.012
14	15	0.0166	0.0911	0.0163	2.012

Table 10.7 Bus Input Data in Per Unit

Bus	Name	Code	V	δ	P_G	P_L	Q_L
1	South	1	1.025	0	—	0	0
2	East	3	1.000	0	0	0	0
3	Main	3	1.000	0	0	0	0
4	North	2	1.025	0	1.7	0	0
5	Rich	3	1.000	0	0	0	0
6	Market	3	1.000	0	0	0	0
7	Ash	3	1.000	0	0	0.25	0.10
8	Cedar	3	1.000	0	0	0.40	0.20
9	Maple	3	1.000	0	0	0.18	0.10
10	Walnut	3	1.000	0	0	1.30	0.40
11	Oak	3	1.000	0	0	1.60	0.50
12	Center	3	1.000	0	0	1.00	0.30
13	West	2	1.035	0	1.2	0	0
14	Woods	3	1.000	0	0	0.50	0.20
15	Green	3	1.000	0	0	0.24	0.10
	Totals					5.47	1.90

Table 10.8 Bus Results

Bus	Name	Code	V	δ	P_G	Q_G
1	South	1	1.025	0	2.68	0.076
2	East	3	1.009	−3.43	0	0
3	Main	3	1.017	−3.65	0	0
4	North	2	1.025	−3.29	1.70	0.443
5	Rich	3	1.002	−6.19	0	0
6	Market	3	1.008	−2.22	0	0
7	Ash	3	0.996	−4.90	0	0
8	Cedar	3	0.998	−5.36	0	0
9	Maple	3	1.009	−4.82	0	0
10	Walnut	3	0.988	−8.81	0	0
11	Oak	3	0.968	−11.0	0	0
12	Center	3	0.994	−12.1	0	0
13	West	2	1.035	−10.0	1.20	1.18
14	Woods	3	0.935	−17.2	0	0
15	Green	3	0.922	−18.6	0	0
	Totals				5.58	1.70

Table 10.9 Final Bus Results

Bus	Name	Code	V	δ	P_G	Q_G
1	South	1	1.025	0	2.68	0.176
2	East	3	1.007	−3.42	0	0
3	Main	3	1.016	−3.64	0	0
4	North	2	1.025	−3.29	1.70	0.571
5	Rich	3	0.998	−6.16	0	0
6	Market	3	1.007	−2.21	0	0
7	Ash	3	0.993	−4.89	0	0
8	Cedar	3	0.994	−5.35	0	0
9	Maple	3	1.007	−4.81	0	0
10	Walnut	3	0.981	−8.79	0	0
11	Oak	3	0.958	−11.0	0	0
12	Center	3	0.975	−12.0	0	0
13	West	2	1.001	−9.76	1.20	0.682
14	Woods	3	0.950	−17.5	0	0
15	Green	3	0.950	−19.0	0	0
	Totals				5.58	1.42

less than the minimum allowable level of 0.95 per unit. These P-Q buses are converted to P-V buses, with their power generations set equal to zero. The voltage magnitudes are set equal to the violated limit V^{min}, or 0.95 per unit. The program is run again to find the generator reactive powers at buses 14 and 15. These are equal to the per-unit MVAR of the capacitors that have to be connected at the two buses to bring the voltages back within limits. These are 0.116 per unit and 0.139 per unit for buses 14 and 15, respectively.

Finally, with the generator voltage at bus 13 set to 1.00 per unit and capacitors of 0.116 and 0.139 per unit MVAR connected to buses 14 and 15, respectively, the program was run again. The final bus results are presented in Table 10.9. It may be noted that all previous constraint violations have been resolved.

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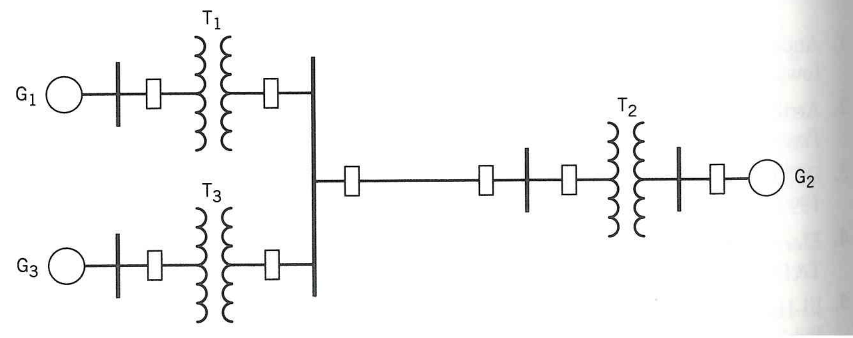
PROBLEMS

10.1 The one-line diagram of an unloaded power system is shown in Fig. 10.12. The reactance of the transmission line is $X = 30\ \Omega$. The generators and transformers are



rated as follows:

G_1: 20 MVA, 12 kV, $X = 1.20$ pu
G_2: 60 MVA, 13.8 kV, $X = 1.40$ pu
G_3: 20 MVA, 13.2 kV, $X = 1.20$ pu
T_1: 25 MVA, 12/69 kV, $X = 0.08$ pu
T_2: 60 MVA, 69/13.8 kV, $X = 0.14$ pu
T_3: 75 MVA, 13.2/69 kV, $X = 0.16$ pu

Choose a power base of 50 MVA and a voltage base of 12 kV in the circuit of generator 1. Form the bus admittance matrix.

10.2 The power system shown in Fig. 10.13 has the following generator and transformer data.

G_1: 50 MVA, 13.2 kV, $X = 1.20$ pu
G_2: 20 MVA, 13.2 kV, $X = 1.10$ pu
T_1: 60 MVA, 13.2/115 kV, $X = 0.16$ pu
T_2: 25 MVA, 34.5/13.2 kV, $X = 0.10$ pu
T_3: 60 MVA, 115/34.5 kV, $X = 0.16$ pu
T_4: 10 MVA, 4.16/34.5 kV, $X = 0.05$ pu

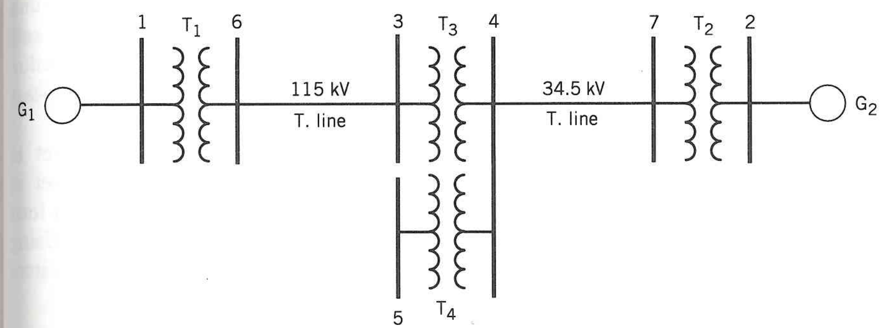


FIGURE 10.13 Power system of Problem 10.2.

The 115-kV transmission line is 50 mi long, and it has a reactance of 0.8 Ω/mi. The 34.5-kV line is 10 mi long, and it has a reactance of 1.2 Ω/mi. Choose a power base of 50 MVA and a voltage base of 115 kV for the high-voltage transmission line.

a. Choose the voltage bases in the other parts of the system in order that per-unit turns ratios of all transformers are 1:1.

b. Draw the one-line impedance diagram showing all impedance values in per unit.

c. Form the bus admittance matrix.

10.3 For the power system shown in Fig. 10.14, bus 1 is selected as the slack or swing bus and its voltage is set to $\mathbf{V}_1 = 1.0\underline{/0^\circ}$ pu. The chosen power base is 100 MVA. Generator 2 delivers a real power of 0.75 pu at a voltage of 1.02 pu. The loads on buses 3 and 4 are $\mathbf{S}_{D3} = (0.40 + j0.30)$ pu and $\mathbf{S}_{D4} = (0.80 + j0.60)$ pu,

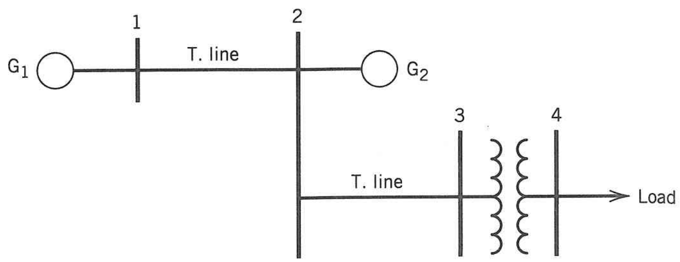


FIGURE 10.14 Power system of Problem 10.3.

respectively. The impedance parameters for the transmission lines referred to the given power base and a voltage base of 115 kV are

$$\text{Line 1–2:} \quad Z = (0.01 + j0.05)\text{ pu}; Y = j0.30\text{ pu}$$

$$\text{Line 2–3:} \quad Z = (0.03 + j0.15)\text{ pu}; Y = j0.90\text{ pu}$$

The transformer is connected between buses 3 and 4, and its reactance is 0.10 pu.

From a flat start, perform one iteration of the Gauss–Seidel iterative technique to find the voltages at buses 2, 3, and 4.

10.4 For the three-bus power system shown in Fig. 10.6, the transmission line per-unit reactances are

$$X_{12} = 0.75 \qquad X_{13} = 0.25 \qquad X_{23} = 0.50$$

Bus 1 is selected as swing bus, and its voltage is used as reference and set at $\mathbf{V}_1 = 1.00\angle 0^\circ$ pu. Bus 2 is a generator bus and its voltage magnitude is set at $V_2 = 1.02$ pu and its real power generation specified as $P_{G2} = 0.6$ pu. Bus 3 is a load bus with real and reactive power demands of $P_{D3} = 0.8$ pu and $Q_{D3} = 0.6$ pu. Using the Gauss–Seidel iterative technique, obtain the power flow solution of this system within a tolerance of 0.01 pu.

10.5 Using the solution to Problem 10.4, calculate the power flow through line 1–3 at both ends.

10.6 Repeat Problem 10.5 for line 2–3.

10.7 Outline the solution of the Newton–Raphson power flow method as it applies to the power system of Problem 10.4.

10.8 For the four-bus power system shown in Fig. 10.8, choose bus 1 as swing bus.

a. For each bus k, specify the bus type and determine which of the variables V_k, δ_k, P_k, and Q_k are the known input data and which are the unknowns.

b. In terms of the partial derivatives, identify the elements of the Jacobian matrix for this particular power system.

c. Set up the linearized system of equations that are solved at each iteration of the Newton's method of power flow solution. *Do not solve* the system of equations.

10.9 For the three-bus power system shown in Fig. 10.6, generator G_1 is chosen as swing generator. The line reactances are given as

$$X_{12} = 0.20\text{ pu} \qquad X_{13} = 0.25\text{ pu} \qquad X_{23} = 0.40\text{ pu}$$

a. Find the bus admittance matrix $\mathbf{Y}_{\text{bus}}$.

b. For each bus k, specify the bus type and determine which of the variables V_k, δ_k, P_k, and Q_k are input data and which are unknowns.

c. Set up the linearized system of equations that are solved at each iteration of the Newton power flow solution method. *Do not solve* the system of equations.

10.10 For the three-bus power system shown in Fig. 10.9, form the $\mathbf{B}'$ and $\mathbf{B}''$ matrices used in the fast-decoupled power flow method.

10.11 The transmission line parameters of a four-bus power system are given as follows:

$$\text{Line 1–2:} \quad Z = (0.020 + j0.060)\text{ pu}; \; Y = j0.080\text{ pu}$$

$$\text{Line 1–3:} \quad Z = (0.015 + j0.045)\text{ pu}; \; Y = j0.060\text{ pu}$$

$$\text{Line 1–4:} \quad Z = (0.008 + j0.024)\text{ pu}; \; Y = j0.030\text{ pu}$$

$$\text{Line 2–3:} \quad Z = (0.010 + j0.030)\text{ pu}; \; Y = j0.040\text{ pu}$$

$$\text{Line 3–4:} \quad Z = j0.100\text{ pu}$$

Bus 2 is a tie-line bus and is chosen as swing bus. Bus 1 is a generator bus, and its voltage magnitude is set to $V_1 = 1.0$ pu. Find the $\mathbf{B}'$ and $\mathbf{B}''$ matrices for use in the fast-decoupled power flow technique.

Eleven

Faults, Protection, and Stability

11.1 INTRODUCTION

The normal operation of the power system at steady state that was described in the previous chapter is affected, sometimes dramatically, by the occurrence of such disturbances as overloads and short circuits. Of primary concern are short circuits, not only because they can cause large damage to the affected system component but also because they may lead to instability of the whole power system. Thus, there is a need to design protection schemes to minimize the risks involved with the occurrences of disturbances. Fault analysis forms the basis for designing such protection systems. The proper coordination of the protective relays and the correct specification of circuit breaker ratings are based on the results of fault calculations. The performance of the power system is simulated in what is called transient stability analysis under a variety of disturbances, such as short circuits, sudden large load changes, and switching operations.

Fault calculations provide currents and voltages in a power system under faulted conditions. The modeling of various power system components, symmetrical components, the interconnections of the sequence networks, and symmetrical and unsymmetrical faults are presented in the second section of this chapter.

Power system protection is an important concern because short circuits present danger of damage to the equipment and loss of synchronism of the synchronous machines. The different types of protection systems and their applications for protecting the various power system components are discussed in the third section of this chapter.

Transient stability studies investigate the ability of the power system to remain in synchronism during major disturbances, such as equipment failure, major load changes, or momentary faults. Basic stability concepts are presented in the last section, along with a brief description of the models for generators, excitation systems, and governor-turbine systems.

11.2 FAULT ANALYSIS

The normal mode of operation of a power system is balanced three-phase AC. However, there are undesirable but unavoidable incidents that may temporarily disrupt normal conditions, as when the insulation of the system fails at any point or when a conducting material comes in contact with a bare conductor. Then we say a fault has occurred. A fault may be caused by lightning, trees falling on the electric wires, vehicular collision with the poles or towers, vandalism, and so forth.

Faults may be classified under four types. The different types of fault are listed here in the order of the frequency of their occurrence.

1. Single line-to-ground fault (SLG)
2. Line-to-line fault (L-L)
3. Double line-to-ground fault (2LG)
4. Balanced three-phase fault

Fault calculations provide information on currents and voltages in a power system during fault conditions. Short-circuit currents are computed for each relay and circuit breaker location and for various system contingency conditions, such as lines or generating units out of service, in order to determine minimum and maximum fault currents. This information is useful to the engineer in selecting circuit breakers for fault interruption, selecting relays for fault detection, and determining the relay settings, which is referred to as relay coordination. The proper selection and setting of protective devices ensure minimum disruption of electrical service and limit possible damage to the faulted equipment.

11.2.1 Three-Phase Fault Analysis

Sufficient accuracy in fault studies can be obtained with certain simplifications in the model of the power system. These assumptions include the following:

a. Shunt elements in the transformer model are neglected; that is, magnetizing currents and core losses are omitted.
b. Shunt capacitances in the transmission line model are neglected.
c. Transformers are set at nominal tap positions.

d. All internal voltage sources are set equal to $1.0\angle 0^\circ$. This is equivalent to neglecting prefault load currents.

Three-phase fault calculations can be performed on a per-phase basis because the power system remains effectively balanced, or symmetrical, during a three-phase fault. Thus, the various power system components are represented by single-phase equivalent circuits wherein all three-phase connections are assumed to be converted to their equivalent wye connections. Calculations are performed using impedances per phase, phase currents, and line-to-neutral voltages.

Fault analysis, like any other power system calculation, is more conveniently performed using per-unit representation. The power system components are described in terms of their per-unit impedances or per-unit admittances. The power base is selected and is used for all parts of the power system. The voltage bases are different for various parts of the system, and they are selected so that the per-unit turns ratio of the transformers is equal to unity. The current base and the impedance base are computed using the specified power base and voltage bases.

EXAMPLE 11.1

A three-phase synchronous generator is rated 50 MVA, 13.2 kV, 0.8 power factor lagging and has a synchronous reactance of 20%. The generator is connected to a 60-MVA, 13.4/138-kV, three-phase transformer having a reactance of 15%. The generator is initially operating at no load and rated terminal voltage. A three-phase fault suddenly occurs on the transformer's high-voltage terminals. Determine the short-circuit current supplied by the generator and its terminal voltage.

Solution Choose the generator ratings as base quantities; thus, $S_{\text{base}} = 50$ MVA and $V_{\text{base}} = 13.2$ kV (line-to-line). The base current is calculated as follows.

$$I_{\text{base}} = 50{,}000/(13.2\sqrt{3}) = 2187 \text{ A}$$

The terminal voltage of the generator is taken as reference phasor; thus,

$$\mathbf{V}_t = 1.0\angle 0^\circ \text{ pu}$$

Because the generator is initially operating at no load, the internal voltage is equal to the terminal voltage.

$$\mathbf{E}_a = \mathbf{V}_t = 1.0\angle 0^\circ \text{ pu}$$

When a three-phase fault occurs at the high side of the transformer, the internal voltage of the generator is assumed to remain constant in determining the short-circuit current. The transformer reactance is converted to per unit referred to the chosen bases as follows.

$$X_t = (0.15)(50/60)(13.4/13.2)^2 = 0.129 \text{ pu}$$

Therefore, the total series reactance X_T is given by

$$X_T = X_s + X_t = 0.20 + 0.129 = 0.329 \text{ pu}$$

Hence, the short-circuit current is found by dividing the internal voltage by the total reactance.

$$\mathbf{I}_{sc} = \frac{\mathbf{E}_a}{jX_T} = \frac{1.0\angle 0^\circ}{j0.329} = 3.04\angle -90^\circ \text{ pu}$$
$$= (3.04)(2187) = 6648 \text{ A}$$

The terminal voltage of the generator is calculated as

$$\mathbf{V}_t = jX_t\mathbf{I}_{sc} = (j0.129)(3.04\angle -90^\circ) = 0.39\angle 0^\circ \text{ pu}$$
$$= (0.39)(13.2) = 5.15 \text{ kV} \quad \text{(line-to-line)}$$

DRILL PROBLEMS

D11.1 A step-up transformer is rated 1000 MVA, 26/345 kV and has a series impedance of $(0.004 + j0.085)$ pu based on its own ratings. It is connected to a 26-kV, 800-MVA generator that can be represented as a constant-voltage source in series with a reactance of $j1.65$ pu based on the generator ratings. The system is initially operating at no load when a three-phase fault occurs at the high-voltage terminals of the transformer. Choose a system power base of 1000 MVA and a voltage base in the generator circuit of 26 kV.

a. Convert the per-unit generator reactance to the system base.

b. Find the short-circuit current, in amperes, at the fault.

c. Find the short-circuit current, in amperes, supplied by the generator.

D11.2 Two 50-MVA, 12-kV, 60-Hz generators G_1 and G_2 are connected to bus A. Each generator has a synchronous reactance of 0.15 pu. A three-phase, 100-MVA, 12/69-kV transformer of reactance 0.10 pu is connected to A. The