REED'S APPLIED HEAT
FOR ENGINEERS

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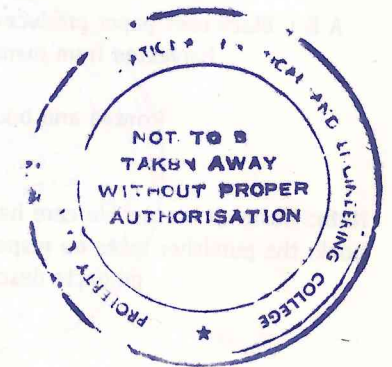
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PREFACE

This book covers the syllabuses in Applied Heat for all classes of the Marine Engineers' Certificates of Competency of the Department of Transport (DTp). The examinations are now administered by the Scottish Vocational Educational Council (SCOTVEC). It is a useful aid to students on Business and Technician Education Council (BTEC) and SCOTVEC engineering courses.

Basic principles are dealt with commencing at a fairly elementary stage. Each chapter has fully worked examples interwoven into the text, test examples are set at the end of each chapter for the student to work out, and finally there are some typical examination questions included. The prefix 'f' is used to indicate those parts of the text, and some test examples, of Class 1 standard.

The author has gone beyond the normal practice of merely supplying bare answers to the test examples and examination questions by providing fully worked step by step solutions leading to the final answers.

This latest revision is a major update in the subject so taking the material for study through the 1990s.

CONTENTS

CHAPTER 1

UNITS AND COMMON TERMS

MASS is the quantity of matter possessed by a body and is proportional to the volume and the density of the body. It is a constant quantity, that is, the mass of a body can only be changed by adding more matter to it or taking matter away from it.

The abbreviation for mass is m and the unit is the kilogramme [kg]. For very large or small quantities, multiples or submultiples of the gramme [g] are used. Large masses are common in engineering and these are measured in megagrammes [Mg]. One megagramme is equal to 10^3 kilogrammes and called a tonne [t].

Mass is proportionally accelerated or retarded by an applied force. To maintain a coherent system of units, a unit of force is chosen which will given unit acceleration to unit mass. This unit of force is called the *newton* [N]. Hence, one newton of force acting on one kilogramme of mass will give it an acceleration of one metre per second per second, therefore:

Accelerating force [N] = mass [kg] × acceleration [m/s²]. In symbols:

$$F = ma$$

FORCE OF GRAVITY. All bodies are attracted towards each other, the force of attraction depending upon the masses of the bodies and their distances apart. Newton's law of gravitation states that this force of attraction is proportional to the product of the masses of the bodies and inversely proportional to the square of the distance apart.

An important example of this is the mass of the earth which attracts all comparatively smaller bodies towards its, the attractive force by which a body tends to be drawn towards the centre of the earth is the force of gravity and is called the *weight* of the body.

If a body is allowed to fall freely, it will fall with an acceleration of 9·81 m/s², this is termed gravitational acceleration and is represented by g. Since one newton is the force which will give one kilogramme of mass an acceleration of one m/s², then the force in newtons to give m kg of mass an acceleration of 9·81 m/s² is m × 9·81. Hence, at the earth's surface, the gravitational force on a mass of m kg is mg newtons, or in other words:

Weight [N] = mass [kg] × g [m/s²].

The further the distance between the centre of gravity of the mass and the centre of gravity of the earth, the less is the attractive force between them. Thus, the weight of a mass measured by a spring balance (not a pair of scales which is merely a means of comparing the weight of one mass with another) will vary slightly at different parts of the earth's surface due to the earth not being a perfect sphere.

If a body is projected in a space-rocket, the attractive force of the earth on the body becomes less as its distance from the earth increases until, in complete outer-space, it becomes nil, that is, it is then weightless. The mass of the body of course remains unchanged.

WORK is done when a force applied on a body causes it to move and is measured by the product of the force and the distance through which the force moves.

The unit of work is the *joule* [J] which is defined as the work done when the point of application of a force of one newton moves through a distance of one metre in the direction in which the force is applied. Hence, one joule is equal to one newton-metre. In symbols, J = Nm.

Work done [J] = force [N] × distance moved [m].

The joule is a small unit. Moderate quantities of work may be expressed in kilojoules [1 kJ = 10^3 J] and larger quantities in megajoules [1 MJ = 10^6 J].

POWER is the rate of doing work, that is, the quantity of work done in a given time. The unit of power is the *watt* [W] which is equal to the rate of one joule of work being done every second. In symbols, W = J/s = Nm/s.

$$\text{Power [W]} = \frac{\text{work done [J]}}{\text{time [s]}}$$

The watt is a small unit and only suitable for small powers. For normal powers in engineering, the kilowatt [1 kW = 10^3 W] and megawatt [1 MW = 10^6 W] are usually more convenient units.

ENERGY is the capacity for doing work and it is measured by the amount of work done. Energy is therefore expressed in the same units as work, that is, joules, kilojoules and megajoules.

Another useful unit of energy is the *kilowatt-hour* [kWh]. This, as its name implies, represents the energy used or the work done when one kilowatt of power is exerted continually for 3600 seconds,

i.e. one hour.

$$\begin{aligned}
\text{Energy} &= \text{power} \times \text{time} \\
1 \text{ kWh} &= 1000 \text{ watts} \times 3600 \text{ seconds} \\
&= 1000 \text{ [J/s]} \times 3600 \text{ [s]} \\
&= 3 \cdot 6 \times 10^6 \text{ J} \\
&= 3 \cdot 6 \text{ MJ}
\end{aligned}$$

EFFICIENCY is the ratio of the work got out of a machine to the work put into it, and, as this is done in the same time, it is also the ratio of the output power to the input power. Since no machine is perfect, the output is always less than the input, due to frictional and other losses, therefore the efficiency is always less than unity.

The symbol for efficiency is η and it may be expressed as a fraction or as a percentage.

$$\eta = \frac{\text{output power}}{\text{input power}}$$

Example. A mass of 1600 kg is lifted by a winch through a height of 25 m in 30 seconds. Calculate (i) the work done, and, if the efficiency of the winch is 60%, find (ii) the input power in kW and (iii) the energy consumed in kWh.

Force [N] to lift mass against gravity

$$\begin{aligned}
&= \text{weight of mass} = mg \\
&= 1600 \times 9 \cdot 81 \text{ newtons}
\end{aligned}$$

$$\begin{aligned}
\text{Work done [J = Nm]} &= \text{force [N]} \times \text{distance [m]} \\
&= 1600 \times 9 \cdot 81 \times 25 \\
&= 392400 \text{ J} = 392 \cdot 4 \text{ kJ} \quad \text{Ans. (i)}
\end{aligned}$$

$$\begin{aligned}
\text{Output power [kW = kJ/s]} &= \frac{\text{work done [kJ]}}{\text{time [s]}} \\
&= \frac{392 \cdot 4}{30} = 13 \cdot 08 \text{ kW}
\end{aligned}$$

$$\text{Efficiency} = \frac{\text{output power}}{\text{input power}}$$

$$\text{Input power} = \frac{13 \cdot 08}{0 \cdot 6} = 21 \cdot 8 \text{ kW} \quad \text{Ans. (ii)}$$

$$\text{Energy consumed} = \frac{392 \cdot 4}{0 \cdot 6} = 654 \text{ kJ}$$

$$1 \text{ kWh} = 3 \cdot 6 \text{ MJ}$$

$$654 \text{ kJ} = \frac{0 \cdot 654}{3 \cdot 6} = 0 \cdot 1817 \text{ kWh} \quad \text{Ans. (iii)}$$

Alternatively,

$$\text{Energy [kWh]} = \text{power [kW]} \times \text{time [h]}$$

$$= 21 \cdot 8 \times \frac{30}{3600}$$

$$= 0 \cdot 1817 \text{ kWh.}$$

PRESSURE is expressed as the intensity of force, that is, the force acting on unit area. The unit of force is the newton [N] and the unit of area is the square metre [m²], therefore the fundamental unit of pressure is newton per square metre [N/m²]. The symbol representing pressure is usually p.

Pressures of liquids and gases reach high values which are expressed in multiples of the basic unit of force. For example, the steam pressure in low pressure boilers is often in the region of 8×10^5 N/m² and in high pressure boilers it could be 6×10^6 N/m², the former can be conveniently written 800 k/Nm² and the latter 600 kN/m² or 6 MN/m². Another very convenient unit of pressure commonly used is the *bar*, this has the advantage of being easy to "think" in these units since one bar is approximately equal to one atmosphere of pressure (1 atm = 1·013 bar). One bar is 10^5 N/m², which is 100 kN/m², hence the working pressure of the boilers given as an example above would be stated as 8 bar and 60 bar, respectively.

Pressures in internal combustion engines vary from a little above or below one bar during the air charging period to about 100 bar (= 100×10^5 N/m² = 10 000 kN/m²) during combustion.

Low pressures and vacua are usually measured in millimetres of mercury [mmHg], very small pressures in millimetres of water [mm water]. The instrument used is the *manometer*. This is a glass U-tube partially filled with mercury or water, one end is connected to the source of pressure and the other end is open to the atmosphere. The difference between the levels of liquid in the two legs indicates the difference in pressure between the source and the atmosphere.

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Considering the manometer containing mercury, if we take the density of mercury as $13 \cdot 6 \times 10^3$ kg/m³ and the force of gravity on a mass of one kilogramme as 9·80665 newtons (a more accurate figure for the standard value of gravitational acceleration than 9·81 which is usually acceptable in engineering), then the weight of one cubic metre of mercury is $13 \cdot 6 \times 10^3 \times 9 \cdot 80665$ newtons = 133·3 kN. Hence a column of mercury one metre high exerts a pressure of 133·3 kN on one square metre, or a column of mercury one millimetre high is equivalent to a pressure of 133·3 N/m².

Similarly, each millimetre of water pressure is equal to 9·80665 N/m² which is usually taken as 9·81 N/m².

Small pressures may also be expressed in millibars [mbar]: One mbar = 1 bar $\times 10^{-3} = 10^5 \times 10^{-3}$ N/m² = 100 N/m².

The mercurial *barometer* works on the principle of the atmospheric pressure supporting a column of mercury. The vertical column of mercury left standing up the tube, perfect vacuum above, is supported by the outside atmospheric pressure and is therefore a measure of the pressure of the atmosphere. As the atmospheric pressure rises and falls, the level of the supported column of mercury rises and falls accordingly.

For example, if the column of mercury supported by the atmospheric pressure is 760 mm, then the atmospheric pressure will be:
$$760 \times 133 \cdot 3 = 1 \cdot 013 \times 10^5 \text{ N/m}^2 = 1 \cdot 013 \text{ bar or } 101 \cdot 3 \text{ kN/m}^2.$$

GAUGE PRESSURE AND ABSOLUTE PRESSURE. Most pressure recording instruments, including the ordinary pressure gauge and the open-ended manometer, measure the pressure from the level of atmospheric pressure. The pressure so recorded is termed the *gauge pressure* and the word "gauge" should follow the units of pressure. Thus, if a pressure gauge reads 2000 kN/m² the pressure should be stated as 2000 kN/m² gauge, meaning that this is the pressure over and above the atmospheric pressure.

The true pressure is measured above a perfect vacuum and called the *absolute pressure* and this is the value which is used in calculations. The absolute pressure is therefore obtained by adding the atmospheric pressure to the gauge pressure, the gauge pressure being read from the pressure gauge and the atmospheric pressure obtained from the barometric reading.

As an example, if the pressure of a fluid is 550 kN/m² gauge, and the barometer stands at 758 mmHg then:

Atmospheric pressure $= 758 \times 133 \cdot 3$
$$= 1 \cdot 01 \times 10^5 \text{ N/m}^2 = 101 \text{ kN/m}^2$$
Absolute pressure $=$ gauge pressure + atmospheric pressure
$$= 550 + 101$$
$$= 651 \text{ kN/m}^2$$

This could be written 651 kN/m² absolute. However it is usual to omit the word absolute and take it for granted that if the word gauge does not follow the value of the pressure then it means that it is an absolute pressure. This will be the practice throughout this book.

Pressure gauges are not always perfectly accurate and, in any case, it is difficult to read to an accuracy of one or two kN/m². It is therefore quite common when exact accuracy is not essential to assume the atmospheric pressure to be 100 kN/m².

In the above example, if the barometer reading was not known, the absolute pressure would be taken as:
$$550 + 100 = 650 \text{ kN/m}^2$$
with very little difference in the final result of a calculation.

If the manometer was to be used as a vacuum gauge, say for a steam condenser, the level of the mercury in the leg connected to the condenser will be higher than the level in the leg open to atmosphere. The difference in level indicates the pressure *below* atmospheric and is written "mmHg vacuum".

For example, if the gauge reads 600 mmHg on vacuum and the barometer stands at 758 mmHg then:
Pressure below atmospheric
$$= 600 \times 133 \cdot 3 = 8 \times 10^4 \text{ N/m}^2$$
Atmospheric pressure
$$= 758 \times 133 \cdot 3 = 1 \cdot 01 \times 10^5 \text{ N/m}^2 = 101 \text{ kN/m}^2$$
Therefore absolute pressure in condenser is 80 kN/m² *below* 101 kN/m² which is 21 kN/m² or, more simply calculated:
Abs. press. $=$ (barometer mmHg – vacuum gauge mmHg) $\times$ 133·3
$$= (758 - 600) \times 133 \cdot 3$$
$$= 158 \times 133 \cdot 3$$
$$= 2 \cdot 1 \times 10^4 \text{ kN/m}^2 = 21 \text{ kN/m}^2$$

VOLUME has the basic unit of the cubic metre [m³]. A common submultiple is the litre (l) and this is equal in volume to one cubic decimetre and is used only for fluid measure.

$$1 \text{ m}^3 = 10^3 \text{ dm}^3 \text{ therefore } 10^3 \text{ litres} = 1 \text{ m}^3$$

The millilitre [ml] is 1×10^{-3} litre and therefore equal in volume to one cubic centimetre. Whereas the basic unit of density is kilogramme per cubic metre [kg/m³], densities of liquids are sometimes expressed in grammes per millilitre [g/ml] and densities of solids in grammes per cubic centimetre [g/cm³].

SPECIFIC VOLUME is the volume occupied by unit mass and the basic unit is cubic metre per kilogramme [m³/kg] thus, the specific volume is the reciprocal of density. In certain cases, specific volume may be expressed in cubic metres per tonne [m³/t] and litres per kilogramme [l/kg].

TEMPERATURE is an indication of hotness or coldness and therefore is a measure of the intensity of heat.

The most common temperature measuring instrument is the mercurial thermometer. This consists of a glass tube of very fine bore with a bulb at its lower end, the bulb and tube are exhausted of air, partially filled with mercury and hermetically sealed at the top end. When the thermometer is placed in a substance whose temperature is to be measured, the mercury takes up the same temperature and expands (if heated) or contracts (if cooled) and the level, which rises or falls in consequence indicates on the termometer scale the degree of heat intensity.

The Celsius scale (formerly known as Centigrade) is used in the SI system of measuring and specifying temperatures. The point at which pure water freezes into ice is marked zero on the Celsius scale and that point at which pure water boils into steam at atmospheric pressure is assigned the number 100. The former is sometimes referred to as the *lower fixed point* or *ice point*, the latter as the *upper fixed point* or *steam point*. The unit representing a temperature reading on the Celsius scale is °C and the symbol for temperature is θ.

The advantages of mercury are that it does not wet the bore of the glass and therefore none sticks to the glass as the temperature falls. It can be used over a wide useful range of temperature as its freezing point is low, about –38°C, and boiling point high, about 358°C.

Alcohol is sometimes used but requires to be coloured. Although its freezing point is in the region of –115°C and can be useful for measuring very low temperature, its boiling point is also low, about 78°C, and therefore has a limited use.

The Farenheit scale was used in the past and to convert a *temperature interval* from Farenheit to Celsius:

Interval on Celsius scale $= \dfrac{5}{9} \times$ interval on Farenheit scale.

For example, if a body is heated through 153°F, this is

$$153 \times \frac{5}{9} = 85°C$$

To convert a *temperature reading* from Farenheit to Celsius:

Reading on the Celsius scale $= (F - 32) \times \dfrac{5}{9}$

For example, a temperature of 77 °F is equivalent to

$$(77 - 32) \times \frac{5}{9} = 25°C$$

ABSOLUTE TEMPERATURE. All gases expand at practically the same rate when heated through the same range of temperature, and contract at the same rate when cooled.

The rate of expansion or contraction of a perfect gas is (very nearly) 1⁄273 of its volume at 0°C when heated or cooled at constant pressure through one degree Celsius. Hence, if a gas initially at 0°C could be cooled at constant pressure until its temperature is 273 Celsius degrees below 0°C, the volume would contract until there was nothing left and no further reduction of temperature would be possible, that is, the gas would then have reached its *absolute zero of temperature*. In practice of course, it is not possible to cool a gas down to the absolute zero. As the absolute zero of temperature is approached the gas will change into a liquid and the laws of gases are then no longer applicable.

From the above, temperatures can be expressed as *absolute* quantities, that is, stating the degrees of temperature above the level of Absolute Zero, by adding 273 to the ordinary Celsius thermometer reading.

Absolute temperature is often referred to as *thermodynamic temperature*, the symbol for this is *T* and the unit is the kelvin which is represented by K (often °K), thus:

Thermodynamic temperature = Celsius temperature + 273.
In symbols,

$$T[K] = \theta[°C] + 273$$

VOLUME FLOW is the volume of a fluid flowing past a given point in unit time. The basic unit is cubic metres per second [m³/s],

other convenient units are cubic metres per hour [m³/h], cubic metres per minute [m³/min] and litres per hour [l/h].

For example, if a fluid is flowing at a velocity of *v* metres per second full bore through a pipe of internal diameter *d* metres, the quantity flowing in cubic metres per second is:

Volume flow [m³/s] $=$ area [m²] $\times$ velocity [m/s]
$$= 0.7854 \, d^2 \times v$$

$\dot{V}$ is used to indicate volume flow rate

MASS FLOW is the mass of fluid flowing past a given point in unit time, the basic unit being kilogrammes per second [kg/s]. Since density is the mass per unit volume, then:

Mass flow [kg/s] = volume flow [m³/s] $\times$ density [kg/m³].

Mass flow may also be expressed in other convenient units such as tonnes per hour [t/h] and kilogrammes per hour [kg/h]; $\dot{m}$ is used to indicate mass flow rate.

SWEPT VOLUME OR STROKE VOLUME is the volume swept through by a piston in the cylinder of a reciprocating engine, pump, compressor, etc., it is the product of the piston area [m²] and the stroke of the piston [m].

The space left between the piston at its inner dead centre (top of its stroke) and the cylinder head is termed the *clearance volume*. This may be expressed as the actual volume of the clearance space, or as a fraction of the stroke volume.

SYSTEM. A *system* is the term given to the collection of matter under consideration enclosed within a *boundary*, the region outside the boundary is termed the *surroundings*. The boundary may be an imaginary enclosure, or it may be real such as the cylinder wall, cylinder head and piston of an internal combustion engine which encloses the mixture of gases within. Energy is transferred across a boundary from one system to another.

If there is no transfer of matter across the boundary, that is, if no substance can enter the system or leave it during investigation, it is a *closed system*, energy only being transferred across the boundary, and whatever changes take place to the substance, is termed a **non-flow** process because no matter flows into or out of it.

If there is a flow of matter through the boundary, it is an *open system*. If the mass flow entering the system is equal to the mass

flow leaving so that at any time the quantity of matter within the system is constant, the series of changes to the matter is referred to as a **steady-flow** process.

An example of a closed system within which a non-flow process takes place is in the cylinder of an air compressor and an IC engine.

A turbine is an example of an open system in which a steady-flow process takes place, as is a nozzle and a uniformly cycling reciprocating unit.

A CYCLE is a recurrent period of a complete set of a series of connected processes which a system undergoes, the final state of the system at the end of the cycle being exactly as it was at the beginning of the cycle.

TEST EXAMPLES 1

1.   A pump discharges 50 tonne of water per hour to a height of 8 m the overall efficiency of the pumping system being 69%. Calculate the output power and the input power. Calculate also the energy consumed by the pump in 2 hours, expressed in kWh and in MJ.

2. (a)   Express a pressure of 20 mm water in N/m^2 and mbars.
   (b)   Express a pressure of 750 mmHg in kN/m^2 and bars.

3.   A condenser vacuum gauge reads 715 mmHg when the barometer stands at 757 mmHg. State the absolute pressure in the condenser in kN/m^2 and bars.

4.   Convert the following temperature readings from °F to °C:
        140 °F   5 °F   −31 °F   − 40 °F

5.   Oil flows full bore at a velocity of 2 m/s through a nest of 16 tubes in a single pass cooler. The internal diameter of the tubes is 30 mm and the density of the oil is 0·85 g/ml. Find the volume flow in litres per second and the mass flow in kilogrammes per minute.

CHAPTER 2

HEAT

Heat is a form of energy associated with the movement of the molecules which constitute the heated body. Heat is transferred from one substance to another by temperature difference between the two substances, it is interchangeable with other forms of energy and can be made available for doing work and producing mechanical and electrical power.

The basic unit of all energy, including heat, is the *joule* [J]. Thus, units of heat are expressed in joules or multiples of the joule, the commonest being kilojoules [kJ] and megajoules [MJ]. The symbol representing quantity of heat is Q.

SPECIFIC HEAT (or specific heat capacity) of a substance is the quantity of heat required to raise the temperature of unit mass of the substance by one degree. The units of specific heat are therefore heat units per unit mass per unit temperature. The symbol for specific heat is c. In most cases involving specific heat the kilojoule is the most convenient size of heat unit for unit mass of one kilogramme, hence the specific heat is usually expressed in kilojoules per kilogramme per kelvin, in symbols this is kJ/kg K. Note also that one Celsius degree of temperature interval on the thermometer scale is the same as one kelvin degree of temperature interval on the absolute scale, the above units could therefore be written kJ/kg°C, but rarely are.

It follows from the above definition of specific heat that the quantity of heat energy transferred to a substance to raise its temperature is the product of the mass of the substance, its specific heat, and its rise in temperature.

In symbols:

$$Q[\text{kJ}] = m \text{ [kg]} \times c \text{ [kJ/kg K]} \times (T_2 - T_1) \text{ [K]}$$

Different substances have different specific heat values. Also, the specific heat of any one particular substance is not always a constant value over a large range of temperature, the variation in specific heat however, is small, and an average value over the temperature range under consideration may be taken for most practical purposes. For example, the specific heat of water decreases from 4·21 kJ/kg K at 0°C to 4·178 kJ/kg K at 35°C and increases thereafter with rise in temperature, being 4·219 kJ/kg K at 100°C.

Between 0°C and 100°C a mean value is usually taken as 4·2.

Example. Calculate the quantity of heat to be transferred to 2·25 kg of brass to raise its temperature from 20°C to 240°C, taking the specific heat of the brass as 0·394 kJ/kg K.

Temperature increase $= 240 - 20 = 220°C = 220$ K
$$\begin{aligned} Q \text{ [kJ]} &= m \text{ [kg]} \times c \text{ [kJ/kg K]} \times (T_2 - T_1) \text{ [K]} \\ &= 2·25 \times 0·394 \times 220 \\ &= 195 \text{ kJ Ans.} \end{aligned}$$

The characteristics of gases vary considerably at different temperatures and pressures, and heat may be transferred under an infinite number of different conditions, consequently the specific heat can have an infinite number of different values. Two important conditions are transferring heat to or from a gas which is at constant pressure, and transferring heat while its volume is constant. The specific heat of a gas at constant pressure is represented by c_p and at constant volume by c_v. This is dealt with in detail later.

Note: $(T_2 - T_1) \text{ [K]} = (\theta_2 - \theta_1) \text{ [°C]}$.

MECHANICAL EQUIVALENT OF HEAT is the relationship between mechanical energy and heat energy. This was determined by Joule, one of the first scientists to demonstrate that heat was a form of energy, using apparatus which generated heat by the expenditure of mechanical work. When work is done in overcoming friction, the mechanical energy expended is converted into heat energy. The force required to overcome sliding friction between two bodies is the product of the coefficient of friction (μ) and the normal force between the surfaces of the bodies.

Example. A shaft runs at a rotational speed of 50 rev/s in oil-cooled bearings 178 mm diameter. The force between the surfaces of the shaft journals and bearings is 2·67 kN and the coefficient of friction is 0·04. Find (i) the friction force at the surface of the journals, (ii) the mechanical energy expended in friction per revolution, (iii) the power loss due to friction, (iv) the temperature rise of the oil if the volume flow through the bearings is 18 litre/min, the specific heat of the oil being 2 kJ/kg K and its density 0·9 g/ml.

Friction force $= \mu \times$ normal force between surfaces
$$\begin{aligned} &= 0·04 \times 2·67 \times 10^3 \\ &= 106·8 \text{ N Ans. (i)} \end{aligned}$$

Work done to overcome friction per revolution $[J = Nm]$
$$= \text{friction force [N]} \times \text{circumference of journal [m]}$$
$$= 106 \cdot 8 \times \pi \times 0 \cdot 178$$
$$= 59 \cdot 7 \text{ J Ans. (ii)}$$

Power expended $[W = J/s]$
$$= \text{energy per revolution} \times \text{rev/s}$$
$$= 59 \cdot 7 \times 50$$
$$= 2985 \text{ W} = 2 \cdot 985 \text{ kW Ans. (iii)}$$

Density of oil $= 0 \cdot 9$ g/ml $= 0 \cdot 9$ kg/litre

Mass flow of oil [kg/s] = volume flow [l/s] × density [kg/l]
$$= \frac{18}{60} \times 0 \cdot 9 = 0 \cdot 27 \text{ kg/s}$$

$$Q \text{ [kJ/s]} = m \text{ [kg/s]} \times c \text{ [kJ/kgK]} \times (T_2 - T_1) \text{ [K]}$$

$$2 \cdot 985 = 0 \cdot 27 \times 2 \times \text{temp. rise}$$

$$\text{Temp. rise} = 5 \cdot 527 \text{ K or } 5 \cdot 527°\text{C Ans. (iv)}$$

WATER EQUIVALENT of a mass of a substance is the mass of water that would require the same heat transfer as the mass of that substance to cause the same change of temperature.

For example, taking an aluminium vessel of mass 2 kg, and the specific heat of aluminium as $0 \cdot 912$ kJ/kgK:

$$Q = \text{mass} \times \text{specific heat} \times \text{temperature change}$$

$$\text{For water, } Q_W = m_W \times c_W \times (T_2 - T_1)_W$$
$$\text{For aluminium, } Q_A = m_A \times c_A \times (T_2 - T_1)_A$$

Since Q is to be the same quantity of heat,
$$Q_W = Q_A$$
$$m_W \times c_W \times (T_2 - T_1)_W = m_A \times c_A \times (T_2 - T_1)_A$$
and the temperature change is to be the same,

$$m_W \times c_W = m_A \times c_A$$

$$\therefore \quad m_W = m_A \times \frac{c_A}{c_W}$$

Taking the specific heat of water as $4 \cdot 2$ kJ/kgK, the water equivalent of this mass of aluminium is

$$m_W = 2 \times \frac{0 \cdot 912}{4 \cdot 2}$$

$$= 0 \cdot 4343 \text{ kg}$$

That is to say, $0 \cdot 4343$ kg of water would require the same amount of heat transferred to it as the 2 kg of aluminium to raise it through the same range of temperature.

It is useful to know the water equivalent of laboratory calorimeters. When water is contained in a vessel, the temperature of the vessel is the same as that of the water inside, and when the temperature of the water is changed, the temperature of the vessel changes with it. The vessel can therefore be considered as an extra mass of water equal to the water equivalent of the vessel.

Example. The mass of a copper calorimeter is $0 \cdot 28$ kg and it contains $0 \cdot 4$ kg of water at 15°C. Taking the specific heat of copper as $0 \cdot 39$ kJ/kgK, calculate the heat required to raise the temperature to 20°C.

Water equivalent of calorimeter

$$= 0 \cdot 28 \times \frac{0 \cdot 39}{4 \cdot 2} = 0 \cdot 026 \text{ kg}$$

Heat received by water and calorimeter,
$$Q \text{ [kJ]} = m \text{ [kg]} \times c \text{ [kJ/kgK]} \times (T_2 - T_1) \text{ [K]}$$
$$= (0 \cdot 4 + 0 \cdot 026) \times 4 \cdot 2 \times (20 - 15)$$
$$= 0 \cdot 426 \times 4 \cdot 2 \times 5$$
$$= 8 \cdot 946 \text{ kJ Ans.}$$

When two substances at different temperatures are mixed together, heat will transfer from the hotter substance to the colder until both become the same temperature. Unless otherwise is stated, it is assumed that no heat is transferred to or from an outside source during the mixing process and therefore the quantity of heat absorbed by the colder substance is all at the expense of the loss of heat by the hotter substance.

Example. In an experiment to find the specific heat of lead, $0 \cdot 5$ kg of lead shot at a temperature of 51°C is poured into an insulated calorimeter containing $0 \cdot 25$ kg of water at 13·5°C and the resultant temperature of the mixture is 15·5°C. If the water equivalent of the calorimeter is $0 \cdot 02$ kg, find the specific heat of the lead.

Heat received by water and calorimeter when their temperature is raised from 13·5°C to 15·5°C:

$$Q = m \times c \times (T_2 - T_1)$$
$$= (0.25 + 0.02) \times 4.2 \times (15.5 - 13.5)$$
$$= 0.27 \times 4.2 \times 2 \text{ kJ}$$

Heat lost by lead in cooling from 51°C to 15.5°C:

$$Q = m \times c \times (T_3 - T_2)$$
$$= 0.5 \times c \times (51 - 15.5)$$
$$= 0.5 \times c \times 35.5 \text{ kJ}$$

Heat transferred from the lead is equal to the heat received by the water and calorimeter:

$$0.5 \times c \times 35.5 = 0.27 \times 4.2 \times 2$$
$$c = \frac{0.27 \times 4.2 \times 2}{0.5 \times 35.5}$$
$$= 0.1278 \text{ kJ/kg K} \quad \text{Ans.}$$

LATENT HEAT is the heat which supplies the energy necessary to overcome some of the binding forces of attraction between the molecules of a substance and is responsible for it changing its physical state from a solid into a liquid, or from a liquid into a vapour, the change taking place without any change of temperature.

The process of changing the physical state from a solid into a liquid is called *melting* or *fusion*, and the quantity of heat required to change unit mass of the substance from solid to liquid at the same temperature is the *latent heat of fusion*.

For example, the latent heat of fusion for ice is 335 kJ/kg at 0°C. This means that one kilogramme of ice at 0°C would require 335 kilojoules of heat transferred to it to completely melt it into one kilogramme of water at 0°C. Also, one kilogramme of water at 0°C would require to lose 335 kilojoules of heat to completely freeze it into ice at 0°C.

The process of changing the physical state of a substance from a liquid into a vapour is called *boiling* or *evaporation* and the quantity of heat to bring about this change at constant temperature to unit mass is the *latent heat of evaporation*.

The latent heat of evaporation of water at atmospheric pressure is 2256.7 kJ/kg. This means that one kilogramme of water at 100°C would require 2256.7 kilojoules of heat to completely boil it into one kilogramme of steam at 100°C. Also, one kilogramme of steam at 100°C would require to lose 2256.7 kilojoules of heat to com-

pletely condense it into one kilogramme of water at 100°C.

The temperature at which a liquid boils and the latent heat of evaporation depend strictly upon the pressure, the higher the pressure, the higher the boiling point and the smaller the amount of latent heat required to evaporate it. For example, at atmospheric pressure, the temperature at which water boils is 100°C and the latent heat of evaporation is 2256.7 kJ/kg, at a pressure of 15 bar (1500 kN/m²) the boiling point is 198.3°C and the latent heat 1947 kJ/kg, at 30 bar (3000 kN/m²) the boiling point is 233.8°C and the latent heat 1795 kJ/kg. These values are obtained from steam tables which are described later.

When heat is transferred to or from a substance which changes only its temperature, and there is no physical change of state, it is sometimes referred to as *sensible heat*. This distinguishes it from latent heat which changes the physical state of the substance without change of temperature.

We shall see later that, for a constant pressure process, the heat energy transferred to a substance is termed *enthalpy*, then, latent heat of fusion is termed *enthalpy of fusion*, and latent heat of evaporation is termed *enthalpy of evaporation*, and so on.

Example. Calculate the heat required to be given to 2 kg of ice at −15°C to change it into steam at atmospheric pressure, taking the values:

Specific heat of ice = 2.04 kJ/kg K
Latent heat of fusion = 335 kJ/kg
Specific heat of water = 4.2 kJ/kg K
Latent heat of evaporation = 2256.7 kJ/kg

Heat to raise the temperature of the ice from −15°C to its melting point of 0°C, *i.e.* a temperature rise of 15°C:

$$\text{Sensible heat} = m \times c \times \text{temp. rise}$$
$$= 2 \times 2.04 \times 15$$
$$= 61.2 \text{ kJ}$$

Heat to change the ice at 0°C into water at 0°C:

$$\text{Latent heat} = 2 \times 335$$
$$= 670 \text{ kJ}$$

Heat to raise the temperature of the water from 0°C to its boiling point of 100°C, *i.e.* a temperature rise of 100°C:

$$\text{Sensible heat} = 2 \times 4.2 \times 100$$
$$= 840 \text{ kJ}$$

Heat to evaporate the water at 100°C into steam at 100°C:

$$\text{Latent heat} = 2 \times 2256 \cdot 7$$
$$= 4513 \cdot 4 \text{ kJ}$$
$$\text{Total heat} = 61 \cdot 2 + 670 + 840 + 4513 \cdot 4$$
$$= 6084 \cdot 6 \text{ kJ Ans.}$$

Working is simplified by finding the total heat transfer required per unit mass and then finally multiplying by the total mass, thus,

$$Q = 2(2 \cdot 04 \times 15 + 335 + 4 \cdot 2 \times 100 + 2256 \cdot 7)$$
$$= 2(30 \cdot 6 + 335 + 420 + 2256 \cdot 7)$$
$$= 2 \times 3042 \cdot 3$$
$$= 6084 \cdot 6 \text{ kJ}$$

Change of temperature, change of quantity of heat energy etc., are often written $\Delta\theta$, ΔQ, etc.

TEST EXAMPLES 2

1. A water brake coupled to an engine on test absorbs 70 kW of power. Find the heat generated at the brake per minute and the mass flow of fresh water through the brake, in kg/min if the temperature increase of the water is 10°C. Assume all the heat generated is carried away by the cooling water.

2. The effective radius of the pads in a single collar thrust block is 230 mm and the total load on the thrust block is 240 kN when the shaft is running at 93 rev/min. Taking the coefficient of friction between thrust collar and pads as 0·025, find (i) the power lost due to friction, (ii) the heat generated per hour, (iii) the mass flow of oil in kilogrammes per hour through the block assuming all the heat is carried away by the oil, allowing an oil temperature rise of 20°C and taking the specific heat of the oil as 2 kJ/kgK.

3. To ascertain the temperature of flue gases, 1·8 kg of copper of specific heat 0·395 kJ/kgK was suspended in the flue until it attained the temperature of the gases, and then dropped into 2·27 kg of water at 20°C. If the resultant temperature of the copper and water was 37·2°C, find the temperature of the flue gases.

4. In an experiment to find the specific heat of iron, 2·15 kg of iron cuttings at 100°C are dropped into a vessel containing 2·3 litre of water at 17°C and the resultant temperature of the mixture is 24·4°C. If the water equivalent of the vessel is 0·18 kg, determine the specific heat of the iron.

5. 0·5 kg of ice at −5°C is put into a vessel containing 1·8 kg of water at 17°C and mixed together, the result being a mixture of ice and water at 0°C. Calculate the final masses of ice and water, taking the water equivalent of the vessel as 0·148 kg, specific heat of ice 2·04 kJ/kgK, latent heat of fusion 335 kJ/kg.

CHAPTER 3

THERMAL EXPANSION

EXPANSION OF METALS

The effect of increasing the temperature of metals is generally to cause their dimensions to increase. Most metals expand when they are heated and contract when they are cooled, the amount of expansion per degree rise of temperature differs with different metals. Some alloys are manufactured to have a minimum amount of expansion over a considerable working temperature range, these are usually for special purposes such as measuring instruments and gauges.

Although the expansion is in all directions so that there is an increase in all dimensions, it is sometimes only relevant to consider the expansion in one direction.

LINEAR EXPANSION. When a linear dimension is under consideration, the amount that a metal will expand lengthwise is expressed by its *coefficient of linear expansion*. This is the increase in length per unit length per degree increase in temperature. For instance, if the coefficient of linear expansion of copper is given as $1 \cdot 7 \times 10^{-5}/°C$ (which is 0·000017 per degree Celsius) it means that each metre of length will increase in length by $1 \cdot 7 \times 10^{-5}$ metre when heated through one degree Celsius. This coefficient may be represented by α. Hence, representing the original length by l, and temperature rise by $(\theta_2 - \theta_1)$,

Increase in length $= \alpha \times l \times (\theta_2 - \theta_1)$.
The new length of the metal will then be:

$$\begin{aligned} \text{new length} &= \text{original length} + \text{increase in length} \\ &= l + \alpha l\,(\theta_2 - \theta_1), \\ &= l\{1 + \alpha(\theta_2 - \theta_1)\} \end{aligned}$$

Example. A main steam pipe is 6·5 m long when fitted at a temperature of 15°C. Calculate how much allowance should be made for its increase in length if it is subjected to a steam temperature of 300°C, taking the coefficient of linear expansion of the material as $1 \cdot 2 \times 10^{-5}/°C$.

$$\begin{aligned} \text{Increase in length} &= \alpha \times l \times (\theta_2 - \theta_1) \\ &= 1 \cdot 2 \times 10^{-5} \times 6 \cdot 5 \times (300 - 15) \\ &= 0 \cdot 02223 \text{ m} \\ &= 22 \cdot 23 \text{ mm Ans.} \end{aligned}$$

Example. A brass liner is 270 mm diameter when the temperature is 17°C. Take the coefficient of linear expansion of the brass as $1 \cdot 9 \times 10^{-5}/°C$ and find the temperature to which the liner should be heated in order to increase the diameter by 2 mm.

Diameter is a linear dimension and therefore the same rule can be applied as for length. Note that the diameter and increase in diameter must be expressed in the same units.

$$\begin{aligned} \text{Increase in diameter} &= \alpha \times d \times (\theta_2 - \theta_1) \\ 2 &= 1 \cdot 9 \times 10^{-5} \times 270 \times (\theta_2 - \theta_1) \end{aligned}$$

$$\text{Increase in temperature} = \frac{2 \times 10^5}{1 \cdot 9 \times 270} = 389 \cdot 7°C$$

$$\begin{aligned} \therefore \text{ Required temperature} &= 17 + 389 \cdot 7 \\ &= 406 \cdot 7°C \text{ Ans.} \end{aligned}$$

SUPERFICIAL EXPANSION refers to increase in area. The coefficient of superficial expansion is the increase in area per unit area per degree increase in temperature. Therefore, if A represents original area, and $(\theta_2 - \theta_1)$ the increase in temperature, then,

Increase in area = coeff. of superficial expansion $\times A \times (\theta_2 - \theta_1)$. Consider an area of metal of unit length and unit breadth (Fig. 1) and let this be heated through one degree.
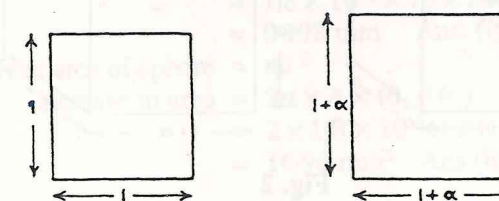

Fig.1

The length and breadth will each increase by an amount equal to α, the coefficient of linear expansion.

$$\begin{aligned} \text{original area} &= 1 \times 1 = 1 \\ \text{new length and new breadth} &= 1 + \alpha \\ \text{new area} &= (1 + \alpha)^2 = 1 + 2\alpha + \alpha^2 \end{aligned}$$

$$\begin{aligned}\text{Increase in area} &= \text{new area} - \text{original area}\\ &= 1 + 2\alpha + \alpha^2 - 1\\ &= 2\alpha + \alpha^2\end{aligned}$$

α is a very small quantity for any metal (such as about $1 \cdot 2 \times 10^{-5}$ for steel), therefore α^2 being the second order of smallness is a very small quantity indeed and is completely negligible as a quantity to be added for all practical purposes. We can therefore take the increase to be 2α. As this is the increase in area per unit area for one degree increase in temperature, it is the value of the coefficient of superficial expansion, hence,

Coeff. of superficial expansion = 2 × coeff. of linear expansion

therefore,

$$\text{Increase in area} = 2\alpha \times A \times (\theta_2 - \theta_1).$$

CUBICAL EXPANSION refers to the increase in volume. The *coefficient of cubical* (or *volumetric*) *expansion* is the increase in volume per unit volume per degree increase in temperature. Therefore, if V represents the original volume, and $(\theta_2 - \theta_1)$ increase in temperature, then:

Increase in volume = coeff. of cubical expansion × $V \times (\theta_2 - \theta_1)$.

Consider a block of metal of unit length, unit breadth, and unit thickness (Fig. 2) and let this be heated through one degree.
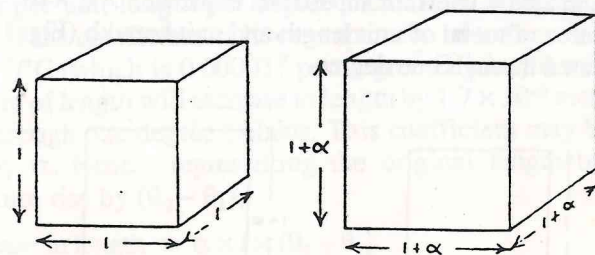
Fig. 2

The length, breadth and thickness will each increase by an amount equal to α, the coefficient of linear expansion.

$$\begin{aligned}\text{Original volume} &= 1 \times 1 \times 1 = 1\\ \text{New volume} &= (1 + \alpha)^3\\ &= 1 + 3\alpha + 3\alpha^2 + \alpha^3\end{aligned}$$

$$\begin{aligned}\text{Increase in volume} &= \text{new volume} - \text{original volume}\\ &= 1 + 3\alpha + 3\alpha^2 + \alpha^3 - 1\\ &= 3\alpha + 3\alpha^2 + \alpha^3\end{aligned}$$

α^2 and α^3 being the second and third order of smallness respectively, are negligible quantities for addition, hence the increase may be taken as 3α. As this is the increase in volume per unit volume per degree increase in temperature, it is the value of the coefficient of cubical expansion.

Coeff. of cubical expansion = 3 × coeff. of linear expansion.

Therefore,

Increase in volume = $3\alpha \times V \times (\theta_2 - \theta_1)$.

Note that *linear* refers to any linear dimension such as length, breadth, thickness, diameter, circumference, and so on, and the expression for linear expansion can be applied to any of these dimensions. It is true for internal dimensions as well as external.

The expression for superficial expansion covers any area of the solid, cross-sectional area, surface area, etc., and holds good for internal areas as well as external.

The expression for cubical expansion is also applicable for internal volumes of a hollow vessel.

Example. A metal sphere is exactly 25 mm diameter at 20°C. Find the increase in diameter, increase in surface area, and increase in volume, when heated to 260°C, if the coefficient of linear expansion of the metal is $1 \cdot 8 \times 10^{-5}$/°C.

$$\begin{aligned}\text{Increase in temperature} &= 260 - 20 = 240°C\\ \text{Increase in diameter} &= \alpha \times d \times (\theta_2 - \theta_1)\\ &= 1 \cdot 8 \times 10^{-5} \times 25 \times 240\\ &= 0 \cdot 108 \text{ mm} \quad \text{Ans. (i)}\\ \text{Surface area of sphere} &= \pi d^2\\ \text{Increase in area} &= 2\alpha \times A \times (\theta_2 - \theta_1)\\ &= 2 \times 1 \cdot 8 \times 10^{-5} \times \pi \times 25^2 \times 240\\ &= 16 \cdot 96 \text{ mm}^2 \quad \text{Ans (ii)}\end{aligned}$$

$$\text{Volume of sphere} = \frac{\pi}{6} d^3$$

$$\begin{aligned}\text{Increase in volume} &= 3\alpha \times V \times (\theta_2 - \theta_1)\\ &= 3 \times 1 \cdot 8 \times 10^{-5} \times \frac{\pi}{6} \times 25^3 \times 240\\ &= 106 \text{ mm}^3 \quad \text{Ans. (iii)}\end{aligned}$$

EXPANSION OF LIQUIDS

Liquids have no definite shape of their own, therefore no linear dimensions, hence the coefficient of cubical expansion of a liquid is an independent quantity. The coefficient of cubical expansion is usually represented by β.

$$\text{Increase in volume} = \beta \times V \times (\theta_2 - \theta_1)$$

Example. 2500 litres of oil are heated through 50°C. If the coefficient of cubical expansion of this oil is 0·0008/°C, find the increase in volume in cubic metres.

$$2500 \text{ litres} = 2·5 \text{ m}^3$$
$$\begin{aligned}
\text{Increase in volume} &= \beta \times V \times (\theta_2 - \theta_1) \\
&= 0·0008 \times 2·5 \times 50 \\
&= 0·1 \text{ m}^3 \quad \text{Ans.}
\end{aligned}$$

APPARENT CUBICAL EXPANSION. The tank or vessel which contains a liquid will also expand when heated. It is therefore useful to know the expansion of the liquid relative to its container so that the correct allowance can be made for changes of temperature.

The apparent or relative increase in volume of a liquid is the difference between the volumetric expansion of the liquid and the volumetric expansion of its container. If both have the same initial volume and are raised through the same range of temperature, then, letting suffix L represent the liquid and suffix C the container:

Apparent increase in volume of the liquid

$$\begin{aligned}
&= \text{vol. increase of liquid} - \text{vol. increase of container} \\
&= \beta_L \times V \times (\theta_2 - \theta_1) - \beta_C \times V \times (\theta_2 - \theta_1) \\
&= (\beta_L - \beta_C) \times V \times (\theta_2 - \theta_1)
\end{aligned}$$

The difference between the coefficients of cubical expansion of the liquid and its container can therefore be termed the *apparent* coefficient of cubical expansion of the liquid.

RESTRICTED THERMAL EXPANSION

If the natural thermal expansion of a metal is restricted, the metal will be strained from its natural length and the material will be stressed. To enable the effects of restricted expansion to be seen here, it is necessary to revise the relationships between stress, strain and modulus of elasticity.

STRESS is the internal resistance set up in a material when an

external force is applied. It is expressed as the force carried by the material per unit area of its cross-section.

$$\text{Stress [N/m}^2] = \frac{\text{force [N]}}{\text{area [m}^2]}$$

High values of stress are expressed in multiples of the force unit, such as kN/m² (= 10^3 N/m²) and MN/m² (= 10^6 N/m²). In some industries, stress may be expressed in hectobars [hbar], one hectobar = 10^7 N/m².

STRAIN is the change of shape that takes place in a material due to it being stressed. Linear strain is the change of length per unit length, thus,

$$\text{Linear strain} = \frac{\text{change of length}}{\text{original length}}$$

MODULUS OF ELASTICITY of a material is the constant obtained by dividing the stress set up in it by the strain it endures under that stress, provided the elastic limit is not exceeded.

$$\text{Modulus of elasticity} (E) = \frac{\text{stress}}{\text{strain}}$$

Example. A solid steel stay 50 mm diameter and 300 mm long at a temperature of 25°C is firmly secured at each end so that expansion is fully restricted. Find the stress set up in the stay and the equivalent total axial force when it is heated to 150°C, taking the coefficient of linear expansion of steel as $1·2 \times 10^{-5}$/°C and its modulus of elasticity as 206 GN/m².

If the stay was perfectly free to expand without restriction,

$$\begin{aligned}
\text{Free expansion} &= \alpha \times l \times (\theta_2 - \theta_1) \\
&= 1·2 \times 10^{-5} \times 300 \times (150 - 25) \\
&= 0·45 \text{ mm}
\end{aligned}$$

Free and unrestrained length would then be

$$300 + 0·45 = 300·45 \text{ mm}$$

If prevented from expanding the effect is that the stay is compressed from its natural unstrained length of 300·45 mm to 300 mm,

$$\text{Strain} = \frac{\text{change of length}}{\text{original length}}$$

$$= \frac{0.45}{300.45} = 0.001498$$

It is usual, however, to make a slight approximation at this stage by taking the original cold length of 300 mm instead of the heated length of 300·45 mm. The difference in the final result is negligible and it proves much more convenient in solving more complicated problems. It will also be seen that by making this slight approximation the length of the material will not be required because the strain can be obtained direct from:

$$\text{Strain} = \frac{\text{change of length}}{\text{original length}}$$

$$= \frac{\alpha \times l \times (\theta_2 - \theta_1)}{l} = \alpha(\theta_2 - \theta_1)$$

Hence the strain for this stay can be taken as:

$$\text{Strain} = \alpha(\theta_2 - \theta_1)$$
$$= 1.2 \times 10^{-5} \times 125 = 0.0015$$

which is the same as $\dfrac{0.45}{300} = 0.0015$

$$E \,[\text{N/m}^2] = \frac{\text{stress } [\text{N/m}^2]}{\text{strain}}$$

$$\therefore \text{Stress} = \text{strain} \times E$$
$$= 0.0015 \times 206 \times 10^9$$
$$= 3.09 \times 10^8 \text{ N/m}^2$$
$$= 309 \text{ MN/m}^2 \text{ or } 30.9 \text{ hbar} \quad \text{Ans. (i)}$$

$$\text{Stress} \,[\text{N/m}^2] = \frac{\text{force } [\text{N}]}{\text{area } [\text{m}^2]}$$

$$\therefore \text{Total axial force} = \text{stress} \times \text{area}$$
$$= 3.09 \times 10^8 \times 0.7854 \times 0.05^2$$
$$= 6.068 \times 10^5 \text{ N}$$
$$= 606.8 \text{ kN} \quad \text{Ans. (ii)}$$

TEST EXAMPLES 3

1. A steam pipe is 3·85 m long when fitted at a temperature of 18°C. Find the increase in length if free to expand, when carrying steam at a temperature of 260°C, taking the coefficient of linear expansion of the pipe material as 1.25×10^{-5}/°C.

2. A solid cast iron sphere is 150 mm diameter. If 2110 kJ of heat energy is transferred to it, find the increase in diameter, taking the following values for cast iron: density = 7·21 g/cm³, specific heat = 0·54 kJ/kg K, coefficient of linear expansion = 1.12×10^{-5}/°C.

3. A bi-metal control device is made up of a thin flat strip of aluminium and a thin flat strip of steel of the same dimensions, connected together in parallel and separated from each other by two brass distance pieces 2·5 mm long, their centres being 50 mm apart. Find the radius of curvature of the strips when heated through 200°C, taking the following values for the coefficients of linear expansion:

$$\begin{aligned}\text{Aluminium } \alpha &= 2.5 \times 10^{-5}/\text{°C} \\ \text{Steel } \alpha &= 1.2 \times 10^{-5}/\text{°C} \\ \text{Brass } \alpha &= 2.0 \times 10^{-5}/\text{°C}\end{aligned}$$

4. The pipe line of a hydraulic system consists of a total length of steel pipe of 13·7 m and internal diameter 30 mm. If the coefficient of linear expansion of the steel is 1.2×10^{-5}/°C and coefficient of cubical expansion of the oil in the pipe is 9×10^{-4}/°C, calculate the volumetric allowance in litres to be made for oil overflow from the pipe when the temperature rises by 27°C.

5. A straight length of steam pipe is to be fitted between two fixed points with no allowance for expansion. If the compressive stress in the pipe is to be limited to 35 hbar (= 350 MN/m²) calculate the initial tensile stress to be exerted on the pipe when fitted cold at 17°C to allow for a steam temperature of 220°C. Take the coefficient of linear expansion of the pipe material as 1.12×10^{-5}/°C and the modulus of elasticity as 206 GN/m².

CHAPTER 4

HEAT TRANSFER

Heat is transferred from one system to another by one of the three methods known as *Conduction*, *Convection*, and *Radiation*, or by a combination of these.

CONDUCTION

Conduction is the flow of heat energy through a body, or from one body to another in contact with each other, due to difference in temperature. The natural flow of heat takes place from a region of high temperature to a region of lower temperature.

Generally speaking, metals are good conductors of heat. Air, and some materials such as asbestos, cork, glass wool, are very bad conductors, these are called insulators and are used to minimise heat loss. Special plastic base compositions are used to lag boilers, pipes, casings, etc. to reduce loss of heat energy to the colder outside surroundings. Cork and fibre glass are common insulating materials to pack the hollow walls of refrigerating chambers to reduce heat flow into the cold chambers from the warmer outside surroundings.

The quantity of heat conducted through a material in a given time depends upon the thermal conductivity of the material, is proportional to the surface area exposed to the source of heat, is proportional to the temperature difference between the hot and cold ends, and is inversely proportional to the distance or thickness through which the heat is conducted, thus,

$$\text{Quantity of heat varies as } \frac{\text{area} \times \text{time} \times \text{temp. difference}}{\text{thickness}}$$

The thermal conductivity depends upon the nature of the material and its ability to conduct heat. This varies for different materials and sometimes varies slightly for the same material depending upon the temperature range.

The *thermal conductivity (k)* of a material expresses the quantity of heat energy conducted through unit area in a unit time for unit temperature difference between two opposite faces of a material of unit distance apart.

Considering the heat flow through a flat wall, taking the following symbols and basic units, and referring to Fig. 3:
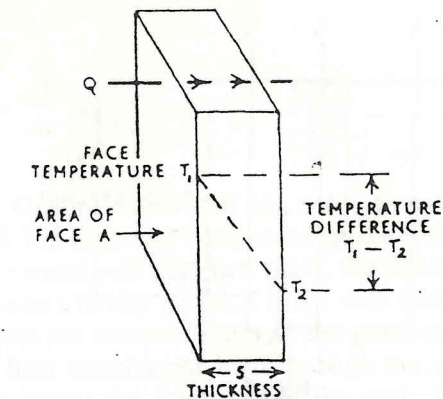
Fig. 3

Q = quantity of heat energy conducted, in joules [J].
A = area through which heat flows, in square metres [m²].
t = time of heat flow, in seconds [s].
$T_1 - T_2$ = temperature difference between the two faces [K].
S = thickness of wall, in metres [m].

Then, the units of k the thermal conductivity are Jm/m²s K. For convenience this is usually shortened by cancelling m into m² and substituting W watts in the place of J/s.

$$\frac{Jm}{m^2 s\,K} = \frac{J}{ms\,K} = \frac{W}{mK} = W/m\,K$$

Hence, the quantity of heat energy transferred by conduction is:

$$Q = \frac{kAt(T_1 - T_2)}{S}$$

Example. Calculate the heat transfer per hour through a solid brick wall 6 m long, 2·9 m high, and 225 mm thick, when the outer surface is at 5°C and the inner surface 17°C, the thermal conductivity of the brick being 0·6 W/m K.

$$Q[J] = \frac{k[W/m\,K] \times A[m^2] \times t[s] \times (T_1 - T_2)[K]}{S[m]}$$

$$= \frac{0 \cdot 6 \times 6 \times 2 \cdot 9 \times 3600 \times (17 - 5)}{0 \cdot 225}$$

$$= 2 \cdot 004 \times 10^6 \text{ J}$$

$$= 2 \cdot 004 \text{ MJ or } 2004 \text{ kJ} \quad \text{Ans.}$$

Note: that T_1 is 17 + 273 = 290 K and T_2 is 5 + 273 = 278 K, their difference, $T_1 - T_2$, is the same as 17°C – 5°C.
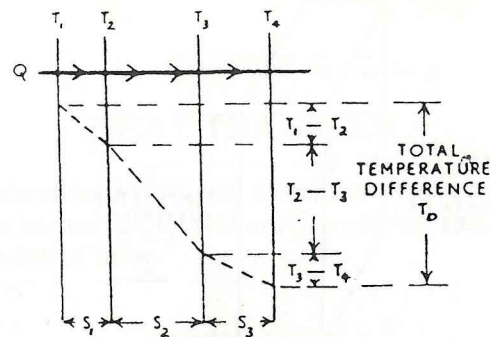
Fig. 4

COMPOSITE WALL. Consider the transfer of heat energy by conduction through a wall made up of a number of layers of different materials, take as an example three slabs of different thicknesses as shown in Fig. 4.

For each thickness:

$$Q = \frac{kAt \times \text{temp. diff.}}{S} \quad \therefore \text{ temp. diff.} = \frac{QS}{kAt}$$

Total drop in temperature across three thicknesses:

$$T_1 - T_4 = \frac{Q_1 S_1}{k_1 A_1 t_1} + \frac{Q_2 S_2}{k_2 A_2 t_2} + \frac{Q_3 S_3}{k_3 A_3 t_3}$$

The same quantity of heat energy is transferred across each layer through the same area in the same time, therefore Q, A and t are common:

$$T_1 - T_4 = \frac{Q}{At}\left\{\frac{S_1}{k_1} + \frac{S_2}{k_2} + \frac{S_3}{k_3}\right\}$$

For any number of layers, let T_D represent the total temperature drop, that is,

$$T_D = (T_1 - T_2) + (T_2 - T_3) + (T_3 - T_4) + \text{etc.}$$

and using the summation sign Σ for the sum of the quantities inside the brackets, that is,

$$\Sigma\left\{\frac{S}{k}\right\} = \frac{S_1}{k_1} + \frac{S_2}{k_2} + \frac{S_3}{k_3} + \text{etc.}$$

then,

$$T_D = \frac{Q}{At}\Sigma\left\{\frac{S}{k}\right\} \quad \text{or} \quad Q = \frac{AtT_D}{\Sigma\left\{\dfrac{S}{k}\right\}}$$

Example. One insulated wall of a cold-storage compartment is 8 m long by 2·5 m high and consists of an outer steel plate 18 mm thick, an inner wood wall 22·5 mm thick, the steel and wood are 90 mm apart to form a cavity which is filled with cork. If the temperature drop across the extreme faces of the composite wall is 15°C, calculate the heat transfer per hour through the wall and the temperature drop across the thickness of the cork. Take the thermal conductivity for steel, cork and wood as 45, 0·045, and 0·18 W/mK respectively.

For the composite wall,

$$T_1 - T_4 = \frac{Q}{At}\left\{\frac{S_1}{k_1} + \frac{S_2}{k_2} + \frac{S_3}{k_3}\right\}$$

where
$$T_1 - T_4 = 15 \text{ K}$$
$$A = 8 \times 2\cdot5 = 20 \text{ m}^2$$
$$t = 3600 \text{ seconds}$$

$$\Sigma\left\{\frac{S}{k}\right\} = \frac{0\cdot018}{45} + \frac{0\cdot09}{0\cdot045} + \frac{0\cdot0225}{0\cdot18}$$

$$= 0\cdot0004 + 2 + 0\cdot125$$

$$= 2\cdot1254$$

$$\therefore \quad 15 = \frac{Q}{20 \times 3600} \times 2\cdot1254$$

$$Q = \frac{15 \times 20 \times 3600}{2\cdot1254}$$

$$= 5\cdot082 \times 10^5 \text{ J}$$

$$= 508\cdot2 \text{ kJ} \quad \text{Ans. (i)}$$

Temperature drop across the cork:

$$= \frac{QS}{Atk}$$

$$= \frac{5\cdot082 \times 10^5 \times 0\cdot09}{20 \times 3600 \times 0\cdot045} = 14\cdot11 \text{ K}$$

$$= 14\cdot11°C \quad \text{Ans. (ii)}$$
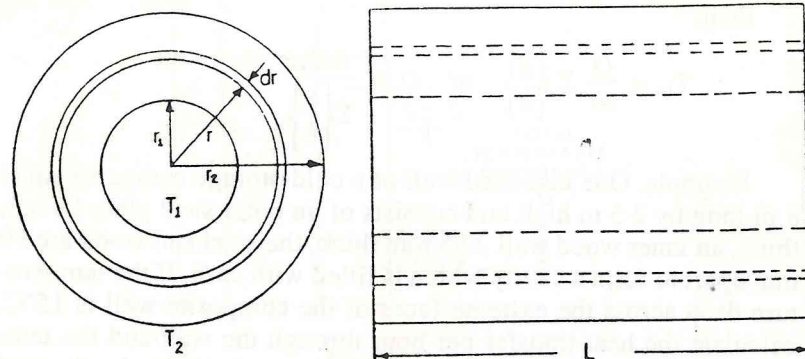
Fig. 5

ƒ CYLINDRICAL WALL. A cylindrical wall, *e.g.* a pipe, could be considered as being made up of many thin cylindrical elements all with the same coefficient of thermal conductivity.

In Fig. 5. If dT is the small temperature difference radially across the cylindrical element whose thickness is dr, length L and area A.

$$\text{Then } Q = kAt\left(-\frac{dT}{dr}\right)$$

$$A = 2\pi r L$$

$$\text{Hence } Q = k2\pi r L\left(-\frac{dT}{dr}\right)$$

Using integration by the use of the calculus to account for the complete cylinder thickness:

$$Q\int_{r_1}^{r_2}\frac{dr}{r} = -k \times 2\pi L t\int_{T_1}^{T_2} dT$$

$$Q \ln\left(\frac{r_2}{r_1}\right) = -2\pi k L t\, (T_2 - T_1)$$

$$Q = \frac{2\pi k L t\, (T_1 - T_2)}{\ln\left(\dfrac{r_2}{r_1}\right)}$$

(ln x, the natural logarithm of x, is often written $\log_e x$)

ƒ Example. A steam pipe is 85 mm external diameter and 25 m long. It carries 1250 kg of steam per hour at a pressure of 20 bar. The steam enters the pipe 0·98 dry and it is to leave the outlet end of the pipe with a minimum dryness fraction of 0·96. The pipe is to be lagged with a material having a thermal conductivity of 0·2 W/m K and outer surface temperature of 26°C. Any temperature drop across the wall of the pipe is to be neglected and the rate of heat transfer through the wall of a thick cylinder is given by:

$$Q = \frac{2\pi k\theta}{\ln\left(\dfrac{D_2}{D_1}\right)} \quad \text{per metre length}$$

where:

k = the thermal conductivity of cylinder material
θ = temperature difference
D_2 = external diameter
D_1 = internal diameter

Determine the thickness of the lagging.

$$\text{Heat loss} = m \times hfg\,(x_i - x_o)$$

$$= \frac{1250 \times 1890 \times (0·98 - 0·96)}{3600}$$

$$= 13·125 \text{ kW}$$

$$Q = \frac{2\pi k\theta}{\ln\left(\dfrac{D_2}{D_1}\right)}$$

$$\frac{13125}{25} = \frac{2\pi \times 0·2 \times (212·4 - 26)}{\ln\left(\dfrac{D_2}{0·085}\right)}$$

$$\ln\left(\frac{D_2}{0·085}\right) = \frac{0·4\pi \times 186·4}{525}$$

$$\lg\left(\frac{D_2}{0·085}\right) = \frac{0·4462}{2·3026}$$

$$D_2 = 0·085 \times 1·563$$

$$= 0·1329 \text{ m}$$

$$\text{Thickness} = \frac{132 \cdot 9 - 85}{2}$$

$$= 23 \cdot 9 \text{ mm} \quad \text{Ans.}$$

Notes: (i)   at 20 bar t_s = 212·4˚C

         (ii)   lg x, the common (base 10) logarithm of x, is often written log x

       (iii)   ratio of diameters is the same as ratio of radii

CONVECTION

Convection is the method of transferring heat through a fluid by the movement of heated particles of the fluid.

Fig. 6 shows a vessel with an inclined tube connected at the bottom. When this contains water, and heat is applied to the tube, the heated particles of the water become less dense and rise, denser particles move to take their place and thus convection currents are set moving resulting in all the water in the vessel and tube becoming heated almost uniformly due to the continuous circulation of the water. This is the principle of the water-tube boiler.

Fig. 6 also shows air in a room heated by convection, the fire, radiator or other heat source being placed at the bottom of the room.

Fig. 6 also illustrates the air in the room cooled by convection, the coolers (such as refrigerator pipes) being situated near the top of the room.

These above are examples of natural circulation of the fluid and is referred to as *free convection*. When the motion of the fluid is produced mechanically, such as by means of a pump or a fan, it is referred to as *forced convection*.
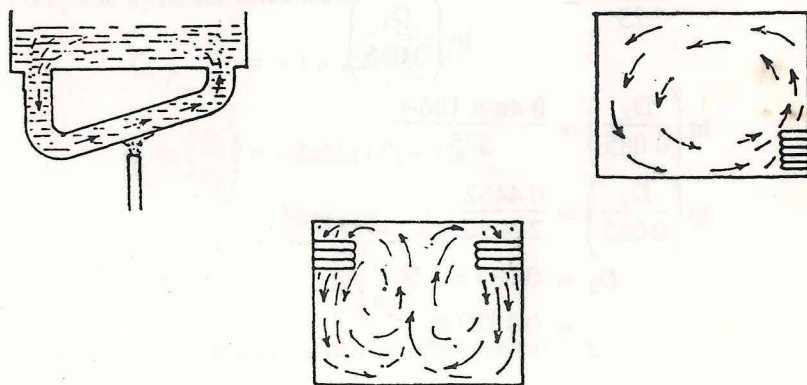
Fig. 6

RADIATION

Radiation is the transfer of heat energy from one body to another through space by rays of electro-magnetic waves. The rays of heat travel in straight lines in all directions at the same velocity as light, that is, at nearly 300 000 kilometres per second.

Some of the radiant heat falling upon a body is reflected in the same manner as light is reflected, the remainder is absorbed. Dark and rough surfaces are good absorbers of radiant heat whereas bright and polished surfaces reflect most of the heat and therefore the amount of absorption is small. A body which is a poor absorber is also a poor radiator, and a good absorber is a good radiator. A perfect absorber and radiator of heat energy is termed a "perfect" black body.

The *emissivity* of a radiating body is the ratio of the heat emitted by that body compared with the heat emitted by a perfect black body of the same surface area and temperature in the same time. Emissivity may be represented by ε, and its value for the ideal radiator is therefore unity.

The STEFAN-BOLTZMANN LAW states that the heat energy radiated by a perfect radiator is proportional to the fourth power of its absolute temperature. Hence Q = quantity of heat energy radiated, A = surface area radiating heat, t = time of radiation, T = absolute temperature, then:

$$Q = AtT^4 \times \text{a constant}$$

The value of the constant depends upon the units employed, and for the basic units of area in square metres, time in seconds, and absolute temperature in degrees Kelvin, the value of the constant to give kilojoules of heat energy is $5 \cdot 67 \times 10^{-11}$ kJ/m²s K⁴.

Thus, the quantity of heat energy radiated from a hot body of absolute temperature T_1 to its surroundings at absolute temperature T_2 is therefore:

$$Q = 5 \cdot 67 \times 10^{-11} \times \varepsilon At \left(T_1^4 - T_2^4\right)$$

Example. The temperature of the flame in a furnace is 1277˚C and the temperature of its surrounds is 277°C. Calculate the maximum theoretical quantity of heat energy radiated per minute per square metre to the surrounding surface area.

$$T_1 = 1550 \text{ K}$$
$$T_2 = 550 \text{ K}$$
$$\begin{aligned}Q &= 5 \cdot 67 \times 10^{-11} \times At(T_1{}^4 - T_2{}^4)\\ &= 5 \cdot 67 \times 10^{-11} \times 1 \times 60 \times (1550^4 - 550^4)\\ &= 5 \cdot 67 \times 10^{-11} \times 1 \times 60 \times 2 \cdot 705 \times 2 \cdot 1 \times 10^{12}\\ &= 1 \cdot 933 \times 10^4 \text{ kJ or } 19 \cdot 33 \text{ MJ} \quad \text{Ans.}\end{aligned}$$

COMBINED MODES

TRANSFER OF HEAT FROM ONE FLUID TO ANOTHER THROUGH A DIVIDING WALL. This is a practical application in many engineering appliances. Consider transfer of heat from a fluid to a flat plate, through the plate, and from the plate to another fluid, as illustrated in Fig. 7. On each side of the plate, a thin film of almost stagnant fluid clings to the surface, the heat transfer through the film is by conduction, convection and radiation.

The quantity of heat transfer through the film of fluid per unit area of surface, in unit time, for unit temperature drop across the thickness of the film, is expressed by the *surface heat transfer coefficient (h)*, and this depends to a large extent upon the velocity of the fluid and the condition of the surfaces.

Since the quantity of heat conducted in a given time is proportional to the surface area and the temperature drop, then the units of h are joules per metre^2 of area per second per degree = $\text{J/m}^2\,\text{s K}$ = $\text{W/m}^2\text{K}$, hence:

$$Q[\text{J}] = h[\text{W/m}^2\text{K}] \times A[\text{m}^2] \times t[\text{s}] \times (T_1 - T_2)[\text{K}]$$

Referring to Fig. 7,
Heat transfer through film of fluid A:
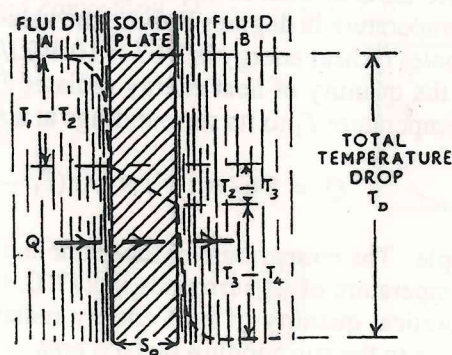
Fig. 7

$$Q = h_A At\,(T_1 - T_2) \qquad \therefore \; T_1 - T_2 = \frac{Q}{h_A At}$$

Heat transfer through solid plate:

$$Q = \frac{k_P At\,(T_2 - T_3)}{S_P} \qquad \therefore \; T_2 - T_3 = \frac{Q S_P}{k_P At}$$

Heat transfer through film of fluid B:

$$Q = h_B At\,(T_3 - T_4) \qquad \therefore \; T_3 - T_4 = \frac{Q}{h_B At}$$

Total drop in temperature:

$$T_D = (T_1 - T_2) + (T_2 - T_3) + (T_3 - T_4)$$
$$= \frac{Q}{At}\left\{\frac{1}{h_A} + \frac{S_P}{k_P} + \frac{1}{h_B}\right\}$$

The quantity inside the brackets may be represented by $1/U$, where U is called the *overall heat transfer coefficient*, then,

$$\frac{1}{U} = \frac{1}{h_A} + \frac{S_P}{k_P} + \frac{1}{h_B}$$

hence,

$$T_D = \frac{Q}{UAt} \qquad \text{or,} \quad Q = UAt T_D$$

Example. A cubical tank of 2 m sides is constructed of metal plate 12 mm thick and contains water at 75°C. The surrounding air temperature is 16°C. Calculate (i) the overall heat transfer coefficient from water to air, and (ii) the heat loss through each side of the tank per minute. Take the thermal conductivity of the metal as 48 W/m K, the heat transfer coefficient of the water 2·5 kW/m^{2}K, and the heat transfer coefficient of the air 16 W/m^{2}K.

$$\frac{1}{U} = \frac{1}{h_W} + \frac{S_P}{k_P} + \frac{1}{h_A}$$
$$= \frac{1}{2 \cdot 5 \times 10^3} + \frac{0 \cdot 125}{48} + \frac{1}{16}$$
$$= 0 \cdot 0004 + 0 \cdot 00025 + 0 \cdot 0625$$
$$= 0 \cdot 06315$$
$$U = 1/0 \cdot 06315 = 15 \cdot 84 \text{ W/m}^2\text{K} \quad \text{Ans. (i)}$$
$$\begin{aligned}Q &= UAt T_D\\ &= 15 \cdot 84 \times 2^2 \times 60 \times (75 - 16)\\ &= 2 \cdot 243 \times 10^5\\ &= 224 \cdot 3 \text{ kJ} \quad \text{Ans. (ii)}\end{aligned}$$

f HEAT EXCHANGERS. Common examples of heat exchangers in which heat is transferred from one fluid to another are: feed heaters, lubricating oil coolers, gas/air heaters, etc.

To determine the amount of heat transferred in a heat exchanger it is necessary to know the overall heat transfer coefficient U for the wall and fluid boundary layers and also the *logarithmic mean temperature difference*.

$$\text{Heat transferred } Q = UAt\,\theta_m \text{ [J]}$$

PARALLEL FLOW   COUNTER FLOW
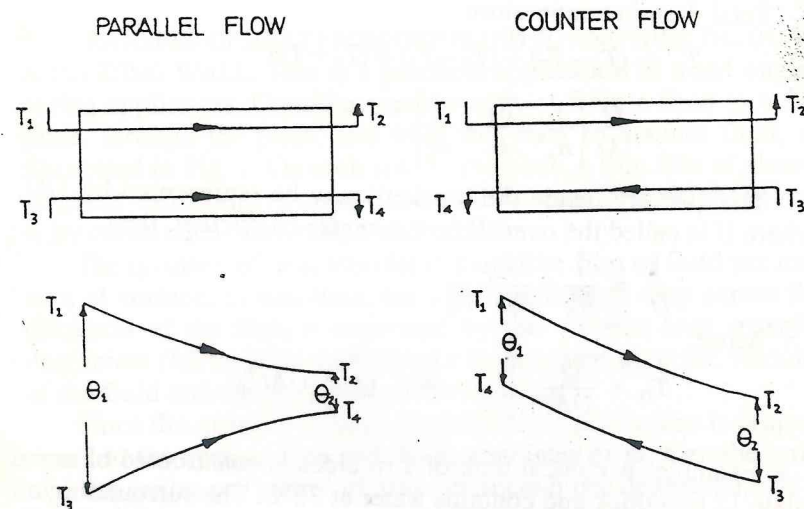
Fig 8

For both heat exchangers shown in Fig. 8:

$$\theta_m = \frac{\theta_1 - \theta_2}{\ln\left(\dfrac{\theta_1}{\theta_2}\right)}$$

Where,

θ_1 = the temperature difference between hot and cold fluid at inlet

θ_2 = the temperature difference between hot and cold fluid at outlet.

θ_m = mean temperature difference between the fluids, usually, called the *logarithmic mean temperature difference*.

f Example. A counter-flow cooler is required to cool 0·5 kg/s of oil from 50°C to 20°C with a cooling water inlet temperature at

10°C and mass flow rate of 0·375 kg/s. There are 200 thin walled tubes of diameter 12·5 mm. Calculate the length of the tube required.

Overall heat transfer coefficient: 70 W/m²K

Specific heat capacities: oil 1400 J/kgK, water 4200 J/kgK

Let T_4 be water outlet temperature.

$$Q = 0.5 \times 1400\,(50 - 20) = 0.375 \times 4200\,(T_4 - 10) = 21\,000 \text{ W}$$

$$T_4 = 13.33°C$$

$$= \frac{\theta_1 - \theta_2}{\ln\left(\dfrac{\theta_1}{\theta_2}\right)}$$

$$= \frac{(T_1 - T_4) - (T_2 - T_3)}{\ln\left(\dfrac{T_1 - T_4}{T_2 - T_3}\right)}$$

$$= \frac{(50 - 13.33) - (20 - 10)}{\ln\left(\dfrac{50 - 13.33}{20 - 10}\right)}$$

$$= \frac{26.67}{1.299}$$

$$= 20.53°C$$

$$Q = UAt\,\theta_m$$

$$21\,000 = 70(\pi \times 0.0125 \times l \times 200) \times 1 \times 20.53$$

$$l = 1.86 \text{ m} \quad \text{Ans.}$$

TEST EXAMPLES 4

1. Calculate the quantity of heat conducted per minute through a duralumin circular disc 127 mm diameter and 19 mm thick when the temperature drop across the thickness of the plate is 5°C. Take the thermal conductivity of duralumin as 150 W/m K.

2. A cold storage compartment is 4·5 m long by 4 m wide by 2·5 m high. The four walls, ceiling and floor are covered to a thickness of 150 mm with insulating material which has a thermal conductivity of $5·8 \times 10^{-2}$ W/m K. Calculate the quantity of heat leaking through the insulation per hour when the outside and inside face temperatures of the material is 15°C and −5°C respectively.

3. One side of a refrigerated cold chamber is 6 m long by 3·7 m high and consists of 168 mm thickness of cork between outer and inner walls of wood. The outer wood wall is 30 mm thick and its outside face temperature is 20°C, the inner wood wall is 35 mm thick and its inside face temperature is −3°C. Taking the thermal conductivity of cork and wood as 0·042 and 0·2 W/m K respectively, calculate (i) the heat transfer per second per square metre of surface area, (ii) the total heat transfer through the chamber side per hour (iii) the interface temperatures.

4. A flat circular plate is 500 mm diameter. Calculate the theoretical quantity of heat radiated per hour when its temperature is 215°C and the temperature of its surrounds is 45°C. Take the value of the radiation constant as $5·67 \times 10^{-11}$ kJ/m²s K⁴.

5. Hot gases at 280°C flow on one side of a metal plate of 10 mm thickness and air at 35°C flows on the other side. The surface heat transfer coefficient of the gases is 31·5 W/m² K and that of air is 32 W/m² K. The thermal conductivity of the metal plate is 50 W/m K. Calculate (i) the overall heat transfer coefficient, and (ii) the heat transfer from gases to air per minute per square metre of plate area.

6. The wall of a cold room consists of a layer of cork sandwiched between outer and inner walls of wood, the wood walls being each 30 mm thick. The inside atmosphere of the room is maintained at −20°C when the external atmospheric temperature is 25°C, and the heat loss through the wall is 42 W/m². Taking the thermal conductivity of wood and cork as 0·2 W/m K and 0·05 W/m K respectively, and the surface heat transfer coefficient between each exposed wood surface and their respective atmospheres as 15 W/m² K, calculate (i) the temperatures of the exposed surfaces, (ii) the

temperatures of the interfaces, and (iii) the thickness of the cork.

ƒ 7. The steam drum of a water-tube boiler has hemispherical ends, the diameter is 1·22 m and the overall length is 6 m. Under steaming conditions the temperature of the shell before lagging was 230°C and the temperature of the surrounds was 51°C. The temperature of the cleading after lagging was 69°C and the surrounds 27°C. Assuming 75% of the total shell area to be lagged and taking the radiation constant as $5·67 \times 10^{-11}$ kJ/m²s K⁴, estimate the saving in heat energy per hour due to lagging.

ƒ 8. A pipe 200 mm outside diameter and 20 m length is covered with a layer, 70 mm thick, of insulation having a thermal conductivity of 0·05 W/m K and a surface heat transfer coefficient of 10 W/m² K at the outer surface.

If the temperature of the pipe is 350°C and the ambient temperature is 15°C calculate:
(i)   the external surface temperature of the lagging
(ii) the heat flow rate from the pipe

Heat flow rate through the lagging per metre length of pipe is
$$\frac{2\pi k T}{\ln(D/d)}$$

where k is the thermal conductivity of the insulation
       T is the temperature difference across the insulation
       D and d are the external and internal diameters of the insulation respectively.

ƒ 9. In an inert gas system the boiler exhaust is cooled from 410°C to 130°C in a parallel flow heat exchanger. Gas flow rate is 0·4 kg/s, cooling water flow rate is 0·5 kg/s, cooling water inlet temperature 10°C. Overall heat transfer coefficient from the gas to the water is 140 W/m² K. Determine the cooling surface area required.

Take c_p for exhaust gas as 1130 J/kg K and c_p for water as 4190 J/kg K.

Note: logarithmic mean temperature difference $\theta_m = \dfrac{\theta_1 - \theta_2}{\ln\left(\dfrac{\theta_1}{\theta_2}\right)}$

Where θ_1 = temperature difference between hot and cold fluid at inlet.
Where θ_2 = temperature difference between hot and cold fluid at outlet.

ƒ 10. Steam at 0·07 bar condenses on the outside of a thin tube 0·025 m diameter and 2·75 m long. Cooling water of density 1000 kg/m³, c_p 4·18 kJ/kg K, flows through the tube at 0·6 m/s rising in temperature from 12°C to 24°C. The surface heat transfer coefficient between steam and tube is 17000 W/m² K. Determine:
>   (a) the overall heat transfer coefficient
>   (b) the surface heat transfer coefficient between water and tube.

CHAPTER 5

LAWS OF PERFECT GASES

When a substance has been evaporated it can exist as a gas or vapour and one of its most important characteristics is its elastic property. For instance, if a certain volume of a liquid is put into a vessel of large volume, the liquid will only partially fill the vessel, taking up no more nor less volume than it did before, but when a gas enters a vessel it immediately fills up every part of that vessel no matter how large it is. Practically speaking, liquids cannot be compressed nor expanded, but gases can be compressed into smaller volumes and expanded to larger volumes.

A perfect gas is a theoretically ideal gas which strictly follows Boyle's and Charles' laws of gases.

Consider a given mass of a perfect gas enclosed in a cylinder by a gas-tight movable piston. When the piston is pushed inward, the gas is compressed to a small volume, when pulled outward the gas is expanded to a larger volume. However, not only is there a change in volume but the pressure and temperature also change. These three quantities, pressure, volume and temperature, are related to each other, and to determine their relationship it is usual to perform experiments with each one of these quantities in turn kept constant while observing the relationship between the other two.

In such basic laws, the pressure, the volume and temperature must all be the absolute values, that is, measured from absolute zero, and not measured from some artificial level. Absolute values were explained in Chapter 1, a brief reminder will suffice:

ABSOLUTE PRESSURE (p) is the pressure measured above a perfect vacuum, obtained by adding the atmospheric pressure to the gauge pressure

"Absolute" need not follow the given pressure, it is to be taken as such unless the pressure is distinctly marked "gauge" to indicate that it is a pressure gauge reading.

VOLUME (V). The volume of the gas is equal to the full volume of the vessel containing it. Note that in the case of the cylinder of an air compressor or reciprocating engine, the total volume of air or gas includes the volume of the clearance space between the piston at its top dead centre and the cylinder cover, as well as the piston swept volume.

ABSOLUTE TEMPERATURE (T). This is the temperature in degrees kelvin measured above the absolute zero of temperature.

$$T[K] = \theta[°C] + 273$$

BOYLE'S LAW

Boyle's law states that the absolute pressure of fixed mass of a perfect gas varies inversely as its volume if the temperature remains unchanged.

$$p \propto \frac{1}{V} \qquad \text{therefore } p \times V = \text{ a constant.}$$

Hence, $p_1 \times V_1 = p_2 \times V_2$

To illustrate this, imagine 2 m³ of gas at a pressure of 100 kN/m² (= 10^5 N/m² = 1 bar) contained in a cylinder with a gas-tight movable piston as illustrated in Fig 9. When the piston is pushed inward the pressure will increase as the gas is compressed to a smaller volume and, provided the temperature remains unchanged, the product of pressure and volume will be a constant quantity for all positions of the piston. From the known initial conditions the constant is calculated:

$$p_1 \times V_1 = \text{ constant}$$

$$100 \times 2 = 200$$

and the pressure at any other volume can be determined:

When the volume is 1·5 m³,

$$p_2 \times 1·5 = 200$$

$$p_2 = 133·3 \text{ kN/m}^2$$

When the volume is 1 m³,

$$p_3 \times 1 = 200$$

$$p_3 = 200 \text{ kN/m}^2$$

When the volume is 0·5 m³,

$$p_4 \times 0·5 = 200$$

$$p_4 = 400 \text{ kN/m}^2$$

And so on.
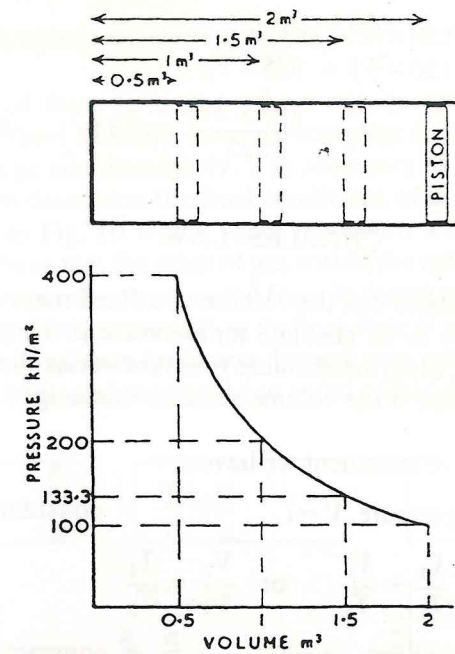
Fig. 9

The variation of pressure with change of volume is shown in the graph below the cylinder in Fig. 9. The graph produced by joining up the plotted points is a rectangular hyperbola, consequently we refer to compression or expansion where pV = constant as hyperbolic compression or hyperbolic expansion. When the temperature is constant as in this example, the operation may also be termed "isothermal".

Note that as the ordinates (vertical measurements) represent pressure, and the abscissae (horizontal measurements) represent volume, and since the product of pressure and volume is constant, then all rectangles drawn from the axes with their corners touching the curve, will be of equal area.

Example. 3·5 m³ of air at a pressure of 20 kN/m² gauge is compressed at constant temperature to a pressure of 425 kN/m² gauge. Taking the atmospheric pressure as 100 kN/m² calculate the final volume of air.

Initial absolute pressure = 20 + 100 = 120 kN/m²
Final absolute pressure = 425 + 100 = 525 kN/m²

$$p_1 V_1 = p_2 V_2$$
$$120 \times 3 \cdot 5 = 525 \times V_2$$
$$V_2 = \frac{120 \times 3 \cdot 5}{525} = 0 \cdot 8 \text{ m}^3 \quad \text{Ans.}$$

CHARLES' LAW

Charles' law states that the volume of a fixed mass of a perfect gas varies directly as its absolute temperature if the pressure remains unchanged, also, the absolute pressure varies directly as the absolute temperature if the volume remains unchanged.

From the above statement we have:

For constant pressure, $V \propto T$ $\quad \therefore \dfrac{V}{T} = \text{constant}$

hence, $\quad \dfrac{V_1}{T_1} = \dfrac{V_2}{T_2}$ or $\dfrac{V_1}{V_2} = \dfrac{T_1}{T_2}$

For constant volume, $p \propto T$ $\quad \therefore \dfrac{p}{T} = \text{constant}$

hence, $\quad \dfrac{p_1}{T_1} = \dfrac{p_2}{T_2}$ or $\dfrac{p_1}{p_2} = \dfrac{T_1}{T_2}$

Example. The pressure of the air in a starting air vessel is 40 bar ($= 40 \times 10^5 \text{N/m}^2$ or 4 MN/m^2) and the temperature is 24°C. If a fire in the vicinity causes the temperature to rise to 65°C, find the pressure of the air. Neglect any increase in volume of the vessel.

As the term "gauge" does not follow the given pressure, it is assumed that this is the initial absolute pressure.

Initial absolute temperature = 24°C + 273 = 297 K
Final absolute temperature = 65°C + 273 = 338 K

$$\frac{p_1}{T_1} = \frac{p_2}{T_2} \quad \therefore p_2 = \frac{p_1 T_2}{T_1}$$

$$p_2 = \frac{40 \times 338}{297} \quad = 45 \cdot 52 \text{ bar}$$

$$= 45 \cdot 52 \times 10^5 \text{ N/m}^2 \text{ or } 4 \cdot 552 \text{ MN/m}^2 \quad \text{Ans.}$$

COMBINATION OF BOYLE'S AND CHARLES' LAWS

Each one of these laws states how one quantity varies with another if the third quantity remains unchanged, but if the three quantities change simultaneously, it is necessary to combine these laws in order to determine the final conditions of the gas.

Referring to Fig. 10 which again represents a cylinder with a piston, gas-tight so that the mass of gas within the cylinder is always the same. Let the gas be compressed from its initial state of pressure p_1 volume V_1 and temperature T_1 to its final state of $p_2 V_2$ and T_2, but to arrive at the final state let it pass through two stages, the first to satisfy Boyle's law and the second to satisfy Charles' law.
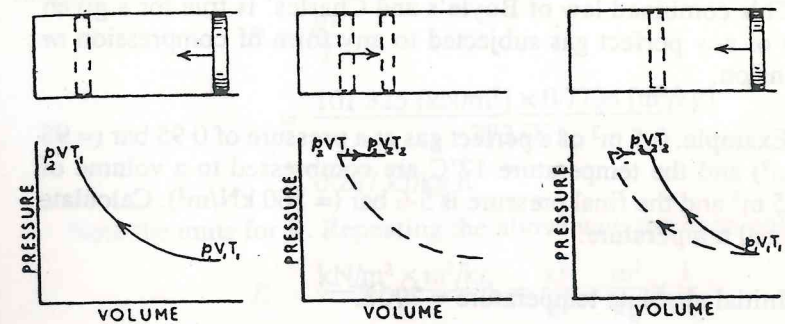
Fig. 10

Imagine the piston pushed inward to compress the gas until it reaches the final pressure of p_2 and let its volume then be represented by V. Normally the temperature would tend to increase due to the work done in compressing the gas, but any heat so generated must be taken away from it during compression so that its temperature remains unchanged at T_1 hence following Boyle's law:

$$p_1 V_1 = p_2 V \qquad \text{(i)}$$

Now apply heat to raise the temperature from T_1 to T_2 and at the same time draw the piston outward to prevent a rise of pressure and keep it constant at p_2. The volume will increase in direct proportion to the increase in absolute temperature according to Charles' law:

$$\frac{V_2}{V} = \frac{T_2}{T_1} \qquad \text{(ii)}$$

By substituting the value of V from (ii) into (i) this quantity will be eliminated:

From (ii) $$V = \frac{V_2 T_1}{T_2}$$

Substituting into (i)

$$p_1 V_1 = p_2 \times \frac{V_2 T_1}{T_2}$$

$$\therefore \frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$$

This combined law of Boyle's and Charles' is true for a given mass of any perfect gas subjected to any form of compression or expansion.

Example. 0.5 m^3 of a perfect gas at a pressure of 0.95 bar (= 95 kN/m^2) and the temperature $17°C$ are compressed to a volume of 0.125 m^3 and the final pressure is 5.6 bar (= 560 kN/m^2). Calculate the final temperature.

Initial absolute temperature = 290 K

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$$

$$\frac{95 \times 0.5}{290} = \frac{560 \times 0.125}{T_2}$$

$$T_2 = \frac{560 \times 0.125 \times 290}{95 \times 0.5}$$

$$= 427.4 \text{ K}$$

$$= 154.4°C \text{ Ans.}$$

CHARACTERISTIC EQUATION

Since pV/T is a constant, its value can be determined for a given mass of any perfect gas. To form a basis on which to work, the constant is calculated on the specific volume (v) of the gas, that is, the volume in cubic metres occupied by a mass of one kilogramme. The constant so obtained is termed the *gas constant*, it is represented

by R and is different for all different gases:

$$\frac{pv}{T} = R \quad \text{or} \quad pv = RT$$

The volume occupied by m kg of mass being represented by V then:

$$pV = mRT$$

This expression is called the *characteristic equation of a perfect gas*.

Taking air as an example, experiments show that at standard atmospheric pressure and temperature, that is, at 101.325 kN/m^2 and $0°C$, the specific volume is $0.7734 \text{ m}^3/\text{kg}$. Inserting these values to find R:

$$R = \frac{pv}{T}$$

$$= \frac{101.325 \, [\text{kN/m}^2] \times 0.7734 \, [\text{m}^3/\text{kg}]}{273 \, [\text{K}]}$$

$$= 0.287 \text{ kJ/kg K}$$

Note the units for R. Repeating the above with the units only:

$$R = \frac{\text{kN/m}^2 \times \text{m}^3/\text{kg}}{\text{K}} = \frac{\text{kN}}{\text{m}^2} \times \frac{\text{m}^3}{\text{kg}} \times \frac{1}{\text{K}}$$

$$= \frac{\text{kN m}}{\text{kg K}} = \text{kN m/kg K} = \text{kJ/kg K}$$

Example. An air compressor delivers 0.2 m^3 of air at a pressure of 850 kN/m^2 and $31°C$ into an air reservoir. Taking the gas constant for air as 0.287 kJ/kg K, calculate the mass of air delivered.

$$pV = mRT$$

$$m = \frac{pV}{RT}$$

$$= \frac{850 \times 0.2}{0.287 \times 304} = 1.948 \text{ kg Ans.}$$

f UNIVERSAL GAS CONSTANT

The *kilogramme-mol* of a substance is a mass of that substance

numerically equal to its molecular weight. The kilogramme-mol may simply be abbreviated to *mol*.

As examples, the molecular weight of oxygen is 32, therefore the mass of one mol of oxygen is 32 kg. The molecular weight of hydrogen is 2 hence one mol of hydrogen is a mass of 2 kg. And so on.

ƒ AVOGADRO'S LAW states that under equal conditions of temperature and pressure, equal volumes of all gases contain the same number of molecules.

Consider two gases, one of mass m_1 of molecular weight M_1 and containing n_1 molecules, the other of mass m_2, of molecular weight M_2 and containing n_2 molecules:

$$\text{Ratio of masses} = \frac{m_1}{m_2} = \frac{n_1 M_1}{n_2 M_2}$$

For equal volumes of the gases at the same temperature and pressure, the gases contain an equal number of molecules, therefore $n_1 = n_2$ and cancel:

$$\frac{m_1}{m_2} = \frac{M_1}{M_2}$$

From the characteristic gas equation $pV = mRT$, substituting for $m = pV/RT$:

$$\frac{p_1 V_1 / R_1 T_1}{p_2 V_2 / R_2 T_2} = \frac{M_1}{M_2} \qquad \therefore \frac{p_1 V_1 R_2 T_2}{p_2 V_2 R_1 T_1} = \frac{M_1}{M_2}$$

pV and T cancel because they are equal, therefore,

$$\frac{R_2}{R_1} = \frac{M_1}{M_2} \quad \text{or} \quad R_1 M_1 = R_2 M_2$$

Hence the product of the gas constant R and the molecular weight M is the same for all gases. This product is termed the *Universal Gas Constant*, it is represented by R_0 and its value has been found by experiment to be 8·314 kJ/mol K.

$$R_0 = RM$$

Any particular gas constant R can therefore be found if its molecular weight is known.

Thus, the molecular weight of nitrogen is 28, therefore the gas constant for nitrogen is:

$$R = \frac{R_0}{M} = \frac{8·314}{28} = 0·2969 \text{ kJ/kg K}$$

ƒ DALTON'S LAW OF PARTIAL PRESSURES

This law states that the pressure exerted in a vessel by a mixture of gases is equal to the sum of the pressures that each separate gas would exert if it alone occupied the whole volume of the vessel.

The pressure exerted by each gas is termed a partial pressure:

Total pressure of the mixture

$$= \begin{array}{c} \text{partial press.} \\ \text{due to gas}_1 \end{array} + \begin{array}{c} \text{partial press.} \\ \text{due to gas}_2 \end{array} + \begin{array}{c} \text{partial press.} \\ \text{due to gas}_3 \end{array} + \text{etc.}$$

$$p = p_1 + p_2 + p_3 + \text{etc.}$$

From the characteristic gas equation $pV = mRT$, substituting for the pressure of each gas, and also for the mixture:

$$\frac{mRT}{V} = \frac{m_1 R_1 T_1}{V_1} + \frac{m_2 R_2 T_2}{V_2} + \frac{m_3 R_3 T_3}{V_3} + \text{etc.}$$

Since it can be considered as though each gas on its own occupied the whole space then the volume V is common throughout. The temperature is also common, therefore V and T being common to all terms will cancel:

$$\frac{mRT}{V} = \frac{T}{V} \left\{ m_1 R_1 + m_2 R_2 + m_3 R_3 + \text{etc.} \right\}$$

$$mR = m_1 R_1 + m_2 R_2 + m_3 R_3 + \text{etc.}$$

where m is the total mass of the mixture and R is the gas constant of the mixture.

ƒ Example. The analysis by mass of a sample of air is 23·14% oxygen, 75·53% nitrogen, 1·28% argon and 0·05% carbon dioxide. Estimate the gas constant for air (to the nearest four figures) taking the molecular weights of O_2, N_2, Ar and CO_2 as 32, 28, 40 and 44 respectively, and the universal gas constant $R_0 = 8·314$ kJ/mol K.

$$R = \frac{R_0}{M}$$

Considering 1 kg of air:

Oxygen $R_1 = \frac{8 \cdot 314}{32} = 0 \cdot 2598$

$m_1 R_1 = 0 \cdot 2314 \times 0 \cdot 2598 = 0 \cdot 06012$

Nitrogen $R_2 = \frac{8 \cdot 314}{28} = 0 \cdot 2969$

$m_2 R_2 = 0 \cdot 7553 \times 0 \cdot 2969 = 0 \cdot 2242$

Argon $R_3 = \frac{8 \cdot 314}{40} = 0 \cdot 2078$

$m_3 R_3 = 0 \cdot 0128 \times 0 \cdot 2078 = 0 \cdot 002661$

Carbon dioxide $R_4 = \frac{8 \cdot 314}{44} = 0 \cdot 1889$

$m_4 R_4 = 0 \cdot 0005 \times 0 \cdot 1889 = 0 \cdot 00009445$

$$\Sigma mR = 0 \cdot 28707545$$

$$R \text{ air} = \frac{0 \cdot 2871}{1} = 0 \cdot 2871 \text{ kJ/kg K} \quad \text{Ans.}$$

f **PARTIAL VOLUMES.** If each gas of a mixture in a closed vessel at the full volume of V and partial pressure $p_1(p_2,$ etc.), is considered as being compressed until its pressure is the full pressure p and its volume is its partial volume V_1 (V_2, etc.), then, since its temperature remains the same:

$$p_1 \times V = p \times V_1 \quad \text{and} \quad p_2 \times V = p \times V_2 \text{ etc.}$$
$$\therefore (p_1 + p_2 + \text{etc.}) \times V = p \times (V_1 + V_2 + \text{etc.})$$

hence, the total volume of the mixture is equal to the sum of the partial volumes of the gases at the same total pressure and temperature.

Also, $\frac{p_1}{p_2} = \frac{V_1}{V_2}$ therefore:

the ratio of the partial volumes is equal to the ratio of the partial pressures.

Dalton's law is also applied for the determination of the quantity of air-leakage into steam condensers. An example is given in Chapter 10 after the study of properties of steam.

SPECIFIC HEATS OF GASES

The specific heat (c) of a substance is defined as the quantity of heat energy required to be transferred to unit mass of that substance to raise its temperature by one degree. Hence, the quantity (Q) of heat in kilojoules required to be given to a mass of m kilogrammes of the substance to raise its temperature from T_1 to T_2 is:

$$Q[kJ] = m[kg] \times c[kJ/kgK] \times (T_2 - T_1) \ [K]$$

Since the characteristics of gases vary considerably at different temperatures and pressures, heat energy may be transferred under an infinite number of conditions, therefore the specific heat can have an infinite number of different values. Consider, however, two important conditions under which heat may be transferred, (i) while the *volume* of the gas remains constant, (ii) while the *pressure* of the gas remains constant. The specific heat of a gas at constant volume is represented by c_V.

The specific heat of a gas at constant pressure is represented by c_P. This is a higher value than c_V because, when the gas is receiving heat it must be allowed to expand in volume to prevent a rise in pressure and, whilst expanding, the gas is expending energy in doing external work, hence extra heat energy must be supplied equivalent to the external work done.

Example. A quantity of air of mass 0·23 kg, pressure 100 kN/m², volume 0·1934 m³ and temperature 20°C is enclosed in a cylinder with a gas-tight movable piston, and heat energy is transferred to the air to raise its temperature to 142°C.

(a) If the piston is prevented from moving during heat transfer so that the volume of the air remains unchanged, calculate (i) the heat supplied, taking the specific heat at constant volume $c_V = 0·718$ kJ/kg K, and (ii) the final pressure.

(b) If the piston moves to allow the air to expand in volume at such a rate as to keep the pressure constant, calculate (i) the heat supplied, taking the specific heat at constant pressure $c_P = 1·005$ kJ/kg K, and (ii) the final volume.

Initial absolute temperature = 293 K
Final absolute temperature = 415 K

Temperature rise = 142 − 20 = 122°C = 122 K

(a) $Q = m \times c_V \times (T_2 - T_1)$
$= 0 \cdot 23 \times 0 \cdot 718 \times 122$

$$= 20.15 \text{ kJ} \quad \text{Ans. (a)(i)}$$

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$$

The volume is constant $\therefore V_2 = V_1$ and cancels,

$$p_2 = \frac{p_1 T_2}{T_1} \quad \text{(which is Charles' law)}$$

$$= \frac{100 \times 415}{293} = 141.6 \text{ kN/m}^2 \quad \text{Ans. (a)(ii)}$$

(b)
$$Q = m \times c_P \times (T_2 - T_1)$$
$$= 0.23 \times 1.005 \times 122$$
$$= 28.19 \text{ kJ} \quad \text{Ans. (b)(i)}$$

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$$

The pressure is constant $\therefore p_1 = p_2$ and cancels

$$V_2 = \frac{V_1 T_2}{T_1} \quad \text{(which is Charles' law)}$$

$$= \frac{0.1934 \times 415}{293} = 0.2738 \text{ m}^3 \quad \text{Ans. (b)(ii)}$$

Note the difference between the quantity of heat energy supplied to the air in the two cases:

$$28.19 - 20.15 = 8.04 \text{ kJ}$$

This amount of heat energy was expended in moving the piston:

Referring to Fig. 11, the piston is pushed forward by the gas at a constant pressure of, say $p[\text{kN/m}^2]$. Let $A[\text{m}^2]$ represent the area of the piston, then the total force on the piston is $p \times A [\text{kN}]$. If the piston moves $S[\text{metres}]$, the work done, being the product of force and distance, is $p \times A \times S [\text{kN m} = \text{kJ}]$. The product of the area A and
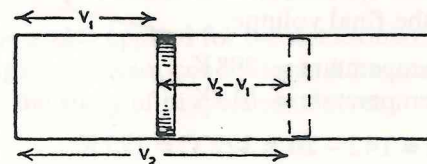

Fig 11

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the distance moved S is the volume swept through by the piston, hence,

$$\text{Work done} = p(V_2 - V_1)$$

In the last example, when the pressure was constant at 100 kN/m^2 the volume of the air increased from 0.1934 m^3 to 0.2738 m^3 so that the piston swept volume is the difference between these.

$$\begin{aligned}
\text{Work done} &= p(V_2 - V_1) \\
&= 100(0.2738 - 0.1934) \\
&= 100 \times 0.0804 \\
&= 8.04 \text{ kJ}
\end{aligned}$$

Showing that the extra heat energy given to the air at constant pressure compared with that at constant volume is the heat energy expended in doing external work by pushing the piston forward.

ENERGY EQUATION (CLOSED SYSTEMS)

The *internal energy* (U) of a gas is the energy contained in it as stored up work by virtue of the movement of its molecules.

JOULE'S LAW states that the internal energy of a gas depends only upon its temperature and is independent of changes in pressure and volume.

By the principle of the conservation of energy, that is, that energy can neither be created nor destroyed, it follows that the total heat energy (Q) transferred to a gas will be the sum of the increase in its internal energy ($U_2 - U_1$) and any work (W) that is done by the gas during the transfer of heat energy to it, thus:

Heat energy transferred to the gas	=	Increase in internal energy of the gas	+	External work done by the gas

$$Q = (U_2 - U_1) + W$$

This is known as the *First Law of Thermodynamics*.

Any of these three terms may be negative i.e. heat rejected (extracted), decrease of internal energy, work done on the gas (compression).

These are non-flow (reversible) processes for perfect gases.

RELATIONSHIP BETWEEN SPECIFIC HEATS. Referring again to the last example, with Fig. 11, and applying the energy equation:

In the first case, heat energy is supplied to the gas to raise its temperature from T_1 to T_2 at *constant volume*, the heat supplied is $mc_V(T_2 - T_1)$ but no work is done because, as the volume is constant, the piston does not move.

$$\text{Heat supplied} = \begin{matrix} \text{Increase in} \\ \text{internal energy} \end{matrix} + \begin{matrix} \text{External} \\ \text{work done} \end{matrix}$$

$$mc_V(T_2 - T_1) = (U_2 - U_1) + 0 \qquad \text{(i)}$$

In the second case, heat energy is supplied to raise the temperature through the same range, from T_1 to T_2 at *constant pressure*. The heat supplied is $mc_P(T_2 - T_1)$ and external work is done equal to $p(V_2 - V_1)$. Expressing the work done in terms of temperature by substituting the value of V from the characteristic gas equation, and inserting into the energy equation,

$$p(V_2 - V_1) = p\left\{\frac{mRT_2}{p} - \frac{mRT_1}{p}\right\} = mR(T_2 - T_1)$$

$$\text{Heat supplied} = \begin{matrix} \text{Increase in} \\ \text{internal energy} \end{matrix} + \begin{matrix} \text{External} \\ \text{work done} \end{matrix}$$

$$mc_P(T_2 - T_1) = (U_2 - U_1) + mR(T_2 - T_1)$$
$$U_2 - U_1 = mc_P(T_2 - T_1) - mR(T_2 - T_1) \qquad \text{(ii)}$$

Since the temperature change is the same in each case then the change in internal energy is the same, hence, from (i) and (ii):

$$mc_V(T_2 - T_1) = mc_P(T_2 - T_1) - mR(T_2 - T_1)$$

m and $(T_2 - T_1)$ are common to all terms and cancel:

$$c_V = c_P - R$$
$$\therefore R = c_P - c_V$$

Inserting the values for air as previously given,

$$R = 1\cdot005 - 0\cdot718 = 0\cdot287 \text{ kJ/kg K}$$

RATIO OF SPECIFIC HEATS. Another important relationship between the specific heats is the ratio c_P/c_V, the symbol denoting this ratio is γ:

$$\gamma = \frac{c_P}{c_V}$$

and for air this ratio is:

$$\gamma = \frac{1\cdot005}{0\cdot718} = 1\cdot4$$

Taking this further:

$$\gamma = \frac{c_P}{c_V} \qquad \therefore c_P = \gamma c_V$$

substituting this into the relationship:

$$R = c_P - c_V$$
$$= \gamma c_V - c_V$$
$$\therefore R = c_V(\gamma - 1)$$

$$\text{or } c_V = \frac{R}{\gamma - 1}$$

These expressions will be found useful in later calculations.

ENTHALPY

The enthalpy (H) of any fluid (liquid, vapour, gas) is a convenient grouping of terms representing (in open systems for reversible steady-flow processes) the sum of two kinds of energy transfer, i.e. internal energy (U) and work transfer (pV).

$$H = U + pV \quad \text{(J)}$$

$$h = u + pv \quad \text{(J/kg) specific enthalpy}$$

ENERGY EQUATION (OPEN SYSTEMS).

Consider mass of fluid m_1, volume V_1, entering system.
Work done against resistance $= p_1V_1$.
Consider mass of fluid m_2, volume V_2, leaving system.
Work done against resistance $= p_2V_2$.
If $m_1 = m_2$ (steady flow process, mass of fluid within system constant)
$$\therefore p_1V_1 = p_2V_2.$$
This is a work, or energy, transfer through the system.
Referring to Fig 12, the mass of fluid within the boundary is constant, *i.e.* the mass entering equals the mass leaving.
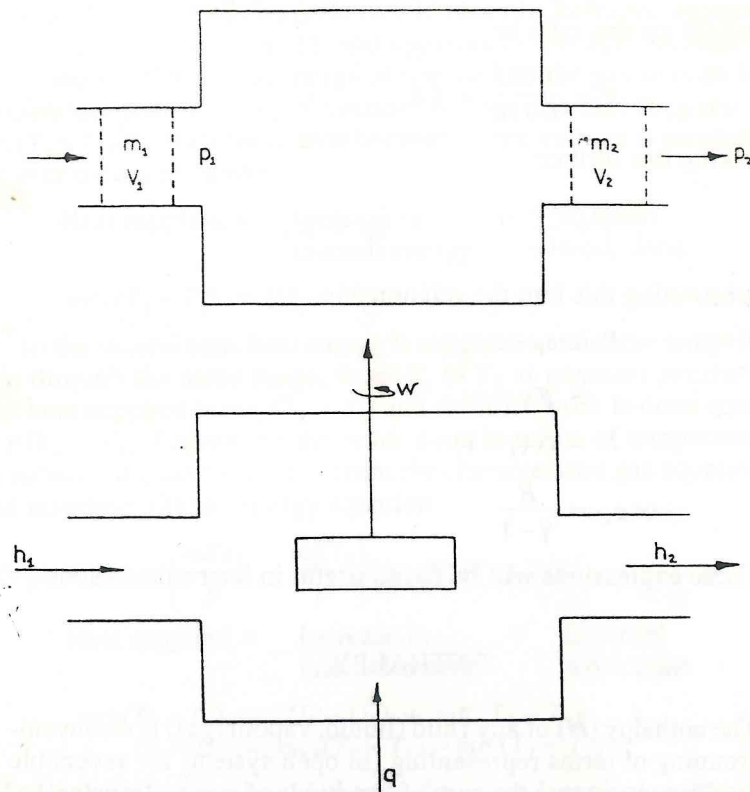
Fig. 12

Let h_1 and h_2 = specific enthalpy of fluid entering and leaving the system respectively (kJ/kg).

Let c_1 and c_2 = velocity of fluid entering and leaving the system respectively (m/s)

Let q = quantity of heat per unit mass crossing the boundary into the system (kJ/kg)

Let w = external work done per unit mass by the fluid (kJ/kg)

Assume no change in potential energy, then:

Kinetic Energy$_1$ + q = h_2 + Kinetic Energy$_2$ + w.

i.e. $h_1 + \tfrac{1}{2}c_1^2 + q = h_2 + \tfrac{1}{2}c_2^2 + w$

This is known as the Steady Flow Energy Equation.

Example. The working fluid passes through a gas turbine at a steady rate of 10 kg/s. It enters with a velocity of 100 m/s and specific enthalpy of 2000 kJ/kg and leaves at 50 m/s with a specific enthalpy of 1500 kJ/kg. If the heat lost to surroundings, as the fluid passes through the turbine, is 40 kJ/kg calculate the power developed.

Steady flow energy equation:

$h_1 + \tfrac{1}{2}c_1^2 + q = h_2 + \tfrac{1}{2}c_2^2 + w$

$\therefore 2000 \times 10^3 + \tfrac{1}{2} \times 100^2 - 40 \times 10^3 = 1500 \times 10^3 + \tfrac{1}{2} \times 50^2 + w$

(*Note*: q is negative as it is heat crossing the boundary out of the system).

$$460 \times 10^3 + \tfrac{1}{2}(100^2 - 50^2) = w$$
$$4 \cdot 6375 \times 10^5 = w$$

$$\therefore \quad w = 4 \cdot 64 \times 10^5 \text{ J}$$
$$\text{power} = 4 \cdot 64 \times 10^5 \text{ J/s}$$
$$= 4 \cdot 64 \text{ MW}$$

Note: For perfect gases:

$$h_2 - h_1 = (u_2 + p_2 v_2) - (u_1 + p_1 v_1)$$
$$= c_V(T_2 - T_1) + R(T_2 - T_1)$$
$$= c_P(T_2 - T_1)$$

TEST EXAMPLES 5

1. Assuming compression according to the law pV = constant:
(i) Calculate the final volume when 1 m³ of gas at 120 kN/m² is compressed to a pressure of 960 kN/m².
(ii) Calculate the initial volume of gas at a pressure of 1·05 bar which will occupy a volume of 5·6 m³ when it is compressed to a pressure of 42 bar.

2. 0·2 m³ of gas at a pressure of 1350 kN/m² and temperature 177°C is expanded in a cylinder to a volume of 0·9 m³ and pressure 250 kN/m². Calculate the final temperature.

3. A receiver containing 2 m³ of air at 10 bar, 20°C has a relief valve set to operate at 20 bar. If 8% of the air were to leak out, calculate the temperature at which the relief valve would operate.
Note: for air R = 287 J/kg K

4. An air reservoir contains 20 kg of air at 3200 kN/m² gauge and 16°C. Calculate the new pressure and heat energy transfer if the air is heated to 35°C. Neglect any expansion of the reservoir, take R for air = 0·287 kJ/kg K, specific heat at constant volume c_V = 0·718 kJ/kg K, and atmospheric pressure = 100 kN/m².

5. The dimensions of a large room are 12 m by 16·5 m by 4 m. The air is completely changed once every 30 minutes and the temperature is maintained at 21°C. If the temperature of the outside atmosphere is 30°C, calculate the quantity of heat required to be extracted from the supply air per hour, and the equivalent power, taking the density of air at atmospheric pressure and 0°C as 1·293 kg/m³ and the specific heat at constant pressure as 1·005 kJ/kg K.

6. In a steady flow process the working fluid enters and leaves a horizontal system with negligible velocity. The temperature drop from inlet to outlet is 480°C and the heat losses from the system are 10 kJ/kg of fluid. Determine the power output from the system for a fluid flow of 1·7 kg/s.
For fluid c_P = 900 J/kg K.

7. Heat energy is transferred to 1·36 kg of air which causes its temperature to increase from 40°C to 468°C. Calculate, for the two separate cases of heat transfer at (a) constant volume, (b) constant pressure:
(i) the quantity of heat energy transferred,
(ii) the external work done,
(iii) the increase in internal energy

Take c_V and c_P as 0·718 and 1·005 kJ/kg K respectively.

ƒ 8. A closed vessel of 500 cm³ capacity contains a sample of flue gas at 1·015 bar and 20°C. If the analysis of the gas by volume is 10% carbon dioxide, 8% oxygen, and 82% nitrogen, calculate the partial pressure and mass of each constituent in the sample.
R for CO_2, O_2 and N_2 = 0·189, 0·26 and 0·297 kJ/kg K respectively.

ƒ 9. A gas is discharged from a horizontal convergent nozzle at a steady rate of 1 kg/s. Conditions at inlet are 10 bar and 200°C and at exit 5 bar and 100°C. The change in specific internal energy passing through the nozzle is 80 kJ/kg and heat lost to the surrounds is negligible. If the gas enters the nozzle at 50 m/s determine the exit velocity.
For the gas M is 30 and R_0 may be taken as 8·314 kJ/mol K.

ƒ 10. A vessel of volume 0·4 m³ contains 0·45 kg of carbon monoxide and 1 kg of air at 15°C. Calculate:
(a) the total pressure in the vessel:
(b) the partial pressure of the nitrogen.

Note: Atomic mass relationships; carbon 12, oxygen 16, nitrogen 14.
Universal Gas Constant = 8·314 kJ/mol K.
Air is 23% oxygen by mass.

CHAPTER 6
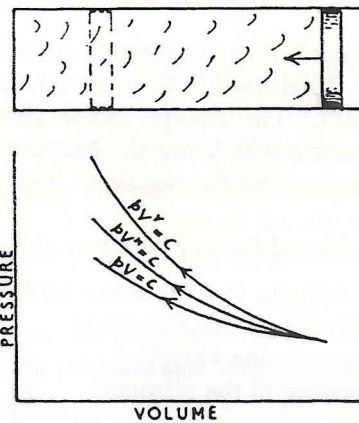
EXPANSION AND COMPRESSION OF PERFECT GASES

Fig. 13

COMPRESSION OF A GAS IN A CLOSED SYSTEM

When a gas is compressed in a cylinder by the inward movement of a gas-tight piston (Fig. 13), the pressure of the gas increases as the volume decreases. The work done *on* the gas to compress it appears as heat energy in the gas and the temperature tends to rise. This effect can readily be seen with a tyre inflator; in pumping up the tyre the discharge end of the inflator gets hot due to compressing the air.

ISOTHERMAL COMPRESSION. Imagine the piston pushed inward slowly to compress the gas and, at the same time, let heat be taken away via the cylinder walls (by a water-jacket or other means) to avoid any rise in temperature. If the gas could be compressed in this manner, *at constant temperature*, the process would be referred to as *isothermal compression* and the relationship between pressure and volume would follow Boyle's law as stated in the previous chapter:

$$pV = \text{constant} \quad \therefore \quad p_1V_1 = p_2V_2$$

ADIABATIC COMPRESSION. Now imagine the piston pushed

inward quickly so that there is insufficient time for any heat energy to be transferred from the gas to the cylinder walls. All the work done in compressing the gas appears as stored up heat energy. The temperature at the end of compression will therefore be high and, for the same ratio of compression as the first case, the pressure will consequently be higher. This form of compression, where no heat energy transfer takes place between the gas and an external source, is known as *adiabatic compression*. The relationship between pressure and volume for adiabatic compression is:

$$pV^\gamma = \text{constant} \quad \therefore \quad p_1V_1{}^\gamma = p_2V_2{}^\gamma$$

where γ (gamma) is the ratio of the specific heat of the gas at constant pressure to the specific heat at constant volume, thus,

$$\gamma = \frac{c_P}{c_V}$$

POLYTROPIC COMPRESSION. In practice, neither isothermal nor adiabatic processes can be achieved perfectly. Some heat energy is always lost from the gas through the cylinder walls, more especially if the cylinder is water cooled, but this is never as much as the whole amount of the generated heat of compression. Consequently, the compression curve representing the relationship between pressure and volume lies somewhere between the two theoretical cases of isothermal and adiabatic. Such compression, where a partial amount of heat energy exchange takes place between the gas and an outside source during the process, is termed *polytropic compression* and the compression curve follows the law:

$$pV^n = \text{constant} \quad \therefore \quad p_1V_1{}^n = p_2V_2{}^n$$

Thus, the law $pV^n = \text{constant}$ may be taken as the general case to cover all forms of compression from isothermal to adiabatic wherein the value of n for isothermal compression is unity, for adiabatic compression n = γ, and for polytropic compression n generally lies somewhere between 1 and γ.

EXPANSION OF A GAS IN A CLOSED SYSTEM

When a gas is expanded in a cylinder (Fig. 14) the pressure falls and the volume increases as the piston is pushed outward by the energy in the gas.
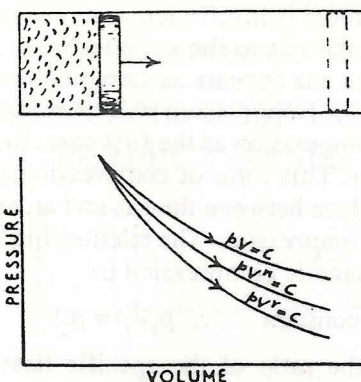
Fig. 14

This is exactly the opposite to compression. Work is done *by* the gas in pushing the piston outward and there is a tendency for the temperature to fall due to the heat energy in the gas being converted into mechanical energy. Therefore to expand the gas *isothermally*, heat energy must be transferred to the gas from an external source during the expansion in order to maintain its temperature constant. The expansion would then follow Boyle's law, pV = constant.

The gas would expand *adiabatically* if no heat energy transfer, to or from the gas, occurs during the expansion, the external work done in pushing the piston forward being entirely at the expense of the stored up heat energy. Therefore the temperature of the gas will fall during the expansion. As for adiabatic compression, the law for adiabatic expansion is pV^γ = constant.

During *polytropic* expansion, a partial amount of heat energy will be transferred to the gas from an outside source but not sufficient to maintain a uniform temperature during the expansion. The law for polytropic expansion is pV^n = constant as it is for polytropic compression.

With reference to Figs. 13 and 14 note that the adiabatic curve is the steepest, the isothermal curve is the least steep, and the polytropic curve lies between the two. Thus, the higher the index of the law of expansion or compression, the steeper will be the curve.

It must also be noted that for any mode of expansion or compression in a closed system, the combination of Boyle's and Charles' laws, and the characteristic gas equation given in the previous chapter, are always true:

$$\frac{pV}{T} = \text{constant} \qquad \therefore \frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$$

$$pV = mRT$$

Example. 0.25 m³ of air at 90 kN/m² and 10°C are compressed in an engine cylinder to a volume of 0.05 m³, the law of compression being $pV^{1.4}$ = constant. Calculate (i) the final pressure, (ii) the final temperature, (iii) the mass of air in the cylinder, taking the characteristic gas constant for air $R = 0.287$ kJ/kg K.

$$p_1 V_1^{1.4} = p_2 V_2^{1.4}$$
$$90 \times 0.25^{1.4} = p_2 \times 0.05^{1.4}$$

$$p_2 = \frac{90 \times 0.25^{1.4}}{0.05^{1.4}}$$

$$= 90 \times 5^{1.4} = 856.7 \text{ kN/m}^2 \quad \text{Ans. (i)}$$

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$$

$$\frac{90 \times 0.25}{283} = \frac{856.7 \times 0.05}{T_2}$$

$$T_2 = \frac{283 \times 856.7 \times 0.05}{90 \times 0.25} = 538.8 \text{ K}$$

$$= 265.8°C \quad \text{Ans. (ii)}$$

$$p_1 V_1 = mRT_1$$

$$m = \frac{90 \times 0.25}{0.287 \times 283} = 0.277 \text{ kg} \quad \text{Ans. (iii)}$$

Example. 0.07 m³ of gas at 4.14 MN/m² is expanded in an engine cylinder and the pressure at the end of expansion is 310 kN/m². If expansion follows the law $pV^{1.35}$ = constant, find the final volume.

$$p_1 V_1^{1.35} = p_2 V_2^{1.35}$$

$$4140 \times 0.07^{1.35} = 310 \times V_2^{1.35}$$

$$V_2 = 0.07 \times \sqrt[1.35]{\frac{4140}{310}} = 0.4774 \text{ m}^3 \quad \text{Ans.}$$

Example. 0.014 m^3 of gas at 3.15 MN/m^2 is expanded in a closed system to a volume of 0.154 m^3 and the final pressure is 120 kN/m^2. If the expansion takes place according to the law pV^n = constant, find the value of n.

$$p_1 V_1^n = p_2 V_2^n$$
$$3150 \times 0.014^n = 120 \times 0.154^n$$
$$\frac{3150}{120} = \left\{ \frac{0.154}{0.014} \right\}^n$$
$$26.25 = 11^n$$
$$n = 1.363 \text{ Ans.}$$

DETERMINATION OF n FROM GRAPH

It will be appreciated that it would be most difficult to obtain two pairs of sufficiently accurate values of pressure and volume from a running engine to enable the law of expansion or compression to be determined as in the previous example. One practical method of finding a fairly close approximation of the law is as follows:

(i) measure a series of connected values of p and V from the curve of an indicator diagram, (ii) reduce the equation $pV^n = C$ to a straight line logarithmic equation, (iii) draw a straight line graph as near as possible through the plotted points of log p and log V to eliminate slight errors of measurement, (iv) determine the law of this graph to obtain the value of n. Thus:

$$p \times V^n = C$$
$$\log p + n \log V = \log C$$
$$\log p = \log C - n \log V$$

This is the same form of equation as,

$$y = a - bx$$

which represents a straight line graph.

The terms log p and log V are the two variables comparable with y and x respectively, and log C and n are constants comparable with a and b respectively. The constant n (like constant b) represents the slope of the straight-line graph and, being a negative value, the line will slope downwards from left to right.

Example. The following related values of the pressure p in kN/m^2 and the volume V in m^3 were measured from the compression curve of an internal combustion engine indicator diagram. Assuming that p and V are connected by the law $pV^n = C$, find the value of n.

p	3450	2350	1725	680	270	130
V	0.0085	0.0113	0.0142	0.0283	0.0566	0.0991

The scales of the graph can be of any convenient choice. In this case both pressure and volume can be expressed in more convenient units, the pressure in bars (1 bar = 10^5 N/m^2 = 10^2 kN/m^2) to
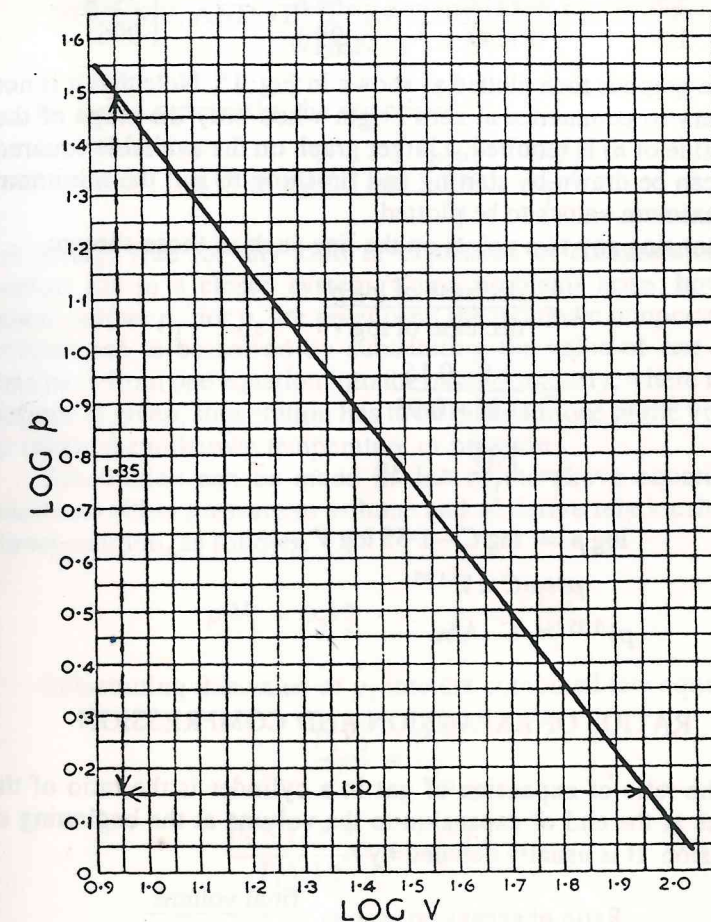

Fig. 15

proportionally reduce the high figures, and the volume in litres (10^3 litres = 1 m³) to express all volumes above unity, thereby avoiding negative logs and so reducing labour. Hence the following tabulated values of p and V are in bars and litres respectively with their corresponding logarithms to obtain graph plotting points from the respective pairs of log p and log V (base 10 used here).

p[bar]	log p	V [litre]	log V
34·5	1·5378	8·5	0·9294
23·5	1·3711	11·3	1·0531
17·25	1·2368	14·2	1·1523
6·8	0·8325	28·3	1·4518
2·7	0·4314	56·6	1·7528
1·3	0·1139	99·1	1·9961

The graph is then plotted as shown in Fig. 15. Note that it is not necessary to commence at zero origin when only the slope of the line (value of n) is required, a larger graph on the available squared paper can be drawn by starting and finishing to suit the minimum and maximum values to be plotted.

Choosing any two points on the line such as those shown:

$$n = \frac{\text{decrease of log } p}{\text{decrease of log } V}$$

$$= \frac{1·5 - 0·15}{1·95 - 0·95}$$

$$= \frac{1·35}{1} = 1·35$$

$$\log p = \log C - 1·35 \log V$$

$$p = C \times V^{-1·35}$$

$$pV^{1·35} = C \quad \text{Ans.}$$

RATIOS OF EXPANSION AND COMPRESSION

The ratio of expansion of gas in a cylinder is the ratio of the volume at the end of expansion to the volume at the beginning of expansion. It is usually denoted by r.

$$\text{Ratio of expansion} = r = \frac{\text{final volume}}{\text{initial volume}}$$

The ratio of compression is the ratio of the volume of the gas at the beginning of compression to the volume at the end of compression. This can also be denoted by r.

$$\text{Ratio of compression} = r = \frac{\text{initial volume}}{\text{final volume}}$$

It will be seen that in each of the above ratios, it is the larger volume divided by the smaller, therefore, the ratio of expansion and ratio of compression is always greater than unity.

RELATIONSHIPS BETWEEN TEMPERATURE AND VOLUME, AND TEMPERATURE AND PRESSURE, WHEN $pV^n = C$

As stated previously, the equations,

$$p_1 V_1^n = p_2 V_2^n \quad \text{and} \quad \frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$$

are always true for any kind of expansion or compression of a perfect gas in a closed system. Some problems arise, however, where neither p_1 nor p_2 are given, and the unknown temperature or volume has to be solved by substituting the value of one of the pressures from one equation into the other. Similarly, where neither volume is given, substitution has to be made for one of the volumes to obtain the unknown temperature or pressure.

Substitution can be made in one of the above equations to eliminate either pressure or volume and so derive relationships for direct solution, as follows.

$$p_1 V_1^n = p_2 V_2^n \qquad \therefore p_1 = \frac{p_2 V_2^n}{V_1^n}$$

Substituting this value of p_1 into the combined law equation:

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$$

$$\frac{p_2 \times V_2^n \times V_1}{T_1 \times V_1^n} = \frac{p_2 V_2}{T_2}$$

$$T_1 \times V_1^n \times p_2 \times V_2 = T_2 \times p_2 \times V_2^n \times V_1$$

$$\frac{T_1}{T_2} = \frac{p_2 \times V_2^n \times V_1}{p_2 \times V_2 \times V_1^n}$$

p_2 cancels,

dividing V_2^n by $V_2 = V_2^n \div V_2 = V_2^{n-1}$

dividing V_1^n by $V_1 = V_1^n \div V_1 = V_1^{n-1}$

$$\frac{T_1}{T_2} = \frac{V_2^{n-1}}{V_1^{n-1}}$$

$$\frac{T_1}{T_2} = \left\{\frac{V_2}{V_1}\right\}^{n-1} \qquad \text{(i)}$$

Again from $p_1 V_1^n = p_2 V_2^n$ $\qquad V_1^n = \dfrac{p_2 V_2^n}{p_1}$

$$\therefore \; V_1 = p_2^{1/n} \times \frac{V_2}{P_1^{1/n}}$$

Substituting this value of V_1 into the combined law equation:

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$$

$$\frac{p_1 \times p_2^{1/n} \times V_2}{T_1 \times p_1^{1/n}} = \frac{p_2 \times V_2}{T_2}$$

$$T_1 \times p_1^{1/n} \times p_2 \times V_2 = T_2 \times p_1 \times P_2^{1/n} \times V_2$$

$$\frac{T_1}{T_2} = \frac{p_1 \times p_2^{1/n} \times V_2}{p_1^{1/n} \times p_2 \times V_2}$$

V_2 cancels,

dividing p_1 by $p_1^{1/n} = p_1 \;\div\; p_1^{1/n} = p_1^{1-1/n}$

dividing p_2 by $p_2^{1/n} = p_2 \;\div\; p_2^{1/n} = p_2^{1-1/n}$

$$\frac{T_1}{T_2} = \frac{p_1^{1-1/n}}{p_2^{1-1/n}}$$

$$\frac{T_1}{T_2} = \left\{\frac{p_1}{p_2}\right\}^{1-1/n}$$

$$\frac{T_1}{T_2} = \left\{\frac{p_1}{p_2}\right\}^{\frac{n-1}{n}} \qquad \text{(ii)}$$

From (i) and (ii) we have the very useful relationship:

$$\frac{T_1}{T_2} = \left\{\frac{V_2}{V_1}\right\}^{n-1} = \left\{\frac{p_1}{p_2}\right\}^{\frac{n-1}{n}}$$

For an adiabatic process, the adiabatic index γ is substituted for the polytropic index n.

Example. Air is expanded adiabatically from a pressure of 800 kN/m² to 128 kN/m². If the final temperature is 57°C, calculate the temperature at the beginning of expansion, taking $\gamma = 1.4$.

$$\frac{T_1}{T_2} = \left\{\frac{p_1}{p_2}\right\}^{\frac{\gamma-1}{\gamma}}$$

$$\frac{T_1}{330} = \left\{\frac{800}{128}\right\}^{2/7}$$

$$T_1 = 330 \times 6.25^{2/7}$$

$$= 557.1 \text{ K}$$

$$= 284.1°\text{C} \quad \text{Ans.}$$

Example. The ratio of compression in a petrol engine is 9 to 1. Find the temperature of the gas at the end of compression if the temperature at the beginning is 24°C, assuming compression to follow the law $pV^n =$ constant, where $n = 1.36$.

$$\frac{T_1}{T_2} = \left\{\frac{V_2}{V_1}\right\}^{n-1} \quad \text{or} \quad \frac{T_2}{T_1} = \left\{\frac{V_1}{V_2}\right\}^{n-1}$$

$$T_2 = 297 \times 9^{0.36} = 655.1 \text{ K}$$

$$= 382.1°\text{C} \quad \text{Ans.}$$

Example. The volume and temperature of a gas at the beginning of expansion are 0.0056 m³ and 183°C, at the end of expansion the values are 0.0238 m³ and 22°C respectively. Assuming expansion follows the law $pV^n = C$, find the value of n.

$$\frac{T_1}{T_2} = \left\{\frac{V_2}{V_1}\right\}^{n-1}$$

$$\frac{456}{295} = \left\{\frac{0\cdot0238}{0\cdot0056}\right\}^{n-1}$$

$$1\cdot546 = 4\cdot25^{n-1}$$

$$n = 1\cdot301 \text{ Ans.}$$

WORK TRANSFER

Firstly, consider a gas expanding at *constant pressure* in a cylinder fitted with a gas-tight piston (also refer to Fig. 11 with notes in Chapter 5).

Work done [kJ = kN m] = force [kN] × distance [m]

The total force [kN] on the piston is the product of the pressure p [kN/m^2] of the gas and the area A [m^2] of the piston and, if the piston moves through a distance of S metres as the gas expands, then,

$$\text{Work done} = p \times A \times S$$

The product of the piston area A and the distance it moves S, is the volume swept through by the piston, this is also the increase in volume of the gas in the cylinder. If V_1 is the volume of the gas at the beginning of the expansion, and V_2 is the volume at the end of expansion, the $A \times S$ is equal to $V_2 - V_1$ cubic metres, hence,

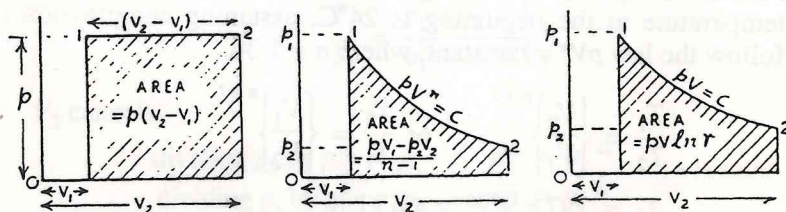$$\text{Work done} = p\,(V_2 - V_1)$$

Fig. 16

Fig. 16 shows the pressure-volume diagram representing work done at constant pressure. The graph is a straight horizontal line and the area under it is a rectangle. The area of a rectangle is height × length which, in this case, is $p \times (V_2 - V_1)$. Hence *the area under the pressure-volume line represents work done.*

Now consider cases where the pressure falls during the expansion of the gas. The formula giving the area under the polytropic curve representing the general relationship between pressure and volume, i.e. $pV^n = C$ can only be derived satisfactorily by the use of the calculus. The expression is illustrated in Fig. 16 and derived in the following example:

f Example. A quantity of gas undergoes a non-flow process from an initial pressure of p_1 and volume V_1 to a final pressure p_2 and volume V_2.

(a) Given that the area beneath the pV curve is given by:

$$\int_1^2 p\,dV$$

show that the total area beneath the pV curve may be written as:

$$\frac{p_2 V_2 - p_1 V_1}{1 - n}$$

(b) During the process a quantity of heat q per unit mass is transferred. Show that q is given by:

$$q = \left(c_v + \frac{R}{1-n}\right)\left(T_2 - T_1\right)$$

$$pV^n = C$$

$$p = \frac{C}{V^n}$$

$$\text{Area} = \int_1^2 p\,dV$$

$$= C \int_{V_1}^{V_2} V^{-n}\,dV$$

$$= pV^n \left[\frac{V^{-n+1}}{-n+1}\right]_{V_1}^{V_2}$$

$$= \frac{pV^n (V_2^{1-n} - V_1^{1-n})}{1 - n}$$

$$\text{Area} = \frac{p_2 V_2 - p_1 V_1}{1 - n} \quad \text{Ans. (a)}$$

Heat transfer = Change of internal energy + External work

$$q = c_v (T_2 - T_1) + \frac{p_2 v_2 - p_1 v_1}{1 - n}$$

$$pv = RT$$

$$q = c_v (T_2 - T_1) + \frac{R (T_2 - T_1)}{1 - n}$$

$$q = \left(c_v + \frac{R}{1 - n} \right)(T_2 - T_1) \quad \text{Ans. (b)}$$

$$\frac{\text{Work done during}}{\text{polytropic expansion}} = \frac{p_1 V_1 - p_2 V_2}{n - 1}$$

This is the general expression for work done. For adiabatic expansion, n is replaced by γ. For isothermal expansion however, since the value of n is 1, and as $p_1 V_1 = p_2 V_2$ then substitution in this expression for work will produce $0 \div 0$ which is indeterminate. A different expression is therefore necessary to obtain work during isothermal expansion (see Fig. 16):

$$pV = C$$

$$p = \frac{C}{V}$$

$$\text{Area} = \int_1^2 p\,dV$$

$$= C \int_{V_1}^{V_2} \frac{dV}{V}$$

$$= C \,(\ln V_2 - \ln V_1)$$

$$= pV \ln \left(\frac{V_2}{V_1} \right)$$

$$\frac{\text{Work done during}}{\text{isothermal expansion}} = pV \ln r$$

where r is the ratio of expansion.

The above expressions give the work done *by* the gas during expansion. The same expressions give the work done *on* the gas during compression.

In the case of expansion the initial condition of $p_1 V_1$ will exceed the final condition of $p_2 V_2$ and the expression for work will produce a *positive* result, indicating that work is done *by* the gas in pushing the piston forward. Conversely, for compression, the initial condition of $p_1 V_1$ will be less than the final condition of $p_2 V_2$ and therefore a *negative* result will be obtained, indicating that work is done *on* the gas by the piston.

It is important to bear in mind that in calculating work, the units must be consistent. For example, to express work in kilojoules, the pressure must be in kN/m² and the volume in m³, thus,

$$kN/m^2 \times m^3 = kN\ m = kJ$$

Work is transfer of energy, therefore the above can be referred to as *work transfer* from the closed system within the boundary of the cylinder to the external mechanism, or vice-versa. In the first case, when the gas expands, work is being transferred from the energy in the gas to the piston, which, in turn, transmits the work through connecting mechanism to the crank shaft, the work transfer in this case is referred to as being positive. In the second case, when the gas is compressed, work is being transferred from the crank shaft, through the connecting mechanism and piston to the gas, thereby increasing the energy in the gas, and this work transfer is called negative.

Further, since $pV = mRT$, the expressions for work may be stated in terms of mRT instead of pV:

Polytropic expansion

$$\text{Work} = \frac{p_1 V_1 - p_2 V_2}{n - 1} = \frac{mR (T_1 - T_2)}{n - 1}$$

Isothermal expansion

$$\text{Work} = pV \ln r = mRT \ln r$$

Example. 0·04 m³ of gas at a pressure of 1482 kN/m² is expanded isothermally until the volume is 0·09 m³. Calculate the work done during the expansion.

$$\text{Work done} = pV \ln r$$

$$r = \frac{\text{final volume}}{\text{initial volume}} = \frac{0 \cdot 09}{0 \cdot 04} = 2 \cdot 25$$

$$\text{Work done} = 1482 \times 0 \cdot 04 \times 0 \cdot 81809$$

$$= 48 \cdot 08 \text{ kJ} \quad \text{Ans.}$$

Example. 7·08 litres of air at a pressure of 13·79 bar and temperature 335°C are expanded according to the law $pV^{1 \cdot 32} =$ constant, and the final pressure is 1·206 bar. Calculate (i) the volume at the end of expansion, (ii) the work transfer from the air, (iii) the temperature at the end of expansion, (iv) the mass of air in the system, taking $R = 0 \cdot 287 \text{ kJ/kg K}$.

$$p_1 V_1^{1 \cdot 32} = p_2 V_2^{1 \cdot 32}$$

$$1379 \times 0 \cdot 00708^{1 \cdot 32} = 120 \cdot 6 \times V_2^{1 \cdot 32}$$

$$V_2 = 0 \cdot 00708 \times \sqrt[1 \cdot 32]{\frac{1379}{120 \cdot 6}}$$

$$= 0 \cdot 04484 \text{ m}^3 \text{ or } 44 \cdot 84 \text{ litres} \quad \text{Ans. (i)}$$

Note that if the units of pressure and volume are of the same kind on each side of the equation, the units cancel each other out and hence any convenient units can be used. The above could therefore be worked in bars of pressure and litres of volume in which the question data is given. However, as pointed out, it is essential to work in fundamental units in such expressions as used in parts (ii) and (iv) of this problem, it is preferential to use fundamental units throughout by expressing the pressure in kN/m^2 and the volume in m^3.

$$\text{Work} = \frac{p_1 V_1 - p_2 V_2}{n - 1}$$

$$= \frac{1379 \times 0 \cdot 00708 - 120 \cdot 6 \times 0 \cdot 04484}{1 \cdot 32 - 1}$$

$$= 13 \cdot 61 \text{ kJ} \quad \text{Ans. (ii)}$$

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$$

$$\frac{1379 \times 0 \cdot 00708}{608} = \frac{120 \cdot 6 \times 0 \cdot 04484}{T_2}$$

$$T_2 = \frac{608 \times 120 \cdot 6 \times 0 \cdot 04484}{1379 \times 0 \cdot 00708}$$

$$T_2 = 336 \cdot 6 \text{ K}$$

$$= 63 \cdot 6°C \quad \text{Ans. (iii)}$$

$$p_1 V_1 = mRT_1$$

$$m = \frac{1379 \times 0 \cdot 00708}{0 \cdot 287 \times 608}$$

$$= 0 \cdot 05595 \text{ kg} \quad \text{Ans. (iv)}$$

Example. A perfect gas is compressed in a cylinder according to the law $pV^{1 \cdot 3} =$ constant. The initial condition of the gas is 1·05 bar, 0·34 m^3 and 17°C. If the final pressure is 6·32 bar, calculate (i) the mass of gas in the cylinder, (ii) the final volume, (iii) the final temperature, (iv) the work done to compress the gas, (v) the change in internal energy, (vi) the transfer of heat between the gas and cylinder walls.

Take $c_v = 0 \cdot 7175 \text{ kJ/kg K}$ and $R = 0 \cdot 287 \text{ kJ/kg K}$.

$$p_1 V_1 = mRT_1$$

$$m = \frac{105 \times 0 \cdot 34}{0 \cdot 287 \times 290}$$

$$= 0 \cdot 4289 \text{ kg} \quad \text{Ans. (i)}$$

$$p_1 V_1^{1 \cdot 3} = p_2 V_2^{1 \cdot 3}$$

$$1 \cdot 05 \times 0 \cdot 34^{1 \cdot 3} = 6 \cdot 32 \times V_2^{1 \cdot 3}$$

$$V_2 = 0 \cdot 34 \times \sqrt[1 \cdot 3]{\frac{1 \cdot 05}{6 \cdot 32}}$$

$$= 0 \cdot 08549 \text{ m}^3 \quad \text{Ans. (ii)}$$

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$$

$$\frac{1 \cdot 05 \times 0 \cdot 34}{290} = \frac{6 \cdot 32 \times 0 \cdot 08549}{T_2}$$

$$T_2 = \frac{290 \times 6 \cdot 32 \times 0 \cdot 08549}{1 \cdot 05 \times 0 \cdot 34}$$

$$T_2 = 438 \cdot 8 \text{ K}$$

$$= 165 \cdot 8°C \quad \text{Ans. (iii)}$$

Alternatively, the final temperature could be obtained from

$$\frac{T_1}{T_2} = \left\{\frac{p_1}{p_2}\right\}^{\frac{n-1}{n}}$$

$$\text{Work done} = \frac{p_1 V_1 - p_2 V_2}{n - 1}$$

$$= \frac{105 \times 0 \cdot 34 - 632 \times 0 \cdot 08549}{1 \cdot 3 - 1}$$

$$= -61 \cdot 1 \text{ kJ}$$

Alternatively, the work done could be obtained from:

$$\frac{mR(T_1 - T_2)}{n - 1}$$

Note that the minus sign indicates that work is done *on* the gas.

Work to compress gas = 61·1 kJ   Ans. (iv)

Increase in internal energy:

$$U_2 - U_1 = mc_V(T_2 - T_1)$$

$$= 0 \cdot 4289 \times 0 \cdot 7175 \times (438 \cdot 8 - 290)$$

$$= 45 \cdot 78 \text{ kJ} \quad \text{Ans. (v)}$$

$$\begin{array}{c}\text{Heat supplied} \\ \text{to the gas}\end{array} = \begin{array}{c}\text{Increase in} \\ \text{internal energy}\end{array} + \begin{array}{c}\text{Work done} \\ \text{by the gas}\end{array}$$

$$= 45 \cdot 78 - 61 \cdot 1$$

$$= -15 \cdot 32 \text{ kJ}$$

The minus sign means that heat is rejected by the gas during compression, that is, this amount of heat energy is transferred *from* the gas *to* the cylinder wall surrounds.

Transfer of heat = 15·32 kJ Ans. (vi)

A RELATIONSHIP BETWEEN HEAT ENERGY SUPPLIED AND WORK DONE

Consider polytropic expansion of a gas in a cylinder from initial conditions represented by state-point 1 to the final conditions of

state-point 2, and apply the energy equation:

$$\begin{array}{c}\text{Heat supplied} \\ \text{to the gas}\end{array} = \begin{array}{c}\text{Increase in} \\ \text{internal energy}\end{array} + \begin{array}{c}\text{Work done} \\ \text{by the gas}\end{array}$$

$$= mc_V(T_2 - T_1) + \frac{mR(T_1 - T_2)}{n - 1}$$

Substituting $c_V = \dfrac{R}{\gamma - 1}$

$$\text{Heat supplied} = \frac{mR(T_2 - T_1)}{\gamma - 1} + \frac{mR(T_1 - T_2)}{n - 1}$$

$$Q = \frac{mR(T_1 - T_2)}{n - 1} - \frac{mR(T_1 - T_2)}{\gamma - 1}$$

$$= \frac{mR(T_1 - T_2)}{n - 1}\left\{1 - \frac{n - 1}{\gamma - 1}\right\}$$

$$= \frac{mR(T_1 - T_2)}{n - 1}\left\{\frac{\gamma - 1 - n + 1}{\gamma - 1}\right\}$$

$$= \text{Work} \times \frac{\gamma - n}{\gamma - 1}$$

During expansion of gas, work is done *by* the gas and is a positive quantity, therefore a positive quantity of heat energy is supplied from the cylinder walls to the gas.

In compression of a gas, work is done *on* the gas which is negative work done by the gas, the result of the above expression is negative heat supplied, meaning that heat energy is transferred from the gas to the cylinder walls.

The relations between the properties of a perfect gas in its initial and final states are the same for both reversible (ideal, frictionless) steady-flow (open) and the non-flow (closed) processes as detailed in this chapter.

TEST EXAMPLES 6

1. Gas is expanded in an engine cylinder, following the law $pV^n =$ constant where the value of n is 1·3. The initial pressure is 2550 kN/m^2 and the final pressure is 210 kN/m^2. If the volume at the end of expansion is 0·75 m^3, calculate the volume at the beginning of expansion.

2. The ratio of compression in a petrol engine is 8·6 to 1. At the beginning of compression the pressure of the gas is 98 kN/m^2 and the temperature is 28°C. Find the pressure and temperature at the end of compression, assuming it follows the law $pV^{1·36}$ = constant.

3. Gas is expanded in an engine cylinder according to the law pV^n = C. At the beginning of expansion the pressure and volume are 1750 kN/m^2 and 0·05 m^3 respectively, and at the end of expansion the respective values are 122·5 kN/m^2 and 0·375 m^3. Calculate the value of n.

4. The temperature and pressure of the air at the beginning of compression in a compressor cylinder are 20°C and 101·3 kN/m^2, and the pressure at the end of compression is 1420 kN/m^2. If the law of compression is $pV^{1·35}$ find the temperature at the end of compression.

5. 0·014 m^3 of a gas at 66°C is expanded adiabatically in a closed system and the temperature at the end of expansion is 2°C. Taking the specific heats of the gas at constant pressure and constant volume as 1·005 and 0·718 $kJ/kg\,K$ respectively, calculate the volume at the end of the expansion.

6. Air is compressed in a diesel engine from 1·17 bar to 36·55 bar. If the temperatures at the beginning and end of compression are 32°C and 500°C respectively, find the law of compression assuming it is polytropic.

7. One kg of air at 20 bar, 200°C is expanded to 10 bar, 125°C by a process which is represented by a straight line on the pV diagram.
    Calculate for the air:
    (a) the work transfer;
    (b) the change in internal energy;
    (c) the heat transfer;
    (d) the change in enthalpy.

Note: for air R = 287 J/kg K and c_p = 1005 J/kg K.

f 8. A closed and insulated vessel contains 1 kg of air at 213°C and 1 bar. A second vessel, which is also insulated, has a volume of 0·2 m^3 and contains air at 6 bar and 412°C. The two vessels are then connected by a pipe of negligible volume.
    Calculate:
    (a) the final pressure of air in the vessels;
    (b) the final temperature of the air in the vessels.
For air: R = 287 *J/kg K*.

f 9. A cylinder fitted with a piston contains 0·1 m^3 of air at 1 bar 15°C. Heat is supplied to the air until the temperature reaches 500°C whilst the piston is fixed. The piston is then released and the air expands according to the law $pV^{1·5}$ = C until the pressure is 1 bar.
    Calculate:
    (a) the final temperature of the air;
    (b) the work done during expansion;
    (c) the heat transferred during each process.
For the gas c_v = 718 J/kg K, R = 287 J/kg K.

f 10. A gas is expanded in a cylinder behind a gas-tight piston. At the beginning of expansion the pressure is 36 bar, volume 0·125 m^3, and temperature 510°C. At the end of expansion the volume is 1·5 m^3 and temperature 40°C. Taking R = 0·284 $kJ/kg\,K$ and c_v = 0·71 $kJ/kg\,K$, calculate (i) the pressure at the end of expansion, (ii) the index of expansion, (iii) the mass of gas in the cylinder, (iv) change of internal energy, (v) work done by the gas, (vi) heat transfer during expansion.

CHAPTER 7

I.C. ENGINES – ELEMENTARY PRINCIPLES

Internal combustion engines are so named because combustion of the fuel takes place *inside* the engine. When the fuel burns inside the engine cylinder, it gives out heat which is absorbed by the air previously taken into the cylinder, the temperature of the air is therefore increased with a consequent increase in pressure and/or volume, thus energy is imparted to the piston. The reciprocating motion of the piston is converted into a rotary motion at the crank shaft by connecting rod and crank.

The method of igniting the fuel varies. In diesel engines the air in the cylinder is compressed to a high pressure so that it attains a high temperature, and when oil fuel is injected into this high temperature air the fuel immediately ignites. When the ignition of the fuel is caused solely by the heat of compression, the engine is classed as a *compression-ignition* engine. In petrol and paraffin engines the fuel is usually taken in with the charge of air, compressed and then ignited by an electric spark.

THE FOUR-STROKE DIESEL ENGINE

In this type of engine it takes four strokes of the piston (*i.e.* two revolutions of the crank) to complete one working cycle of operations, hence the name *four-stroke* cycle.

Fig. 17 illustrates each of these four strokes in one cylinder. One the cylinder head is shown the fuel valve (or injector) which lifts to admit oil fuel (under pressure) into the cylinder, the air-induction valve through which air is drawn in, and the exhaust valve through which the exhaust gases are expelled from the cylinder. There are two more valves which are not shown here because they do not operate during the normal working cycle; one is the relief valve which opens against the compression of its spring when the pressure in the cylinder rises too high, the other is the air-starting valve which opens to admit high pressure air into the cylinder to move the piston and start the engine (on large engines).
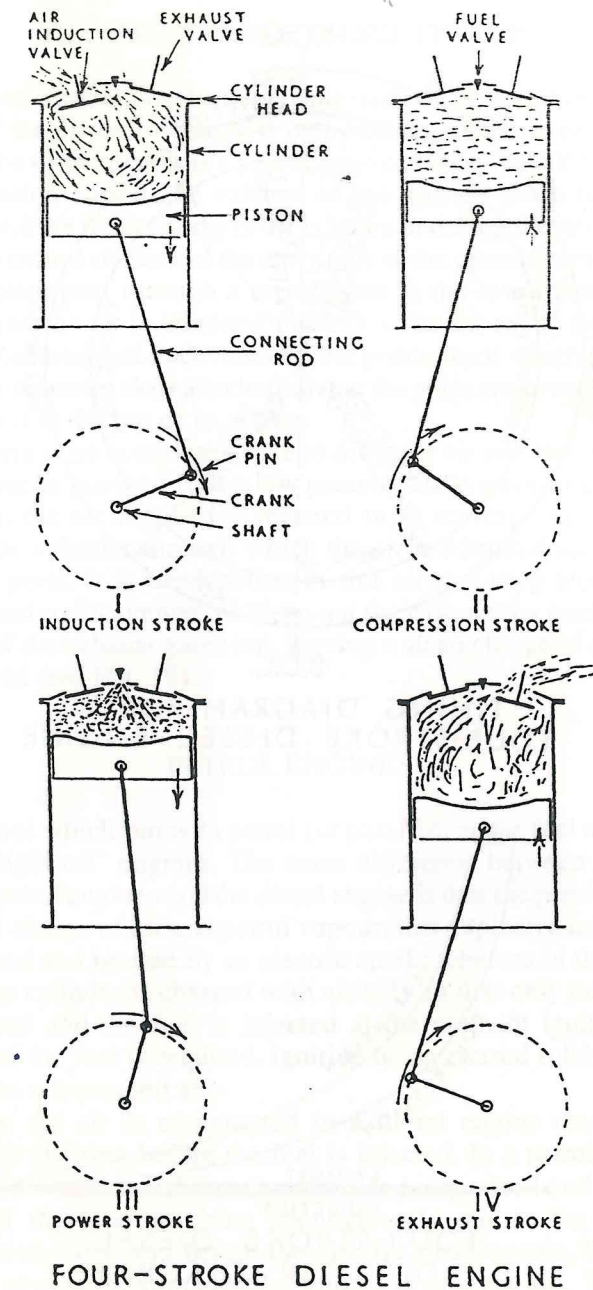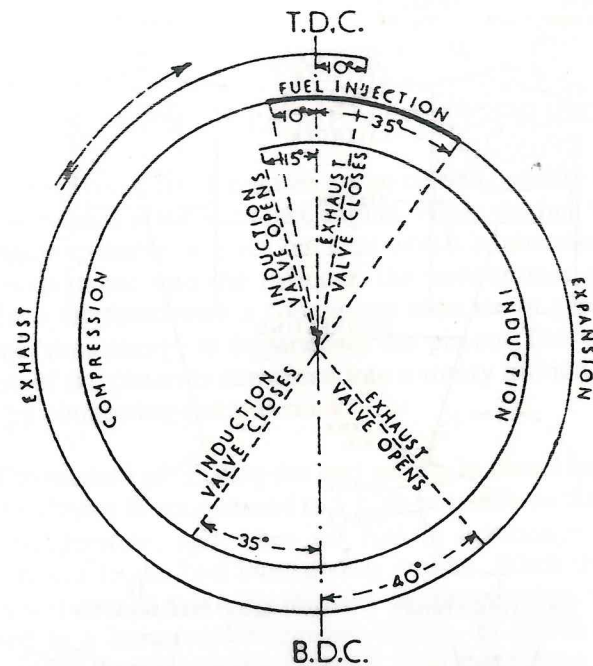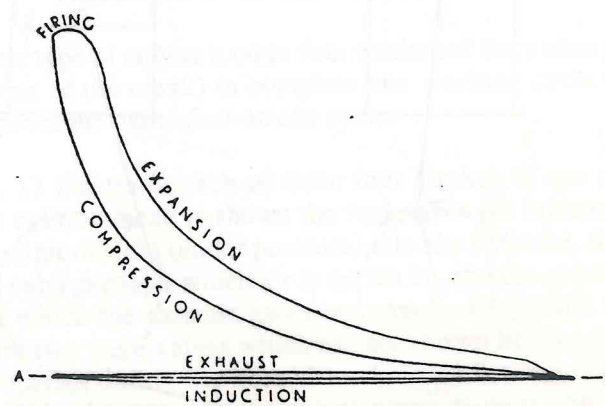

FOUR-STROKE DIESEL ENGINE

Fig. 17

TIMING DIAGRAM FOR A
FOUR-STROKE DIESEL ENGINE

FOUR-STROKE DIESEL
INDICATOR DIAGRAM

Fig. 18

THE TWO-STROKE DIESEL ENGINE

The two-stroke diesel engine is so named because it takes two strokes of the piston to complete one working cycle. Every downward stroke of the piston is a power stroke, every upward stroke is a compression stroke, the exhaust of the burned gases from the cylinder and the fresh charge of air is taken in during the late period of the downward stroke and the early part of the upward stroke. The exhaust gases pass through a set of ports in the lower part of the cylinder and the air is admitted through a similar set of ports, the ports are covered and uncovered by the piston itself which must be a long one or have a skirt attached so that the ports are covered when the piston is at the top of its stroke.

As there is no complete stroke to draw the air into the cylinder, the air must be pumped in at a low pressure from a scavenge pump or blower, the air supplied is referred to as scavenge-air and the ports in the cylinder through which the air is admitted are termed scavenge ports. It is the function of this air to sweep around the cylinder and so "scavenge" or clean out the cylinder by pushing the remains of the exhaust gases out, leaving a clean charge of air to be compressed (see Fig. 19).

PETROL ENGINES

Engines which run with petrol (or paraffin) as the fuel are often termed "light oil" engines. The main difference between the majority of petrol engines and the diesel engine is that the petrol engine takes in a charge of air and petrol vapour, this explosive mixture is compressed and ignited by an electric spark; whereas in the diesel engine the cylinder is charged with air only so that only pure air is compressed and the fuel is injected at the moment ignition and burning of the fuel is required, ignition being caused solely by the heat of the compressed air.

When the air is compressed in a diesel engine there is no possibility of firing before the fuel is injected. In a petrol engine, an explosive mixture of petrol and air is compressed and there is danger of the mixture firing spontaneously due to the heat of compression alone and before the electric spark occurs, therefore the ratio of compression must be limited to prevent this. The ratio of compression in diesel engines can be high, such as twelve to one and upwards whereas the ratio of compression in petrol engines is much less, in the region of eight or nine to one.
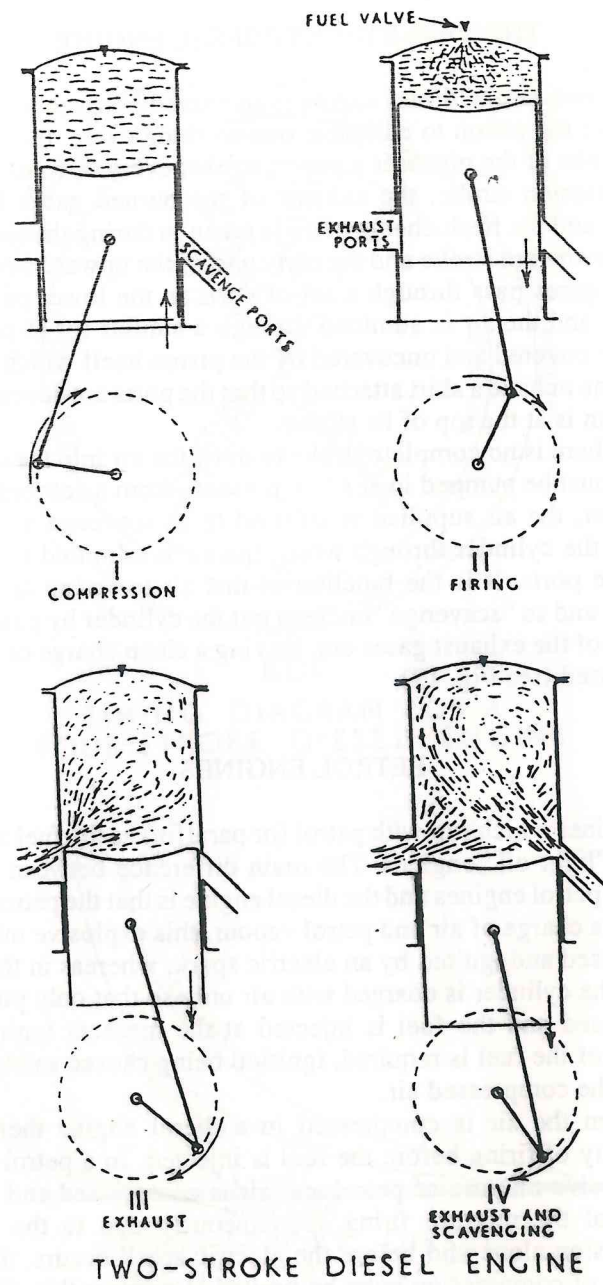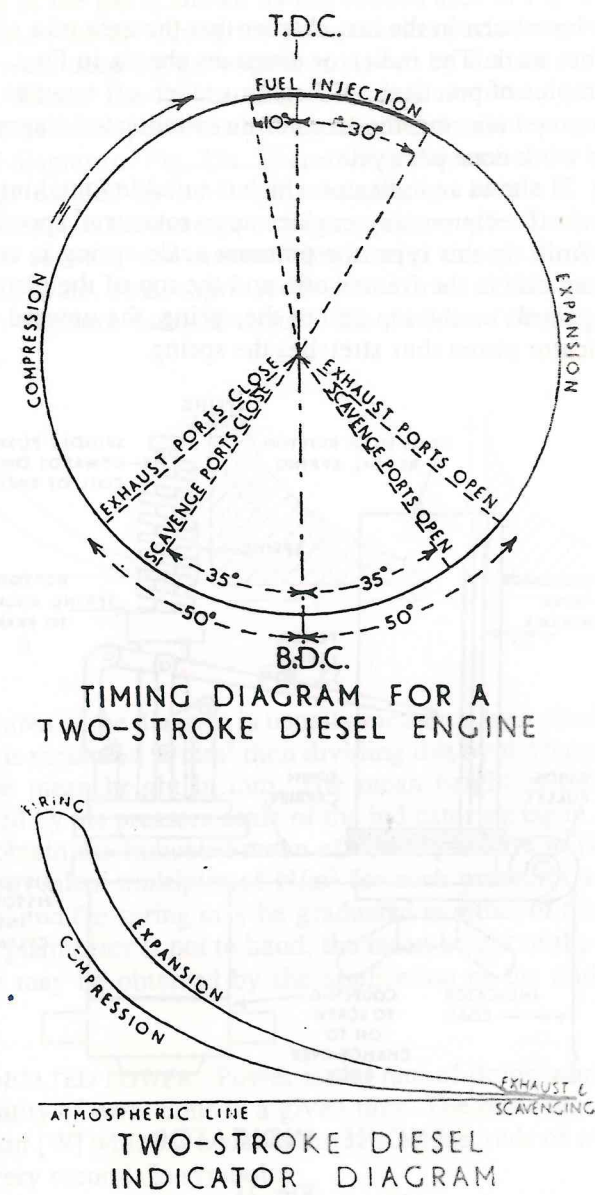
COMPRESSION

FUEL VALVE

EXHAUST PORTS

SCAVENGE PORTS

FIRING

III
EXHAUST

IV
EXHAUST AND
SCAVENGING

TWO-STROKE DIESEL ENGINE

Fig. 19

T.D.C.

FUEL INJECTION

COMPRESSION

EXPANSION

EXHAUST PORTS CLOSE

SCAVENGE PORTS CLOSE

EXHAUST PORTS OPEN

SCAVENGE PORTS OPEN

B.D.C.

TIMING DIAGRAM FOR A
TWO-STROKE DIESEL ENGINE

FIRING

EXPANSION

COMPRESSION

ATMOSPHERIC LINE

EXHAUST &
SCAVENGING

TWO-STROKE DIESEL
INDICATOR DIAGRAM

Fig. 20

MEAN EFFECTIVE PRESSURE AND POWER

We have seen in the last chapter that the area of a *pV* diagram represents work. The indicator diagrams shown in Figs. 18 and 20 are examples of practical *pV* diagrams taken off engines by means of an engine indicator, the areas of these indicator diagrams represent the work done per cycle.

Fig. 21 shows an indicator which is suitable for taking indicator diagrams off reciprocating engines up to rotational speeds of about 300 rev/min. In this type, the pressure scale spring is anchored at its bottom end to the framework, and the top of the piston spindle bears upwards on the top coil of the spring, the upward motion of the indicator piston thus stretches the spring.
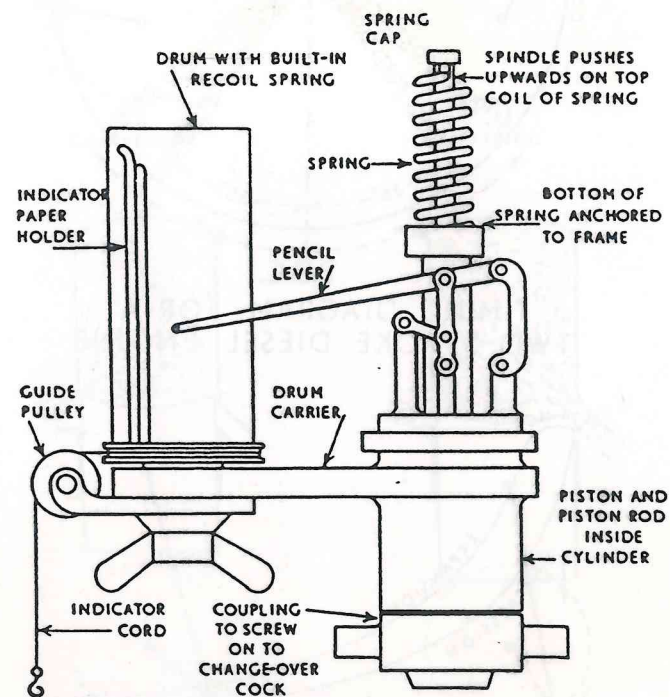
ENGINE INDICATOR

Fig. 21

MEAN EFFECTIVE PRESSURE. Consider the two-stroke diesel engine indicator diagram shown in Fig. 22. The positive work done

in one cycle of operations *by* the gas during the burning period and expansion of the gas is shown by the shaded area of Fig. 22b. The work done *on* the air during the compression period, representing negative work done by the engine is shown by the shaded area of Fig. 22c. Hence the net useful work done in one cycle is the difference between positive and negative work and represented by the actual diagram of Fig. 22a. Therefore, if the area of the indicator diagram is divided by its length, the average height is obtained which, to scale, is the average or mean pressure effectively pushing the piston forward and transmitting useful energy to the crank during one cycle. This, expressed in N/m² or a suitable multiple of the basic pressure unit, is termed the *indicated mean effective pressure.*
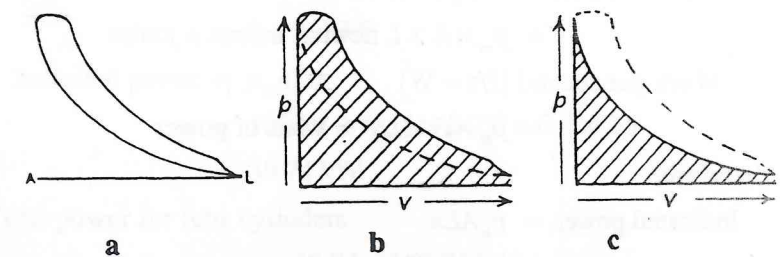

Fig. 22

The area of the diagram is usually measured by a planimeter. If the area is measured in mm² then dividing this by the length in mm gives the mean height in mm. The mean height in mm is now multiplied by the pressure scale of the indicator spring in N/m² per mm to obtain the indicated mean effective pressure in N/m². The usual convenient multiples of N/m² for such pressures are kN/m² and bars, and the spring may be graduated in either of these units.

If a planimeter is not to hand, the mean height of the indicator diagram may be obtained by the application of the mid-ordinate rule.

INDICATED POWER. Power is the rate of doing work, that is, the quantity of work done in a given time. The basic unit of power is the *watt* [W] which is equal to the rate of one joule of work being done every second. In symbols:

$$1 \text{ W} = 1 \text{ J/s} = 1 \text{ N m/s}$$

The watt is a small unit and only suitable for expressing the power of small machines. For normal powers in marine engineering.