mechanical, electrical or hydraulic, the kilowatt [kW] is usually a more convenient size, and large powers may be expressed in megawatts [MW].

Let p_m = mean effective pressure [N/m^2]
A = area of piston [m^2]
L = length of stroke [m]
n = number of power strokes per second

then,

Average force [N] on piston

$$= p_m \times A \text{ newtons}$$

Work done [J] in one power stroke

$$= p_m \times A \times L \text{ newton-metres} = \text{joules}$$

Work per second [J/s = W]

$$= p_m \times A \times L \times n \text{ watts of power}$$

therefore,

$$\text{Indicated power} = p_m ALn$$

This is the power indicated in one cylinder. The total power of a multi-cylinder engine is that multiplied by the number of cylinders, if the mean effective pressure is the same for all cylinders.

Note that when the mean effective pressure is in N/m^2 the power obtained by the above expression is in watts. If the mean effective pressure in kN/m^2 is inserted, the result will be the power in kW, and this is usually more convenient.

The value of n, the number of power strokes per second, depends upon the working cycle of the engine (two-stroke or four-stroke), its rotational speed, and whether it is a single-acting or double-acting engine.

Referring to single-acting engines, wherein the cycle of operations takes place only on the top side of the piston:

In the four stroke cycle, there is one power stroke in every four strokes, that is, one power stroke in every two revolutions, hence,

$$n = \text{rev/s} \div 2$$

In the two-stroke cycle, there is one power stroke in every two strokes, that is, one power stroke in every revolution, thus,

$$n = \text{rev/s}$$

Example. The area of an indicator diagram taken off one cylinder of a four-cylinder, four-stroke, single-acting internal combustion engine is 378 mm^2, the length is 70 mm, and the indicator spring scale is 1 mm = 2 bar. The diameter of the cylinders is 250 mm, stroke 300 mm, and rotational speed 5 rev/s. Calculate the indicated power of the engine assuming all cylinders develop equal power.

$$\text{Mean height of diagram} = \text{area} \div \text{length}$$
$$= 378 \div 70 = 5{\cdot}4 \text{ mm}$$

$$\text{Indicated } p_m = \text{mean height} \times \text{spring scale}$$
$$= 5{\cdot}4 \times 2 = 10{\cdot}8 \text{ bar}$$

$$10{\cdot}8 \text{ bar} \times 10^2 = 1080 \text{ kN/m}^2$$

$$n = \text{rev/s} \div 2$$
$$= 5 \div 2 = 2{\cdot}5$$

$$\text{Indicated power} = p_m ALn$$
$$= 1080 \times 0{\cdot}7854 \times 0{\cdot}25^2 \times 0{\cdot}3 \times 2{\cdot}5$$
$$= 39{\cdot}74 \text{ kW}$$

Total power for four cylinders

$$= 4 \times 39{\cdot}74 = 158{\cdot}96 \text{ kW Ans.}$$

Example. The diameter of the cylinders of a six-cylinder, single-acting, two-stroke diesel engine, is 635 mm and the stroke is 1010 mm. Indicator diagrams taken off the engine when running at 2·2 rev/s give an average area of 563 mm^2, the length of the diagram being 80 mm and the scale of the indicator spring 1 mm = 160 kN/m^2. Calculate the indicated power.

Indicated mean effective pressure

$$= \text{mean height of diagram} \times \text{spring scale}$$

$$= \frac{\text{area of diagram}}{\text{length of diagram}} \times \text{spring scale}$$

$$= \frac{563}{80} \times 160 = 1126 \text{ kN/m}^2$$

For a single-acting two-stroke

$$n = \text{rev/s} = 2{\cdot}2$$

For a six-cylinder engine,

$$\text{Indicated power} = p_m ALn \times 6$$
$$= 1126 \times 0{\cdot}7854 \times 0{\cdot}635^2 \times 1{\cdot}01 \times 2{\cdot}2 \times 6$$
$$= 4756 \text{ kW} \quad \text{Ans.}$$

BRAKE POWER AND MECHANICAL EFFICIENCY

Power is absorbed in overcoming frictional resistances at the various rubbing surfaces of the engine, such as at the piston rings, crosshead, crank and shaft bearings, therefore only part of the *indicated power* (ip) developed in the cylinders is transmitted as useful power at the engine shaft. The power absorbed in overcoming friction is termed the *friction power* (fp). The power available at the shaft is termed *shaft power* (sp) or, as this is measured by means of a brake it is also called *brake power* (bp).

Brake power = indicated power – friction power

The *mechanical efficiency* is the ratio of the brake power to the indicated power:

$$\text{Mechanical efficiency} = \frac{\text{brake power}}{\text{indicated power}}$$

Since the brake power is always less than the indicated power, the above expresses the mechanical efficiency as a fraction less than unity. It is common practice to state the efficiency as a percentage, by multiplying the fraction by 100.

Brake power is measured by applying a resisting torque as a brake on the shaft, the heat generated by the friction at the brake being transferred to and carried away by circulating water.

Let F = resisting force of brake, in newtons, applied at a radius of R metres when the rotational speed is in revolutions per second, then:

$$\text{Work absorbed per revolution [Nm = J]}$$
$$= \text{force [N]} \times \text{circumference [m]}$$
$$= F \times 2\pi R$$

$$\text{Work absorbed per second [J/s]} = \text{power absorbed [W]}$$
$$= F \times 2\pi R \times \text{rev/s}$$
$$F \times R = \text{torque in Nm} = T$$
$$\therefore \text{brake power} = T \times 2\pi \times \text{rev/s}$$

$$2\pi \times \text{rev/s} = \text{angular velocity in radians/second}$$
$$= \omega$$
$$\therefore \text{brake power} = T\omega$$

Common types of brakes for measuring brake power are, for small engines, a loaded rope or steel band around a flywheel on the shaft, and, for large engines, a hydraulic dynamometer.

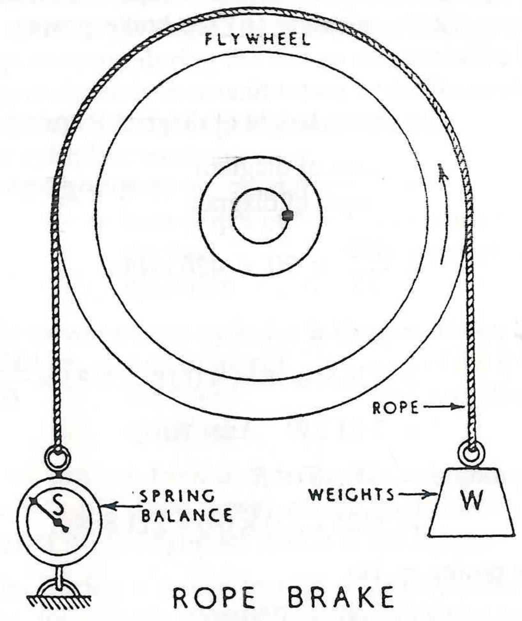


Fig. 23

Fig. 23 illustrates a simple rope brake. The rope passes over the flywheel with one end of the rope anchored to the engine base and the other end hanging freely and loaded against the direction of rotation of the flywheel, the amount of loading depending upon the desired speed.

If W = weight of the load in newtons, and S = reading of spring balance in newtons, then the effective tangential braking force on the flywheel rim is $(W - S)$ newtons. If R = effective radius in metres from centre of shaft to centre of rope, then the braking torque in newton-metres is:

$$T = (W - S) \times R$$

Example. In a single-cylinder four-stroke single-acting gas engine, the cylinder diameter is 180 mm and the stroke 350 mm. When running at 250 rev/min the mean area of the indicator diagrams taken off the engine is 355 mm², length of diagram 75 mm, scale of the indicator spring 90 kN/m² per mm, and the number of explosions was counted to be 114 per minute. Calculate (i) the indicated power. If the effective radius of the rope brake on the flywheel is 600 mm, load on free end of rope 425 N, and reading of spring balance 72 N, calculate (ii) the brake power, and (iii) the mechanical efficiency.

Indicated mean effective pressure

$$= \text{mean height of diagram} \times \text{spring scale}$$

$$= \frac{\text{area of diagram}}{\text{length of diagram}} \times \text{spring scale}$$

$$= \frac{355}{75} \times 90 = 426 \text{ kN/m}^2$$

$$\text{Indicated power} = p_m A L n$$

$$= 426 \times 0{\cdot}7854 \times 0{\cdot}18^2 \times 0{\cdot}35 \times \frac{114}{60}$$

$$= 7{\cdot}21 \text{ kW} \quad \text{Ans. (i)}$$

$$\text{Braking torque} = (W - S) \times R$$

$$= (425 - 72) \times 0{\cdot}6 = 211{\cdot}8 \text{ Nm}$$

$$\text{Brake power} = T\omega$$

$$= 211{\cdot}8 \times \frac{250 \times 2\pi}{60}$$

$$= 5546 \text{ W} = 5{\cdot}546 \text{ kW} \quad \text{Ans. (ii)}$$

$$\text{Mech. efficiency} = \frac{\text{brake power}}{\text{indicated power}}$$

$$= \frac{5{\cdot}546}{7{\cdot}21}$$

$$= 0{\cdot}7693 \text{ or } 76{\cdot}93\% \quad \text{Ans. (iii)}$$

MORSE TEST

In multi-cylinder internal combustion engines wherein all cylinders are of the same cubic capacity, a reasonable estimate of the indicated power developed in each cylinder can be made by the *morse test*. This is most useful in small high speed engines where indicator diagrams cannot be taken satisfactorily by the standard mechanical indicator.

The test consists of measuring the brake power at the shaft when all cylinders are firing and then measuring the brake power of the remaining cylinders when each one is "cut out" in turn. Cutting out the power of each cylinder is done in petrol engines by shorting the sparking plug, and in diesel engines by by-passing the cylinder fuel supply. The speed of the engine and the petrol throttle or fuel pump setting is kept constant during the test so that friction and pumping losses are approximately constant.

Taking a four-cylinder engine as an example:

With all four cylinders working,

$$\text{Total bp} = \text{total ip} - \text{total fp}$$

$$= \text{sum of ip's of the four cylinders} - \text{sum of the fp's of the four cylinders}$$

When the power of one cylinder is cut out,

$$\text{Total bp} = \text{sum of ip's of the three cylinders} - \text{sum of the fp's of the four cylinders}$$

Hence, we can see from the above that, when one cylinder is cut out, the loss of *brake* power at the shaft is the loss of the *indicated* power of that cylinder which is not firing.

Example. During a morse test on a four-cylinder four-stroke petrol engine, the throttle was set in a fixed position and the speed maintained constant at 35 rev/s by adjusting the brake, and the following powers in kW were measured at the brake,

With all cylinders working, bp developed	=	57
With sparking plug of no. 1 cyl. shorted, bp	=	38·5
.. .. .. no. 2 cyl. .. .. bp	=	37
.. .. .. no. 3 cyl. .. .. bp	=	37·5
.. .. .. no. 4 cyl. .. .. bp	=	38

Estimate the indicated power of the engine and the mechanical efficiency.

ip of no. 1 cyl	=	57 – 38·5	=	18·5
.. no. 2 cyl	=	57 – 37	=	20
.. no. 3 cyl	=	57 – 37·5	=	19·5
.. no. 4 cyl	=	57 – 38	=	19
		total ip	=	77·0 kW Ans. (i)

$$\text{Mech. efficiency} = \frac{\text{brake power}}{\text{indicated power}}$$

$$= \frac{57}{77} = 0{\cdot}74 \text{ or } 74\% \quad \text{Ans. (ii)}$$

THERMAL EFFICIENCY

In engine trials it is usually most convenient to base calculations on a running time of one hour. Also, to enable comparisons to be made on the quantity of fuel oil to run the engine under different conditions, or comparisons of one engine with another, the fuel consumption is expressed per unit power developed. The fuel consumed in unit time per unit power developed is termed the *specific fuel consumption* and commonly stated in the units kilogrammes of fuel per kilowatt-hour [kg/kW h].

The thermal efficiency of an engine is the relationship between the quantity of heat energy converted into work and the quantity of heat energy supplied:

$$\text{Thermal efficiency} = \frac{\text{heat energy converted into work}}{\text{heat energy supplied}}$$

In internal combustion engines the heat is supplied directly into the cylinders by the burning of the injected fuel. The heat energy given off during complete combustion of unit mass of the fuel is termed the *calorific value* and may be expressed in kilojoules of heat energy given off during the burning of one kilogramme of fuel [kJ/kg]. However, the calorific value of fuel oil ranges from about 40 000 to 44 000 kJ/kg and is therefore more conveniently expressed in megajoules per kilogramme [MJ/kg], that is, 40 to 44 MJ/kg.

Hence the heat supplied in megajoules is the product of the mass of fuel burned in kilogrammes and its calorific value in megajoules per kilogramme. Therefore, on a basis of one kilowatt-hour:

$$\text{Thermal effic.} = \frac{\text{heat energy equivalent of 1 kW h [MJ/kW h]}}{\text{spec. fuel cons. [kg/kW h]} \times \text{cal. value [MJ/kg]}}$$

The heat energy equivalent of one kilowatt-hour is:

$$\begin{aligned} \text{Energy} &= \text{power} \times \text{time} \\ &= 1000\ [\text{W}] \times 3600\ [\text{s}] \\ &= 3{\cdot}6 \times 10^6 \text{ J or } 3{\cdot}6 \times 10^3 \text{ kJ or } 3{\cdot}6 \text{ MJ} \end{aligned}$$

Thermal efficiency may be based on the heat energy supplied to develop 1 kW of indicated power in the cylinders, or the heat energy supplied to obtain 1 kW of brake power at the shaft. In the former, the specific fuel consumption (indicated) is expressed as the kilogrammes of fuel per indicated kilowatt-hour [kg/ind. kWh] and the efficiency is the *indicated thermal efficiency*. In the latter, the specific fuel consumption (brake) is expressed as the kilogrammes of fuel per brake kilowatt-hour [kg/brake kWh] and the efficiency is the *brake thermal efficiency*.

Thus, on the basis of one kilowatt-hour:

$$\text{Indicated thermal effic.} = \frac{3{\cdot}6\ [\text{MJ/kW h}]}{\text{kg fuel/ind. kW h} \times \text{cal. value [MJ/kg]}}$$

$$\text{Brake thermal effic.} = \frac{3{\cdot}6\ [\text{MJ/kW h}]}{\text{kg fuel/brake kW h} \times \text{cal. value [MJ/kg]}}$$

The brake thermal efficiency is also the product of the indicated thermal efficiency and the mechanical efficiency.

The symbol to represent efficiency is η.

Example. The following data were taken during a one-hour trial run on a single-cylinder, single-acting, four-stroke diesel engine of cylinder diameter 175 mm and stroke 225 mm, the speed being constant at 1000 rev/min:

Indicated mean effective pressure	= 5·5 bar
Effective diameter of rope brake	= 1066 mm
Load on brake	= 400 N
Reading of spring balance	= 27 N
Fuel consumed	= 5·7 kg
Calorific value of fuel	= 44·2 MJ/kg

Calculate the indicated power, brake power, specific fuel consumption per indicated kWh and per brake kWh, mechanical efficiency, indicated thermal efficiency and brake thermal efficiency.

$$\begin{aligned} \text{ip} &= p_m ALn \\ &= 5{\cdot}5 \times 10^2 \times 0{\cdot}7854 \times 0{\cdot}175^2 \times 0{\cdot}225 \times \frac{1000}{60 \times 2} \\ &= 24{\cdot}8 \text{ kW} \quad \text{Ans. (i)} \end{aligned}$$

$$\text{bp} = T\omega$$

$$= (400 - 27) \times \frac{1{\cdot}066}{2} \times \frac{1000 \times 2\pi}{60}$$

$$= 2{\cdot}082 \times 10^4 \text{W} = 20{\cdot}82 \text{ kW} \quad \text{Ans. (ii)}$$

Spec. fuel cons. (indicated)

$$= \frac{5{\cdot}7}{24{\cdot}8} = 0{\cdot}2298 \text{ kg/ind. kW h} \quad \text{Ans. (iii)}$$

Spec. fuel cons. (brake)

$$= \frac{5{\cdot}7}{20{\cdot}82} = 0{\cdot}2738 \text{ kg/brake kWh} \quad \text{Ans. (iv)}$$

$$\text{Mechanical effic.} = \frac{\text{brake power}}{\text{indicated power}}$$

$$= \frac{20{\cdot}82}{24{\cdot}8}$$

$$= 0{\cdot}8395 \text{ or } 83{\cdot}95\% \quad \text{Ans. (v)}$$

$$\text{Ind. thermal effic.} = \frac{3{\cdot}6 \text{ [MJ/kW h]}}{\text{kg fuel/ind. kW h} \times \text{cal. value [MJ/kg]}}$$

$$= \frac{3.6}{0{\cdot}2298 \times 44{\cdot}2}$$

$$= 0{\cdot}3544 \text{ or } 35{\cdot}44\% \quad \text{Ans. (vi)}$$

$$\text{Brake. therm. effic.} = \frac{3{\cdot}6 \text{ [MJ/kW h]}}{\text{kg fuel/brake kW h} \times \text{cal. value [MJ/kg]}}$$

$$= \frac{3.6}{0{\cdot}2738 \times 44{\cdot}2}$$

$$= 0{\cdot}2975 \text{ or } 29{\cdot}75\% \quad \text{Ans. (vii)}$$

Alternatively,

$$\text{Brake therm. effic.} = \text{indicated thermal effic.} \times \text{mech. effic.}$$

$$= 0{\cdot}3544 \times 0{\cdot}8395$$

$$= 0{\cdot}2975 \text{ (as above)}$$

HEAT BALANCE

Of the total heat energy supplied to an engine, only a small proportion is converted into useful work. The heaviest losses are those due to the heat energy transferred to and carried away by the cooling water, and the heat energy remaining in the gases which are released from the cylinders and exhausted up the flue. A clear picture of the distribution of heat is shown by constructing a heat balance chart, based on taking the heat supplied in the fuel as 100%.

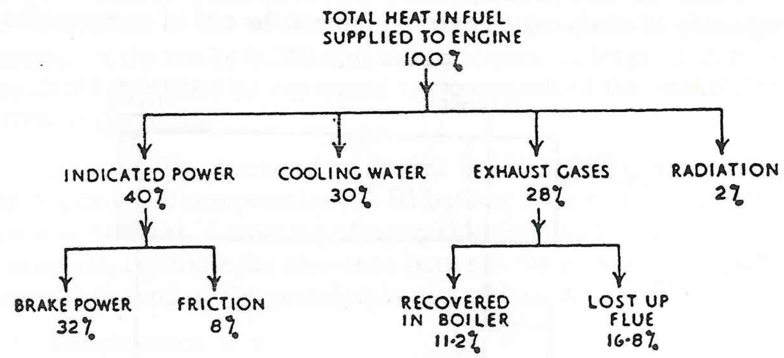


Fig. 24

Fig. 24 illustrates an example of a heat balance of a four-stroke diesel engine. In this particular case the exhaust gases were passed through the tubes of a waste-heat steam boiler before escaping up the funnel, and 40% of the heat energy in the exhaust gases was usefully recovered in generating steam. The radiation loss is usually very small and in most cases it is included with the other losses. Exhaust gas driven turbo-showers further improve efficiencies.

CLEARANCE AND STROKE VOLUME

Mechanical clearance is necessary between the inner face of the cylinder cover and the top of the piston when the piston is at the top of its stroke to avoid contact, and this is measured as the minimum distance between those two parts.

The *clearance volume* is the volume of the enclosed space above the piston when at the top of its stroke, including all cavities up to the valve faces when the valves are closed. In internal combustion engines the clearance volume is the combustion space to accommodate sufficient air for the complete combustion of the fuel and to limit the rise of temperature during burning. It is designed as near a spherical space as practicable, in many cases by concave piston tops and concave cylinder covers.

The *stroke volume*, sometimes termed the *swept volume*, is the volume swept out by the piston as it moves through one complete stroke. It is, therefore, equal to the product of the cross-sectional area of the cylinder and the length of the stroke.

Since the ratio of compression is the ratio of the volume at the beginning of compression to the volume at the end of compression,

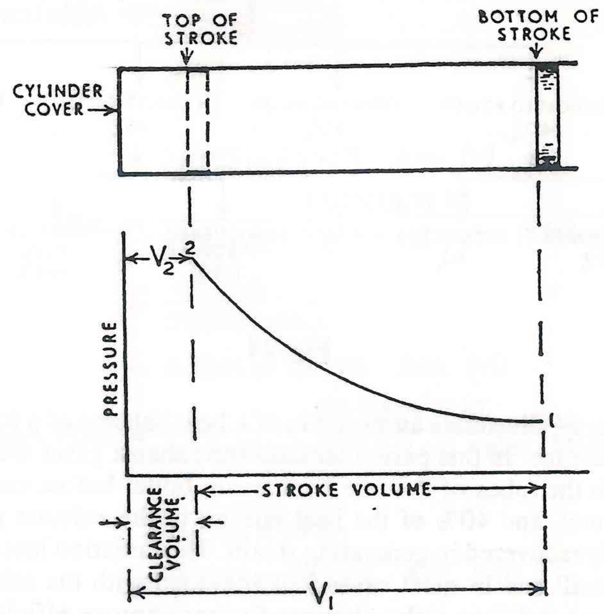


Fig. 25

then, referring to Fig. 25,

$$r = \frac{\text{initial volume}}{\text{final volume}}$$

$$= \frac{V_1}{V_2} = \frac{\text{clearance vol.} + \text{stroke vol.}}{\text{clearance volume}}$$

Therefore, the magnitude of the clearance volume affects the ratio of compression (and expansion) and can be adjusted by shims under the foot of the connecting rod or plates in the clearance space.

Dividing stroke-volume by the cross-sectional area of the cylinder gives the length of the stroke. Similarly, dividing clearance-volume by the cross-sectional area of the cylinder gives the clearance in terms of length. Hence, for convenience, if the stroke is expressed in millimetres, the clearance may be expressed as a length in millimetres. Alternatively, the clearance may be expressed as "a fraction of the stroke" or "a percentage of the stroke", for instance, if the stroke is 200 mm and the clearance length is 20 mm, the clearance could be expressed as "one-tenth of the stroke", or, "10% of the stroke".

Example. The stroke of an engine is 450 mm. The pressure at the beginning of compression is 1·01 bar and at the end of compression it is 11·1 bar. Assuming compression follows the law $pV^{1 \cdot 36}$ = a constant, calculate the clearance between the piston and cylinder cover at the end of compression in mm of length.

Let clearance = c

$$V_1 = \text{stroke} + \text{clearance} = 450 + c$$

$$V_2 = \text{clearance} = c$$

$$p_1 V_1^{1 \cdot 36} = p_2 V_2^{1 \cdot 36}$$

$$\left\{\frac{V_1}{V_2}\right\}^{1 \cdot 36} = \frac{p_2}{p_1}$$

$$\frac{V_1}{V_2} = \sqrt[1 \cdot 36]{\frac{p_2}{p_1}}$$

$$\frac{450 + c}{c} = \sqrt[1 \cdot 36]{\frac{11 \cdot 1}{1 \cdot 01}}$$

$$\frac{450 + c}{c} = 5 \cdot 826$$

$$450 + c = 5 \cdot 826c$$

$$450 = 4 \cdot 826c$$

$$c = 93 \cdot 26 \text{ mm} \quad \text{Ans.}$$

TEST EXAMPLES 7

1. The area of an indicator diagram taken off a four-cylinder, single-acting, four-stroke, internal combustion engine when running at 5·5 rev/s is 390 mm^2, the length is 70 mm, and the scale of the indicator spring is 1 mm = 1·6 bar. The diameter of the cylinders is 150 mm and the stroke is 200 mm. Calculate the indicated power of the engine assuming all cylinders develop equal power.

2. The cylinder diameters of an eight-cylinder, single-acting, four-stroke diesel engine are 750 mm and the stroke is 1125 mm. The indicated mean effective pressure in the cylinders is 1172 kN/m^2 when the engine is running at 110 rev/min. Calculate the indicated power and the brake power if the mechanical efficiency is 86%.

3. Calculate the cylinder diameters and stroke of a six-cylinder, single-acting, two-stroke diesel engine to develop a brake power of 2250 kW at a rotational speed of 2 rev/s when the indicated mean effective pressure in each cylinder is 10 bar. Assume a mechanical efficiency of 84% and the length of the stroke 25% greater than the diameter of the cylinders.

4. The flywheel of a rope brake is 1·22 m diameter and the rope is 24 mm diameter. When the engine is running at 250 rev/min the load on the brake is 480 N on one end of the rope and 84 N on the other end. Calculate the brake power. If the rise in temperature of the brake cooling water is 18 K, calculate the quantity of water flowing through the brake in litres per hour assuming that the water carries away 90% of the heat generated at the brake. Take the specific heat of the water as 4·2 kJ/kg K.

5. During a Morse test on a four-cylinder petrol engine, the speed was kept constant at 24·5 rev/s by adjusting the brake and the following readings taken:

With all cylinders firing, torque at brake	= 193·8 Nm
.. no. 1 cyl. cut out	= 130·8 ..
.. no. 2 cyl. cut out	= 130·2 ..
.. no. 3 cyl. cut out	= 129·9 ..
.. no. 4 cyl. cut out	= 131·1 ..

Calculate the bp, ip, and mechanical efficiency.

6. When developing a certain power, the specific fuel consumption of an internal combustion engine is 0·255 kg/kW h (brake) and the mechanical efficiency is 86%. Calculate (i) the indicated thermal efficiency, and (ii) the brake thermal efficiency, taking the calorific value of the fuel as 43·5 MJ/kg. If 35 kg of air are supplied per kg of fuel, the air inlet being at 26°C and exhaust at 393°C, find (iii) the heat energy carried away in the exhaust gases as a percentage of the heat supplied, taking the specific heat of the gases as 1·005 kJ/kg K.

7. A diesel engine uses 27 tonne of fuel per day when developing 4960 kW indicated power and 4060 kW brake power. Of the total heat supplied to the engine, 31·7% is carried away by the cooling water and radiation, and 30·8% in the exhaust gases. Calculate the indicated thermal efficiency, mechanical efficiency, overall efficiency, specific fuel consumption (indicated), and the calorific value of the fuel.

8. The stroke of a petrol engine is 87·5 mm and the clearance is equal to 12·5 mm. A compression plate is now fitted which has the effect of reducing the clearance to 10 mm. Assuming the compression period to be the whole stroke, the pressure at the beginning of compression as 0·97 bar, and the law of compression $pV^{1.35} = C$, calculate the pressure at the end of compression before and after the compression plate is fitted.

f 9. The mean effective pressure measured from the indicator diagram taken off a single-cylinder four-stroke gas engine was 3·93 bar when running at 5 rev/s and developing a brake power of 4·33 kW. The number of explosions per minute was 123 and the gas consumption 3·1 cubic metres per hour. The diameter of the cylinder is 180 mm, stroke 300 mm, and calorific value of the gas 17·6 MJ per cubic metre. Calculate the indicated power, mechanical efficiency, indicated thermal efficiency and brake thermal efficiency.

f 10. A two-stroke cycle compression-ignition engine has a stroke of 1·5 m, mean piston speed 6 m/s and brake mean effective pressure of 7 bar.

A similar engine with the same stroke/bore rate of 2:1 is to develop 370 kW. Both engines have the same power to swept volume ratio and operate at the same speed.

For the engine which is to develop 370 kW, calculate:

(a) the cylinder bore;

(b) brake mean effective pressure.

CHAPTER 8

IDEAL CYCLES

Theoretical cycles are reversible with isentropic (frictionless adiabatic) – sometimes isothermal – processes and can be considered for gas and vapour power cycles. The efficiency of such cycles is called the ideal cycle (thermal) efficiency. By introducing process efficiencies, e.g. utilising polytropics or modifying isentropic to actual compression/expansion utilising isentropic efficiencies, it is possible to estimate the actual cycle efficiencies. The ratio of actual to ideal cycle efficiency is called the efficiency ratio.

Consider now gas power cycles which can be classified into two main groups – IC reciprocating engines (non-flow) and gas turbines (steady-flow). Performance can be assessed using air as the working fluid, i.e. air standard cycles and air standard efficiency.

In any cycle:

Heat converted into work = heat supplied – heat rejected

and;

$$\text{Thermal efficiency} = \frac{\text{heat converted into work}}{\text{heat supplied}}$$

$$= \frac{\text{heat supplied} - \text{heat rejected}}{\text{heat supplied}}$$

$$= 1 - \frac{\text{heat rejected}}{\text{heat supplied}}$$

(Note that the heat converted into work, i.e. work done, is represented by the area of the pV diagram and m.e.p. can be calculated by dividing by the length equivalent – using correct units i.e. kN/m^2 and m^3).

CONSTANT VOLUME CYCLE

This is also known as the Otto cycle and is the basis on which petrol, paraffin, and gas engines usually work.

Designating in sequence, the four cardinal state points of the cycle as 1,2 3 and 4 respectively (Fig. 26), the cycle of operations commence with a volume of air V_1 at pressure p_1 and temperature T_1. The piston moves inward and the air is compressed adiabatically to a volume V_2 and the pressure and temperature rise to p_2 and T_2.

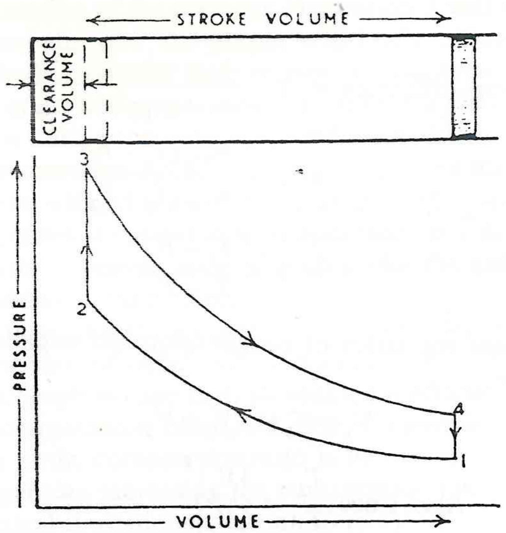


Fig. 26

Heat energy is now given from some outside source and it is assumed that the air receives this heat instantaneously so the there is no time for any change of volume to occur. The pressure and temperature consequently rise to p_3 and T_3 while the volume remains unchanged and, therefore, V_3 is equal to V_2. Adiabatic expansion of the air now takes place while the piston is pushed outward on its power stroke, the volume increasing to V_4 which is the same as the initial volume V_1 and the pressure and temperature during expansion falling to p_4 and T_4. Finally, the cycle is completed by the air rejecting heat (theoretically instantaneously) at constant volume, to an outside source, which causes the pressure and temperature to fall to their initial values of p_1 and T_1.

It should be noted that, since the compression and expansion of the air is adiabatic, then there is no exchange of heat during these operations. This means that all the heat supplied takes place at constant volume between the state points 2 and 3, and all the heat rejected takes place at constant volume between the state points 4 and 1.

Heat supplied or rejected = mass × spec. ht. × temp. change hence,

$$\text{heat supplied} = m \times c_V \times (T_3 - T_2)$$

$$\text{heat rejected} = m \times c_V \times (T_4 - T_1)$$

therefore,

$$\text{Ideal thermal efficiency} = 1 - \frac{\text{heat rejected}}{\text{heat supplied}}$$

$$= 1 - \frac{mc_V\,(T_4 - T_1)}{mc_V\,(T_3 - T_2)}$$

$$= 1 - \frac{T_4 - T_1}{T_3 - T_2}$$

In this case the ratios of compression and expansion are the same because:

$$V_1 = V_4 \text{ and } V_2 = V_3$$

$$\text{Also, since } \frac{T_2}{T_1} = \left\{\frac{V_1}{V_2}\right\}^{\gamma-1} = r^{\gamma-1}$$

$$\text{and, } \frac{T_3}{T_4} = \left\{\frac{V_4}{V_3}\right\}^{\gamma-1} = r^{\gamma-1}$$

$$\text{then, } \frac{T_2}{T_1} = \frac{T_3}{T_4} = r^{\gamma-1}$$

$$\text{hence, } T_3 = T_4 r^{\gamma-1} \text{ and } T_2 = T_1 r^{\gamma-1}$$

$$\text{therefore, } T_3 - T_2 = r^{\gamma-1}\,(T_4 - T_1)$$

Substituting this value of $(T_3 - T_2)$ into the general expression for the ideal thermal efficiency, $(T_4 - T_1)$ cancels, leaving:

$$\textbf{Ideal thermal efficiency} = \frac{1}{r^{\gamma-1}}$$

If γ is taken as 1·4 (for air) then this is also the Air Standard Efficiency.

$$\text{also, since } r^{\gamma-1} = \frac{T_2}{T_1} = \frac{T_3}{T_4}$$

$$\text{then, Ideal thermal efficiency} = 1 - \frac{T_1}{T_2}$$

$$= 1 - \frac{T_4}{T_3}$$

On examination of the previous expression it will be seen that the greater the value of r, the greater will be the efficiency, hence the trend for higher ratios of compression in modern petrol engines. The majority of petrol engines, however, take in a mixture of petrol vapour and air during the induction stroke and this is compressed during the compression stroke. Being an explosive mixture it will burst into flame without the assistance of an electric spark or other means if it reaches it temperature of spontaneous ignition. Therefore, if the ratio of compression is too high for the grade of petrol used, preignition can take place.

Fig. 27 shows the relationship between the ideal thermal efficiency and the ratio of compression in a constant volume cycle.

From the graph we see that, although the efficiency increases with higher compression ratios, the rate of increase in efficiency becomes less as the compression ratio is increased, and there is no appreciable gain by increasing the compression ratio above about 16 to 1 (practical limitations to about 10 to 1)

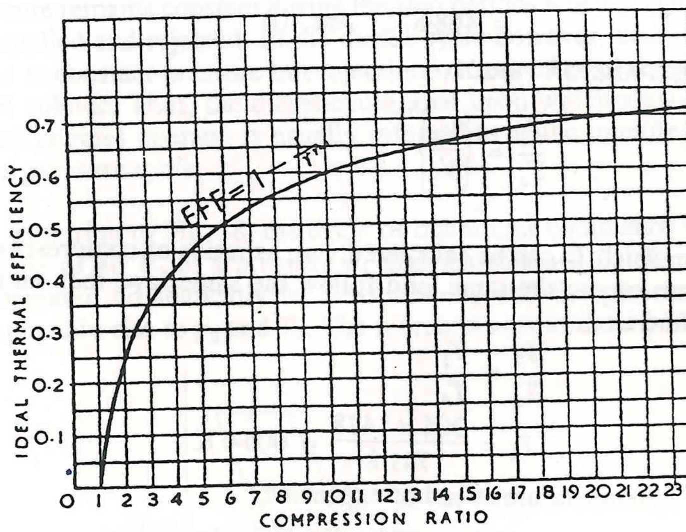


Fig. 27

Example. The compression ratio of an engine working on the constant volume cycle is 9·3 to 1. At the beginning of compression the temperature is 31°C and at the end of combustion the temperature is 1205°C. Taking compression and expansion to be adiabatic and the value of γ as 1·4, calculate (i) the temperature at the end of compression, (ii) the temperature at the end of expansion, (iii) the ideal thermal efficiency.

Referring to Fig. 26

$$V_1 = 9{\cdot}3 \text{ and } V_4 = 9{\cdot}3$$
$$V_2 = 1 \text{ and } V_3 = 1$$
$$T_1 = 304 \text{ K}$$
$$T_3 = 1478 \text{ K}$$

COMPRESSION PERIOD :

$$\frac{T_2}{T_1} = \left\{\frac{V_1}{V_2}\right\}^{\gamma-1}$$

$$\therefore T_2 = 304 \times 9{\cdot}3^{0{\cdot}4} = 741{\cdot}8 \text{ K}$$

$\therefore$ temperature at end of compression

$$= 468{\cdot}8\text{°C} \quad \text{Ans. (i)}$$

EXPANSION PERIOD :

$$\frac{T_3}{T_4} = \left\{\frac{V_4}{V_3}\right\}^{\gamma-1}$$

from which T_4 can be calculated, but, as ratios of compression and expansion are the same, and follow the same law, then an easier method is:

$$\frac{T_2}{T_1} = \frac{T_3}{T_4}$$

$$T_4 = \frac{304 \times 1478}{741{\cdot}8} = 605{\cdot}6 \text{ K}$$

$\therefore$ temperature at end of expansion

$$= 605{\cdot}6 - 273 = 332{\cdot}6\text{°C} \quad \text{Ans. (ii)}$$

The ideal thermal efficiency can now be calculated from any of the expressions given above, thus,

$$\text{Ideal thermal efficiency} = 1 - \frac{T_4 - T_1}{T_3 - T_2} \quad \text{or} \quad 1 - \frac{1}{r^{\gamma-1}}$$

$$\text{or} \quad 1 - \frac{T_1}{T_2} \quad \text{or} \quad 1 - \frac{T_4}{T_3}$$

Taking the last expression,

$$\text{Ideal thermal efficiency} = 1 - \frac{605{\cdot}6}{1478} = 1 - 0{\cdot}4099$$

$$= 0{\cdot}5901 \text{ or } 59{\cdot}01\% \quad \text{Ans. (iii)}$$

f DIESEL CYCLE

The term *constant pressure cycle* refers to one wherein the pressure remains constant during the two periods when heat energy is supplied and rejected. In the diesel cycle however, heat is supplied at constant pressure but rejection of heat takes place at constant volume. Thus, the diesel cycle, that upon which slow-speed diesel engines operate, is usually referred to as the *modified constant pressure cycle.*

Referring to Fig. 28, the cycle of operations commence with a volume of air V_1 at a pressure p_1 and temperature T_1. The air is compressed adiabatically to a volume V_2 and the pressure and temperature rise to p_2 and T_2. The piston is now at the top (inward

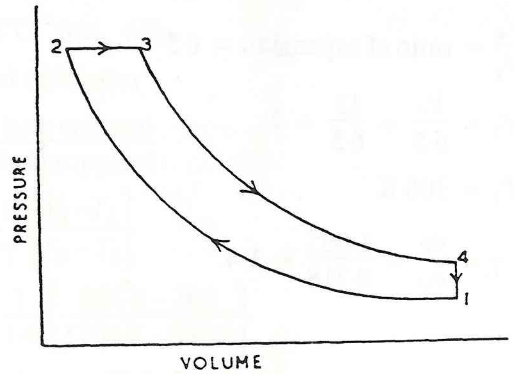


Fig. 28

end) of its stroke and heat is supplied at such a rate to maintain the pressure constant as the piston moves down the cylinder for a fraction of the power stroke. At the end of the heat supply period the volume is V_3, the temperature has been further increased to T_3, and the pressure represented by p_3 is the same as p_2. The air now expands adiabatically for the remainder of the power stroke until the final volume V_4 is the same as the initial volume V_1, the pressure and temperature falling during expansion to p_4 and T_4. Finally, the cycle is completed by the rejection of heat at constant volume to the initial conditions.

$$\text{Ideal thermal efficiency} = 1 - \frac{\text{heat rejected}}{\text{heat supplied}}$$

$$= 1 - \frac{mc_V(T_4 - T_1)}{mc_P(T_3 - T_2)}$$

$$= 1 - \frac{1}{\gamma}\left\{\frac{T_4 - T_1}{T_3 - T_2}\right\}$$

Example. The compression ratio in a diesel engine is 13 to 1 and the ratio of expansion is 6·5 to 1. At the beginning of compression the temperature is 32°C. Assuming adiabatic compression and expansion, calculate the temperatures at the three remaining cardinal points of the cycle, and the ideal thermal efficiency, taking the specific heats at constant pressure and constant volume as 1·005 and 0·718 kJ/kg K respectively.

Referring to Fig. 28,

$$V_1 = 13 \qquad V_4 = 13 \qquad V_2 = 1$$

$$\frac{V_4}{V_3} = \text{ratio of expansion} = 6{\cdot}5$$

$$\therefore\ V_3 = \frac{V_4}{6{\cdot}5} = \frac{13}{6{\cdot}5} = 2$$

$$T_1 = 305\text{ K}$$

$$\gamma = \frac{c_P}{c_V} = \frac{1{\cdot}005}{0{\cdot}718} = 1{\cdot}4$$

FIRST STAGE, ADIABATIC COMPRESSION:

$$\frac{T_2}{T_1} = \left\{\frac{V_1}{V_2}\right\}^{\gamma-1}$$

$$\therefore\ T_2 = 305 \times 13^{0{\cdot}4} = 850{\cdot}9\text{ K}$$

$\therefore$ temperature at end of compression

$= 577{\cdot}9$°C Ans. (i)

SECOND STAGE, HEATING AT CONSTANT PRESSURE:

$$\frac{T_3}{T_2} = \frac{V_3}{V_2} \quad \text{(Charles' law)}$$

$$T_3 = 850{\cdot}9 \times 2 = 1701{\cdot}8\text{ K}$$

$\therefore$ temperature at end of combustion

$= 1428{\cdot}8$°C Ans. (ii)

THIRD STAGE, ADIABATIC EXPANSION:

$$\frac{T_4}{T_3} = \left\{\frac{V_3}{V_4}\right\}^{\gamma-1}$$

$$T_4 = 1701{\cdot}8 \times \left\{\frac{2}{13}\right\}^{0{\cdot}4}$$

$$= \frac{1701{\cdot}8}{6{\cdot}5^{0{\cdot}4}} = 804{\cdot}8\text{ K}$$

$\therefore$ temperature at end of expansion

$= 531{\cdot}8$°C Ans. (iii)

Ideal thermal efficiency

$$= 1 - \frac{\text{heat rejected}}{\text{heat supplied}}$$

$$= 1 - \frac{1}{\gamma}\left\{\frac{T_4 - T_1}{T_3 - T_2}\right\}$$

$$= 1 - \frac{1}{1{\cdot}4}\left\{\frac{804{\cdot}8 - 305}{1701{\cdot}8 - 850{\cdot}9}\right\}$$

$$= 1 - \frac{1}{1{\cdot}4} \times \frac{499{\cdot}8}{850{\cdot}9}$$

$$= 1 - 0{\cdot}4196$$

$$= 0{\cdot}5804 \text{ or } 58{\cdot}045\% \quad \text{Ans. (iv)}$$

In the ideal diesel cycle, where the compression and expansion are both adiabatic, the ideal thermal efficiency can be expressed in terms of the ratio of compression and a comparison can then be made with the efficiency of a constant volume cycle of the same ratio of compression.

Expressing all temperatures in terms of T_1, substituting and simplifying:

$$\frac{T_2}{T_1} = \left\{\frac{V_1}{V_2}\right\}^{\gamma-1}$$

$$\therefore T_2 = T_1 r^{\gamma-1}$$

$$\frac{T_3}{T_2} = \frac{V_3}{V_2}$$ let this ratio of burning period volumes be represented by ρ, then,

$$\frac{T_3}{T_2} = \rho$$

$$\therefore T_3 = T_2\rho = T_1 r^{\gamma-1}\rho$$

$$\frac{T_4}{T_3} = \left\{\frac{V_3}{V_4}\right\}^{\gamma-1}$$

Since $\dfrac{V_3}{V_2} = \rho$ and $\dfrac{V_4}{V_2} = r$, then $\dfrac{V_3}{V_4} = \dfrac{\rho}{r}$

$$\therefore \frac{T_4}{T_3} = \left\{\frac{\rho}{r}\right\}^{\gamma-1}$$

$$T_4 = T_3 \times \left\{\frac{\rho}{r}\right\}^{\gamma-1}$$

$$= T_1 r^{\gamma-1}\rho \times \left\{\frac{\rho}{r}\right\}^{\gamma-1}$$

$$= T_1\rho^{\gamma}$$

Ideal thermal efficiency

$$= 1 - \frac{1}{\gamma}\left\{\frac{T_4 - T_1}{T_3 - T_2}\right\}$$

$$= 1 - \frac{1}{\gamma}\left\{\frac{T_1\rho^{\gamma} - T_1}{T_1 r^{\gamma-1}\rho - T_1 r^{\gamma-1}}\right\}$$

$$= 1 - \frac{1}{\gamma} \times \frac{1}{r^{\gamma-1}}\left\{\frac{\rho^{\gamma} - 1}{\rho - 1}\right\}$$

If $\gamma = 1{\cdot}4$, this is also the Air Standard Efficiency.

Comparing this expression with the ideal thermal efficiency of the constant volume cycle in terms of r, it will be seen that, for the same ratio of compression, the constant volume cycle has the higher thermal efficiency. This does not mean, however, that a petrol engine working on the constant volume cycle is more efficient than a diesel engine working on the modified constant pressure cycle, because, in the former, an explosive mixture is compressed and there is a limit to the ratio of compression, whereas air only is compressed in a diesel engine and the ratio of compression can be as high as required.

DUAL COMBUSTION CYCLE

In most high-speed compression-ignition engines, combustion takes place partly at constant volume and partly at constant pressure and therefore the cycle is referred to as *dual-combustion* (or mixed).

Fig. 29 shows the ideal dual-combustion cycle. Commencing with a volume of air, V_1 at pressure p_1 and temperature T_1, the air is compressed adiabatically to a volume V_2 and the pressure and temperature rise to p_2 and T_2. Heat energy is now supplied at constant volume, the pressure and temperature are increased to p_3 and T_3 while the volume remains unchanged so that V_3 is equal to V_2. The supply of heat energy is continued at such a rate as to maintain the pressure constant while the piston moves outward until the volume is V_4, the temperature is further increased to T_4 and the pressure p_4 remains the same as p_3. Now adiabatic expansion takes place until the volume V_5 is the same as the initial volume V_1, the pressure and temperature falling due to expansion to p_5 and T_5. Finally, heat is rejected at constant volume and the pressure and temperature fall to the initial conditions of p_1 and T_1.

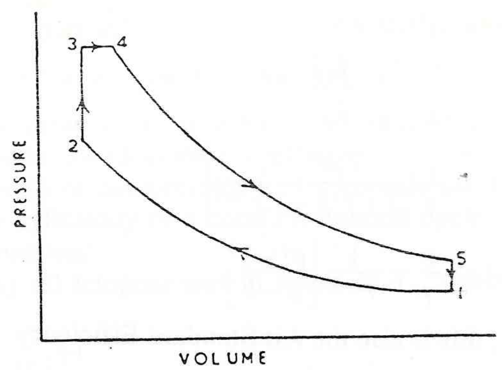


Fig. 29

Ideal thermal efficiency

$$= 1 - \frac{\text{heat rejected}}{\text{heat supplied}}$$

$$= 1 - \frac{mc_V\,(T_5 - T_1)}{mc_V\,(T_3 - T_2) + mc_P\,(T_4 - T_3)}$$

$$= 1 - \frac{(T_5 - T_1)}{(T_3 - T_2) + \gamma\,(T_4 - T_3)}$$

The above expression can be converted into terms of the ratio of compression in a similar manner as the previous cycles, thus,

If r = ratio of compression = V_1/V_2
γ = ratio of specific heats = c_P/c_V
ρ = ratio of burning period, or "cut-off" ratio = V_4/V_3
α = ratio of pressure increase at constant volume = p_3/p_2

Ideal thermal efficiency

$$= 1 - \frac{1}{r^{\gamma-1}}\left\{\frac{\alpha\rho^{\gamma} - 1}{(\alpha - 1) + \gamma\,\alpha\,(\rho - 1)}\right\}$$

Note (i) if $\alpha = 1$, the above becomes a pure diesel cycle.
(ii) if $\rho = 1$, it becomes a pure constant-volume cycle.

Since compression-ignition oil engines depend upon the temperature of the air at the end of compression to ignite the fuel injected into the cylinder, the compression-ratio must be fairly high, usually not less than about 12 to give the necessary temperature rise during compression.

The higher compression pressures developed in this type of engine limit the use of constant volume combustion, since the maximum pressure in the cycle is limited by the consideration of strength.

As the maximum pressure is limited, increasing the compression ratio reduces the amount of fuel burned at constant volume, so that more must be burned at constant pressure and thus the gain due to increased compression ratio is partly nullified.

f CARNOT CYCLE

This is a purely theoretical cycle devised by the French scientist Sadi Carnot. Although it is not possible from practical considerations for an engine to work on this cycle, it has a higher theoretical thermal efficiency than any other working between the same temperature limits and, therefore, provides a useful standard for comparing the performance of other heat engines.

Referring to the *pV* diagram, Fig. 30, it is usual to explain this cycle by commencing at state point A. Gas has been previously compressed in the cylinder by the piston moving inward and, at A, the piston is at the "top" of its stroke. The pressure and temperature are high, the value of the latter being represented by T_1. As the piston is pushed outward, doing work, heat is supplied to the gas from an external hot source at such a rate as to maintain its temperature

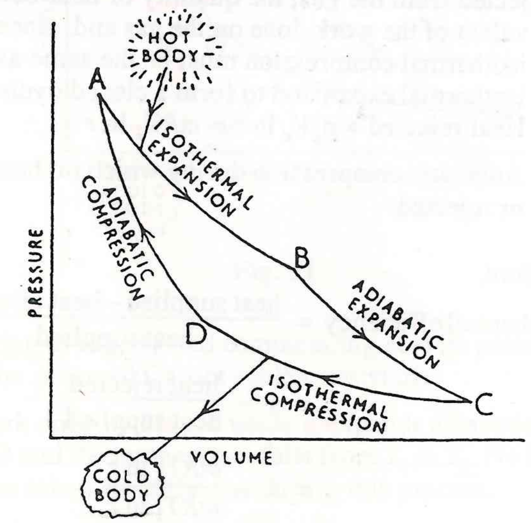


Fig. 30

constant, and during this period the gas therefore expands isothermally until point B is reached. At this point the heat supply is cut off and no heat is given to or rejected from the gas as the piston moves on to the end of the stroke at C. Hence during this period, the gas expands adiabatically as it does work and therefore the temperature falls. The temperature of the gas at point C is represented by T_2. The piston now moves inward to compress the gas from C to D and during this period it is assumed that any generated heat due to compression can flow out of the gas into a cold "sink". That is, the gas rejects heat energy to a cold external source at such a rate to maintain the temperature constant at T_2. This is isothermal compression. At point D, the flow of heat out of the gas is stopped and from D to A the gas is compressed adiabatically while the piston completes its stroke, the temperature of the gas rising to the initial temperature T_1.

Hence, the four stages of the Carnot cycle are briefly as follows:

A to B Isothermal expansion of the gas during which the amount of heat supplied is equal to the work done. Letting r = ratio of isothermal expansion.
Heat supplied = $p_A V_A \ln r = mRT_1 \ln r$

B to C Adiabatic expansion of the gas during which no heat is supplied or rejected.

C to D Isothermal compression. During this period heat is rejected from the gas, the quantity of heat being the equivalent of the work done on the gas and, since the ratio of isothermal compression must be the same as the ratio of isothermal expansion to form a closed cycle, then,
Heat rejected = $p_c V_c \ln r = mRT_2 \ln r$

D to A Adiabatic compression during which no heat is supplied or rejected.

Therefore,

$$\text{Ideal thermal efficiency} = \frac{\text{heat supplied} - \text{heat rejected}}{\text{heat supplied}}$$

$$= 1 - \frac{\text{heat rejected}}{\text{heat supplied}}$$

$$= 1 - \frac{mRT_2 \ln r}{mRT_1 \ln r}$$

$$= 1 - \frac{T_2}{T_1}$$

$$= \frac{T_1 - T_2}{T_1}$$

This expression for the Carnot Efficiency shows that, to obtain the highest efficiency, heat should be taken in at the highest possible temperature (T_1) and rejected at the lowest possible temperature (T_2). This conclusion is applicable in the design of any heat engine.

f REVERSED CARNOT CYCLE

The Carnot cycle is theoretically reversible and if applied in reverse manner would act as a refrigerator by taking heat from a cold region and maintaining it at a low temperature as follows:

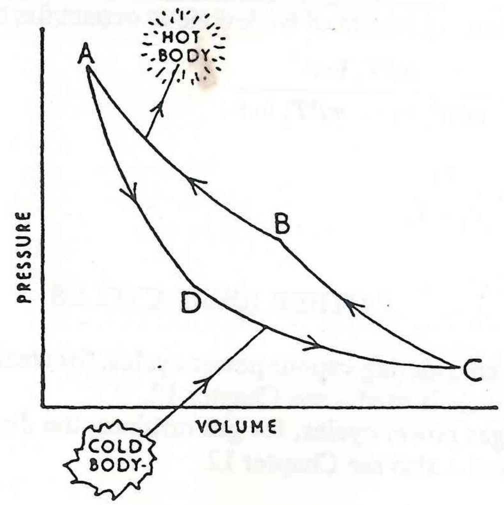


Fig. 31

Referring to Fig. 31 and commencing at state point A, the four stages of the reversed Carnot cycle consist of:

(i) Work done by the gas while it expands adiabatically from A to D and the temperature falls from T_1 to T_2. No heat is given to or taken from the gas during this process.

(ii) Further work done by the gas as it expands isothermally from D to C, a quantity of heat is taken in by the gas (from the cold body) equal to the work done, to maintain the temperature constant at T_2.

(iii) Adiabatic compression of the gas from C to B, no heat being given to or taken from the gas, therefore the temperature increases from T_2 to T_1.

(iv) Isothermal compression from B to A during which heat is rejected from the gas (to the hot body) to maintain the temperature constant at T_1.

Thus an engine working on the reversed Carnot cycle would require to be driven and, as heat would be continually taken from a cold region and sent out to a hotter region, it would therefore act as a refrigerating machine. The measure of the "efficiency" of refrigeration is known as the *coefficient of performance*, and its theoretical value is:

$$\frac{\text{Quantity of heat extracted}}{\text{Heat equivalent of work done to extract the heat}}$$

$$= \frac{mRT_2 \ln r}{mRT_1 \ln r - mRT_2 \ln r}$$

$$= \frac{T_2}{T_1 - T_2}$$

f OTHER IDEAL CYCLES

When considering vapour power cycles, for steam turbines, the **Rankine** cycle is used – see Chapter 12.

With gas power cycles, for gas turbines, the **Joule (Brayton)** cycle is used – also see Chapter 12

Operation on other cycles includes Stirling (constant volumes and isothermals), Ericsson (constant pressures and isothermals), Atkinson (modified constant volume – see Test Example 7) and theoretical variations (see Test Example 8)

f MEAN EFFECTIVE PRESSURE

The ideal cycles can be analysed to calculate the indicated

m.e.p. from the diagram. Consider, as example, the Diesel cycle and refer to Fig. 28.

$$\text{m.e.p.} = \frac{p_2(V_3 - V_2) + \dfrac{p_3V_3 - p_4V_4}{\gamma - 1} - \dfrac{p_2V_2 - p_1V_1}{\gamma - 1}}{V_1 - V_2}$$

(The m.e.p. will be in kN/m^2 if the work done i.e. the area of the diagram is in kJ i.e. kN/m^2 × m^3 and the "length" of the diagram in m^3. Pressures can be in bars, and volumes represented by stroke lengths or ratios giving the m.e.p. in bars – the work done is not then, of course, in consistent units).

f NON-IDEAL CYCLES

The ideal cycle is sometimes used as the model for calculation of properties at cardinal points, evaluation of m.e.p., etc. utilising a diagram and polytropic processes (see Test Examples 9 and 10).

TEST EXAMPLES 8

1. The compression ratio of a petrol engine working on the constant volume cycle is 8·5. The pressure and temperature at the beginning of compression are 1 bar and 40°C and the maximum pressure of the cycle is 31 bar. Taking compression for the air-petrol gas mixture to follow the adiabatic law $pV^{1 \cdot 35} = C$, calculate (i) the pressure at the end of compression, (ii) temperature at end of compression, (ii) temperature at end of combustion.

2. In an ideal constant volume cycle the temperature at the beginning of compression is 50°C. The volumetric compression ratio is 5:1. If the heat supplied during the cycle is 930 kJ/kg of working fluid, calculate:
   (a) the maximum temperature attained in the cycle,
   (b) work done during the cycle/kg of working fluid, and
   (c) the ideal thermal efficiency of the cycle.
   Take $\gamma = 1 \cdot 4$ and $c_V = 0 \cdot 717$ kJ/kg K.

f 3. In an air-standard Otto cycle the pressure and temperature of the air at the start of compression are 1 bar and 330 K respectively. The compression ratio is 8:1 and the energy added at constant volume is 1250 kJ/kg.
   (a) Calculate:
      (i) the maximum temperature in the cycle;
      (ii) the maximum pressure in the cycle.
   (b) Draw the cycle:
      (i) on a pressure-volume diagram;
      (ii) on a temperature-entropy diagram.
   For air: $c_P = 1005$ *J/kg K.* $c_V = 718$ *J/kg K.*

f 4. In an air-standard Diesel cycle the pressure and temperature of the air at the start of compression are 1 bar and 330 K respectively. The compression ratio is 16:1 and the energy added at constant pressure is 1250 kJ/kg.
   Calculate:
   (a) the maximum pressure in the cycle;
   (b) the maximum temperature in the cycle;
   (c) the cycle efficiency;
   (d) the mean effective pressure.
   For air: $c_P = 1005$ *J/kg K.* $c_V = 718$ *J/kg K*

f 5. The compression ratio of an engine working on the dual-combustion cycle is 10·7. The pressure and temperature of the air at the

beginning of compression is 1 bar and 32°C. The maximum pressure and temperature during the cycle is 41 bar and 1593°C. Assuming adiabatic compression and expansion, calculate the pressures and temperatures at the remaining cardinal points of the cycle and the ideal thermal efficiency. Take the values, $c_V = 0 \cdot 718$, and $c_P = 1 \cdot 005$ kJ/kg K.

f 6. A heat engine is to be operated, using the Carnot cycle, with maximum and minimum temperatures of 1027°C and 27°C respectively.
   (a) Calculate the efficiency of the cycle;
   (b) The working fluid within the cycle is to be steam;
      (i) state the difficulty associated with the practical operation of the cycle;
      (ii) state the modification required to the cycle to allow practical operation.

f 7. Gas initially at a pressure of 1 bar and temperature 60°C undergoes the following cycle.
   (a) Adiabatic compression through a compression ratio of 4·5:1.
   (b) Heating at constant volume through a pressure ratio of 1·35:1.
   (c) Adiabatic expansion to initial pressure.
   (d) Constant pressure cooling to initial volume.

   If $c_P = 1000$ J/kg K and $c_V = 678$ J/kg K for the gas determine the thermal efficiency of the cycle.

f 8. One kilogram of air at a pressure and temperature of 1 bar and 15°C initially, undergoes the following processes in a cycle: 1. Isothermal compression to 2 bar. 2. Polytropic compression from 2 bar to 4 bar. 3. Isentropic expansion from 4 bar to initial condition.

   Sketch the *p-V* diagram of the cycle and calculate for each process (a) the work transfer (b) the heat transfer.
   $R = 287$ J/kg K and $\gamma = 1 \cdot 4$ for air.

f 9. The compression ratio of a diesel engine is 15 to 1. Fuel is admitted for one-tenth of the power stroke and combustion takes place at constant pressure. Exhaust commences when the piston has travelled nine-tenths of the stroke. At the beginning of compression the temperature of the air is 41°C. Assuming compression and expansion to follow the law $pV^n = C$ where $n = 1 \cdot 34$, calculate the temperatures at the end of compression, end of combustion, and beginning of exhaust.

f 10. A two-stroke, single cylinder engine, operating on "diesel cycle", has a volumetric compression ratio of 12:1 and a stroke volume of 0·034 m^3.

The pressure and temperature, at the start of compression are 1 bar and 80°C respectively, while the maximum temperature at the end of heat reception is 1650°C.

If the compression is according to the law $pV^{1·36}$ = constant and the expansion follows the law $pV^{1·4}$ = constant, calculate:

(a) the indicated mean effective pressure (m.e.p.) for the cycle,

(b) the power developed at 200 cycles/min.

CHAPTER 9

RECIPROCATING AIR COMPRESSORS

Compressed air, at various pressures, is used for many purposes such as scavenging, supercharging, and starting diesel engines, and as the operating fluid for many automatic control systems. Air compressors to produce medium and high pressures are usually of the reciprocating type and may be single or multi-stage. Rotary types are common for large quantities of air at low pressures.

Fig. 32 shows diagrammatically a single-stage, single-acting reciprocating compressor, and its *pV* diagram illustrating the cycle.

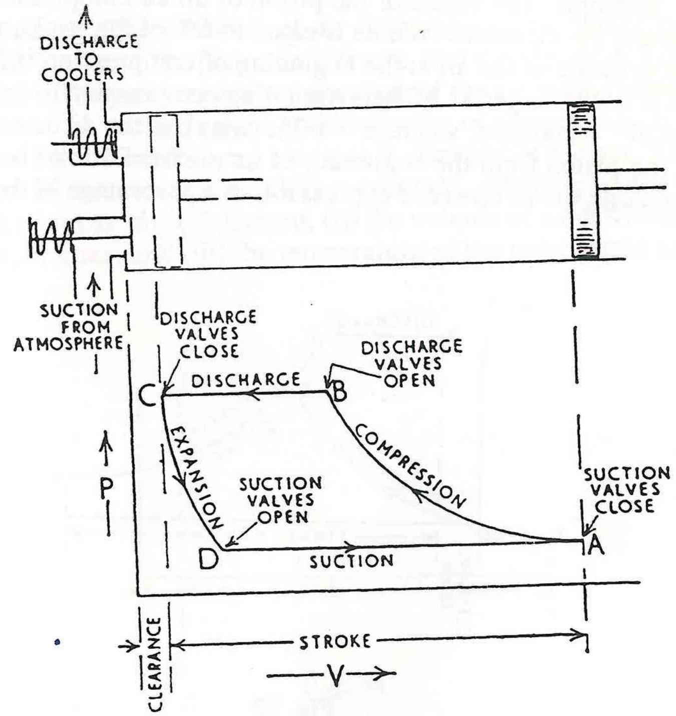


Fig. 32

Commencing at point A, the cycle of operations is as follows: A to B, compression period; with all valves closed the piston moves inward and the air which was previously drawn into the cylinder is compressed. Compression continues until the air pressure is sufficiently high to force the discharge valves open against their pe-set compression springs. Thus, at point B, the discharge valves open, and the compressed air is discharged at constant pressure for the remainder of the inward stroke, *i.e.*, from B to C. At point C the piston has completed its inward stroke and changes direction to move outward. Immediately the piston begins to move back there is a drop in pressure of the compressed air left in the clearance space, the discharge valves close, and from C to D this air expands. At point D the pressure has fallen to less than the atmospheric pressure and the lightly-sprung suction valves are opened by the greater pressure of the atmospheric air. Air is drawn into the cylinder for the remainder of the outward stroke, *i.e.*, from D to A.

Example. The stroke of the piston of an air compressor is 250 mm and the clearance volume is equal to 6% of the stroke volume. The pressure of the air at the beginning of compression is 0·98 bar and it is discharged at 3·8 bar. Assuming compression to follow the law pV^n = constant, where n = 1·25, calculate the distance moved by the piston from the beginning of its pressure stroke before the discharge valves open and express this as a percentage of the stroke.

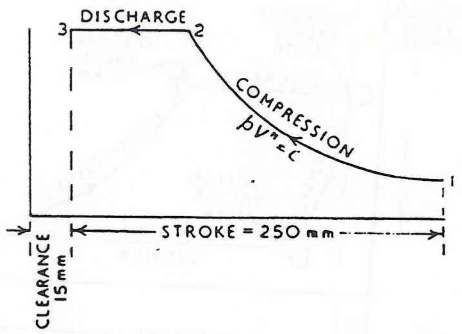


Fig. 33

$$\text{Clearance length} = 6\% \text{ of } 250 = 15$$

$$p_1 = 0{\cdot}98 \quad p_2 = 38 \quad V_1 = 250 + 15 = 265$$

$$p_1 V_1^{1\cdot25} = p_2 V_2^{1\cdot25}$$

$$0{\cdot}98 \times 265^{1\cdot25} = 3{\cdot}8 \times V_2^{1\cdot25}$$

$$V_2^{1\cdot25} = \frac{0{\cdot}98 \times 265^{1\cdot25}}{3{\cdot}8}$$

$$V_2 = 265 \times \sqrt[1\cdot25]{\frac{0{\cdot}98}{3{\cdot}8}} = 89{\cdot}6$$

Distance moved by piston from beginning of stroke to point where discharge valves open is represented by $V_1 - V_2$

$$V_1 - V_2 = 265 - 89{\cdot}6 = 175{\cdot}4 \text{ mm} \quad \text{Ans. (i)}$$

Expressed as a percentage of the stroke of 250 mm

$$= \frac{175{\cdot}4}{250} \times 100 = 70{\cdot}16\% \quad \text{Ans. (ii)}$$

Example. The diameter of an air compressor cylinder is 140 mm, the stroke of the piston is 180 mm, and the clearance volume is 77 cm³. The pressure and temperature of the air in the cylinder at the end of the suction stroke and beginning of compression is 0·97 bar and 13°C. The delivery pressure is constant at 4·2 bar. Taking the law of compression as $pV^{1\cdot3}$ = constant, calculate (i) for what length of stroke air is delivered, (ii) the volume of air delivered per stroke, in litres, and (iii) the temperature of the compressed air.

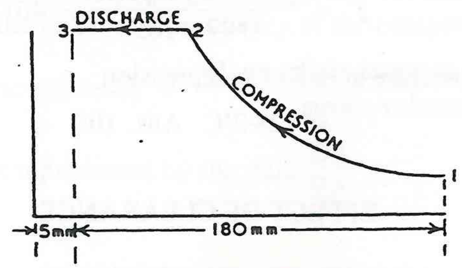


Fig. 34

$$\text{Clearance length [mm]} = \frac{\text{clearance volume [mm}^3]}{\text{area of cylnder [mm}^2]}$$

$$= \frac{77 \times 10^3}{0{\cdot}7854 \times 140^2} = 5 \text{ mm}$$

$$p_1 V_1^{1\cdot3} = p_2 V_2^{1\cdot3}$$

$$0{\cdot}97 \times (180 + 5)^{1{\cdot}3} = 4{\cdot}2 \times V_2^{1{\cdot}3}$$

$$V_2^{1{\cdot}3} = \frac{0{\cdot}97 \times 185^{1{\cdot}3}}{4{\cdot}2}$$

$$V_2 = 185 \times \sqrt[1{\cdot}3]{\frac{0{\cdot}97}{4{\cdot}2}}$$

$$= 59{\cdot}94 \text{ mm}$$

$$\text{Delivery period} = V_2 - V_3 = 59{\cdot}94 - 5$$

$$= 54{\cdot}94 \text{ mm of stroke. Ans. (i)}$$

$$\text{Volume delivered} = \text{area} \times \text{length}$$

$$= 0{\cdot}7854 \times 140^2 \times 54{\cdot}94$$

$$= 8{\cdot}457 \times 10^5 \text{ mm}^3$$

10^6 mm^3 = 1 litre

$$\text{Volume delivered} = 0{\cdot}8457 \text{ litre Ans. (ii)}$$

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$$

$$\frac{0{\cdot}97 \times 185}{286} = \frac{4{\cdot}2 \times 59{\cdot}94}{T_2}$$

$$T_2 = \frac{286 \times 4{\cdot}2 \times 59{\cdot}94}{0{\cdot}97 \times 185} = 401{\cdot}2 \text{ K}$$

∴ Temperature at end of compression

$$= 128{\cdot}2^\circ\text{C Ans. (iii)}$$

EFFECT OF CLEARANCE

Clearance is necessary between the piston face and the cylinder head and valves to avoid contact. This should be kept to a minimum because the volume of compressed air left in the clearance space at the end of the inward stroke must be expanded on the outward stroke to below atmospheric pressure before the suction valves can open, thus affecting the volume of air drawn into the cylinder during the suction stroke. This effect is illustrated below.

An ideal compressor would have, theoretically, no clearance as in Fig. 35a. The suction valves would open immediately the piston began to move on its outward stroke and air would be drawn into the cylinder for the whole stroke.

Fig. 35b. shows the effect of a small clearance. The small volume of compressed air left in the clearance space is quickly expanded to sub-atmospheric pressure to allow the suction valves to open, and air is drawn into the cylinder from D to A which is a large proportion of the full stroke E to A.

Fig. 35c shows the effect of excessive clearance. There is a comparatively large volume of air left in the clearance space and it requires a considerable movement of the piston on its outward stroke to expand this air to a pressure below atmospheric. The suction period D to A is an inefficient proportion of the stroke E to A.

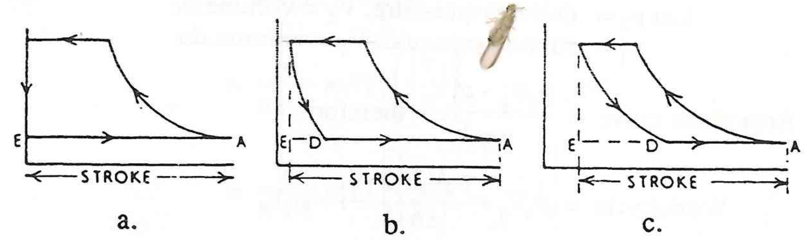


Fig. 35

The ratio between the volume of air drawn into the cylinder during the suction stroke and the full stroke volume swept out by the piston, is the *volumetric efficiency* of the compressor.

$$\text{Volumetric efficiency} = \frac{\text{volume of air drawn in per stroke}}{\text{stroke volume}}$$

and this is represented by the ratio $\frac{\text{DA}}{\text{EA}}$

WORK DONE PER CYCLE

NEGLECTING CLEARANCE

As previously shown, the area of a pV diagram represents work done, if the pressure is in kN/m^2 and the volume in cubic metres then the area of the pV diagram and the work done per cycle is in kJ.

Referring to Fig. 36 (neglecting clearance):

Net area = Net work done on the air per cycle

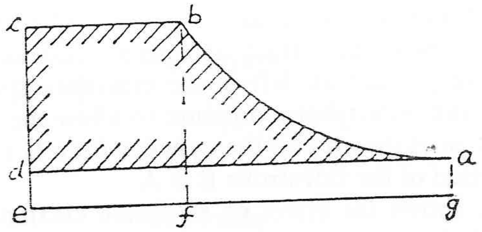


Fig. 36

Area $abcd$ = Area $bcef$ + Area $abfg$ – Area $adeg$

Let p_2 = delivery pressure, V_2 = volume bc
p_1 = suction pressure, V_1 = volume da

$$\text{Area under curve} = \frac{p_2V_2 - p_1V_1}{n-1} \text{ therefore:}$$

$$\text{Work/cycle} = p_2V_2 + \frac{p_2V_2 - p_1V_1}{n-1} - p_1V_1$$

$$= \frac{P_2V_2(n-1) + p_2V_2 - p_1V_1 - p_1V_1(n-1)}{n-1}$$

$$= \frac{np_2V_2 - p_2V_2 + p_2V_2 - p_1V_1 - np_1V_1 + p_1V_1}{n-1}$$

$$= \frac{n}{n-1}(p_2V_2 - p_1V_1)$$

$$p_1V_1 = mRT_1 \qquad p_2V_2 = mRT_2$$

$$\frac{T_2}{T_1} = \left\{\frac{P_2}{p_1}\right\}^{\frac{n-1}{n}} = \left\{\frac{V_1}{V_2}\right\}^{n-1}$$

By substitution, the work per cycle can be expressed in other terms to suit available data, such as:

$$\text{Work/cycle} = \frac{n}{n-1}(p_2V_2 - p_1V_1)$$

$$= \frac{n}{n-1} mR\,(T_2 - T_1)$$

$$= \frac{n}{n-1} mRT_1 \left\{\frac{T_2}{T_1} - 1\right\} \text{ or}$$

$$= \frac{n}{n-1} p_1V_1 \left\{\frac{T_2}{T_1} - 1\right\}$$

$$= \frac{n}{n-1} mRT_1 \left[\left\{\frac{p_2}{p_1}\right\}^{\frac{n-1}{n}} - 1\right] \text{ or}$$

$$= \frac{n}{n-1} p_1V_1 \left[\left\{\frac{p_2}{p_1}\right\}^{\frac{n-1}{n}} - 1\right]$$

$$= \frac{n}{n-1} mRT_1 \left[\left\{\frac{V_1}{V_2}\right\}^{n-1} - 1\right] \text{ or}$$

$$= \frac{n}{n-1} p_1V_1 \left[\left\{\frac{V_1}{V_2}\right\}^{n-1} - 1\right]$$

Example. The cylinder of a single-acting compressor is 225 mm diameter and the stroke of the piston is 300 mm. It takes in air at 0·96 bar and delivers it at 4·8 bar and makes four delivery strokes per second. Assuming that compression follows the law pV^n =

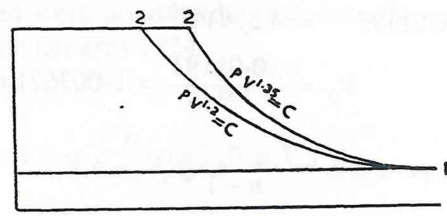


Fig. 37

constant, and neglecting clearance, calculate the theoretical power required to drive the compressor when the value of the index of the law of compression is, (i) 1·2, (ii) 1·35.

$$V_1 = 0{\cdot}7854 \times 0{\cdot}225^2 \times 0{\cdot}3 = 0{\cdot}01193 \text{ m}^3$$

When n = 1·2:

$$p_1V_1^{1\cdot2} = p_2V_2^{1\cdot2}$$

$$0{\cdot}96 \times 0{\cdot}01193^{1\cdot2} = 4{\cdot}8 \times V_2^{1\cdot2}$$

$$V_2^{1\cdot 2} = \frac{0\cdot 96 \times 0\cdot 01193^{1\cdot 2}}{4\cdot 8} = \frac{0\cdot 01193^{1\cdot 2}}{5}$$

$$V_2 = \frac{0\cdot 01193}{\sqrt[1\cdot 2]{5}} = 0\cdot 00312 \text{ m}^3$$

$$\text{Work done per cycle} = \frac{n}{n-1}(p_2V_2 - p_1V_1)$$

$$= \frac{1\cdot 2}{0\cdot 2}(4\cdot 8 \times 10^2 \times 0\cdot 00312 - 0\cdot 96 \times 10^2 \times 0\cdot 01193)$$

$$= 2\cdot 114 \text{ kJ}$$

Power [kW] = work done per second [kJ/s]
= kJ/cycle × cycle/s
= $2\cdot 114 \times 4 = 8\cdot 456$ kW. Ans. (i)

Note that although any convenient units can be used for pressure and volume when they appear on both sides of an equation, such as pressure in bars, and volume represented by length on a pV diagram (as in the two previous examples), it is again emphasised that the pressure must be in kN/m² (1 bar = 10² kN/m²) and the volume in m³ when kJ of work is required by their product.

When $n = 1\cdot 35$

$$p_1V_1^{1\cdot 35} = p_2V_2^{1\cdot 35}$$

$$0\cdot 96 \times 0\cdot 01193^{1\cdot 35} = 4\cdot 8 \times V_2^{1\cdot 35}$$

$$V_2 = \frac{0\cdot 01193}{\sqrt[1\cdot 35]{5}} = 0\cdot 003621 \text{ m}^3$$

$$\text{Work done per cycle} = \frac{n}{n-1}(p_2V_2 - p_1V_1)$$

$$= \frac{1\cdot 35}{0\cdot 35}(4\cdot 8 \times 10^2 \times 0\cdot 003621 - 0\cdot 96 \times 10^2 \times 0\cdot 01193)$$

$$= 2\cdot 287 \text{ kJ}$$

Power = $2\cdot 287 \times 4 = 9\cdot 148$ kW. Ans. (ii)

Note that the *mass* of air delivered per stroke is the same in each case. The greater volume indicated by the value of V_2 when n is $1\cdot 35$ is due purely to the air being at a higher temperature at the end of compression than it is when $n = 1\cdot 2$.

It can also be seen from the above that the nearer isothermal compression can be approached, the less work will be required to compress and deliver a given mass of air.

WORK DONE PER CYCLE

f INCLUDING CLEARANCE

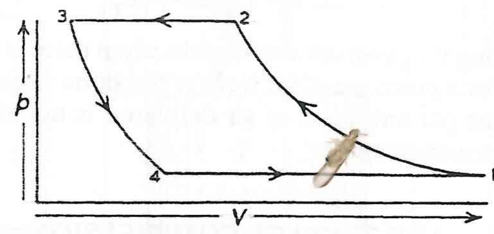


Fig. 38

Referring to Fig. 38 at the end of the compression and delivery stroke, the clearance space of volume V_3 is full of air at the delivery pressure of p_2 and temperature T_2. As the piston moves outward, the air does work on the piston as it expands to volume V_4 and the pressure falls to p_4, theoretically when p_4 is equal to p_1 the suction valves lift to begin the suction period.

The indicated work done by the piston on the air is therefore represented by the net area 1234.

Work/cycle

$$= \frac{n}{n-1}(p_2V_2 - p_1V_1) - \frac{n}{n-1}(p_3V_3 - p_4V_4)$$

$$= \frac{n}{n-1}p_1V_1\left[\left\{\frac{p_2}{p_1}\right\}^{\frac{n-1}{n}} - 1\right] - \frac{n}{n-1}p_4V_4\left[\left\{\frac{p_3}{p_4}\right\}^{\frac{n-1}{n}} - 1\right]$$

Taking the index of expansion to be equal to the index of compression and assuming $p_4 = p_1$ $p_3 = p_2$ $T_4 = T_1$ $T_3 = T_2$ then,

Work/cycle

$$= \frac{n}{n-1}p_1V_1\left[\left\{\frac{p_2}{p_1}\right\}^{\frac{n-1}{n}} - 1\right] - \frac{n}{n-1}p_1V_4\left[\left\{\frac{p_2}{p_1}\right\}^{\frac{n-1}{n}} - 1\right]$$

$$= \frac{n}{n-1} p_1(V_1 - V_4)\left[\left\{\frac{p_2}{p_1}\right\}^{\frac{n-1}{n}} - 1\right]$$

$V_1 - V_4$ is the volume of air taken in per cycle at the pressure p_1 and temperature T_1, letting m = mass of this air:

$$\text{Work/cycle} = \frac{n}{n-1} mRT_1\left[\left\{\frac{p_2}{p_1}\right\}^{\frac{n-1}{n}} - 1\right]$$

Comparing this with the work/cycle when there is no clearance, we see that for a given mass the work is the same in each case, thus, the work done per unit mass of air delivered is not affected by the size of the clearance space.

MULTI-STAGE COMPRESSION

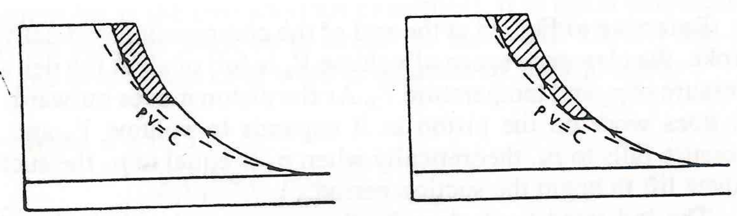


Fig. 39

By compressing the air in more than one stage and intercooling between stages, the practical compression curve can approach the isothermal more closely, hence reducing the work required per kg of air compressed. Fig. 39 shows the pV diagrams neglecting clearance for two-stage and three-stage compression respectively, the shaded areas representing the work saved in each case compared with single-stage compression.

To obtain maximum efficiency from a multi-stage compressor, that is, to do the least work to compress and deliver a given mass of air, (i) the air should be intercooled to as near the initial temperature as possible, and (ii) the pressure ratio in each stage should be the same.

Multi-stage compressors may consist simply of separate compressor cylinders, or may be arranged in tandem. Fig. 40 shows a diagrammatic arrangement of a three-stage tandem air compressor.

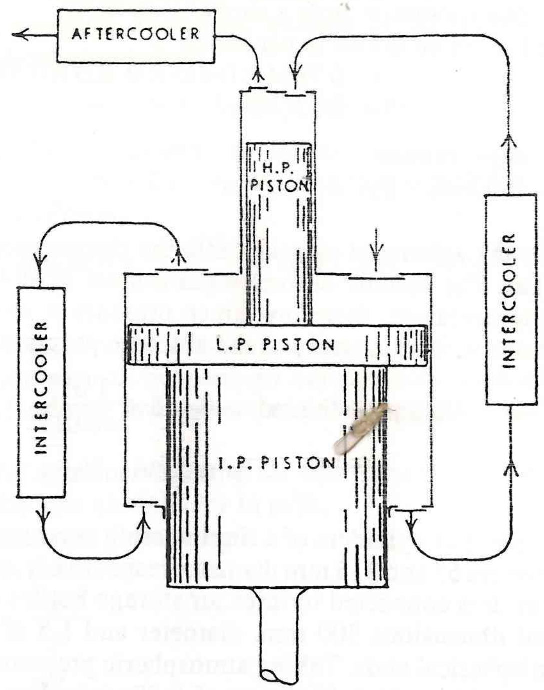


Fig. 40

It is important to note that, when calculating the volume of atmospheric air drawn into the low-pressure cylinder of a tandem compressor, the *effective* area of suction is the annulus between the area of the low pressure and the area of the high pressure. Thus, if D = diameter of low pressure piston, and d = diameter of high pressure, then:

Effective area = $0{\cdot}7854\,(D^2 - d^2)$

Example. In a single-acting three-stage tandem air compressor, the piston diameters are 70, 335 and 375 mm diameter respectively, the stroke is 380 mm, and it is driven directly from a motor running at 250 rev/min. The suction pressure is atmospheric (1·013 bar) and the discharge is 45 bar gauge pressure. Assuming that the air delivered to the reservoirs is cooled down to the initial suction temperature and taking the volumetric efficiency as 90%, calculate the volume of compressed air delivered to the reservoirs per minute.

$$\begin{aligned}\text{Effective area of L.P.} &= 0{\cdot}7854\,(0{\cdot}375^2 - 0{\cdot}07^2)\\ &= 0{\cdot}7854 \times 0{\cdot}445 \times 0{\cdot}305\ \text{m}^2\end{aligned}$$

Stroke volume = area × stroke

∴ volume [m³] of air drawn in per stroke

$= 0{\cdot}7854 \times 0{\cdot}445 \times 0{\cdot}305 \times 0{\cdot}38 \times 0{\cdot}9$

$= 0{\cdot}03646 \text{ m}^3$

air drawn in per minute

$= 0{\cdot}03646 \times 250 = 9{\cdot}115 \text{ m}^3$

This is the volume of air taken into the compressor per minute at 1·013 bar. The volume delivered per minute at 46·013 bar is at the same temperature, therefore, since pressure × volume = constant, volume varies inversely as the absolute pressure, then:

$$\text{Volume delivered} = 9{\cdot}115 \times \frac{1{\cdot}013}{46{\cdot}013}$$

$$= 0{\cdot}2006 \text{ m}^3/\text{min. Ans.}$$

Example. The cylinders of a single-acting two-stage tandem air compressor are 55 and 215 mm diameter respectively, and the stroke is 230 mm. It is connected to three air storage bottles of equal size of internal dimensions 300 mm. diameter and 1·5 m long overall with hemispherical ends. Taking atmospheric pressure as 1 bar and assuming a volumetric efficiency of 0·88, calculate the time required to pump up the bottles to a pressure of 31 bar gauge from empty, when running at 125 rev/min.

Length of cylindrical part of bottles

= overall length − diameter

$= 1{\cdot}5 - 0{\cdot}3 = 1{\cdot}2 \text{ m}$

$$\text{Volume of three bottles} = 3\left(\frac{\pi}{6} \times 0{\cdot}3^3 + \frac{\pi}{4} \times 0{\cdot}3^2 + 1{\cdot}2\right)$$

$$= 3\pi \times 0{\cdot}3^2 (0{\cdot}05 + 0{\cdot}3)$$

$$= 0{\cdot}297 \text{ m}^3$$

To produce 0·297 m³ of air at an absolute pressure of 31 + 1 = 32 bar, the volume of atmospheric pressure air (termed "free air") required, at the same temperature, is inversely proportional to the pressure:

Volume of atmospheric air

$$= 0{\cdot}297 \times \frac{32}{1} = 9{\cdot}504 \text{ m}^3$$

However, "empty" bottles contain their own volume of air at atmospheric pressure, therefore, volume of air to be taken into compressor from atmosphere

$= 9{\cdot}504 - 0{\cdot}207 = 9{\cdot}207 \text{ m}^3$

Volume of air taken into compressor per minute

$= 0{\cdot}7854\,(0{\cdot}215^2 - 0{\cdot}055^2) \times 0{\cdot}23 \times 0{\cdot}88 \times 125$

$= 0{\cdot}8584 \text{ m}^3/\text{min}$

$$\therefore \text{Time required} = \frac{9{\cdot}207}{0{\cdot}8584} = 10{\cdot}73 \text{ min. Ans.}$$

FREE AIR DELIVERY It is convenient to use free air delivery as a practical comparison between air compressors. Free air delivery is the rate of volume flow measured at inlet pressure and temperature.

At inlet (*i.e.* before entering the compressor) $pV = mRT$

If $\dot{V}$ is the free air delivery in m³/s.

If $\dot{m}$ is the mass flow in kg/s.

$$\therefore p\dot{V} = \dot{m}RT$$

$$\text{or} \quad \dot{V} = \frac{\dot{m}RT}{p}$$

$$\text{or} \quad \dot{m} = \frac{p\dot{V}}{RT}$$ either may be required when solving problems.

e.g. Power input/cycle $= \dfrac{n}{n-1}\,\dot{m}R\,(T_2 - T_1)$ [kW]

ISOTHERMAL EFFICIENCY It has been stated that the nearer the isothermal compression can be approached the less will be the work required to compress and deliver the air. Hence another compressor comparison is Isothermal Efficiency.

$$\text{Isothermal Efficiency} = \frac{\text{Power input with isothermal compression}}{\text{Actual power input}} \times 100\%$$

INDICATED MEAN EFFECTIVE PRESSURE Taking the work per cycle as at Fig. 36:

$$\text{m.e.p. (kN/m}^2\text{)} = \frac{\text{area of diagram (kJ)}}{\text{length of diagram (m}^3\text{)}}$$

MINIMUM WORK For reversible polytropic compression the total work required is a minimum when the work is equally divided between the stages. For a two stage machine with perfect intercooling, by differentiating the total work expression with respect to the intermediate pressure p_2, it can be shown that:

$$p_2 = \sqrt{p_1 p_3}$$

f Example. A two-stage single acting air compressor takes air at 1 bar, 15°C and delivers 0·075 kg/s at 9 bar. The compression and expansion in both stages is according to the law $pV^{1.3}$ = constant. The compressor is designed for minimum work with perfect intercooling.

Calculate:
(a) the power input to the compressor;
(b) the heat rejected in the intercooler.
For air: R = 287 J/kgK, c_p = 1005 J/kg K.

$$p_2 = \sqrt{p_1 p_3} = \sqrt{9} = 3$$

$$\frac{T_2}{T_1} = \left\{\frac{p_2}{p_1}\right\}^{\frac{n-1}{n}}$$

$$T_2 = 288 \times 3^{\frac{0.3}{1.3}} = 370.9 \text{ K}$$

$$\text{Work done per stage} = \frac{n}{n-1}\dot{m}R(T_2 - T_1) = \frac{1.3}{0.3} \times 0.075 \times 0.287\,(370.9 - 288) = 7.732 \text{ kW}$$

$$\text{Power input to compressor} = 2 \times 7.732 = 15.464 \text{ kW} \quad \text{Ans. (a)}$$

$$\text{Heat rejected in intercooler} = \dot{m}c_p\,(370 - 288) = 0.075 \times 1.005 \times 82.9 = 6.249 \text{ kW} \quad \text{Ans. (b)}$$

TEST EXAMPLES 9

1. In a single-stage air compressor the diameter of the cylinder is 250 mm, the stroke of the piston is 350 mm, and the clearance volume is 900 cm³. Air is drawn in at a pressure of 0·986 bar and delivered at 4·1 bar. Taking the law of compression to be $pV^{1.25} = C$, calculate the distance travelled by the piston from the beginning of its compression stroke when the delivery valves open.

2. A four cylinder single acting compressor of 152 mm bore and 105 mm stroke runs at 12 rev/s and compresses air from 1 bar to 8 bar. The clearance volume is 8% of the swept volume and both compression and expansion are according to the law $pV^{1.3}$ = constant.
   (a) Sketch the cycle on a pressure volume diagram.
   (b) Calculate:
      (i) the volumetric efficiency;
      (ii) the free air delivery.

3. A single stage air compressor of 180 mm stroke and 140 mm bore compresses air from 1 bar, 15°C to 8·5 bar. The mass of air in the cylinder at the beginning of the compression stroke is 0·0035 kg. The air in the clearance volume expands according to the law $pV^{1.32}$ = constant.
   (a) Sketch the cycle on a pV diagram;
   (b) Calculate:
      (i) the clearance volume;
      (ii) the volumetric efficiency.
   For air R = 287 J/kg K.

4. A three cylinder single stage compressor with negligible clearance volume and 100 mm bore, 120 mm stroke produces a free air delivery of 1·2 m³/min. It compresses air from 1 bar to 6·6 bar and the compression process may be assumed to be polytropic with an index of 1·3. The mechanical efficiency is 90%.
   (a) Sketch the process on a pV diagram.
   (b) Calculate:
      (i) the operating speed;
      (ii) the power input required.

5. A motor driven three-stage single-acting tandem air compressor runs at 170 rev/min. The high pressure and low pressure cylinder diameters are 75 mm and 350 mm diameter respectively, and the stroke is 300 mm. Find the time to pump up air reservoirs of 22 m³ total capacity from 19 bar gauge to 30 bar gauge, taking the

volumetric efficiency as 0·92 and atmospheric pressure as 1·01 bar.

f 6. In a single-acting air compressor, the clearance volume is 364 cm^3, diameter of cylinder 200 mm, stroke 230 mm, and it runs at 2 rev/s. It receives air at 1 bar and delivers it at 5 bar, the index of compression and expansion being 1·28. Calculate the indicated power, the mean indicated pressure, and the volumetric efficiency.

f 7. The pressure and temperature of the air at the beginning of compression in a single-stage double-acting air compressor are 0·98 bar and 24°C, the pressure ratio is 4·55 and the index of compression and expansion is 1·25. The stroke is 1·2 × cylinder diameter, clearance equal to 5% of the stroke, and the compressor runs at 8 rev/s. If it takes air from the atmosphere at 1·013 bar, 16°C, at the rate of 5 m^3/min, calculate (i) the compressor power, (ii) volumetric efficiency, (iii) the dimensions of the cylinder.

f 8. A single acting two stage air compressor delivers air at 16 bar. Inlet conditions are 1 bar, 33°C and the free air delivery is 17 m^3/min.

If the compressor is designed for minimum work and complete intercooling determine if the compression law is $pV^{1.3}$ = constant:

(a) the power required to drive the first stage.
(b) the heat rejected in the intercooler per minute.

Note: c_p = 1005 J/kg K and R = 287 J/kg K.

f 9. The free air delivery of a single stage, double acting, reciprocating air compressor is 0·6083 m^3/s measured at 1·013 bar and 15°C. Compressor suction conditions are 0·97 bar and 27°C, delivery pressure is 4·85 bar. Clearance volume is 6% of stroke volume, stroke and bore are equal.

If the index of compression and expansion is 1·32 and the machine runs at 5 rev/s determine:

(a) the required cylinder dimensions.
(b) the isothermal efficiency of the compressor.

R = 287 J/kg K.

f 10. A multi-stage air compressor is fitted with perfect intercoolers and is designed for minimum work. It takes in air at 1 bar, 35°C and delivers it at 100 bar. If the maximum temperature at any point during compression is not to exceed 95°C and the compression and expansion in each stage obeys the law $pV^{1.3}$ = constant, determine:

(a) the minimum number of stages required.
(b) the indicated power input required for a mass flow rate of 0·1 kg/s.

Take R = 0·287 kJ/kg K for air.

CHAPTER 10

STEAM

Under normal conditions of a steam engine plant, the engines consume steam at the same rate at which it is generated in the boilers, therefore the steam is generated at constant pressure.

The temperature at which water changes into steam, that is, the boiling point of water depends strictly upon the pressure exerted on it. A few examples are as follows:

PRESSURE [bar]	0·04	1·013	10	20	30
BOILING POINT [°C]	29	100	179·9	212·4	233·8

The temperature of the steam produced at any given pressure is the same as the temperature of the boiling point at the same pressure. Thus, if a boiler is working at a pressure of 30 bar the water begins to boil when its temperature reaches 233·8°C and the steam is generated at the same temperature.

DEFINITIONS. Steam which is in physical contact with the boiling water from which it has been generated is termed *saturated steam*, its temperature is the same as the boiling water and this is referred to as the *saturation temperature* (t_{sat} or t_s). The boiling water is referred to as *saturated liquid* because it is at the saturation temperature corresponding to that particular pressure (p_{sat}) under which evaporation takes place.

If the vapour produced is pure steam, it is called *dry saturated steam*. If the steam contains water (usually very fine particles held in suspension in the form of a mist) it is called *wet saturated steam* which is more often briefly referred to as *wet steam*. The quality of wet steam is expressed by its *dryness fraction* (x) which is the ratio of the mass of pure steam in a given mass of the steam-plus-water mixture.

In order to increase the temperature of steam above its saturation temperature, the steam must be taken away from direct contact with the water from which it was generated and heated externally, usually as it passes through superheater elements heated by high temperature flue gases. If the steam is still wet as it enters the superheaters, the particles of water must first be evaporated to produce dry steam before the temperature is increased. Steam whose temperature is higher than its saturation temperature (which depends upon its pressure) is termed *superheated steam*, and the difference in temperature between the superheated steam and its saturation temperature is referred to as the *degree of superheat*.

FORMATION OF STEAM. Consider generating unit mass of steam in a boiler, at constant pressure p, from the initial stage of pumping in unit mass of the feed water.

Firstly, the water is forced into the boiler at pressure p[kN/m^2] and, representing the volume of unit mass [1 kg] of the water as v_1 [m^3] then the flow energy given to the water to enable it to enter the boiler is pv_1 [kJ].

Heat energy is then transferred to the water and its temperature is raised from feed temperature to evaporation temperature (boiling point). Most of the heat energy received by the water is to increase its temperature and therefore increase its stored up energy, called *internal energy* and represented by u, but some is used to do the work of increasing its volume from v_1 to v_f as it expands against the pressure p, the work done being $p(v_f - v_1)$. If u_1 represents the internal energy initially in the feed water, u_f the internal energy in the water at evaporation temperature, and h_f the heat energy supplied then,

$$\text{heat energy transferred} = \text{increase in internal energy} + \text{external work done}$$

$$h_f = (u_f - u_1) + p(v_f - v_1)$$

On reaching the boiling point, the water evaporates into steam as it absorbs more heat energy, thus changing its physical state from a liquid into a vapour. During the evaporating process the pressure and temperature remain constant, and the steam produced is saturated steam at the same pressure and temperature as the boiling point of water. During the change, considerable expansion takes place, the volume of the steam being many times greater than the volume occupied by the water from which it was generated. Some of the heat energy transferred during this stage is to evaporate the water into steam, freeing the molecules and increasing its internal energy from u_f to u_g. The remainder of the heat energy is used in expanding the volume from v_f to v_g against the pressure p, the work done being $p(v_g - v_f)$. Hence, if h_{fg} represents the heat energy transferred then,

$$h_{fg} = (u_g - u_f) + p(v_g - v_f)$$

ENTHALPY. For a constant pressure process, the heat energy transferred, which is the sum of the internal energy and the work

done, is termed *enthalpy*. Thus, enthalpy is an energy function of a constant pressure process and, in quantities per unit mass, is defined by the equation

$$h = u + pv$$

where h = specific enthalpy [kJ/kg]
u = specific internal energy [kJ/kg]
v = specific volume [m^3/kg]
p = absolute pressure [kN/m^2]

For a mass of m kg, the appropriate symbols are written in capitals:

$$H = U + pV$$

Suffixes distinguish between the properties of liquid and vapour, some of these have already been introduced in the previous chapter:

f refers to saturated liquid
g refers to saturated vapour
fg refers to evaporation from liquid to vapour

Thus,

h_f = specific enthalpy of saturated liquid, that is, when the liquid is at its saturation temperature.
h_g = specific enthalpy of dry saturated vapour.
h_{fg} = $h_g - h_f$ = specific enthalpy of evaporation, that is, the change of specific enthalpy to evaporate unit mass of saturated liquid to dry saturated vapour.
h = specific enthalpy of either liquid or vapour at any other state.

Since there is no absolute value for internal energy, there is none for enthalpy, but changes of the property can be measured and the convenient datum of water at 0°C is chosen from which to measure enthalpy and other properties of water and steam. These values are set out in steam tables.

STEAM TABLES

Much experimental work has been done on the properties of steam and the resuts are published in various forms. Those used in this book are:

THERMODYNAMIC AND TRANSPORT PROPERTIES OF FLUIDS, SI UNITS arranged by Y. R. Mayhew and G. F. C. Rogers, published by Basil Blackwell, Oxford.

The student should have a copy by his side for reference while studying the following examples. Note that in these tables, p represents the pressure in bars. 1 bar = 10^5 N/m^2 = 10^2 kN/m^2.

The properties of saturated water and steam are tabulated on pages 2 to 5. The first table, on page 2, is for the convenience of reference to temperature up to 100°C. The remainder are set out with reference to pressure.

The properties of superheated steam at various pressures and temperatures are tabulated on pages 6 to 8.

WATER. The properties of liquids depend almost entirely on the temperature, therefore when looking up the properties of water, we look at its particular temperature and disregard its pressure.

Example. To read from the tables the specific enthalpy of water at,

(a)	pressure of	1	bar and temperature		60°C
(b)	...	... 5	bar ...	...	88°C
(c)	...	... 10	bar ...	...	130°C
(d)	...	... 12	bar ...	...	188°C

(a) 60°C: see page	2,	h = 251·1 kJ/kg
(b) 88°C:	3,	h = 369 kJ/kg
(c) 130°C:	4,	h = 546 kJ/kg
(d) 188°C:	4,	h_f = 798 kJ/kg

Note that for case (d) we see that the temperature of 188°C is the saturation temperature corresponding to its pressure of 12 bar, therefore we use the suffix f to signify that the water is at its saturation temperature.

STEAM. Example. To find the temperature and specific enthalpy of saturated water and dry saturated steam at pressures of (a) 1·01325 bar, (b) 10 bar, (c) 20 bar, (d) 42 bar.

(a) 1·01325 bar,	page	2,	t_s = 100°C	h_f = 419·1	h_g = 2675·8
(b) 10 bar,	...	4,	t_s = 179·9°C,	h_f = 763,	h_g = 2778
(c) 20 bar,	...	4,	t_s = 212·4°C,	h_f = 909,	h_g = 2779
(d) 42 bar,	...	5,	t_s = 253·2°C,	h_f = 1102,	h_g = 2800

WET STEAM. When steam is "wet" it means that it is a mixture of dry saturated steam and saturated water. Thus, if the dryness fraction is represented by x, then 1 kg of wet steam is composed of

x kg of dry saturated steam, and the remainder, which is $(1 - x)$ kg is water. Hence only x kg out of each 1 kg has been evaporated and the change of enthalpy to do this is x × specific enthalpy of evaporation, written xh_{fg}.

Therefore, specific enthalpy of wet steam of dryness fraction x is:

$$h = h_f + xh_{fg}$$

Example. Find the enthalpy and volume of 1 kg of wet saturated steam at a pressure of 0·2 bar and dryness fraction 0·85, and find also the additional heat energy required to completely dry the steam.

0·2 bar, page 3,

$$\begin{aligned} h &= h_f + xh_{fg} \\ &= 251 + 0{\cdot}85 \times 2358 \\ &= 251 + 2004{\cdot}3 \\ &= 2255{\cdot}3 \text{ kJ/kg} \quad \text{Ans. (i)} \end{aligned}$$

v_g is the volume occupied by 1 kg of dry saturated steam, therefore the volume occupied by x kg of dry saturated steam is xv_g. The value of v_g is given on page 3 as 7·648 m^3 for a pressure of 0·2 bar.

x_f is the volume occupied by 1 kg of saturated water, therefore the volume occupied by $(1 - x)$ kg of saturated water is $(1 - x)v_f$. The value of v_f is not listed on page 3 but a near figure can be found on page 10 which gives $v_f \times 10^2 = 0{\cdot}1017$ for a pressure of 0·1992 bar.

Hence, volume of 1 kg of wet steam is

$$\begin{aligned} v &= xv_g + (1 - x)v_f \\ &= 0{\cdot}85 \times 7{\cdot}648 + 0{\cdot}15 \times 0{\cdot}001\,017 \\ &= 6{\cdot}5008 + 0{\cdot}000\,152\,55 \end{aligned}$$

We see that the volume of the water is comparatively very small and therefore, for most practical cases can be neglected. Hence, the specific volume of wet steam is usually taken as,

$$v = xv_g$$

and for this example it is:

$$\begin{aligned} v &= 0{\cdot}85 \times 7{\cdot}648 \\ &= 6{\cdot}5008 \text{ m}^3\text{/kg} \quad \text{Ans. (ii)} \end{aligned}$$

To dry the steam, $(1 - 0{\cdot}85)$ kg of water is to be evaporated,

$$\text{Increase of enthalpy} = 0{\cdot}15 \times h_{fg} = 0{\cdot}15 \times 2358 = 353{\cdot}7 \text{ kJ} \quad \text{Ans. (iii)}$$

Alternatively, it is the difference between the enthalpy of dry saturated steam and wet steam:

$$= h_g - h = 2609 - 2255{\cdot}3 = 353{\cdot}7 \text{ kJ}$$

Example. If the specific enthalpy of wet saturated steam at a pressure of 11 bar is 2681 kJ/kg, find its dryness fraction.

The specific enthalpy of wet steam of dryness x is:

$$h = h_f + xh_{fg}$$

$$\text{hence, } x = \frac{xh_{fg}}{h_{fg}} = \frac{h - h_f}{h_{fg}}$$

$$= \frac{2681 - 781}{2000} \text{ (page 4)}$$

$$= \frac{1900}{2000} = 0{\cdot}95 \quad \text{Ans.}$$

Example. Find the heat transfer required to convert 5 kg of water at a pressure of 20 bar and temperature 21°C into steam of dryness fraction 0·9, at the same pressure.

Water 21°C, page 2, $h = 88$ kJ/kg
Steam 20 bar, page 4,

$$h = h_f + xh_{fg} = 909 + 0{\cdot}9 \times 1890 = 2610 \text{ kJ/kg}$$

Change of enthalpy, water to steam

$$= 2610 - 88 = 2522 \text{ kJ/kg}$$

Heat transfer = total change of enthalpy (for 5 kg)

$$= 5 \times 2522 = 12610 \text{ kJ} \quad \text{Ans.}$$

SUPERHEATED STEAM. As previously stated, steam is said to be *superheated* when its temperature is higher than the saturation temperature corresponding to its pressure. In practice, steam is superheated at constant pressure, the saturated steam being taken from the boiler steam space and passed through the superheater tubes where it receives additional heat energy to dry the steam and raise its temperature. The properties of superheated steam are given on pages 6 to 8 of the tables.

Example. Find the specific volume and specific enthalpy of superheated steam (a) at a pressure of 10 bar and temperature 250°C, (b) at a pressure of 100 bar and temperature 400°C.

10 bar 250°C, page 7,

$$\left.\begin{aligned} v &= 0{\cdot}2328 \text{ m}^3\text{/kg} \\ h &= 2944 \text{ kJ/kg} \end{aligned}\right\} \text{Ans. (a)}$$

100 bar 400°C, page 8

$$\left.\begin{aligned} v &= 0{\cdot}02639 \text{ m}^3\text{/kg} \\ h &= 3097 \text{ kJ/kg} \end{aligned}\right\} \text{Ans. (b)}$$

Note that at the higher pressures of 80 bar and upwards (page 8) the values of the specific volume are given as $v \times 10^2$, that is, the listed values are $10^2 \times$ actual value.

Example. Find the specific volume and specific enthalpy of steam at a pressure of 7 bar having 85°C of superheat, and determine the mean specific heat of the superheated steam over this range.

7 bar, page 7, $t_{sat} = 165$°C, $h_g = 2764$ kJ/kg

Temperature of superheated steam

$$= 165 + 85 = 250°\text{C}$$

$$v = 0{\cdot}3364 \text{ m}^3\text{/kg} \quad \text{Ans. (i)}$$

$$h = 2955 \text{ kJ/kg} \quad \text{Ans. (ii)}$$

Heat energy [kJ] added to superheat the steam

$$= \text{mass [kg]} \times c_p\text{[kJ/kgK]} \times \theta\text{[K]}$$

$$2955 - 2764 = 1 \times c_p \times 85$$

$$c_p = \frac{191}{85} = 2{\cdot}247 \text{ kJ/kg K} \quad \text{Ans. (iii)}$$

INTERPOLATION. If properties are required at an intermediate pressure or temperature to those given in the tables, an estimate can be made by taking values from the nearest listed pressure or temperature above and below that required and assume a linear variation between the two. The following demonstrate the usual methods.

Example. Find the enthalpy per kg of steam at a pressure of 8 bar and temperature 270°C.

Properties at 8 bar are given on page 7 but 270°C lies between the listed temperatures of 250 and 300°C.

At 8 bar 300°C, $h = 3057$ kJ/kg
At 8 bar 250°C, $h = 2951$

Difference, increase 50°C, $h = 106$ increase
Given temperature of 270°C is 20° higher than 250°C,

difference for 20°C, $= \frac{20}{50} \times 106 = 42{\cdot}4$ kJ/kg

$\therefore$ At 8 bar, 270°C, $h = 2951 + 42{\cdot}4$
$= 2993{\cdot}4$ kJ/kg Ans.

Example. Find the specific enthalpy of steam at a pressure of 12 bar and temperature 350°C.

Values of pressures of 10 bar and 15 bar are given on page 7, temperature of 350°C is listed.

At 10 bar 350°C, $h = 3158$ kJ/kg
At 15 bar 350°C, $h = 3148$
Difference, increase 5 bar $h =$ 10 kJ/kg *decrease*
Given pressure of 12 bar is 2 bar greater than 10 bar

Difference for 2 bar increase $= \frac{2}{5} \times 10 = 4$ kJ/kg less

$\therefore$ at 12 bar 350°C, $h = 3158 - 4$
$= 3154$ kJ/kg. Ans.

Example. Find the specific enthalpy of steam at a pressure of 25 bar and temperature 380°C.

Nearest listed pressures and temperatures are 20–30 bar and 350–400°C, on page 7, this requires double interpolation.

20 bar 400°C, $h = 3158$ kJ/kg
30 bar 400°C, $h = 3231$
For 10 bar increase $h = 17$ kJ/kg decrease
For 5 bar increase $h = \frac{5}{10} \times 17 = 8{\cdot}5$ decrease

$\therefore$ At 25 bar 400°C, $h = 3248 - 8{\cdot}5$
$= 3239{\cdot}5$ kJ/kg ... ... (i)

20 bar 350°C, $h = 3138$ kJ/kg
30 bar 400°C, $h = 3117$

For 10 bar increase $h = 21$ kJ/kg decrease
For 5 bar increase $h = \frac{5}{10} \times 21 = 10{\cdot}5$ decrease

$\therefore$ At 25 bar 350°C, $h = 3138 - 10{\cdot}5$
$= 3127{\cdot}5$ kJ/kg ... ... (ii)

From (i) 25 bar 400°C, $h = 3239{\cdot}5$
From (ii) 25 bar 350°C, $h = 3127{\cdot}5$
For increase of 50°C, $h = 112$ kJ/kg increase
For increase of 30°C, $h = \frac{30}{50} \times 112 = 67{\cdot}2$ increase

$\therefore$ At 25 bar 380°C, $h = 3127{\cdot}5 + 67{\cdot}2$
$= 3194{\cdot}7$ kJ/kg Ans.

MIXING STEAM AND WATER

In Chapter 2, examples were given involving the mixing of ice and water, and metals in liquids at different temperature, and now further examples will be given to include steam. These have many practical applications as will be seen later.

The same principles apply, namely, that unless otherwise stated it is assumed there is no transfer of heat energy from or to an outside source during the mixing process. That is, the quantity of heat energy absorbed by the colder substance is equal to the quantity of heat energy lost by the hotter substance. In other words, when steam and water are mixed together, the total enthalpy of the water and steam before mixing is equal to the total enthalpy of the resultant mixture.

Example. 3 kg of wet steam at 14 bar and dryness fraction 0·95 are blown into 100 kg of water at 22°C. Find the resultant temperature of the water.

Water 22°C, page 2 of tables, $h = 92{\cdot}2$ kJ/kg
Steam 14 bar, page 4,

$$h = h_f + xh_{fg}$$
$$= 830 + 0{\cdot}95 \times 1960 = 2692 \text{ kJ/kg}$$

Before mixing:
enthalpy of 100 kg water $= 100 \times 92{\cdot}2 = 9220$ kJ
enthalpy of 3 kg steam $= 3 \times 2692 = 8076$
total enthalpy $= 17296$ kJ

After mixing there are 100 + 3 = 103 kg of water of total enthalpy 17296 kJ, therefore specific enthalpy of resultant water is:

$$h = \frac{17296}{103} = 167{\cdot}9 \text{ kJ/kg}$$

From page 2 of tables, 167·5 kJ/kg (which is sufficiently close) corresponds to a water temperature of 40°C.

hence, resultant temperature = 40°C. Ans.

An alternative method is to work on the principle of heat energy absorbed by the cold water is equal to the heat energy lost by the steam.

When heat is taken from the steam, it first condenses at its saturation temperature of 195°C (page 4 of tables, at 14 bar) into water at the same temperature, the heat energy lost being the change of enthalpy of condensation which is mass × xh_{fg}. This water condensate falls in temperature from 195°C to the final temperature of the mixture, the further heat energy lost being the product of the mass, specific heat, and fall in temperature.

The heat energy gained by the cold water is the product of its mass, specific heat, and rise in temperature.

Thus, let θ = final temperature of the water mixture, and assuming the mean specific heat of water to be 4·2 kJ/kg K, then:

$$\begin{aligned}
\text{Heat energy gained by water} &= \text{Heat energy lost by steam} \\
100 \times 4{\cdot}2\,(\theta - 22) &= 3\{0{\cdot}95 \times 1960 + 4{\cdot}2(195 - \theta)\} \\
420\,\theta - 9240 &= 3\{1862 + 819 - 4{\cdot}2\,\theta\} \\
420\,\theta - 9240 &= 8043 - 12{\cdot}6\,\theta \\
432{\cdot}6\,\theta &= 17283 \\
\theta &= 39{\cdot}95^\circ\text{C}
\end{aligned}$$

Example. Steam is tapped from an intermediate stage of a steam turbine at a pressure of 2·5 bar, its dryness fraction being 0·95, and passed to a contact feed heater. The hot water at 48°C is pumped into the heater where it mixes with the heating steam. Estimate the temperature of the feed water leaving the heater when the amount of steam tapped off is 9% of the steam supplied to the turbine.

Referring to Fig. 41, let the mass of steam supplied to the engine be 1 kg, then 0·09 kg is tapped off to the heater. This leaves (1 – 0·09) = 0·91 kg of steam to continue through the engine, into the condenser, as water pumped into the feed heater. Being a *contact* heater, the 0·09 kg of heating steam makes contact and mixes with the 0·91 kg of condensate, making 1 kg of feed water to be pumped back into the boilers.

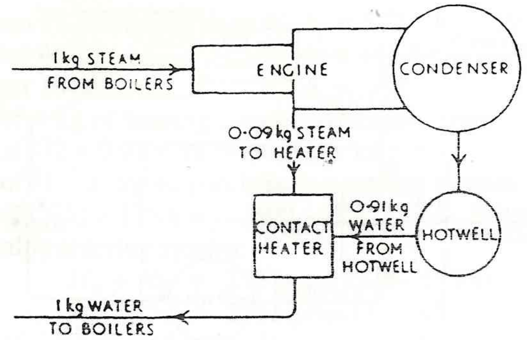


Fig. 41

Tables page 4, steam 2·5 bar, $h_f = 535$, $h_{fg} = 2182$
Tables page 2, water 48°C, $h = 200{\cdot}9$

Enthalpy of steam entering heater:

$$= 0{\cdot}09(h_f + xh_{fg})$$
$$= 0{\cdot}09(535 + 0{\cdot}95 \times 2182) = 234{\cdot}7 \text{ kJ}$$

Enthalpy of water entering heater:

$$= 0{\cdot}91 \times 200{\cdot}9 = 182{\cdot}8 \text{ kJ}$$

Total enthalpy of steam and water entering heater:

$$= 234{\cdot}7 + 182{\cdot}8 = 417{\cdot}5 \text{ kJ}$$

Total enthalpy of water leaving heater is that of 1 kg of feed water, therefore the *specific* enthalpy of the feed water is 417·5 kJ/kg which we now look for in the tables and find the near figure of 417 kJ/kg on page 3 for a temperature of 99·6°C.

∴ Temperature of feed water = 99·6°C Ans.

Alternatively for the latter part, we could assume a mean specific heat of 4·2 kJ/kg K for the feed water and take the enthalpy of water at θ°C as the heat energy to be transferred to it to raise its temperature from 0 to θ°C (mass × spec. heat × temp. rise), then:

$$1 \times 4{\cdot}2 \times \theta = 417{\cdot}5$$

$$\theta = \frac{417{\cdot}5}{4{\cdot}2} = 99{\cdot}4^\circ\text{C}$$

Example. Steam is bled from the main steam pipe line at a pressure of 17 bar and dryness fraction 0·98, to a surface feed heater, and the remainder passes through the engine. The condensate at 42°C from the engine condenser, and the drain from the feed heater, passes to the hotwell. Calculate the percentage of the main steam bled off to the heater if the temperature of the feed water to the

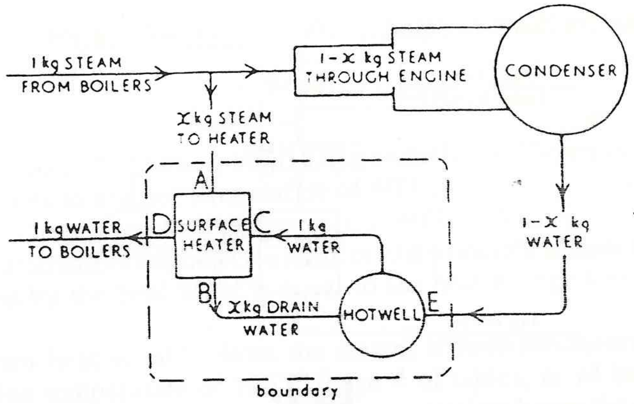


Fig. 42

boilers is 104·8°C.

The arrangement is illustrated in Fig. 42. Being a *surface* heater, there is no mixing of the heating steam and feed water. In this type, the water passes through nests of tubes which are heated on the outside by the steam.

Let H_A = enthalpy of steam entering heater
H_B = ... drain water leaving heater
H_C = ... water entering heater from hotwell
H_D = ... water leaving heater (feed to boilers).
H_E = ... water entering hotwell from condenser.

Enthalpy of drain water and condensate entering hotwell
= Enthalpy of water leaving hotwell

$H_B + H_E = H_C$ .... (i)

Enthalpy of heating steam and hotwell water entering heater
= Enthalpy of drain and feed water leaving heater

$H_A + H_C = H_B + H_D$ ....(ii)

Substituting value of H_C from (i) into (ii):

$H_A + H_B + H_E = H_B + H_D$

$\therefore H_A + H_E = H_D$

From this we can see that, since there is no heat energy transfer to or from an external source, the heater and hotwell with their connections can be considered as one common system as shown by the dotted boundary line on Fig. 42.

Let the mass of steam supplied from the boilers = 1 kg, then 1 kg of water is fed back to the boilers.

Let x kg of the supply steam be the amount bled off to the heater, then (1 − x) kg is the mass of steam passed on to the engine.

Steam 17 bar, tables page 4, $h_f = 872$ $h_{fg} = 1923$
Water 42°C, ... 2, $h = 175·8$
Water 104·8°C ... ... 4, $h = 439$

Enthalpy of x kg of heating steam entering system:

$H_A = x(872 + 0·98 \times 1923) = 2757x$ kJ

Enthalpy of $(1 - x)$ kg of condensate entering system:

$H_E = (1 - x) \times 175·8 = 175·8 - 175·8x$ kJ

Total enthalpy entering system:

$H_A + H_E = 2757x + 175·8 - 175·8x$
$= 2581·2x + 175·8$ kJ ... ... (i)

Enthalpy of 1 kg of feed water leaving system:

$H_D = 439$ kJ ... ... (ii)

Total entry $H_A + H_E$ = Total exit H_D

$2581·2x + 175·8 = 439$
$2581·2x = 263·2$
$x = 0·102$

As a percentage of the main steam supply,

Bled steam = 10·2% Ans.

Note: Feed heating increases cycle efficiency.

THROTTLING OF STEAM

Throttling is the process of passing the steam through a restricting orifice or a partially opened valve which causes a "wire-drawing" effect and reduces the pressure. There is not sufficient time for any appreciable heat transfer to take place between the steam and its surrounds and, as there are no moving parts, expansion takes place freely and no work is done by the steam. The difference in the velocity of the steam before and after is sufficiently small to be negligible. Consequently, if there is no transfer or conversion of heat energy, then there is no change in the enthalpy of the steam, that is, enthalpy after throttling is equal to enthalpy before throttling.

One important application of throttling takes place in the steam reducing valve. This is a specially designed valve connected to the high pressure steam range, the restricted valve opening is spring controlled and set to maintain a near steady reduced pressure at the outlet, the reduced pressure steam being more suitable for some pumps and other auxiliaries.

Example. Steam is passed through a reducing valve and reduced in pressure from 20 bar to 7 bar. Find the effect of throttling on the temperature and quality of the steam if the high pressure steam was (a) wet, having a dryness fraction of 0·9, (b) dry saturated, (c) superheated to a temperature of 300°C.

Tables pages 4 and 7

7 bar,	$t_s = 165°C$,	$h_f = 697$,	$h_{fg} = 2067$,	$h_g = 2764$
20 bar,	$t_s = 212{\cdot}4°C$,	$h_f = 909$,	$h_{fg} = 1890$,	$h_g = 2799$, $h_{sup} = 3025$

(a) 20 bar, dryness 0·9:

$$h = h_f + xh_{fg}$$
$$= 909 + 0{\cdot}9 \times 1890 = 2610$$

2610 is less than h_g for 7 bar, therefore reduced steam is wet

$\therefore$ 7 bar, dryness x, $h = h_f + xh_{fg}$

enthalpy before = enthalpy after

$$2610 = 697 + x \times 2067$$
$$2067x = 1913$$
$$x = 0{\cdot}9253$$

Effect of throttling:

Temperature is reduced from 212·4°C to 165°C
Dryness is increased from 0·9 to 0·9253 } Ans. (a)

(b) 20 bar dry sat. $h_g = 2799$
7 bar dry sat. $h_g = 2764$
enthalpy before = enthalpy after

hence, enthalpy at 7 bar is 2799, which is 35 kJ/kg higher than its h_g and is therefore superheated.

From page 7 of tables, for 7 bar, $h = 2799$, lies between $h_g = 2764$ at 165°C and $h = 2846$ at 200°C

by interpolation,

7 bar,	$h = 2846$	for 200°C
	$h = 2764$	for 165°C
difference in	$h = 82$ kJ	for 35°C

$$\text{difference for 35 kJ} = \frac{35}{82} \times 35 = 14{\cdot}94° \text{ say } 15°$$

$\therefore$ reduced pressure steam has 15°C of superheat, its temperature being 165 + 15 = 180°C.

Effect of throttling:

Temperature is reduced from 212·4°C to 180°C
Steam has 15° of superheat } Ans. (b)

(c) 20 bar, temperature 300°C, $h = 3025$
(steam has 300 − 212·4 = 87·6°C of superheat)

$h = 3025$ is more than h_g for 7 bar, reduced pressure steam is therefore superheated.

enthalpy before = enthalpy after

From page 7 of tables, for 7 bar, $h = 3025$, lies between $h = 2955$ at 250°C and $h = 3060$ at 300°C.

	$h = 3060$	for 300°C
	$h = 2955$	for 250°C
difference in	$h = 105$ kJ	for 50°C

$\therefore$ difference in temperature for (3025 − 2955) = 70 kJ is

$$\frac{70}{105} \times 50 = 33{\cdot}3°C$$

$\therefore$ temperature = 250 + 33·3 = 283·3°C

Degree of superheat = 283·3 − 165
= 118·3°C

Effect of throttling:

Temperature is reduced from 300°C to 283·3°C
Degree of superheat is increased from 87·6 to 118·3 } Ans. (c)

Note that in every case the effect of throttling is to (i) reduce the pressure, (ii) reduce the temperature, (iii) increase the quality, that is, either to produce drier steam or to increase the degree of superheat.

With regard to finding the degree of superheat at the lower pressure, if the mean specific heat of superheated steam over this range were given, the degree of superheat could be obtained from,

Heat energy to raise temperature = mass × spec. heat × temp. rise

therefore, for 1 kg

Increase of enthalpy above h_g = mass × spec. heat × temp. rise

or, $h - h_g = c_P(t - t_{sat})$

where t = temperature of the steam

Taking the last case (c) as an example, if the mean specific heat was given as 2·21 kJ/kg K:

$$h - h_g = c_P \times \text{degree of superheat}$$

$$\text{degree of superheat} = \frac{3025 - 2764}{2{\cdot}21} = 118{\cdot}1°C$$

THROTTLING CALORIMETER

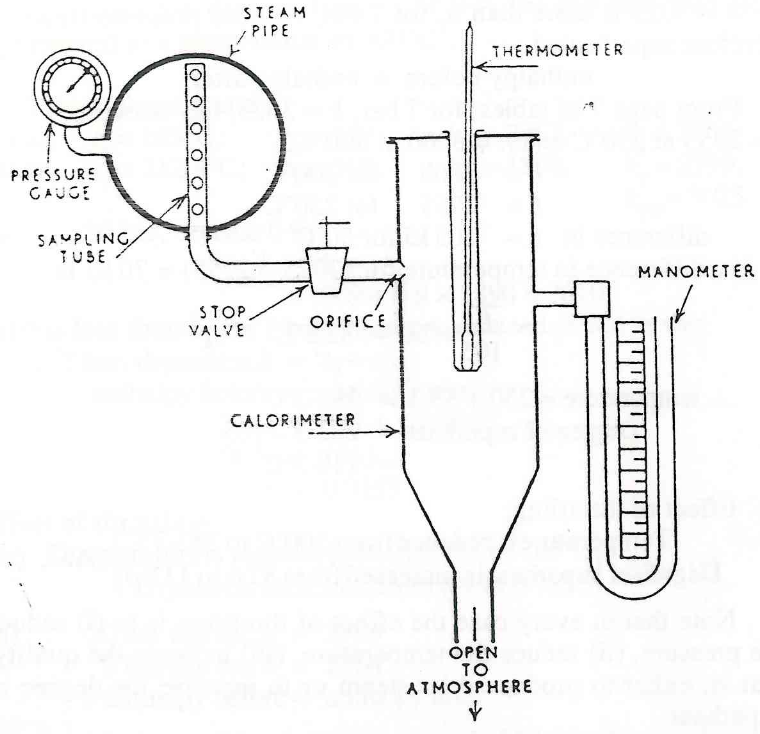


Fig. 43

The throttling calorimeter, an instrument for measuring the dryness fraction of steam, is illustrated diagrammatically in Fig. 43.

The principle of operation depends upon the fact that if steam is reduced in pressure by a throttling process, the enthalpy after throttling is equal to the enthalpy before throttling, as previously explained.

Example. Steam at a pressure of 14 bar from the main steam pipe is passed through a throttling calorimeter. The pressure in the calorimeter is 1·2 bar and the temperature 119°C. Taking the specific heat of the low pressure superheated steam as 2·0 kJ/kg K calculate the dryness fraction of the main steam.

From tables page 4,

1·2 bar, $t_s = 104{\cdot}8$, $h_g = 2683$

14 bar, $h_f = 830$, $h_{fg} = 1960$

$$\begin{aligned}\text{Enthalpy before throttling} &= \text{Enthalpy after}\\ 830 + x \times 1960 &= 2683 + 2(119 - 104{\cdot}8)\\ 830 + 1960\,x &= 2683 + 28{\cdot}4\\ 1960\,x &= 1881{\cdot}4\\ x &= 0{\cdot}9599 \text{ say } 0{\cdot}96. \quad \text{Ans.}\end{aligned}$$

It will be noted that the throttling calorimeter has its limitations. The dryness fraction can only be determined when the temperature of the throttled steam in the calorimeter is higher than its saturation temperature corresponding to its pressure, that is, when it is superheated. If the throttled steam was not superheated, the thermometer would record the saturation temperature whether it was dry or wet and its condition would not be known.

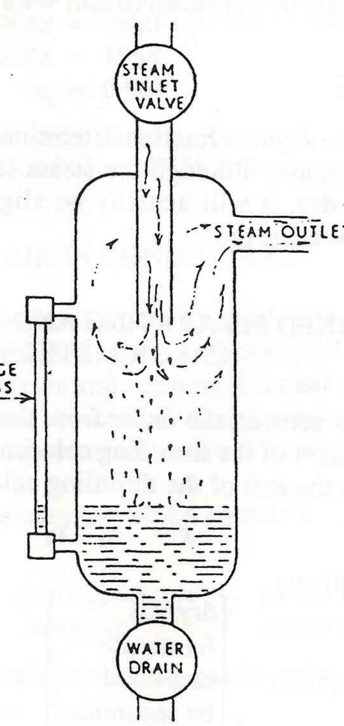


Fig. 44

SEPARATING CALORIMETER

When very wet steam samples are to be tested an *approximate* value for the dryness fraction may be determined by passing the steam through a *separating calorimeter* (Fig. 44).

The separated water collects in the bottom of the separator which is later drained off and measured, let this be m_1. The remaining steam passes out through the outlet and is led to a small condenser where it is condensed into water, collected and measured, let this be m_2:

From the sample of wet steam taken,

m_1 = mass of water separated from the sample
m_2 = mass of steam (assumed dry) remaining

Dryness fraction

$$= \frac{\text{mass of pure steam}}{\text{total mass of wet steam (steam + water mixture)}}$$

$$= \frac{m_2}{m_2 + m_1}$$

However, the dryness fraction determined in this manner is only approximate because, although the steam leaving the separator is assumed to be dry, it will actually be slightly wet since perfect separation is not achieved.

COMBINED SEPARATING AND THROTTLING CALORIMETER

In this arrangement, the outlet from the separator is connected directly to the inlet of the throttling calorimeter, and the condenser is connected to the exit of the throttling calorimeter.

$$x = x_1 \times x_2$$

In words this is:

$$\text{Dryness fraction} = \left\{\begin{matrix}\text{dryness}\\ \text{fraction as}\\ \text{measured}\\ \text{by separating}\\ \text{calorimeter}\end{matrix}\right\} \times \left\{\begin{matrix}\text{dryness}\\ \text{fraction as}\\ \text{measured}\\ \text{by throttling}\\ \text{calorimeter}\end{matrix}\right\}$$

Example. Steam at a pressure of 8 bar was tested by passing a sample through a combined separating and throttling calorimeter. The mass of water collected in the separator was 0·25 kg and the mass of condensate collected from the condenser after throttling was 2·5 kg. The pressure of the steam in the throttling calorimeter was 1·1 bar and its temperature 106·8°C. Taking the specific heat of the superheated steam after throttling as 2·0 kJ/kgK, find the dryness fraction of the steam sample.

From tables page 4,

8 bar, $h_f = 721$ $\quad h_{fg} = 2048$
1·1 bar, $t_s = 102{\cdot}3$°C, $\quad h_g = 2680$

Dryness fraction by separating calorimeter:

$$x_1 = \frac{m_2}{m_2 + m_1} = \frac{2{\cdot}5}{2{\cdot}5 + 0{\cdot}25} = 0{\cdot}909$$

Dryness fraction by throttling calorimeter:

$$\begin{aligned}\text{Enthalpy before throttling} &= \text{Enthalpy after}\\ 721 + x_2 \times 2048 &= 2680 + 2(106{\cdot}8 - 102{\cdot}3)\\ x_2 \times 2048 &= 1968\\ x_2 &= 0{\cdot}961\end{aligned}$$

Dryness fraction of steam sample:
$x = x_1 \times x_2 = 0{\cdot}909 \times 0{\cdot}961 = 0{\cdot}8736$. Ans.

f AIR IN CONDENSERS

It was stated in Chapter 5 under Dalton's law of partial pressures that the pressure exerted in a vessel occupied by a mixture of gases, or a mixture of gases and vapours, is equal to the sum of the pressures that each would exert if it alone occupied the whole volume of the vessel. The pressure exerted by each gas is termed a *partial pressure*.

Applying this to condensers which contain a mixture of leakage air and steam:

$$\text{Total pressure of mixture} = \text{partial pressure due to air} + \text{partial pressure due to steam}$$

This enables an estimate to be made on the amount of air leakage into condensers.

Example. The pressure in a condenser is 0·12 bar and the temperature is 43·8°C. If the internal volume of the condenser is 8·5 m^3, estimate the mass of air in the condenser, taking R for air = 0·287 kJ/kg K. If the dryness fraction of the steam is 0·9, calculate the mass of steam present.

Since there is always some water condensate in the condenser and the steam is in contact with this water, then the steam is at its saturation temperature and the pressure of the steam in the condenser depends upon this temperature. From the tables, page 3, we see that for a saturation temperature of 43·8°C the pressure of the steam is 0·09 bar:

$$\begin{aligned}\text{Partial pressure due to air} &= \text{total pressure} - \text{steam pressure}\\ &= 0{\cdot}12 - 0{\cdot}09\\ &= 0{\cdot}03 \text{ bar} = 3 \text{ kN/m}^2\end{aligned}$$

From $pV = mRT$, mass of air present:

$$m = \frac{3 \times 8{\cdot}5}{0{\cdot}287 \times (43{\cdot}8 + 273)}$$

$$= 0{\cdot}2804 \text{ kg.} \quad \text{Ans. (i)}$$

Also from tables page 3, for saturation temperature of 43·8°C, specific volume of dry saturated steam $v_g = 16{\cdot}2$ m³/kg.

For wet steam

$$v = xv_g = 0{\cdot}9 \times 16{\cdot}2 = 14{\cdot}58 \text{ m}^3\text{/kg}$$

∴ mass of steam in the condenser volume of 8·5 m³

$$= \frac{8{\cdot}5}{14{\cdot}58} = 0{\cdot}583 \text{ kg} \quad \text{Ans. (ii)}$$

ABSOLUTE HUMIDITY is the ratio of the mass of water vapour to the mass of dry air (sometimes called specific humidity or moisture content).

$$\omega = \frac{m_{WV}}{m_A} = \frac{\rho_{WV}}{\rho_A}$$

$$\omega\% = \frac{v_A}{v_{WV}} \times 100$$

(Relative humidity is sometimes used - ratio of partial pressures of vapour i.e. actual to saturated).

TEST EXAMPLES 10

1. Steam at a pressure of 9 bar is generated in an exhaust gas boiler from feed water at 80°C. If the dryness fraction of the steam is 0·96, determine the heat transfer per kg of steam produced.

2. A turbo-generator is supplied with superheated steam at a pressure of 30 bar and temperature 350°C. The pressure of the exhaust steam from the turbine is 0·06 bar with a dryness fraction of 0·88.
(i) Calculate the enthalpy drop per kg of steam through the turbine.
(ii) If the tubine uses 0·5 kg of steam per second, calculate the power equivalent of the total enthalpy drop.

3. Steam enters the superheaters of a boiler at a pressure of 20 bar and dryness 0·98, and leaves at the same pressure at a temperature of 350°C. Find (i) the heat energy supplied per kg of steam in the superheaters, and (ii) the percentage increase in volume due to drying and superheating.

4. One kg of wet steam at a pressure of 8 bar and dryness 0·94 is expanded until the pressure is 4 bar. If expansion follows the law pV^n = a constant, where n = 1·2, find the dryness fraction of the steam at the lower pressure.

5. Dry saturated steam at a pressure of 2·4 bar is tapped off the inlet branch of a low pressure turbine to supply heating steam in a contact feed heater. The temperature of the feed water inlet to the heater is 42°C and the outlet is 99·6°C. Find the percentage mass of steam tapped off.

6. Wet saturated steam at 16 bar and dryness 0·98 enters a reducing valve and is throttled to a pressure of 8 bar. Find the dryness fraction of the reduced pressure steam.

7. Steam of mass 0·2 kg trapped within a cylinder is allowed to expand at constant pressure from an initial temperature 165°C and dryness fraction of 0·75 until its volume is doubled.
Calculate:
(a) the final temperature;
(b) the work done;
(c) the heat energy transferred.

8. A combined separating and throttling calorimeter was connected to a main steam pipe carrying steam at 15 bar and the following data recorded:

Mass of water collected in separator = 0·55 kg

Mass of condensate after throttling = 10 kg
Press. of steam in throttling calorimeter = 1·1 bar
Temp. ... ... = 111°C

Taking the specific heat of the throttled superheated steam as 2·0 kJ/kg K, find the dryness fraction of the main steam.

f 9. During the process of raising steam in a boiler, when the pressure was 1·9 bar gauge the temperature inside the boiler was 130°C, and when the pressure was 6·25 bar gauge the temperature was 165°C. If the volume of the steam space is constant at 4·25 m^3 calculate the masses of steam and air present in each case. Take R for air = 0·287 kJ/kg K, atmospheric pressure = 1 bar, and assume the steam is dry in each case.

f 10. A sample of air is saturated with water vapour (i.e. the water vapour in the air is saturated vapour) at 20°C, the total pressure being 1 bar.

Calculate:

(a) the partial pressures of the oxygen, nitrogen and water vapour;

(b) the absolute humidity.

Air is 21% O_2 and 79% N_2 by volume and for air R = 287 J/kg K.

Note: Absolute humidity is the mass of water vapour per unit mass of dry air.

f CHAPTER 11

ENTROPY

Mechanical energy, being the product of two quantities can be represented as an area on rectangular axes. Fig. 45 is a diagram in which the ordinates represent pressure in kN/m^2 and the abscissae represent volume in m^3, the area enclosed represents mechanical work, kN/m^2 × m^3 = kNm = kJ. The availability to do work depends upon the magnitude of the pressure.

In a similar manner, heat energy can be represented as an area. The availability to transfer heat energy depends upon the temperature and therefore the ordinates of a heat energy diagram represent absolute temperature. The abscissae are termed *entropy*. The symbol for entropy is S, change of entropy may be written ΔS and the units are kilojoule per kelvin [kJ/K]. Specific entropy, which is entropy per unit mass, is represented by the symbol s and the units are kilojoule per kilogramme kelvin [kJ/kg K].

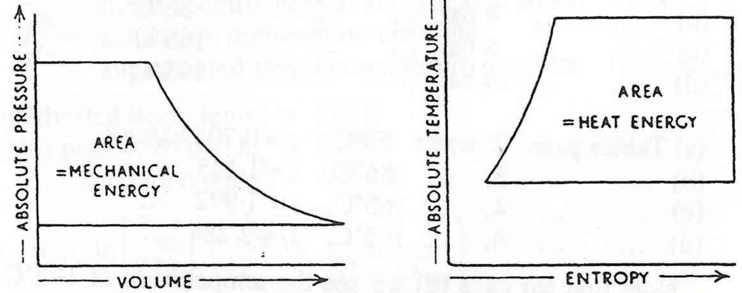


Fig. 45

Thus a diagram whose area represents heat energy, the ordinates represent absolute temperatue (T) and the abscissae represent entropy (S), is referred to as a temperature-entropy (T–S) diagram, as shown in Fig. 45.

ENTROPY OF WATER AND STEAM

Subscripts to distinguish the specific entropy of saturated liquids and vapours are used as for other properties, explained in the previous chapter, thus:

s_f = specific entropy of saturated liquid
s_g = ... vapour
s_{fg} = $s_g - s_f$ = specific entropy of evaporation from liquid to vapour
s = specific entropy of either liquid or vapour at any other state.

As for internal energy and enthalpy, the values of specific entropy measured from the datum of water at 0°C are listed in the steam tables. (Thermodynamic and Transport Properties of Fluids, SI units, by Y. R. Mayhew and G. F. C. Rogers).

WATER. The properties of liquids depend almost entirely on temperature, therefore when looking up the properties of water we look for those against its particular temperature, disregarding its pressure unless it is saturated.

Example. To read from the tables the specific entropy of water at (a) pressure of 1 bar and temperature 50°C
(b) ... ... 4 bar ... ... 86°C
(c) ... ... 8 bar ... ... 165°C
(d) ... ... 14 bar ... ... 195°C

(a) Tables page 2, water 50°C, $s = 0{\cdot}704$ kJ/kg K
(b) ... ... 3, ... 86°C, $s = 1{\cdot}145$ ...
(c) ... ... 4, ... 165°C, $s = 1{\cdot}992$ ...
(d) ... ... 4, ... 195°C, $s_f = 2{\cdot}284$ ...

Note that for case (d) we see the temperature of 195°C is the saturation temperature corresponding to its pressure of 14 bar, therefore we use the suffix f to signify that the water is at its saturation temperature.

STEAM. Example. To find the specific entropy of saturated water and dry saturated steam at pressures (a) 0·1 bar, (b) 5 bar, (c) 20 bar, (d) 50 bar.

(a) Tables page 2, 0·1 bar, $s_f = 0{\cdot}649$, $s_g = 8{\cdot}149$
(b) ... ... 4, 5 bar, $s_f = 1{\cdot}860$, $s_g = 6{\cdot}822$
(c) ... ... 4, 20 bar, $s_f = 2{\cdot}447$, $s_g = 6{\cdot}340$
(d) ... ... 5, 50 bar, $s_f = 2{\cdot}921$, $s_g = 5{\cdot}973$

Example. To find the specific entropy of wet steam (a) pressure 0·2 bar, dryness 0·8 (b) pressure 6 bar, dryness 0·9 (c) pressure 44 bar, dryness 0·95.

Specific entropy of wet steam of dryness fraction x is obtained in a similar manner as for specific enthalpy, previously explained:

$$s = s_f + xs_{fg}$$

(a) Tables page 3, $s = 0{\cdot}832 + 0{\cdot}8 \times 7{\cdot}075 = 6{\cdot}492$
(b) ... ... 4, $s = 1{\cdot}931 + 0{\cdot}9 \times 4{\cdot}830 = 6{\cdot}278$
(c) ... ... 5, $s = 2{\cdot}849 + 0{\cdot}95 \times 3{\cdot}180 = 5{\cdot}870$

Example. Using the following formula, calculate the entropy per kg of superheated steam at a pressure of 20 bar and temperature 300°C, taking the mean specific heat of water as 4·25 kJ/kg K and the mean specific heat of superheated steam as 2·58 kJ/kg K.

Compare the result with the value of entropy given in the tables.

$$s = c_w \ln\left(\frac{T_{sat}}{273}\right) + \frac{h_{fg}}{T_{sat}} + c_{sup} \ln\left(\frac{T}{T_{sat}}\right)$$

c_w = mean specific heat of water
c_{sup} = mean specific heat of superheated steam
T_{sat} = saturation temperature absolute
T = superheated steam temperature absolute

Superheated steam temp. = 573 K
Tables page 4, 20 bar, h_{fg} = 1890
Saturation temp. = 485·4 K

$$s = c_w \ln\left(\frac{T_{sat}}{273}\right) + \frac{h_{fg}}{T_{sat}} + c_{sup} \ln\left(\frac{T}{T_{sat}}\right)$$

$$= 4{\cdot}25 \ln\left(\frac{485{\cdot}4}{273}\right) + \frac{1890}{485{\cdot}4} + 2{\cdot}58 \ln\left(\frac{573}{485{\cdot}4}\right)$$

$$= 4{\cdot}25 \ln 1{\cdot}778 + 3{\cdot}894 + 2{\cdot}58 \ln 1{\cdot}18$$

$$= 2{\cdot}446 + 3{\cdot}894 + 0{\cdot}427$$

$$= 6{\cdot}767 \text{ kJ/kg K} \quad \text{Ans. (i)}$$

From tables, page 7, press. 20 bar, temp. 300°C:

s = 6·768 kJ/kg K. Ans. (ii)

A TEMPERATURE–ENTROPY (T–S) DIAGRAM is a clear method of illustrating the process of generating steam.

Referring to Fig. 46, consider 1 kg of water commencing at a

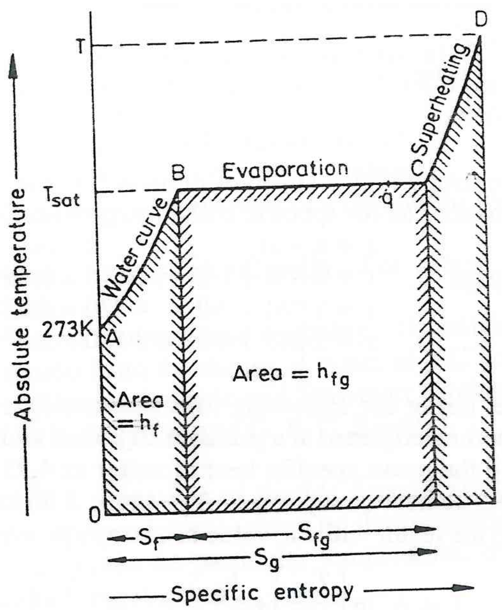


Fig. 46

temperature of 0°C = 273 K, as heat energy is transferred to the water its temperature rises and its entropy is increased, the heating of the water up to its boiling point is represented by the water curve AB. At any point on this water curve, the ordinate is the absolute temperature of the water, and the abscissa is its specific entropy (i.e. for 1 kg here) above that for water at 0°C. The heat energy transferred to the water to raise its temperature is the area of the diagram under the curve AB.

On reaching the saturation temperature corresponding to its pressure, point B, evaporation commences and this continues at constant temperature as the entropy is increased, therefore the evaporation process appears as a horizontal straight line. At point C evaporation is complete and dry saturated steam is produced. The latent heat energy of evaporation is represented by the area of the diagram under the evaporation line BC. Any intermediate point, such as q, along the evaporation line represents incomplete evaporation, that is, the steam is wet, the dryness fraction (x) is represented by the ratio Bq/BC and the heat energy of evaporation is the area under that part of the evaporation line, which is xh_{fg}.

When further heat energy is transferred to dry saturated steam at constant pressure, it becomes superheated and the temperature

rises as its entropy increases. This is shown by the superheat curve CD, the area of the diagram under this curve is the heat energy added to superheat the steam.

TEMPERATURE-ENTROPY CHART FOR STEAM

A temperature-entropy (T–S) chart is a complete diagram drawn up within the range of temperatures normally required. A simplified chart, for steam, is illustrated in Fig. 47. The temperature scale here is in °C but this is not usual. Entropy here is specific.

The WATER LINE gives the relation between the temperature of the water and its specific entropy. Curve AB on Fig. 46 is part of this line.

The DRY SATURATED STEAM LINE is produced by drawing a curve through points scaled off to the right from the water line, representing the increase in entropy for complete evaporation of one kg of water into steam at the various temperatures. Any point on this line represents dry saturated steam conditions at the given level of temperature. Point C on Fig. 46 lies on this line. Inside this line the steam is wet, outside this line the steam is superheated.

SUPERHEAT LINES slope steeply upwards from points on the dry saturated steam line. These give the relation between the temperature of the steam and its specific entropy when the steam is superheated at constant pressure. CD on Fig. 46 is one of these lines.

LINES OF CONSTANT DRYNESS FRACTIONS are drawn within the wet steam region, that is, between the two main boundary lines of the water and dry saturated steam. These are plotted by scaling off the values of xs_{fg} from the water line. For instance, if 0·9 of the values of specific entropy of complete evaporation for a series of temperatures were scaled off from the water line, the line drawn through these points is the dryness fraction line of 0·9. Lines for dryness fractions of 0·8, 0·7, etc., down to 0·1 are obtained in a similar manner.

CONSTANT VOLUME LINES are also included. Each constant volume line represents one particular volume occupied by one kg of saturated steam under varying conditions of quality from dry to very wet.

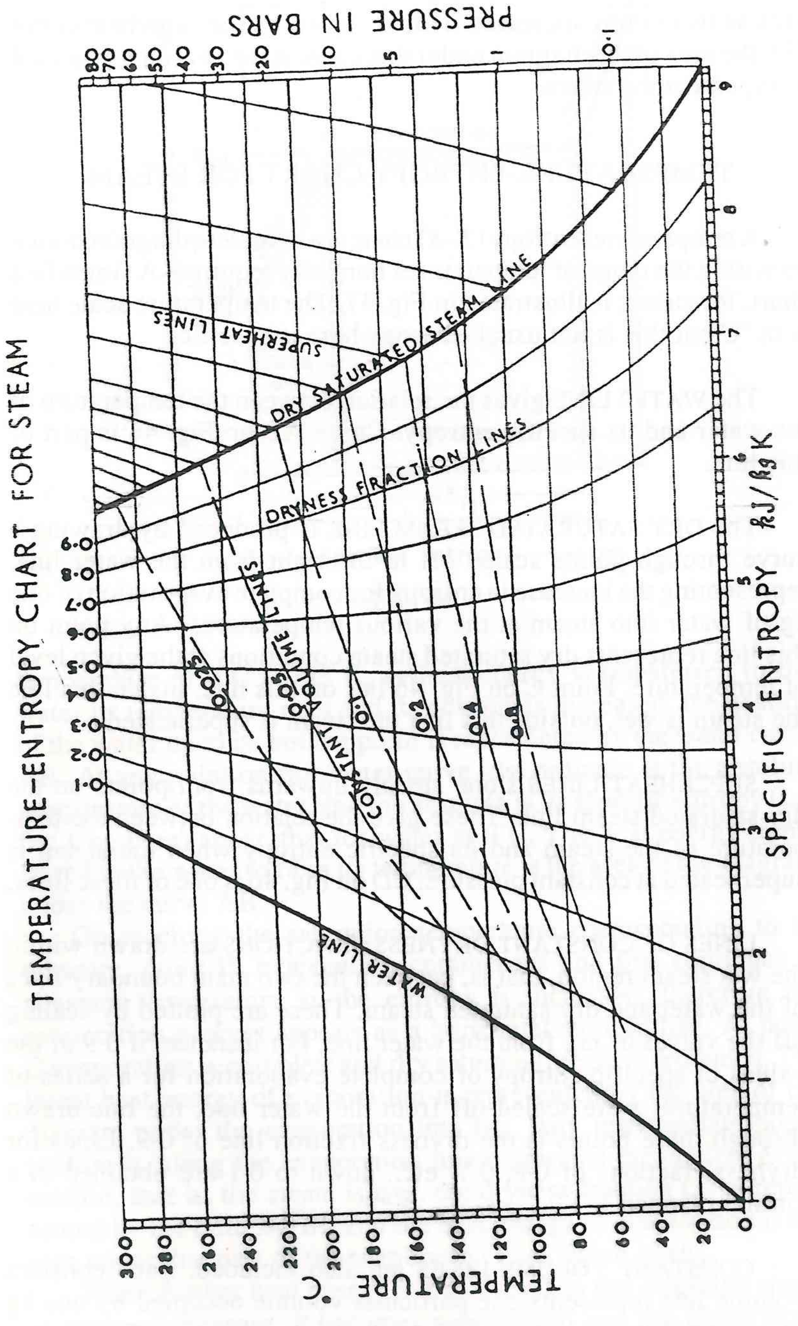


Fig. 47

Taking a volume of 0·1 m^3 as an example, we read from the tables that 0·1 m^3 is the volume occupied by 1 kg of dry saturated steam when its temperatue is 212·2°C. The curve for a volume of 0·1 m^3 therefore begins at this point on the dry saturated steam line at the level of 212·2°C of temperature.

Now taking some other temperature, say 195°C, we note that the specific volume of dry saturated steam is given as 0·1408 m^3/kg hence, for 1 kg of steam of 195°C to occupy a volume of 0·1 m^3 it must be wet, and its dryness fraction will be 0·1 ÷ 0·1408 = 0·7103. At another temperature, say 165°C, v_g = 0·2728 m^3/kg for 1 kg of steam at 165°C to occupy a volume of 0·1 m^3 the dryness fraction will be 0·1 ÷ 0·2728 = 0·3665. Plotting these three corresponding points of temperature and dryness fractions and drawing a curve through them, the constant volume line for 0·1 m^3 is produced.

The above neglects the volume occupied by the water in the wet steam as being negligible, and obviously more than three plotted points are needed to obtain a true curve. Constant volume lines for various volumes are given covering such a range as likely to be met in practice.

ISOTHERMAL AND ISENTROPIC PROCESSES

During an isothermal process the temperature remains constant, therefore an isothermal operation is represented on a temperature-entropy diagram by a straight horizontal line.

During an adiabatic process, no heat transfer takes place to or from the surroundings, a reversible adiabatic process is *isentropic*, that is, it takes place without change of entropy. Therefore, the entropy after the process is equal to the entropy before, hence, an isentropic process appears on the temperature-entropy diagram as a straight vertical line. The Carnot cycle for a vapour, which is the same for a gas (see Fig. 48), shows these processes.

EFFECT OF ISENTROPIC EXPANSION ON QUALITY. By drawing vertical lines down from the initial steam temperature on the temperature-entropy chart to represent isentropic expansion, it will be seen that dry saturated steam becomes wet during isentropic expansion, steam of normal wetness becomes wetter, and steam which is very wet initially becomes slightly dryer. The T–s diagram of the Rankine cycle (Fig. 61 of Chapter 12) illustrates this.

Example. Steam at a pressure of 14 bar expands isentropically to a pressure of 2·7 bar, find the dryness fraction at the end of expansion if the dryness of the steam at the beginning of expansion is (a) 1·0, (b) 0·8, (c) 0·6, (d) 0·4.

Tables page 4,

14 bar, $s_f = 2{\cdot}284$ $s_{fg} = 4{\cdot}185$ $s_g = 6{\cdot}469$

2·7 bar, $s_f = 1{\cdot}634$ $s_{fg} = 5{\cdot}393$

Entropy after expansion = Entropy before

(a) $1{\cdot}634 + x \times 5{\cdot}393 = 6{\cdot}469$

$x \times 5{\cdot}393 = 4{\cdot}835$

$x = 0{\cdot}8964$ Ans. (a)

(b) $1{\cdot}634 + x \times 5{\cdot}393 = 2{\cdot}284 + 0{\cdot}8 \times 4{\cdot}185$

$x \times 5{\cdot}393 = 3{\cdot}998$

$x = 0{\cdot}7415$ Ans. (b)

(c) $1{\cdot}634 + x \times 5{\cdot}393 = 2{\cdot}284 + 0{\cdot}6 \times 4{\cdot}185$

$x \times 5{\cdot}393 = 3{\cdot}161$

$x = 0{\cdot}5861$ Ans. (c)

(d) $1{\cdot}634 + x \times 5{\cdot}393 = 2{\cdot}284 + 0{\cdot}4 \times 4{\cdot}185$

$x \times 5{\cdot}393 = 2{\cdot}324$

$x = 0{\cdot}4309$ Ans. (d)

The above results can be checked graphically on the temperature-entropy chart, by drawing a vertical line from the temperature level corresponding to 14 bar, which is 195°C, and the appropriate dryness, down to the temperature level corresponding to 2·7 bar, which is 130°C, and reading off the dryness fraction at the lower temperature.

Example. Superheated steam at a pressure of 20 bar and temperature 300°C is expanded isentropically. (i) At what pressure will the steam be just dry and saturated? (ii) What will be the dryness fraction if it is expanded to a pressure of 0·04 bar?

Tables page 7, 20 bar 300°C, $s = 6{\cdot}768$

Tables page 3, 0·04 bar, $s_f = 0{\cdot}422$ $s_{fg} = 8{\cdot}051$

(i) Since entropy at 20 bar 300°C is 6·768, dry saturated steam at a lower pressure is to have the same entropy.

Tables page 4, gives $s_g = 6{\cdot}761$ for 6 bar, therefore, pressure when steam is dry sat. = practically 6 bar. Ans. (i).

(ii) Final entropy = Initial entropy

$0{\cdot}422 + x \times 8{\cdot}051 = 6{\cdot}768$

$x \times 8{\cdot}051 = 6{\cdot}346$

$x = 0{\cdot}7881$ Ans. (ii)

T–s DIAGRAM FOR GASES

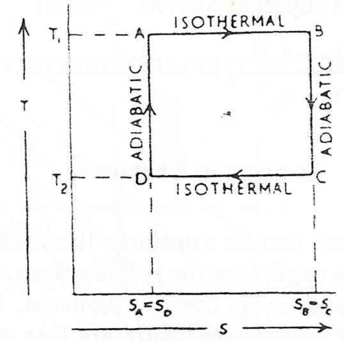


Fig. 48

This provides a good illustration of isothermal and isentropic processes.

As an example, Fig. 30 in Chapter 8 shows the pressure-volume diagram of the Carnot cycle for a gas, this on a temperature-entropy diagram appears as shown in Fig. 48, and, referring to this:

$$\text{Efficiency} = \frac{\text{heat supplied} - \text{heat rejected}}{\text{heat supplied}}$$

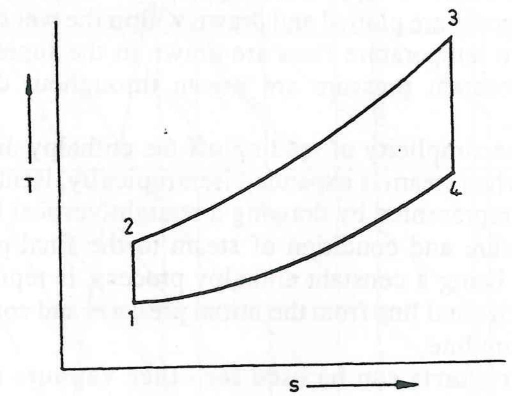


Fig. 49

$$\text{Efficiency} = 1 - \frac{\text{heat rejected}}{\text{heat supplied}}$$

$$= 1 - \frac{T_2(s_C - s_D)}{T_1(s_B - s_A)}$$

$$= 1 - \frac{T_2}{T_1} \quad \text{(as given in Chapter 8)}$$

Other ideal cycles can be similarly illustrated - for example, Fig. 49 is for the Otto cycle (see the p-V diagram, Fig. 26 of Chapter 8 (also see for the Joule cycle the T–s diagram, Fig. 67 of Chapter 12; there the lines of constant pressure are less steep than the lines of constant volume here).

ENTHALPY–ENTROPY CHART FOR STEAM

A diagram drawn with specific entropy of steam as the base, and the vertical ordinates representing specific enthalpy, is a most useful chart for solving problems or checking results of calculations. Fig. 50 is a simplified chart. Complete full-size enthalpy-entropy (h–s) charts to scale are used.

That part under the saturation line is referred to as the wet steam region, and above it represents superheated conditions. Lines of constant dryness are plotted and drawn within the wet steam region, and constant temperature lines are drawn in the superheat region. Lines of constant pressure are drawn throughout the complete diagram.

Note the simplicity of reading off the enthalpy drop and final condition when steam is expanded isentropically, isentropic expansion being represented by drawing a straight vertical line from the initial pressure and condition of steam to the final pressure line. Throttling, being a constant enthalpy process, is represented by a straight horizontal line from the initial pressure and condition to the final pressure line.

Similar charts can be used for other vapours (e.g. refrigerants) and gases. The theoretical basis is also the same.

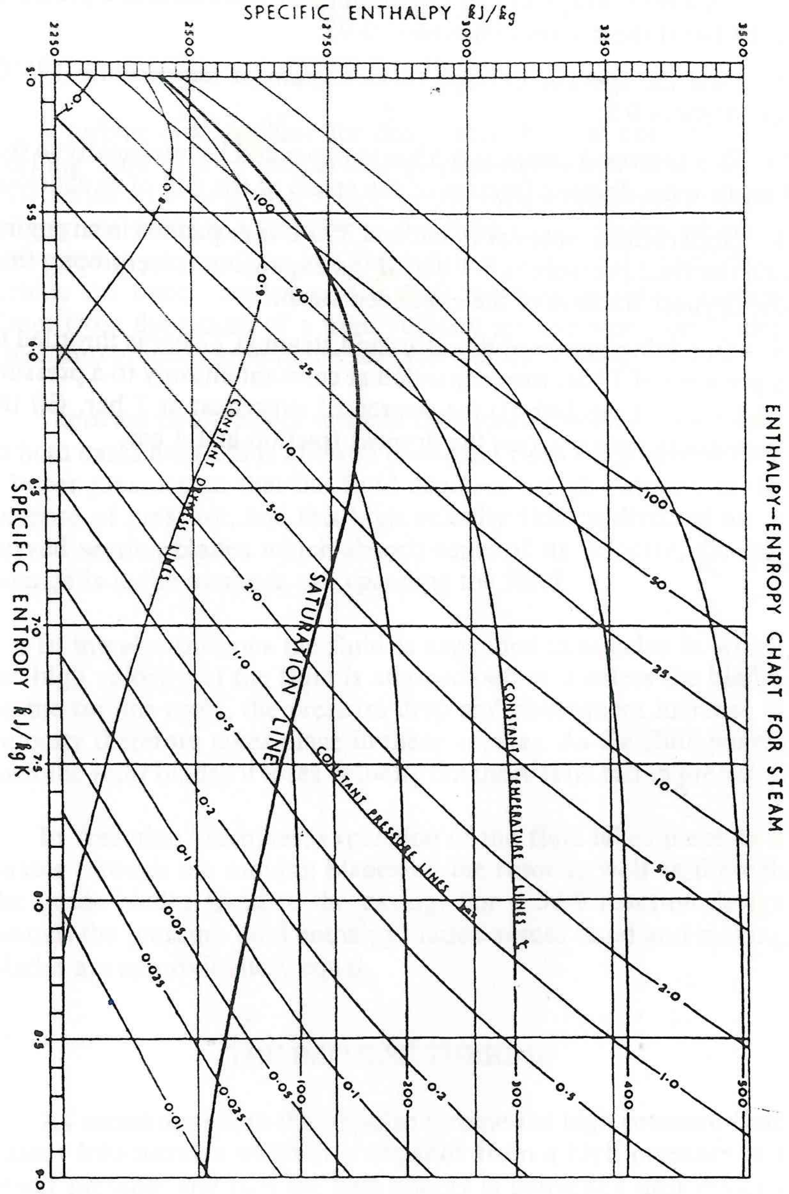


Fig. 50

f TEST EXAMPLES 11

1. Find the entropy of one kilogramme of wet steam at a pressure of 17 bar if the dryness fraction is 0·95.

2. Find the specific entropy of wet steam of temperature 195°C and dryness 0·9.

3. Dry saturated steam at 5·5 bar is expanded isentropically to 0·2 bar, find the dryness fraction of the steam at the end of expansion.

4. Superheated steam at 17 bar and 350°C is expanded in an engine and the final pressure is 1·7 bar. If the expansion is isentropic, find the dryness fraction of the expanded steam.

5. One kilogramme of dry saturated steam at 22 bar is throttled to a pressure of 7 bar, then expanded at constant entropy to a pressure of 1·4 bar. Calculate (i) the degree of superheat at 7 bar, (ii) the increase in entropy, (iii) the dryness fraction at 1·4 bar.

CHAPTER 12

TURBINES

A turbine is a machine for converting the heat energy in the working fluid (gas or steam) into mechanical energy at the shaft. This series of events is reversed in the rotary compressor. Input drive gives increased velocity to the fluid in the blades which is converted to pressure rise in a fixed diffuser ring. In the axial-flow turbine the rotor coupled to the shaft receives its rotary motion direct from the action of a high velocity jet impinging on blades fitted into grooves around the periphery of the rotor.

There are two types of turbine, the impulse and the "reaction". In both cases the fluid is allowed to expand from a high pressure to a lower pressure so that the fluid acquires a high velocity at the expense of pressure, and this high velocity fluid is directed on to curved section blades which absorb some of its velocity; the difference is in the methods of expanding the fluid.

In impulse turbines the fluid is expanded in nozzles in which the high velocity of the fluid is attained before it enters the blades on the turbine rotor, the pressure drop and consequent increase in velocity therefore takes place in these nozzles. As the fluid passes over the rotor blades it loses velocity but there is no fall in pressure.

In "reaction" turbines, expansion of the fluid takes place as it passes through the moving blades on the rotor as well as through the guide blades fixed to the casing. For a 50% reaction design (usual) the pressure (and enthalpy) ratios across fixed and moving blades are approximately equal.

THE IMPULSE TURBINE

As stated above, in the impulse turbine the high pressure fluid passes into nozzles wherein it expands from a high pressure to a lower pressure and thus the heat energy is converted into velocity energy (kinetic energy). The high velocity jet is directed on to blades fitted around the turbine wheel, the blades being of curved section so that the direction of the steam is changed thereby imparting a force to the blades to push the wheel around. The simplest form of impulse is the single-stage De-Laval. A number of such stages in series is a Rateau turbine; velocity compounding is a Curtis stage.

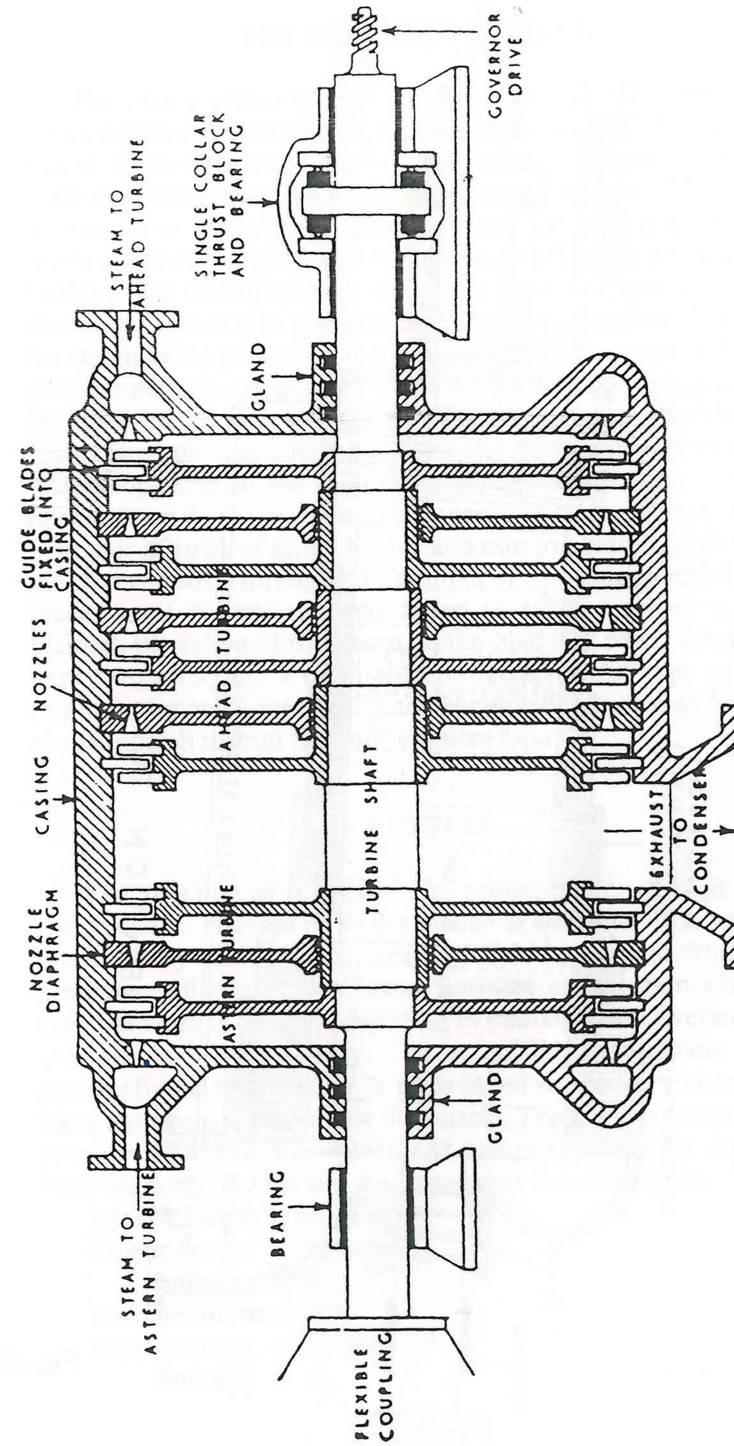
The best efficiency is obtained when the linear speed of the blades is half of the velocity of the fluid entering the blades, thus, when one set of nozzles is used to expand the fluid from its high supply pressure right down to the final low pressure, the resultant velocity of the fluid leaving the nozzles is very high, say about 1200 m/s. To obtain a high efficiency it means therefore that the wheel should run at a very high speed so that the linear velocity of the blades approaches 600 m/s, for example, in the case of a turbine wheel diameter of one metre, the speed would be about 191 rev/s. Lower speeds, which are more suitable, can be obtained by pressure-compounding, or velocity-compounding, or a combination of these termed pressure-velocity-compounding.

In the pressure-compounded impulse turbine, the drop in pressure is carried out in stages, each stage consisting of one set of nozzles and one bladed turbine wheel, the series of wheels being keyed to the one shaft with nozzle plates fixed to the casing between the wheels.

In the velocity-compounded impulse turbine, the complete drop in pressure takes place in one set of nozzles but the drop in velocity is carried out in stages, by absorbing only a part of the velocity in each row of blades on separate wheels and having guide blades fixed to the casing at each stage between the wheels to guide the fluid in the proper direction on to the moving blades.

The pressure-velocity-compounded turbine is a combination of the two.

CONSTRUCTION. In marine engines where the shaft is required to run in the astern direction as well as ahead, a separate astern turbine is necessary. Fig. 51 shows the principal parts of a pressure-velocity-compounded impulse steam turbine in which there are four pressure stages consisting of four sets of nozzles and four wheels in the ahead turbine, and two similar pressure stages in the astern turbine. Each wheel carries two rows of blades and there is one row of guide blades fixed to the casing protruding radially inwards between each row of moving blades to drop the velocity in two steps from each set of nozzles.



PRESSURE-VELOCITY COMPOUNDED IMPULSE TURBINE

Fig. 51

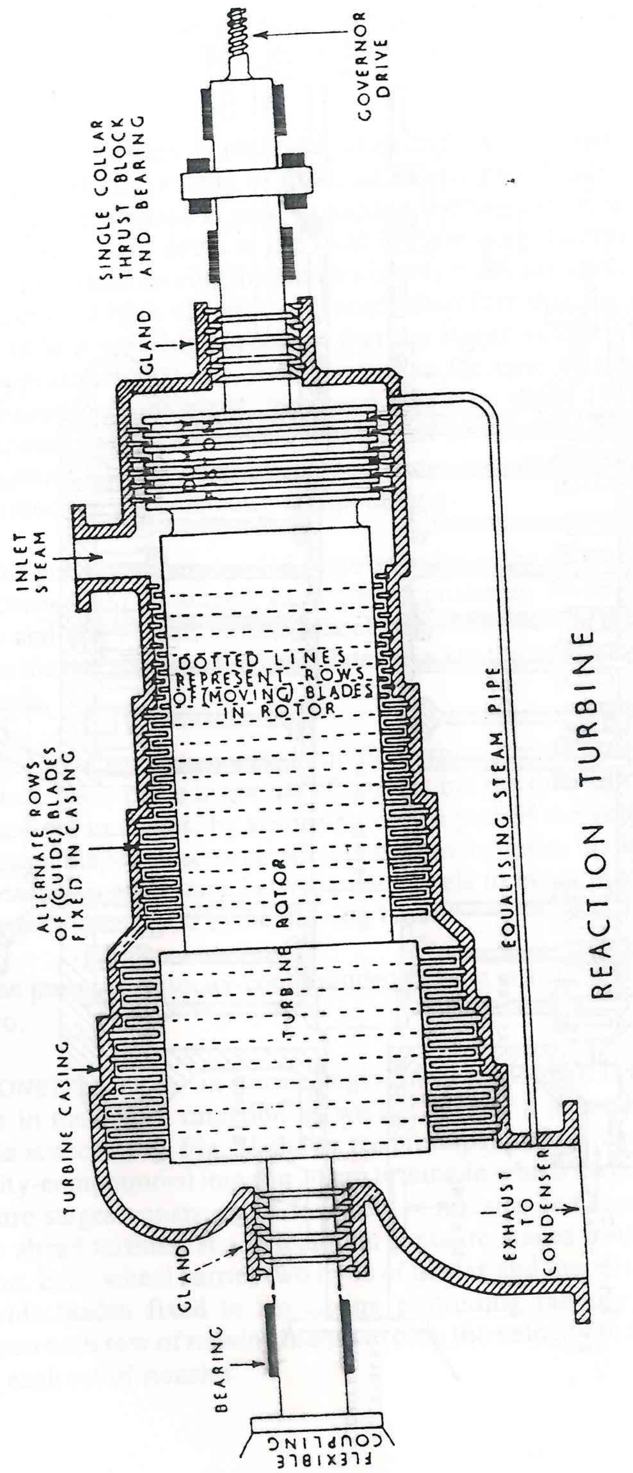


Fig. 52

THE REACTION TURBINE

This, for a steam turbine, is shown in Fig. 52. The steam is expanded continuously through guide blades fixed to the casing and also as it passes through the moving blades on the rotor, on its way from the inlet end to the exhaust end of the turbine. There are no nozzles as in the impulse turbine. When the high pressure steam enters the reaction turbine, it is first passed through a row of guide blades in the casing through which the steam is expanded slightly, causing a little drop in pressure with a resulting increase in velocity, the steam being guided on to the blades in the first row of the rotor gives an impulse effect to these blades. As the steam passes through the rotor blades it is allowed to expand further so that the steam issues from them at a high relative velocity in a direction approximately opposite to the movement of the blades, thus exerting a further force due to reaction. This operation is repeated through the next pair of rows of guide blades and moving blades, then through the next and so on throughout a number of rows of guide and moving blades until the pressure has fallen to exhaust pressure. As explained, the action of the steam on the blades is partly impulse and partly reaction, and a more correct name for this type of turbine might be "impulse-reaction" but it is generally known as "reaction" to distinguish it from the pure impulse type.

NOZZLES

The function of a nozzle is to produce a jet of high velocity which can be directed on to the blades of a turbine. The velocity is produced by allowing the working fluid to expand from a high pressure at the inlet to a lower pressure at the open exit, as no external work is done the decrease in enthalpy is converted into an increase in kinetic energy. The expansion is adiabatic because practically no heat energy is transferred in the very short time it takes the steam to pass along the nozzle. The ideal expansion would produce the maximum velocity at exit, this would be obtained if there were no friction and the expansion was isentropic.

Let conditions at entrance be:

Kinetic energy $= \frac{1}{2} m v_1^2$

Enthalpy $= H_1$

and conditions at exit:

Kinetic energy $= \frac{1}{2} m v_2^2$

Enthalpy $= H_2$

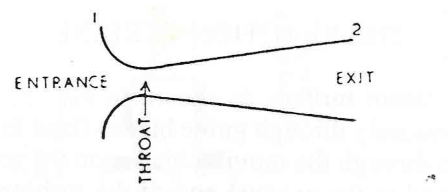


Fig. 53

then, working in joules of energy,

$$\text{Gain in kinetic energy} = \text{Enthalpy drop}$$

$$\tfrac{1}{2}\, m\,(v_2^2 - v_1^2) = H_1 - H_2$$

for unit mass,

$$\tfrac{1}{2}\,(v_2^2 - v_1^2) = h_1 - h_2$$

where h = specific enthalpy.

The inlet velocity is usually small compared with the exit velocity, therefore v_1 is negligible,

$$\tfrac{1}{2} v_2^2 = h_1 - h_2$$

$$v_2 = \sqrt{2(h_1 - h_2)}$$

$$\therefore v\ [\text{m/s}] = \sqrt{2 \times \text{spec. enthalpy drop [J/kg]}}$$

Since $\sqrt{2 \times 10^3} = 44{\cdot}72$, the above may be written

$$v = 44{\cdot}72\sqrt{\text{spec. enthalpy drop [kJ/kg]}}$$

MASS AND VOLUME FLOW. At any point along the nozzle:

$$\text{Volume flow [m}^3\text{/s]} = \text{area [m}^2\text{]} \times \text{velocity [m/s]}$$

$$\text{mass flow [kg/s]} = \frac{\text{volume flow [m}^3\text{/s]}}{\text{specific volume [m}^3\text{/kg]}}$$

$\therefore$ area [m²] × velocity [m/s]

= mass flow [kg/s] × spec. vol. [m³/kg].

Example. Dry saturated steam enters a nozzle at 7 bar and leaves at 4 bar 0·98 dry. Find the velocity of the steam at exit. If the exit area of the nozzle is 300 mm², calculate the mass flow rate.

Tables page 4, 7 bar, $h_g = 2764$

4 bar, $h_f = 605$ $h_{fg} = 2134$

Spec. enthalpy drop $= 2764 - (605 + 0{\cdot}98 \times 2134)$

$= 2764 - 2696$

$= 68$ kJ/kg $= 68 \times 10^3$ J/kg

$$v = \sqrt{2 \times 68 \times 10^3} = 368{\cdot}7 \text{ m/s Ans.}$$

Volume flow [m³s] = area [m²] × velocity [m/s]

$= 300 \times 10^{-6} \times 368{\cdot}7$

$= 0{\cdot}1106$ m³/s

Tables page 4, 4 bar $v_g = 0{\cdot}4623$ m³/kg

Spec. volume at 0·98 dry

$= 0{\cdot}98 \times 0{\cdot}4623 = 0{\cdot}453$ m³/kg

$$\text{mass flow} = \frac{0{\cdot}1106}{0{\cdot}453} = 0{\cdot}2442 \text{ kg/s Ans.}$$

Care must be taken to avoid confusion with the symbols. v usually represents velocity, and the same symbol is used to represent specific volume. In such cases where both velocity and specific volume would appear in the one expression or equation it is therefore advisable to write out the words fully or use an understandable abbreviation.

f Example. Dry saturated steam enters a convergent-divergent nozzle at a pressure of 3·5 bar. The specific enthalpy drop between entrance and throat is 97 kJ/kg and the pressure there is 2·0 bar. The pressure at exit from the nozzle is 0·1 bar, the total possible specific enthalpy drop from entrance to exit is 534 kJ/kg and this is reduced by 12% due to the effect of friction in the divergent part of the nozzle. Calculate the nozzle area at the throat and the mouth to pass 0·113 kg of steam per second.

Tables page 4, 3·5 bar, $h_g = 2732$ $v_g = 0{\cdot}5241$

2 bar, $h_f = 505$ $h_{fg} = 2202$ $v_g = 0{\cdot}8856$

page 3, 0·1 bar, $h_f = 192$ $h_{fg} = 2392$ $v_g = 14{\cdot}67$

h of steam at 2 bar $= h_g$ at 3·5 bar $- 97$

$505 + x \times 202 = 2732 - 97$

$x \times 2202 = 2130$

dryness at throat $x = 0{\cdot}9674$

Spec. vol. of steam at throat

$= 0{\cdot}9674 \times 0{\cdot}8856 = 0{\cdot}8567$ m³/kg

Velocity through throat [m/s]

$= \sqrt{2 \times \text{spec. enthalpy drop [J/kg]}}$

$= \sqrt{2 \times 10^3 \times 97} = 440{\cdot}4$ m/s

area [m²] × velocity [m/s] = mass flow [kg/s] × spec. vol. [m³/kg]

$$\therefore \text{ Area [mm}^2] = \frac{0{\cdot}113 \times 0{\cdot}8567}{440{\cdot}4} \times 10^6$$
$$= 219{\cdot}9 \text{ mm}^2 \quad \text{Ans. (i)}$$

Spec. enthalpy drop between entrance and exit
$$= 0{\cdot}88 \times 534 = 469{\cdot}9 \text{ kJ/kg}$$
$$h \text{ of steam at } 0{\cdot}1 \text{ bar} = h \text{ at } 3{\cdot}5 \text{ bar} - 469{\cdot}9$$
$$192 + x \times 2392 = 2732 - 469{\cdot}9$$
$$x \times 2392 = 2070$$
$$\text{dryness at exit } x = 0{\cdot}8654$$

Spec. vol. of steam at exit
$$= 0{\cdot}8654 \times 14{\cdot}67 = 12{\cdot}7 \text{ m}^3\text{/kg}$$
$$\text{velocity at exit} = \sqrt{2 \times 10^3 \times 469{\cdot}9}$$
$$\text{area [m}^2] \times \text{velocity [m/s]} = \text{mass flow [kg/s]} \times \text{spec. vol. [m}^3\text{/kg]}$$
$$\therefore \text{ Area [mm}^2] = \frac{0{\cdot}113 \times 12{\cdot}7}{969{\cdot}4} \times 10^6$$
$$= 1480 \text{ mm}^2 \quad \text{Ans. (ii)}$$

CRITICAL PRESSURE RATIO. Fig. 53 shows a convergent (entrance to throat) divergent nozzle. In the convergent part the fluid (steam, air or gas) is expanding according to the law pV^n = constant gaining in velocity as the area of the nozzle is reducing. Eventually a point is reached when any further reduction in nozzle area does not increase the velocity. The pressure ratio p_2/p_1, where p_2 (or p_T) is the pressure at the throat and p_1 is the inlet pressure, at which the minimum area of nozzle is reached is called the *critical pressure ratio*. At this point mass flow per unit area of nozzle is a maximum.

$$\text{Critical pressure ratio } \frac{p_T}{p_1} = \left(\frac{2}{n+1}\right)^{\frac{n}{n-1}}$$

This may vary from about 0·57 to 0·48 depending upon the value of n. If $n = \gamma = 1{\cdot}4$ for air then

$$\frac{p_T}{p_1} = 0{\cdot}528$$

ISENTROPIC EFFICIENCY

$$\text{Isentropic efficiency} = \frac{\text{actual KE at nozzle exit}}{\text{isentropic KE at nozzle exit}}$$

$$\text{also, isentropic efficiency} = \frac{\text{actual enthalpy change}}{\text{isentropic enthalpy change}}$$
$$\text{and for a perfect gas } h_1 - h_2' = c_P (T_1 - T_2')$$
$$h_1 - h_2 = c_P (T_1 - T_2)$$
$$\text{hence isentropic efficiency} = \frac{c_P (T_1 - T_2')}{c_P (T_1 - T_2)}$$

f Example. Steam at 5·5 bar, 250°C expands isentropically to 1 bar through a convergent-divergent nozzle. The flow is in equilibrium throughout and the inlet velocity is negligible. Determine, if the critical pressure ratio is 0·546, the throat area of the nozzle for a mass flow rate of 0·15 kg/s and the condition of the steam at the exit.

$$p_T = 5{\cdot}5 \times 0{\cdot}546 = 3 \text{ bar}$$

from an h–s chart
$$h_1 = 2960 \text{ kJ/kg}$$
$$h_2 = 2830 \text{ kJ/kg}$$

To determine h_1 and h_2 using tables; h_1 can be read directly, interpolation of entropy values is used for h_2 i.e.:
Isentropic expansion to throat $\therefore s_1 = s_2 = 7{\cdot}227$ kJ/kg K
If t = degrees of superheat.

$$\text{then } s_1 = s_{150} + \frac{t}{50}(s_{200} - s_{150}) \text{ at 3bar}$$
$$\text{is } 7{\cdot}227 = 7{\cdot}078 + \frac{t}{50}(7{\cdot}312 - 7{\cdot}078)$$
$$\therefore t = 50\,\frac{(7{\cdot}227 - 7{\cdot}078)}{(7{\cdot}312 - 7{\cdot}078)} = 31{\cdot}84°\text{C}$$
$$h_2 = h_{150} + \frac{t}{50}(h_{200} - h_{150})$$
$$= 2762 + \frac{31{\cdot}84}{50}(2866 - 2762)$$
$$= 2828{\cdot}2 \text{ kJ/kg}$$

$$\text{Velocity at throat} = \sqrt{2(h_1 - h_2)}$$
$$= \sqrt{2\,(2962 - 2828) \times 10^3}$$
$$= 513{\cdot}8 \text{ m/s}$$

Specific volume at throat by interpolation

$$= v_{150} + \frac{t}{50}(v_{200} - v_{150})$$

$$= 0{\cdot}6342 + \frac{31{\cdot}84}{50}(0{\cdot}7166 - 0{\cdot}6342)$$

$$= 0{\cdot}6867 \text{ m}^3/\text{kg}$$

$\therefore$ Nozzle area $\times$ 513·8 = 0·6867 $\times$ 0·15
Nozzle area = 0·0002 m² or 2 cm². Ans.

Isentropic expansion to inlet $\therefore s_1 = s_2 = s_3$
At 1 bar $s_1 = s_f + xs_{fg}$
$7{\cdot}227 = 1{\cdot}303 + x \times 6{\cdot}056$
$x = 0{\cdot}98$ dry. Ans.

VELOCITY DIAGRAMS FOR IMPULSE TURBINES

Fig. 54 illustrates the vector diagram of velocities at the entrance side of the rotor blades ("moving" blades). v_1 represents the absolute velocity of the fluid (illustrated here as steam) directed towards the blades at an angle α_1 (as close as practicable) to the direction of their movement. The linear velocity of the blades is represented by u. Drawing the vectors of the steam velocity and blade velocity towards a common point, the vector joining these to form a closed figure is the velocity of the steam relative to the moving blades, represented by v_{r1}.

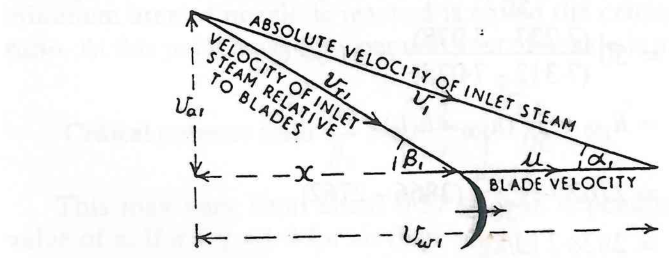


Fig. 54

The relative direction of the steam to the blades is β_1 and in order that the steam should glide on to the blades without shock, the entrance edge of the blades must be in this direction. Therefore β_1 is the *entrance angle of the blades*.

$v_{w1} = v_1 \cos \alpha_1$ is the component of the velocity of the steam jet in the direction of the blade movement, and is referred to as the *velocity of whirl at entrance*.

$v_{a1} = v_1 \sin \alpha_1$ this is the axial component of the steam jet, that is, the component in the direction of the axis of the turbine.

Fig. 55 is the vector diagram of velocities at the exit side of the moving blades. β_2 is the *exit angle of the blades* and the relative velocity of the exit steam v_{r2} is in this direction. In impulse turbines, since there is no fall in steam pressure as it passes over the rotor blades, if friction is neglected, the relative velocity at exit is the same magnitude as the relative velocity at entrance, that is, $v_{r2} = v_{r1}$. Friction between the steam and the blade surface reduces the velocity and, to take friction into account, $v_{r2} = kv_{r1}$ where the velocity

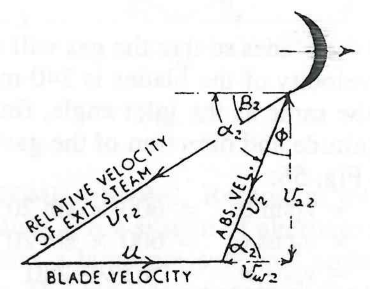


Fig. 55

coefficient k is in the region of 0·8 to 0·95. Loss of kinetic energy in the steam due to friction over the blade surfaces is converted into heat energy.

In the simple impulse turbine the blades are often symmetrical, whence $\beta_1 = \beta_2$.

u is the vector of the blade velocity, v_2 is the vector of the absolute velocity of the exit steam. The direction of the exit steam is at α_2 to the direction of the blade movement, or at angle ϕ to the axis of the turbine.

$v_{w2} = v_2 \cos \alpha_2$ this is the component of the exit steam velocity in the direction of blade movement and is referred to as the *velocity of whirl at exit*.

The axial component of the exit steam is $v_{a2} = v_2 \sin \alpha_2$

Gas, for steam, would be treated in the same way.

Example. Gas at a velocity of 600 m/s from a nozzle is directed on to the blades at 20° to the direction of blade movement. Calculate

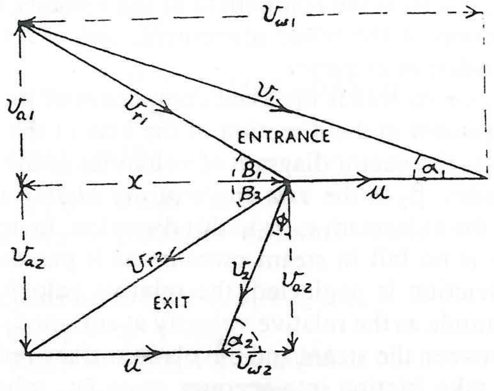


Fig. 56

the inlet angle of the blades so that the gas will enter without shock when the linear velocity of the blades is 240 m/s. If the exit angle of the blades is the same as the inlet angle, find, neglecting blade friction, the magnitude and direction of the gas leaving the blades.

Referring to Fig. 56:

$$v_{a1} = v_1 \sin \alpha_1 = 600 \times \sin 20^\circ = 205{\cdot}2 \text{ m/s}$$
$$v_{w1} = v_1 \cos \alpha_1 = 600 \times \sin 20^\circ = 563{\cdot}9 \text{ m/s}$$
$$x = v_{w1} - u = 563{\cdot}9 - 240 = 323{\cdot}9 \text{ m/s}$$

$$\tan \beta_1 = \frac{v_{a1}}{x} = \frac{205{\cdot}2}{323{\cdot}9}$$

Inlet angle of blades $\beta_1 = 32°22'$. Ans. (i)

Neglecting friction across the blades, $v_{r2} = v_{r1}$ and since $\beta_2 = \beta_1$ then x is common to both entrance and exit diagrams, and $v_{a2} = v_{a1}$.

$$v_{w2} = x - u$$
$$= 323{\cdot}9 - 240 = 83{\cdot}9 \text{ m/s}$$

$$\tan \phi = \frac{v_{w2}}{v_{a2}} = \frac{83{\cdot}9}{205{\cdot}2} = 0{\cdot}4089$$

$\phi = 22°\ 14'$, and $\alpha_2 = 90° - 22°\ 14' = 67°\ 46'$

$$v_2 = \frac{v_{w2}}{\sin \phi} = \frac{83{\cdot}9}{0{\cdot}3783} = 221{\cdot}8 \text{ m/s}$$

Abs. velocity of gas at exit = 221·8 m/s at 67° 46′ to blade movement or 22° 14′ to turbine axis } Ans. (ii)

Both entrance and exit velocity diagrams contain the vector of

the blade velocity u therefore they can be combined together by using the blade velocity as a common base, as shown in Fig. 57. This is a convenient diagram to solve either graphically or by calculation and will be used for all future problems of this type. A diagram is essential for reference in the calculations, students are advised to draw the diagram to scale rather than just a rough sketch, it will then serve as a check on the calculated results.

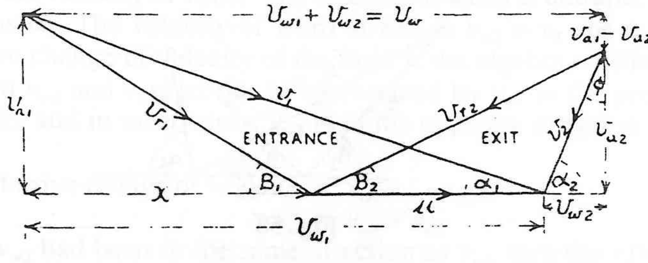


Fig. 57

f VELOCITY COMPOUNDING. Reference was made earlier to velocity compounding, it is a system of alternate moving and fixed blades which enables a large pressure and enthalpy drop to take place in the nozzles. This reduces pressure on the turbine casing and simplifies sealing at the glands, especially for H.P. steam turbines.

The fluid leaves the nozzles with a high velocity and enters a moving row of blades whose velocity is low compared to that of the fluid.

Upon leaving the moving blades at an angle of α_2 to the plane of rotation, the fluid enters a row of stationary blades whose entrance and exit angles are α_2. *i.e.* symmetrical blades. Ideally, as the fluid passes through the fixed blades, there will be no alteration in the pressure, velocity or enthalpy and the fluid will be guided without shock into the next row of moving blades.

f Example. A velocity compounded impulse wheel has two rows of moving and one row of fixed blades, all of which are symmetrical.

Steam leaves the nozzles at 16° to the plane of rotation of the wheel with a velocity of 600 m/s. If the mean blade velocity is 125 m/s and the steam loses 10% of its inlet relative velocity in each of the moving blade passages determine: (a) the blade angles in each of the fixed and moving blades and (b) the direction of the absolute velocity of the final exit steam.

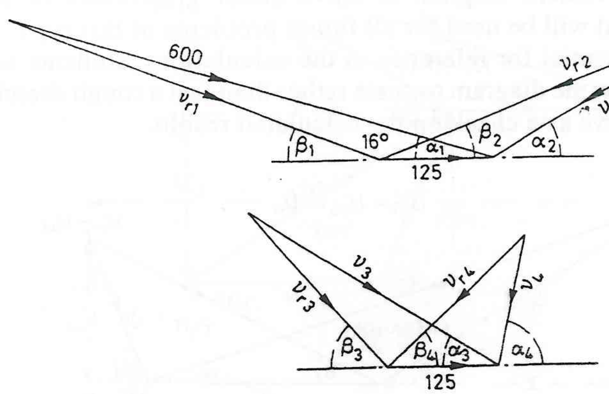


Fig. 58

$$\tan \beta_1 = \frac{600 \sin 16^\circ}{600 \cos 16^\circ - 125}$$

$$\beta_1 = 20{\cdot}1^\circ \quad \beta_1 = \beta_2. \quad \text{Ans. (a) (i)}$$

$$v_{r1} = \frac{600 \sin 16^\circ}{\sin \beta_1} = 481{\cdot}2 \text{ m/s}$$

$$v_{r2} = 0{\cdot}9 \times 481{\cdot}2 = 433{\cdot}1 \text{ m/s}$$

$$\tan \alpha_2 = \frac{433{\cdot}1 \sin 20{\cdot}1^\circ}{433{\cdot}1 \cos 20{\cdot}1^\circ - 125}$$

$$\alpha_2 = 27{\cdot}85^\circ \quad \alpha_2 = \alpha_3 \quad \text{Ans. (a) (ii)}$$

$$v_2 = \frac{433{\cdot}1 \sin 20{\cdot}1^\circ}{\sin 27{\cdot}85^\circ} = 318{\cdot}6 \text{ m/s}$$

$$v_2 = v_3 \quad \text{as no friction in fixed blade (assumed)}$$

$$\tan \beta_3 = \frac{318{\cdot}6 \sin 27{\cdot}85^\circ}{318{\cdot}6 \cos 27{\cdot}85^\circ - 125}$$

$$\beta_3 = 43{\cdot}52^\circ \quad \beta_3 = \beta_4 \quad \text{Ans. (a) (iii)}$$

$$v_{r3} = \frac{318{\cdot}6 \sin 27{\cdot}85^\circ}{\sin 43{\cdot}52^\circ} = 216 \text{ m/s}$$

$$v_{r4} = 0{\cdot}9\, v_{r3} = 194{\cdot}5 \text{ m/s}$$

$$\tan \alpha_4 \quad \frac{194{\cdot}5 \sin 43{\cdot}52^\circ}{194{\cdot}5 \cos 43{\cdot}52^\circ - 125} \quad \alpha_4 = 83^\circ \quad \text{Ans. (b)}$$

FORCE ON BLADES

The effective velocity of the fluid causing motion of the rotor blades is the component in the direction of movement of the blades. This is the velocity of whirl. The velocity of whirl at entrance is $v_{w1} = v_1 \cos \alpha_1$. The velocity of whirl at exit is $v_{w2} = v_2 \cos \alpha_2$. The effective change of velocity of the fluid is the algebraic difference between v_{w1} and v_{w2}, let this be represented by v_w. In the previous example, and in most cases, v_{w2} is in the opposite direction to v_{w1} then

$$\text{effective change of velocity} = v_{w1} - (-v_{w2})$$

$$\therefore v_w = v_{w1} + v_{w2}$$

If v_{w2} had been in the same direction as v_{w1} then the effective change of velocity would be $v_{w1} - v_{w2}$.

Since the motion of the fluid is changed as it passes over the blades, the the *blades* must exert a force *on the fluid* to cause this change. From Newton's third law of motion which states that action and re-action are equal in magnitude and opposite in direction, this force is also the magnitude of the force exerted by the fluid on the *blades*.

$$\text{Force [N]} = \text{mass [kg]} \times \text{acceleration [m/s}^2\text{]}$$
$$= \text{mass} \times \text{change of velocity per second}$$
$$= \text{change of momentum per second}$$

Therefore, if the mass rate of flow [kg/s] of the fluid is represented by $\dot{m}$, then:

$$\text{Force on blades [N]} = \dot{m}\text{[kg/s]} \times \text{effective change of velocity}$$
$$= \dot{m}v_w$$

$$\text{Work done} = \text{force} \times \text{distance}$$

Since distance through which the force acts on the blades is the linear distance moved by the blades, we have:

Work done per second [N m/s] = force on blades [N] × blade velocity [m/s].

Also,

Work done per second [N m/s] = Power [J/s = W]

therefore,

$$\text{Power [W]} = \dot{m}v_w u$$

The work supplied to the blades is the kinetic energy of the fluid jet,

Work supplied per second = $\frac{1}{2}\dot{m}v_1^2$

hence,

$$\text{Diagram efficiency} = \frac{\text{work done on blades [J/s]}}{\text{work supplied [J/s]}}$$

$$= \frac{\dot{m}v_w u}{\frac{1}{2}\dot{m}v_1^2} = \frac{2uv_w}{v_1^2}$$

This is sometimes called the blade efficiency.

The *axial* force on the blades is due to the difference between the axial components of the fluid velocities at entrance and exit, thus,

$$\text{axial thrust} = \dot{m}\,(v_{a1} - v_{a2})$$

Example. Steam leaves the nozzles of a single stage impulse turbine at a velocity of 670 m/s at 19° to the plane of the wheel, and the steam consumption is 0·34 kg/s. The mean diameter of the blade ring is 1070 mm. Find (i) the inlet angle of the blades to suit a rotor speed of 83·3 rev/s. If the velocity coefficient of the steam across the blades is 0·9 and the blade exit angle is 32°, find (ii) the force on the blades, (iii) the power given to the wheel, and (iv) the diagram efficiency.

Referring to Fig. 57:

Linear velocity of blades = mean circumference × rev/s

$u = \pi \times 1{\cdot}07 \times 83{\cdot}3 = 280$ m/s

$v_{a1} = v_1 \sin\alpha_1 = 670 \times \sin 19° = 218{\cdot}1$ m/s

$v_{w1} = v_1 \cos\alpha_1 = 670 \times \cos 19° = 633{\cdot}6$ m/s

$x = v_{w1} - u = 633{\cdot}6 - 280 = 353{\cdot}6$ m/s

$$\tan\beta_1 = \frac{v_{a1}}{x} = \frac{218{\cdot}1}{353{\cdot}6} = 0{\cdot}6169$$

Entrance angle = 31°40′ Ans. (i)

$$v_{r1} = \frac{v_{a1}}{\sin\beta_1} = \frac{218{\cdot}1}{\sin 31°\,40'} = 415{\cdot}6 \text{ m/s}$$

$v_{r2} = 0{\cdot}9v_{r1} = 0{\cdot}9 \times 415{\cdot}6 = 374$ m/s

$v_{w2} = v_{r2}\cos\beta_2 - u = 374 \cos 32° - 280$

$= 317{\cdot}1 - 280 = 37{\cdot}1$ m/s

Effective change of velocity = $v_w = v_{w1} + v_{w2}$

Force on blades [N] = $\dot{m}$[kg/s] × v_w[m/s]

$= 0{\cdot}34 \times 670{\cdot}7$

$= 228{\cdot}1$ N Ans. (ii)

Power [W = J/s = N m/s] = force [N] × linear velocity [m/s]

$= 228{\cdot}1 \times 280$

= 63870 W or 63·87 kW Ans. (iii)

$$\text{Diagram efficiency} = \frac{\text{work done on blades [J/s]}}{\text{work supplied [J/s]}}$$

$$= \frac{\dot{m}v_w u}{\frac{1}{2}\dot{m}v_1^2} = \frac{2uv_w}{v_1^2}$$

$$= \frac{2 \times 280 \times 670{\cdot}7}{670^2}$$

= 0·8368 or 83·68% Ans. (iv)

f **Example. An impulse turbine has a row of nozzles set at angle α_1, to the plane of motion of the moving blades and issues steam at a velocity of *C*. The blades have a tangential velocity of *u* and are symmetrical. Neglecting the effects of friction show that the diagram efficiency is given by:**

$$\eta = \frac{4u\,(C\cos\alpha_1 - u)}{C^2}$$

$v_w = v_{r1}\cos\beta_1 + v_{r2}\cos\beta_2$

$v_{r1} = v_{r2}$ as no blade friction

$\beta_1 = \beta_2$ as blades symmetrical

$v_{r1}\cos\beta_1 = C\cos\alpha_1 - u$

$P = \dot{m}v_w u$

$= \dot{m} \times 2(C\cos\alpha_1 - u) \times u$

Work supplied = $\frac{1}{2}\dot{m}C^2$

$$\therefore \eta = \frac{2\dot{m}u\,(C\cos\alpha_1 - u)}{0{\cdot}5\,\dot{m}C^2}$$

$$= \frac{4u\,(C\cos\alpha_1 - u)}{C^2} \quad \text{Ans.}$$

VELOCITY DIAGRAMS FOR REACTION TURBINES

It has been seen that, in the impulse turbine, the fluid expands whilst passing through fixed nozzles and there is no expansion on