

Fig. 59

its passage through the channels between the moving blades. All generation of velocity takes place in the nozzles.

In the reaction turbine, expansion of the fluid takes place during its passage through the fixed (guide) blades which take the place of nozzles, and it also expands as it passes through the moving blades. Therefore the velocity of the fluid is increased as it passes through the fixed blades, and the relative velocity of the fluid to the moving blades is increased as it passes through the moving blades, so that v_{r2} is greater than v_{r1} .

In the reaction turbine, the fixed and moving blades are often of the same section and reversed in direction. In such cases, the entrance and exit angles of the fixed blades are the same as those of the moving blades, and the velocity vector diagram at entrance is identical with the velocity vector diagram at exit, therefore the combined diagram is symmetrical. See Figs. 59 and 60. Thus the relative velocity of the fluid at exit from the moving blades is equal to the absolute velocity at entrance, $v_{r2} = v_1$ and the absolute velocity of the fluid at exit from the moving blades is equal to the relative velocity at entrance, $v_2 = v_{r1}$. Hence, $\beta_2 = \alpha_1$ and $\alpha_2 = \beta_1$.

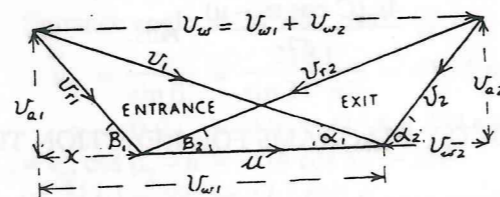


Fig. 60

Example. At one stage of a reaction turbine the velocity of the steam leaving the fixed blades is 90 m/s and the exit angle is 20° . The linear velocity of the moving blades is 60 m/s and the steam consumption is 1.1 kg/s. Assuming the fixed and moving blades to be of identical section, calculate (i) the entrance angle of the blades, (ii) the force on the blades, and (iii) the stage power.

$$v_{a1} = v_1 \sin \alpha_1 = 90 \times \sin 20^\circ = 30.78 \text{ m/s}$$

$$v_{w1} = v_1 \cos \alpha_1 = 90 \times \cos 20^\circ = 84.57 \text{ m/s}$$

$$x = v_{w1} - u = 84.57 - 60 = 24.57 \text{ m/s}$$

$$\tan \beta_1 = \frac{30.78}{24.57} = 1.253$$

Entrance angle = $51^\circ 24'$ Ans. (i)

Effective change of velocity = $v_w = v_{w1} + v_{w2}$
(note that $v_{w2} = x$)

$$v_w = 84.57 + 24.57 = 109.14 \text{ m/s}$$

$$\text{Force on blades [N]} = \dot{m}[\text{kg/s}] \times v_w[\text{m/s}]$$

$$= 1.1 \times 109.19$$

$$= 120 \text{ N Ans. (ii)}$$

$$\text{Power [W = J/s = Nm/s]} = \text{force [N]} \times \text{velocity [m/s]}$$

$$= 120 \times 60$$

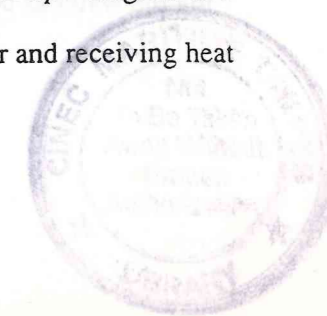
$$= 7200 \text{ W or } 7.2 \text{ kW Ans. (iii)}$$

f IDEAL CYCLES

The Carnot cycle has been described for a gas and a vapour - see Fig. 30 (pressure-volume diagram) of Chapter 8 and Fig. 48 (temperature-entropy) of Chapter 11. The wider the range of temperature the more efficient the cycle. The lowest practical condensing temperature is governed by the coolant temperature and the highest practical temperature is governed by metallurgical limits of pressure and temperature. There are two major reasons why the Carnot cycle is not used as the theoretical basis for vapour (steam) cycles. Firstly, it has a low work ratio and secondly it is difficult to compress a wet vapour efficiently; it is easier to fully condense the vapour and compress the liquid to boiler pressure using a feed pump. The resulting cycle is known as the Rankine cycle.

The Rankine cycle is illustrated in Fig. 61 on a pV diagram and on a $T-s$ diagram and consists of:

- (i) a to b , feed water pumped into the boiler and receiving heat



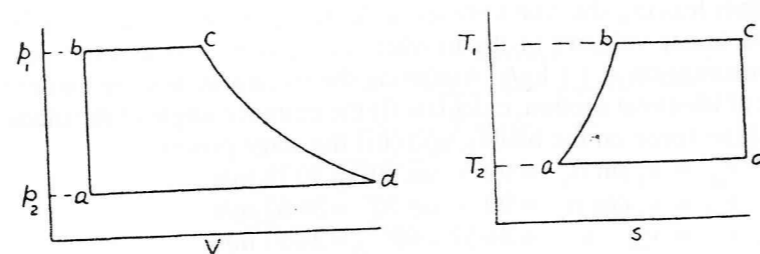


Fig. 61

energy as it is increased in pressure from p_2 to p_1 and in temperature from T_2 to T_1 .

(ii) b to c . The water is completely evaporated into steam in the boiler at constant pressure p_1 and constant temperature T_1 and the steam is supplied to the engine as it is generated.

(iii) c to d . The steam supply to the engine expands isentropically from the highest to the lowest limits of pressure and temperature.

(iv) d to a . The steam is exhausted from the engine into the condenser and condensed into water at constant pressure p_2 and constant temperature T_2 .

In the ideal engine there would be no energy losses due to friction or heat transfer, no leakage of steam, and no undercooling of the condensate in the condenser, *i.e.* the condensate would be at the saturation temperature corresponding to the condenser steam pressure.

Therefore the work which would be done in the ideal engine is equal to the heat energy given up by the steam on its passage through the engine. This is, per unit mass flow, the enthalpy drop, *i.e.* the difference between enthalpy of supply steam (h_1) and the enthalpy of exhaust steam (h_2).

The heat energy supplied per unit mass flow to the feed water in the boiler to produce the steam is the difference between the enthalpy of the supply steam (h_1) and the enthalpy of the feed water (h_2), the feed water temperature being equal to the condensate temperature.

Thus, neglecting the work done by the feed pump in compressing the water as being comparatively small:

$$\text{Rankine efficiency} = \frac{h_1 - h_2}{h_1 - h_2}$$

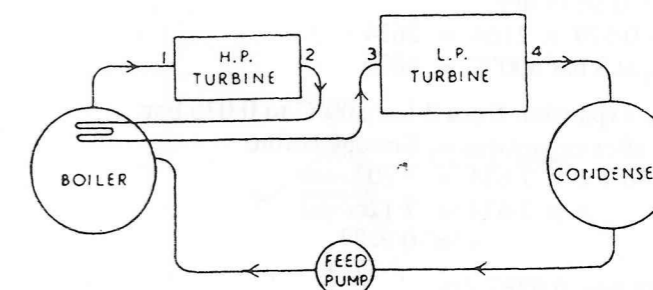


Fig. 62

A cycle operating with steam in the wet region will be limited to a maximum at the critical state (22.1 bar, 374.15°C) which limits efficiency. Superheating raises the temperature (above saturation) without raising the boiler pressure and this increases the ideal cycle efficiency.

It is also common practice to re-heat steam between stages which reduces wetness at low pressures and increases power output for a given size of units. Fig. 62 and the example following illustrate both applications and a resulting Rankine cycle. Examples are often solved more quickly using an $h-s$ chart.

Rankine efficiency

$$\begin{aligned} &= \frac{\text{Heat energy given up by steam through engine}}{\text{Heat energy supplied by boiler and re-heater}} \\ &= \frac{(h_1 - h_2) + (h_3 - h_4)}{(h_1 - h_2) + (h_3 - h_2)} \end{aligned}$$

f Example. In an ideal steam reheat cycle the steam is expanded in the first stage of a turbine from 40 bar 400°C to 3.0 bar. At this pressure the steam is passed through a reheater and its temperature is raised to 300°C at constant pressure. It then passes through the remainder of the turbine and expanded to 0.75 bar. Calculate the Rankine efficiency.

$$h_1 \text{ at 40 bar } 400^\circ\text{C} = 3214$$

Isentropic expansion from 40 bar 400°C to 3 bar:

$$\text{Entropy after expansion} = \text{Entropy before}$$

$$1.672 + x \times 5.321 = 6.769$$

$$x \times 5.321 = 5.097$$

$$x = 0.9579$$

$$\begin{aligned}
 h_2 \text{ at 3 bar, 0.9579 dry} \\
 &= 561 + 0.579 \times 2164 = 2634 \\
 h_3 \text{ at 3 bar } 300^\circ\text{C} &= 3070
 \end{aligned}$$

Isentropic expansion from 3 bar 300°C to 0.075 bar:

$$\begin{aligned}
 \text{Entropy after expansion} &= \text{Entropy before} \\
 0.576 + x \times 7.674 &= 7.702 \\
 x \times 7.674 &= 7.126 \\
 x &= 0.9287
 \end{aligned}$$

$$\begin{aligned}
 h_4 \text{ at 0.075 bar, 0.9287 dry} \\
 &= 169 + 0.9287 \times 2405 = 2403
 \end{aligned}$$

$$h_{f4} \text{ at 0.075 bar} = 169$$

$$\begin{aligned}
 \text{Rankine efficiency} &= \frac{(h_1 - h_2) + (h_3 - h_4)}{(h_1 - h_{f4}) + (h_3 - h_2)} \\
 &= \frac{(3214 - 2634) + (3070 - 2403)}{(3214 - 169) + (3070 - 2634)} \\
 &= \frac{580 + 667}{3045 + 436} = \frac{1247}{3481} \\
 &= 0.3583 \text{ or } 35.83\% \text{ Ans.}
 \end{aligned}$$

ACTUAL STEAM CYCLES

An estimate of the actual vapour (steam) cycle efficiency can be arrived at by amending the isentropic process of expansion (and compression) utilising an efficiency factor. Net work is the turbine (expansion) actual work minus the actual compressor (compression) or pump work.

ISENTROPIC EFFICIENCY. When an actual process is compared with an isentropic process the resulting efficiency is called "isentropic efficiency".

If we consider steam passing through, for example, a turbine. The ratio of the actual enthalpy drop to the isentropic enthalpy drop is the isentropic efficiency. This can be easily shown on a $h-s$ chart.

$$h = \text{Specific Enthalpy kJ/kg. } s = \text{Specific Entropy kJ/kg K.}$$

$$\text{Isentropic efficiency} = \frac{\text{Actual Enthalpy drop}}{\text{Isentropic Enthalpy drop}} \times 100\%$$

$$= \frac{h_a}{h_s} \times 100\%$$

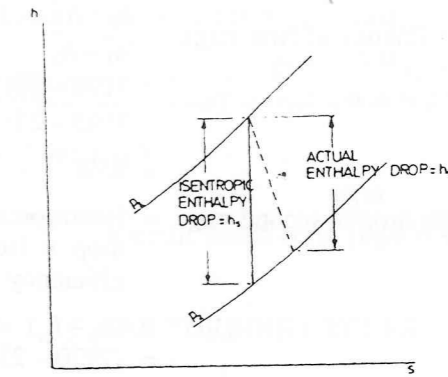


Fig. 63

f Example. Steam at 50 bar and 400°C is expanded in a turbine to 2 bar and 0.96 dry. It is then reheated at constant pressure to 250°C and finally expanded to 0.06 bar with isentropic efficiency of 0.8. Using the $h-s$ chart, find (a) the isentropic efficiency of the first expansion (b) the total power developed for a mass flow of 3 kg/s.

If the reheater was by-passed determine, if its isentropic efficiency was unaltered, the loss of power in the second stage.

Specific enthalpy values read from the chart (Fig. 64) as follows:

$$\begin{aligned}
 h_1 &= 3195 \text{ kJ/kg} & h_3 &= 2970 \text{ kJ/kg} & h_5 &= 2120 \text{ kJ/kg} \\
 h_2' &= 2615 \text{ kJ/kg} & h_4' &= ? \\
 h_2 &= 2512 \text{ kJ/kg} & h_4 &= 2370 \text{ kJ/kg}
 \end{aligned}$$

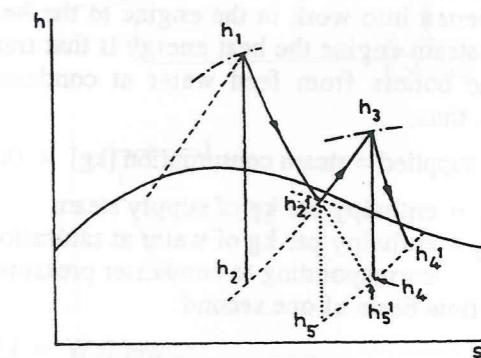


Fig. 64

$$\begin{aligned} \text{Isentropic efficiency of first stage} &= \frac{h_1 - h_2'}{h_1 - h_2} \\ &= \frac{3195 - 2615}{3195 - 2515} \times 100\% \\ &= 85\% \end{aligned}$$

$$\begin{aligned} \text{Actual enthalpy drop in second stage} &= \text{Isentropic enthalpy drop} \times \text{Isentropic efficiency} \\ &= (h_3 - h_4) \times 0.8 \\ &= (2970 - 2370) \times 0.8 \\ &= 480 \text{ kJ/kg} \end{aligned}$$

$$\begin{aligned} \text{Total power developed} &= \dot{m}[(h_1 - h_2') + (h_3 - h_4)] \\ &= 3[580 + 480]/10^3 \\ &= 3.18 \text{ MW.} \end{aligned}$$

With re-heater by-passed

$$\begin{aligned} \text{Actual enthalpy drop in second stage} &= (h_2' - h_3) \times 0.8 \\ &= (2615 - 2120) \times 0.8 \\ &= 396 \text{ kJ/kg} \end{aligned}$$

$$\begin{aligned} \text{Power loss in second stage} &= \text{Original power} - \text{New power} \\ &= 3[480 - 396] \\ &= 252 \text{ kW.} \end{aligned}$$

THERMAL EFFICIENCY

The thermal efficiency of an engine is the ratio of the heat energy converted into work in the engine to the heat energy supplied. For a steam engine the heat energy is that transferred to the steam in the boilers from feed water at condenser condensate temperature, thus,

$$\text{energy [kJ] supplied} = \text{steam consumption [kg]} \times (h_1 - h_{f2}) \text{ [kJ/kg]}$$

where, h_1 = enthalpy per kg of supply steam
 h_{f2} = enthalpy per kg of water at saturation temperature corresponding to condenser pressure.

hence, on a time basis of one second:

$$\text{Thermal efficiency} = \frac{\text{power [kW = kJ/s]}}{\text{steam consumption [kg/s]} \times (h_1 - h_{f2})}$$

or, on a time basis of one hour (3600 seconds):

$$\text{Thermal efficiency} = \frac{\text{power [kW]} \times 3600}{\text{steam consumption [kg/h]} \times (h_1 - h_{f2})}$$

or, on a basis of one kW h:

$$\text{Thermal efficiency} = \frac{3600}{\text{specific steam cons. [kg/kW h]} \times (h_1 - h_{f2})}$$

f GAS TURBINE CYCLES

Marine type gas turbines work on the ideal constant pressure (Joule) cycle. Fig. 65 is a diagrammatic sketch of a simple open cycle gas turbine plant which consists of three essential parts – air compressor, combustion chamber, and turbine. Referring to Fig. 65, air is drawn in from the atmosphere and compressed from $p_1V_1T_1$ to the higher pressure, smaller volume and higher temperature $p_2V_2T_2$. The compressed air is delivered to the combustion chamber. Some of this air is used for burning the fuel which is admitted through the burner into the combustion chamber, the remainder of the air passes through the jacket surrounding the burner housing, mixes with the products of combustion, and is heated at constant pressure while the volume and temperature increases, the conditions now being $p_3V_3T_3$. The mixture of hot air and gases now passes through the

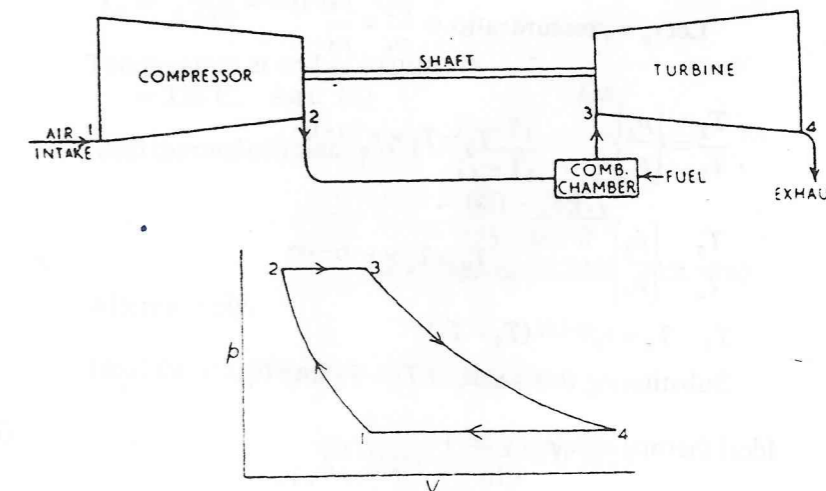


Fig. 65

turbine where it expands to $p_4V_4T_4$ as it does work in driving the rotor. Finally, the gases exhaust at constant pressure.

Of the power developed in the turbine, some is absorbed in driving the compressor, the remainder being available for external use such as for propulsion, or driving an electric generator. A starting motor is fitted at the opposite end of the shaft to that for the external drive.

In the ideal cycle, compression and expansion are isentropic between the same pressures p_2 and p_1 following the law $pV^\gamma = a$ constant. Referring to Fig. 65:

$$\begin{aligned} \text{Ideal thermal efficiency} &= \frac{\text{heat energy converted into work}}{\text{heat energy supplied}} \\ &= \frac{\text{heat supplied} - \text{heat rejected}}{\text{heat supplied}} \\ &= 1 - \frac{\text{heat rejected}}{\text{heat supplied}} \\ &= 1 - \frac{m \times c_p \times (T_4 - T_1)}{m \times c_p \times (T_3 - T_2)} \\ &= 1 - \frac{T_4 - T_1}{T_3 - T_2} \quad \text{(i)} \end{aligned}$$

$$\text{Let } r_p = \text{pressure ratio} = \frac{p_2}{p_1} = \frac{p_3}{p_4}$$

$$\frac{T_2}{T_1} = \left\{ \frac{p_2}{p_1} \right\}^{\frac{\gamma-1}{\gamma}} \quad \therefore T_2 = T_1 \times r_p^{(\gamma-1)/\gamma}$$

$$\frac{T_3}{T_4} = \left\{ \frac{p_3}{p_4} \right\}^{\frac{\gamma-1}{\gamma}} \quad \therefore T_3 = T_4 \times r_p^{(\gamma-1)/\gamma}$$

$$T_3 - T_2 = r_p^{(\gamma-1)/\gamma} (T_4 - T_1)$$

Substituting this value of $T_3 - T_2$ into (i):

$$\text{Ideal thermal efficiency} = 1 - \frac{1}{r_p^{(\gamma-1)/\gamma}} \quad \text{(ii)}$$

Showing that the ideal thermal efficiency depends upon the pressure ratio.

f Example. In a simple gas turbine plant working on the ideal constant pressure cycle, air is taken into the compressor at 1 bar, 16°C , and delivered at 5.5 bar. If the temperature at turbine inlet is 700°C , calculate (i) the temperature at the end of compression, (ii) temperature at exit from the turbine, (iii) the ideal thermal efficiency. Take $\gamma = 1.4$.

$$\text{Pressure ratio } r_p = \frac{p_2}{p_1} = \frac{p_3}{p_4} = \frac{5.5}{1} = 5.5$$

$$\frac{\gamma-1}{\gamma} = \frac{0.4}{1.4} = \frac{2}{7}$$

$$r_p^{(\gamma-1)/\gamma} = 5.5^{2/7} = 1.619$$

$$\frac{T_2}{T_1} = \left\{ \frac{p_2}{p_1} \right\}^{\frac{\gamma-1}{\gamma}}$$

$$T_2 = 289 \times 1.619 = 467.8 \text{ K}$$

$$\text{Temperature at end of compression} = 194.8^\circ\text{C. Ans. (i)}$$

$$\frac{T_4}{T_3} = \left\{ \frac{p_4}{p_3} \right\}^{\frac{\gamma-1}{\gamma}}$$

$$T_4 = \frac{973}{1.619} = 601 \text{ K}$$

$$\text{Temperature at end of expansion} = 328^\circ\text{C. Ans. (ii)}$$

$$\begin{aligned} \text{Ideal thermal efficiency} &= 1 - \frac{T_4 - T_1}{T_3 - T_2} \\ &= 1 - \frac{601 - 289}{973 - 467.8} \\ &= 0.3824 \text{ or } 38.24\% \text{ Ans. (iii)} \end{aligned}$$

Alternatively,

$$\begin{aligned} \text{Ideal thermal efficiency} &= 1 - \frac{1}{r_p^{(\gamma-1)/\gamma}} \\ &= 1 - \frac{1}{1.619} = 0.3823 \end{aligned}$$

HEAT EXCHANGER. By including a heat exchanger, some of the heat energy in the exhaust gases can be utilized by transferring it to the air before it enters the combustion chamber, resulting in less fuel being needed to raise the temperature of the air to the required turbine inlet temperature and therefore increasing the thermal efficiency. A typical arrangement is shown diagrammatically in Fig. 66.

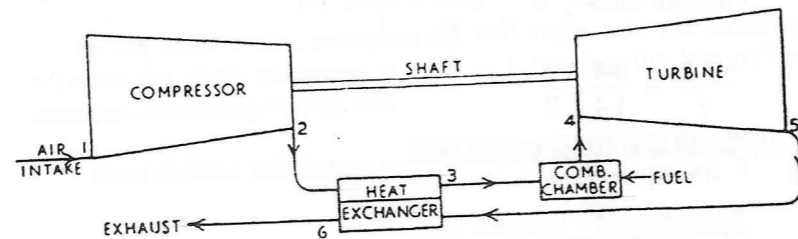


Fig. 66

The lowest temperature entering the heat exchanger is that of the compressed air T_2 and the highest temperature is that of the exhaust gases from the turbine T_5 . The difference $T_5 - T_2$ is the overall temperature range for the exchanger. The air, on its passage through the exchanger is increased in temperature from T_2 to T_3 , that is, an increase of $T_3 - T_2$. The ratio of the increase in air temperature to the overall temperature range is termed the *thermal ratio* or *effectiveness* of the heat exchanger, thus:

$$\text{Thermal ratio of heat exchanger} = \frac{T_3 - T_2}{T_5 - T_2}$$

ACTUAL CYCLE EFFICIENCY. The actual processes of compression and expansion can be estimated by applying the isentropic efficiency to the ideal isentropic processes. Isentropic efficiency is determined from enthalpy change, which, for a perfect gas, is given by:

$$\Delta h = mc_p \Delta T$$

The processes and cycle, for gas turbines are best illustrated, as in Fig. 67, by a T - s diagram.

f Example. An open cycle gas turbine unit has a pressure ratio of 6:1 and a maximum cycle temperature of 760°C . The isentropic efficiency of the compressor and turbine is 0.85. The minimum cycle temperature is 15°C .

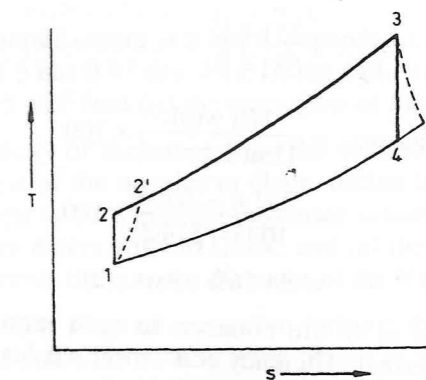


Fig. 67

- (a) sketch the cycle on a temperature-entropy diagram;
 (b) calculate the thermal efficiency of the unit.

For the working substance $c_p = 1005 \text{ J/kg K}$ $\gamma = 1.4$.

Refer to Fig. 67:

Ans. (a)

$$\begin{aligned} \frac{T_2}{T_1} &= \left\{ \frac{p_2}{p_1} \right\}^{\frac{\gamma-1}{\gamma}} \\ T_2 &= 288 \times 6^{0.4} \\ &= 480.4 \text{ K} \\ T_2 - T_1 &= 480.4 - 288 \\ &= 192.3 \text{ K} \\ T_2' - T_1 &= \frac{192.3}{0.85} \\ &= 226.3 \text{ K} \\ T_2' &= 226.3 + 288 \\ &= 514.3 \text{ K} \\ \frac{T_3}{T_4} &= \left\{ \frac{p_3}{p_4} \right\}^{\frac{\gamma-1}{\gamma}} \\ \frac{1033}{T_4} &= 6^{0.4} \\ T_4 &= 619.3 \text{ K} \\ T_3 - T_4 &= 1033 - 619.3 \\ &= 413.7 \text{ K} \\ T_3 - T_4' &= 0.85 \times 413.7 \end{aligned}$$

$$= 351.6 \text{ K}$$

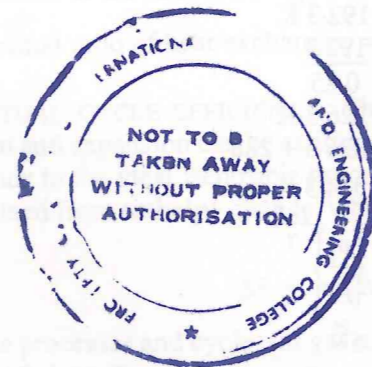
$$T_4' = 681.4 \text{ K}$$

$$\text{Thermal efficiency} = \frac{\text{Net work}}{\text{Heat supplied}} \times 100$$

$$= \frac{351.6 - 226.3}{1033 - 514.3} \times 100$$

$$= 24.16\% \text{ Ans. (b)}$$

Note: c_p and \dot{m} are common to each term in the above expression for thermal efficiency and cancel out leaving temperatures. c_p , although given, is not required.



TEST EXAMPLES 12

1. Dry saturated steam at 8 bar is expanded in turbine nozzles to a pressure of 5 bar 0.97 dry. Find (i) the velocity at exit. If the area at exit is 14.5 cm^2 find (ii) the mass flow of the steam in kg/s.
2. The velocity of the steam from the nozzles of a turbine is 450 m/s, the angle of the nozzles to blade motion is 20° and the blade entrance angle is 33° . Find (i) the linear velocity of the blades so that the steam enters without shock, and (ii) the rotational speed of the rotor in rev/s if the mean diameter of the blade ring is 660 mm.
3. At a certain stage of a reaction turbine, the steam leaves the guide blades and enters the moving blades at an absolute velocity of 243 m/s at an angle of 23° to the plane of rotation, and the blade velocity is 159 m/s. Fixed and moving blades have the same inlet and exit angles and the steam flow is 0.9 kg/s. Calculate (i) the inlet angle of the blades, (ii) the force on the blades, and (iii) the stage power.
4. A single stage impulse steam turbine has symmetrical rotor blades with inlet and outlet angles of 35° . The mean blade diameter is 600 mm, the turbine runs at 100 rev/s and the steam leaves the rotor in the axial direction:
 - (a) Determine:
 - (i) the nozzle angle;
 - (ii) the velocity of steam leaving the nozzle;
 - (iii) the power developed per kg of steam flow.
 - (b) State the major assumption made to solve this problem.
- f5. Dry saturated steam at 14 bar is expanded in a turbine nozzle to 10 bar, expansion following the law $pV^n = \text{constant}$, where the value of n is 1.135, calculate:
 - (i) the dryness fraction of the steam at exit,
 - (ii) the enthalpy drop through the nozzle per kg of steam,
 - (iii) the velocity of discharge,
 - (iv) the area of nozzle exit in mm^2 per kg of steam discharged per second.
- f6. A perfect gas expands from 7 bar 150°C to atmospheric pressure isentropically through a convergent-divergent nozzle. If the mass flow rate of the gas through the nozzle is 0.25 kg/s determine:
 - (a) the velocity of the gas at the nozzle throat
 - (b) the throat area.

Note. $\gamma = 1.67$, $R = 2078.5 \text{ J/kg K}$ and $c_p = 833.9 \text{ J/kg K}$ for the gas.

Also $\frac{p_T}{p_1} = \left[\frac{2}{\gamma + 1} \right]^{\frac{\gamma}{\gamma - 1}}$ is the critical pressure ratio for a gas expanding in a convergent-divergent nozzle according to the law $pV^\gamma = \text{constant}$ where p_T and p_1 are the pressures at throat and inlet respectively.

f 7. In an impulse turbine the theoretical enthalpy drop of the steam through the nozzles is 312.5 kJ/kg and 10% of this is lost in friction in the nozzles. The nozzle angle is 20° , the inlet angle of the blades is 35° , and the absolute velocity of the steam leaving the blades is 204 m/s in the direction of the axis of the turbine. Calculate on the basis of one kg of steam supplied per second:

- (i) blade velocity so that there is no shock at steam entry,
- (ii) blade angle at exit,
- (iii) energy lost due to friction of the steam across the blades,
- (iv) axial thrust,
- (v) power supplied,
- (vi) efficiency of the blading.

f 8. An engine is supplied with steam at a pressure of 15 bar and temperature 250°C , and the pressure of the exhaust is 0.16 bar. Assuming isentropic expansion, find (i) the dryness fraction of the steam after expansion, (ii) the Rankine efficiency.

f 9. Dry saturated steam at 20 bar is throttled to supply a turbine at 10 bar, this is followed by isentropic expansion to 1.4 bar. It is then reheated at constant pressure to 200°C and finally expanded isentropically to 0.4 bar. Using the $h-s$ chart determine per kg of steam.

- (i) changes in enthalpy during each stage.
- (ii) the overall change in entropy.
- (iii) the condition of the steam at the end of expansion.

f 10. Air is drawn into a gas turbine working on the constant pressure cycle, at 1 bar 21°C and compressed to 5.7 bar. The temperature at the end of heat supply is 680°C . Taking expansion and compression to be adiabatic where $c_v = 0.718 \text{ kJ/kg K}$, $c_p = 1.005 \text{ kJ/kg K}$, calculate (i) temperature at end of compression, (ii) temperature at exhaust, (iii) heat energy supplied per kg at constant pressure, (iv) increase in internal energy per kg from inlet to exhaust, (v) ideal thermal efficiency.

CHAPTER 13

BOILERS AND COMBUSTION

There are two types of boiler, (i) the fire-tube, in which the hot gases from the combustion of the fuel in the furnace pass through the tubes, while the water is around the outside of the tubes, and (ii) the water-tube boiler in which the water flows through the tubes while the hot gases pass around the outside of the tubes.

CAPACITY AND EQUIVALENT EVAPORATION

The *capacity* of a boiler is the mass of steam that can be produced by it in a given time, usually per hour. However, since the feed water temperature, and pressure and temperature of the steam varies with different plants, it is necessary for purposes of comparison, to refer the evaporative capacity to a common standard. This is the imaginary condition of assuming the feed water temperature as 100°C , to be converted into dry saturated steam at 100°C , and to reduce this to the mass of steam evaporated by unit mass of fuel burned in the furnace. This basis of comparison is termed the *equivalent evaporation, per kg of fuel, from and at 100°C* . For brevity, "per kg of fuel" is usually omitted.

Thus, if for example a boiler produces 60 Mg of steam per hour at 40 bar and 450°C , from feed water at 130°C , when 4800 kg of fuel are burned per hour, then:

Tables page 7, steam 40 bar 450°C , $h = 3330$
 4, water at 130°C , $h = 546$

Heat energy transferred to each kg of steam
 = $3330 - 546 = 2784 \text{ kJ}$

Actual evaporative capacity is $60 \times 10^3 \text{ kg/h}$ therefore,
 total heat energy transferred to steam
 = $2784 \times 60 \times 10^3 \text{ kJ/h}$

Now suppose the temperature of the feed water was 100°C and dry saturated steam at 100°C was produced, the heat energy required per kg of steam would be h_{fg} at 100°C which (tables page 2) is 2256.7 kJ , therefore the mass of steam that would be produced under these conditions of "from and at 100°C " would be:

$\frac{\text{total heat energy transferred to steam per hour}}{\text{heat energy required for each kg}}$

$$= \frac{2784 \times 60 \times 10^3}{2256.7} = 74 \times 10^3 \text{ kg/h or } 74 \text{ Mg/h}$$

This is the capacity of the boiler from and at 100°C.
Since 4800 kg of fuel are burned per hour, the mass of steam that would be produced by each kg of fuel is,

$$\frac{74 \times 10^3}{4800} = 15.42 \text{ kg steam/kg fuel}$$

This is the equivalent evaporation, per kg of fuel, from and at 100°C.

Summing up,

if m_s = actual mass of steam generated per unit mass of fuel burned

h_1 = enthalpy per kg of steam

h_w = enthalpy per kg of feed water

h_{fg} 100°C = enthalpy of evaporation per kg at 100°C then, equivalent evaporation from and at 100°C

$$= \frac{m_s (h_1 - h_w)}{h_{fg} 100^\circ\text{C}}$$

BOILER EFFICIENCY

The efficiency of a boiler is the ratio of the heat energy transferred to the feed water in converting it into steam, to the heat energy supplied to the boiler by the combustion of the fuel.

The heat energy transferred to the water to produce steam is the difference between the enthalpy of the steam leaving the boiler and the enthalpy of the feed water entering the boiler, thus:

$$\text{mass of steam generated} \times (h_1 - h_w)$$

The heat energy supplied to the boiler is the energy released during combustion of the fuel, which is the product of the mass of fuel burned and its calorific value. The calorific value, as previously explained (Chapter 7) is the heat energy given off during complete combustion of unit mass of the fuel. Therefore,

$$\text{Boiler efficiency} = \frac{\text{Heat energy transferred to water and steam}}{\text{Heat energy supplied by fuel}}$$

$$= \frac{\text{mass of steam} \times (h_1 - h_w)}{\text{mass of fuel} \times \text{calorific value}}$$

or, if we let m_s represent the mass of steam generated per unit mass of fuel burned, then,

$$\text{Boiler efficiency} = \frac{m_s (h_1 - h_w)}{\text{calorific value of the fuel}}$$

Example. An oil-fired boiler working at a pressure of 15 bar generates 14.5 kg of steam per kg of fuel burned. The feed water temperature is 95°C and the steam leaves the boiler 0.98 dry. If the calorific value of the oil is 42 MJ/kg, calculate the thermal efficiency of the boiler and the equivalent evaporation from and at 100°C.

Tables page 4, steam 15 bar, $h_f = 845$ $h_{fg} = 1947$

... .. 2, water 95°C, $h_w = 398$

$$\text{Boiler steam, } h_1 = h_f + x h_{fg} \\ = 845 + 0.98 \times 1947 = 2753$$

Heat energy transferred to steam per kg of fuel burned

$$= m_s (h_1 - h_w) \\ = 14.5 (2753 - 398) \\ = 14.5 \times 2355 \text{ kJ}$$

Heat energy supplied to boiler per kg of fuel burned

$$= 42 \times 10^3 \text{ kJ}$$

$$\text{Boiler efficiency} = \frac{14.5 \times 2355}{42 \times 10^3}$$

$$= 0.8132 \text{ or } 81.32\% \quad \text{Ans. (i)}$$

Equivalent evaporation, per kg of fuel, from and at 100°C

$$= \frac{m_s (h_1 - h_w)}{h_{fg} 100^\circ\text{C}} \\ = \frac{14.5 \times 2355}{2256.7}$$

$$= 15.13 \text{ kg steam/kg fuel. Ans. (ii)}$$

FEED WATER

Every precaution must be taken to maintain water as pure as possible. One of the most common causes of contamination of the water is a leakage of sea water.

There is always a certain amount of loss in the steam circuit through the engines and this loss must be made up to maintain the

working level of the water in the boiler. Make-up feed may be taken direct from the reserve fresh water tanks, or, distilled water may be produced by evaporating sea water in evaporators. It is usual to pass all make-up feed water through evaporators.

The quantity of dissolved solids present in water is expressed in parts of solids per million parts of water, abbreviated to *parts per million* and represented by p.p.m. This is the same ratio as the grammes of solids in one million grammes of water. Note that $10^6 \text{ g} = 10^3 \text{ kg} = 1 \text{ tonne}$ and this is the mass of one cubic metre of pure water. For example, if there are 80 g of dissolved solids in one m^3 of water, it is expressed as 80 p.p.m. As a further example, 5 tonne of water having 150 p.p.m. dissolved solids, the total mass of dissolved solids is

$$5 \times 150 = 750 \text{ g.}$$

Equations dealing with the amount of dissolved solids in evaporators and boilers are dealt with on this basis. Thus, in boilers with a contaminated feed, the equation is built upon the simple principle:

$$\begin{array}{l} \text{initial mass [g] of} \\ \text{solids in boiler} \end{array} + \begin{array}{l} \text{mass [g] of solids} \\ \text{put in with feed} \end{array} = \begin{array}{l} \text{final mass [g] of} \\ \text{solids in boiler} \end{array}$$

$$\begin{array}{l} \text{mass [t] of} \\ \text{water in} \\ \text{boiler} \end{array} \times \begin{array}{l} \text{initial} \\ \text{p.p.m.} \end{array} + \begin{array}{l} \text{mass [t]} \\ \text{of feed} \end{array} \times \begin{array}{l} \text{feed} \\ \text{p.p.m.} \end{array} = \begin{array}{l} \text{mass [t] of} \\ \text{water in} \\ \text{boiler} \end{array} \times \begin{array}{l} \text{final} \\ \text{p.p.m.} \end{array}$$

In evaporators where it is common to have a constant blow-down to maintain a steady pre-determined density of the water in the evaporator, the equation would be based upon

$$\begin{array}{l} \text{mass of solids put in} = \text{mass of solids blown out} \\ \text{mass of feed} \times \text{feed p.p.m.} = \text{mass blown out} \times \text{blow out p.p.m.} \end{array}$$

If the evaporated water contains a given amount of solids this would be included on the right hand side of the above equation.

Example. A boiler initially contains 4 tonne of water of 80 p.p.m. If the evaporation rate is 500 kg/h and the feed water contains 150 p.p.m. of dissolved solids, calculate (i) the density (p.p.m.) of the boiler water after 12 hours, and (ii) the time for the water to reach 2000 p.p.m. from the time of initial condition.

$$\begin{aligned} \text{Amount of feed} &= \text{evaporation rate} \\ &= 500 \text{ kg/h} = 0.5 \text{ tonne/h} \\ &= 0.5 \times 12 \text{ tonne in 12 hours} \end{aligned}$$

Solids in initially + solids put in = solids in finally

$$\begin{array}{l} \text{water in} \\ \text{boiler} \end{array} \times \begin{array}{l} \text{initial} \\ \text{p.p.m.} \end{array} + \begin{array}{l} \text{amount} \\ \text{of feed} \end{array} \times \begin{array}{l} \text{feed} \\ \text{p.p.m.} \end{array} = \begin{array}{l} \text{water in} \\ \text{boiler} \end{array} \times \begin{array}{l} \text{final} \\ \text{p.p.m.} \end{array}$$

$$4 \times 80 + 0.5 \times 12 \times 150 = 4 \times \text{final density}$$

$$320 + 900 = 4 \times \text{final density}$$

$$\text{Final density} = \frac{1220}{4} = 305 \text{ p.p.m. Ans. (i)}$$

Let t = time in hours for boiler water to reach 2000 p.p.m. then feed in t hours = $0.5t$ tonne:

$$\begin{array}{l} \text{water in} \\ \text{boiler} \end{array} \times \begin{array}{l} \text{initial} \\ \text{p.p.m.} \end{array} + \begin{array}{l} \text{amount} \\ \text{of feed} \end{array} \times \begin{array}{l} \text{feed} \\ \text{p.p.m.} \end{array} = \begin{array}{l} \text{water in} \\ \text{boiler} \end{array} \times \begin{array}{l} \text{final} \\ \text{p.p.m.} \end{array}$$

$$4 \times 80 + 0.5t \times 150 = 4 \times 2000$$

$$75t = 7680$$

$$t = 102.4 \text{ hours Ans. (ii)}$$

If an analysis of the feed water is given so that a distinction can be made with regard to the solids that remain in solution and those that precipitate to form scale when the water is heated, then the density in the evaporator or boiler is taken as that due to permanently soluble solids.

The density of sea water and the composition of the dissolved solids vary throughout different parts of the world. A typical sample could be taken as dissolved solids 32280 p.p.m. and mass analysis of the solid matter:

Sodium chloride	79.3%
Magnesium chloride	10.2%
Magnesium sulphate	6.1%
Calcium sulphate	3.8%
Calcium bicarbonate	0.6%

Scale is formed from magnesium sulphate, calcium sulphate and calcium bicarbonate. All scales are bad heat conductors and therefore hinder transfer of heat, the result being overheating of the metal heating surfaces with consequent loss of strength and possibility of collapse. The remainder of the solids may be regarded as permanently soluble solids.

PRINCIPLES OF COMBUSTION

Combustion of fuel is the chemical combination, at high temperature, of the combustible elements in the fuel with oxygen, heat energy being released in the process.

In furnaces of boilers and cylinders of internal combustion engines, the oxygen is obtained from an air supply, air being composed of approximately 23% oxygen and 77% nitrogen, by mass. The oxygen is the active element. Nitrogen, being an inert gas, takes no active part, it acts as a moderator, dilutes the products of combustion and, as it absorbs some of the heat energy produced it reduces the temperature of combustion.

The principal combustible elements in fuels are carbon and hydrogen, others may be present in small quantities, such as sulphur.

The air must be intimately mixed with the fuel and the amount of fuel which can be burned depends upon the quantity of air supplied. An excess over the theoretical minimum quantity of air for complete combustion is always necessary, the amount of excess depending upon the design of the combustion space and conditions under which the fuel is burned. If there is an insufficient air supply, combustion will not be complete, one indication of this being black smoke. If too much air is supplied, an unnecessary amount of heat energy will be carried away to waste. Each case represents a loss of efficiency.

Taking the composition of air by mass as 23% oxygen and 77% nitrogen, then 23 kg of oxygen will be obtained from 100 kg of air. In the same proportion, to obtain 1 kg of oxygen, the supply of air must be $\frac{100}{23}$ kg. Thus, the theoretical minimum mass of air required

is $\frac{100}{23}$ times the mass of oxygen needed for complete combustion.

Elements are substances in their simplest form and consist of molecules which are identical. They are represented by a symbol, usually the first letter of their names except when necessary to distinguish between two whose names have the same first letter. The molecules of some elements consist of single atoms, such as carbon (C) and sulphur (S). Other elements consist of molecules of two atoms each and these are distinguished by the subscript 2, examples of these are hydrogen (H₂), nitrogen (N₂) and oxygen (O₂).

Compounds are chemical combinations of different elements and denoted by placing together the symbols of the elements which

constitute the compound. Thus, each molecule of water (and steam) is composed of two atoms of hydrogen and one atom of oxygen, and is therefore written H₂O. Carbon dioxide is represented by CO₂ which signifies that each molecule of carbon dioxide is composed of one atom of carbon and two atoms of oxygen. The number of molecules, when more than one, is represented by placing that number in front, thus 3CO₂ represents three molecules of carbon dioxide.

Atomic weights are pure numbers representing the relative masses of the atoms. For instance, if the atomic weight of hydrogen is 1 and the atomic weight of sulphur is 32, it means that the mass of one atom of sulphur is 32 times greater than the mass of one atom of hydrogen. These numbers are purely relative and neither the actual weight nor actual mass.

The *molecular weight* is the sum of the atomic weights of the atoms of which the molecules is composed.

A list of the relative masses of the substances involved in calculations on the combustion of fuels is given below, to the nearest whole number which is sufficiently accurate for practical purposes.

Substance	Symbol	Atomic weight	Molecular weight
ELEMENTS			
Hydrogen	H ₂	1	2 × 1 = 2
Carbon	C	12	1 × 12 = 12
Nitrogen	N ₂	14	2 × 14 = 28
Oxygen	O ₂	16	2 × 16 = 32
Sulphur	S	32	1 × 32 = 32
COMPOUNDS			
Water, Steam	H ₂ O		2 × 1 + 1 × 16 = 18
Carbon dioxide	CO ₂		1 × 12 + 2 × 16 = 44
Sulphur dioxide	SO ₂		1 × 32 + 2 × 16 = 64
Carbon monoxide	CO		1 × 12 + 1 × 16 = 28

STOICHIOMETRY. Is the determination of the exact proportion of elements to make pure chemical compounds. In examples that follow the expression "*stoichiometric air requirement*" is sometimes used. The stoichiometric air requirement is the exact amount of air required for complete combustion.



CALORIFIC VALUES. The calorific value (c.v.) of a substance is the amount of heat energy released during complete combustion of unit mass of that substance and, for fuels, it is usually expressed in megajoules of energy per kilogramme of mass [MJ/kg].

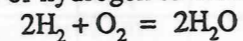
When hydrogen burns it combines with oxygen to form steam and the heat energy released is about 144 MJ per kg of hydrogen.

Carbon, if supplied with sufficient oxygen, will burn completely to carbon dioxide and in doing so, about 33.7 MJ of heat energy is released per kg of carbon. If there is a deficiency of oxygen, some or all of the carbon will burn to carbon monoxide and the heat energy released by each kg of carbon is then only about one-third of that when carbon dioxide is formed. Thus there is a great loss when carbon monoxide is produced due to an insufficient air supply to the fuel.

In the burning of sulphur, it chemically combines with oxygen to form sulphur dioxide and about 9.3 MJ of heat energy is released per kg of sulphur.

CHEMICAL EQUATIONS represent the proportions in which the elements combine and, by substituting the atomic weights, the relative mass of oxygen required for the burning of each combustible element can be calculated.

Combustion of hydrogen to steam,



inserting atomic weights,

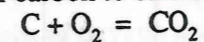
$$2 \times (2 \times 1) + 2 \times 16 = 36$$

$$4 \text{ kg H}_2 + 32 \text{ kg O}_2 = 36 \text{ kg H}_2\text{O}$$

$$1 \text{ kg H}_2 + 8 \text{ kg O}_2 = 9 \text{ kg H}_2\text{O}$$

That is, 1 kg of hydrogen requires 8 kg of oxygen to burn it completely, and this produces 9 kg of steam.

Combustion of carbon to carbon dioxide,



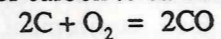
$$1 \times 12 + 2 \times 16 = 44$$

$$12 \text{ kg C} + 32 \text{ kg O}_2 = 44 \text{ kg CO}_2$$

$$1 \text{ kg C} + 2\frac{2}{3} \text{ kg O}_2 = 3\frac{2}{3} \text{ kg CO}_2$$

Thus, 1 kg of carbon requires $2\frac{2}{3}$ kg of oxygen to burn it completely and this produces $3\frac{2}{3}$ kg of carbon dioxide.

Combustion of carbon to carbon monoxide,



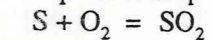
$$2 \times 12 + 2 \times 16 = 2 \times 28$$

$$24 \text{ kg C} + 32 \text{ kg O}_2 = 56 \text{ kg CO}$$

$$1 \text{ C} + 1\frac{1}{3} \text{ O}_2 = 2\frac{1}{3} \text{ CO}$$

Thus, 1 kg of carbon if supplied with $1\frac{1}{3}$ kg of oxygen gives incomplete combustion which produces $2\frac{1}{3}$ kg of carbon monoxide.

Combustion of sulphur to sulphur dioxide,



$$1 \times 32 + 2 \times 16 = 64$$

$$32 \text{ kg S} + 32 \text{ kg O}_2 = 64 \text{ kg SO}_2$$

$$1 \text{ kg S} + 1 \text{ kg O}_2 = 2 \text{ kg SO}_2$$

Hence, 1 kg of sulphur requires 1 kg of oxygen to burn it and 2 kg of sulphur dioxide is produced.

* The analysis of some fuels show that a little oxygen is present, in these cases it is usual to assume that this oxygen is combined with some of the hydrogen in the form of water and therefore this portion of hydrogen cannot be burned. We have seen above that hydrogen and oxygen combine in the proportion of 1 to 8, hence the amount of hydrogen which is not available for combustion is one-eighth of the mass of oxygen present in the fuel. The remaining hydrogen in the fuel is referred to as *available hydrogen*, thus,

$$\text{Available hydrogen} = \text{H}_2 - \frac{\text{O}_2}{8}$$

From the foregoing notes we can now write the expressions:

$$\text{Calorific value} = 33.7 \text{ C} + 144 \left(\text{H}_2 - \frac{\text{O}_2}{8} \right) + 9.3 \text{ S MJ/kg}$$

$$\text{Oxygen required} = 2\frac{2}{3} \text{ C} + 8 \left(\text{H}_2 - \frac{\text{O}_2}{8} \right) + \text{S kg O}_2/\text{kg fuel}$$

$$\text{Minimum air reqd. (stoichiometric)} = \frac{100}{23} \times \text{oxygen required}$$

$$= \frac{100}{23} \left\{ 2\frac{2}{3} \text{ C} + 8 \left(\text{H}_2 - \frac{\text{O}_2}{8} \right) + \text{S} \right\} \text{ kg air/kg fuel}$$

Example. An oil fuel is composed of 86% carbon, 11% hydrogen, 2% oxygen and 1% impurities. Calculate the calorific value and stoichiometric mass of air required to burn 1 kg of the fuel, taking the calorific values of carbon and hydrogen as 33.7 and 144 MJ/kg respectively.

$$\text{Available hydrogen} = \text{H}_2 - \frac{\text{O}_2}{8}$$

$$= 0.11 - \frac{0.02}{8} = 0.1075 \text{ kg}$$

$$\begin{aligned} \text{Heat energy from 0.86 kg of carbon} &= 0.86 \times 33.7 = 28.98 \text{ MJ} \\ \text{Heat energy from 0.1075 kg of hydrogen} &= 0.1075 \times 144 = 15.48 \text{ MJ} \\ \text{Total heat energy per kg of fuel} &= 28.98 + 15.48 = 44.46 \text{ MJ/kg}^* \text{ Ans. (i)} \end{aligned}$$

$$\begin{aligned} \text{Oxygen required to burn the carbon} &= 2\frac{2}{3} \text{ kg} \times 0.86 = 2.293 \text{ kg} \\ \text{Oxygen required to burn the available hydrogen} &= 8 \times 0.1075 = 0.86 \text{ kg} \\ \text{Total oxygen} &= 2.293 + 0.86 = 3.153 \text{ kg} \\ \text{Air required (stoichiometric)} &= \frac{100}{23} \times 3.153 = 13.71 \text{ kg air/kg fuel} \text{ Ans. (ii)} \end{aligned}$$

HIGHER AND LOWER CALORIFIC VALUES. H_2O formed by the hydrogen in the fuel cannot exist as water in the high temperatures of boiler flue gases or the exhaust gases from internal combustion engines, it exists in the form of steam, and this steam passing away in the waste gases carries enthalpy of evaporation which is not available as heat energy to the boiler or engine.

Therefore, the theoretical calorific value of a fuel containing hydrogen, as calculated in the above example, is termed the *higher* (or *gross*) *calorific value* and, when the unavailable heat energy is subtracted from this, it is termed the *lower* (or *net*) *calorific value*. It has been recommended that the amount of heat energy to be considered as not available should be 2.442 MJ per kg of steam in the products of combustion. The amount of steam, as shown above, is nine times the mass of hydrogen in the fuel.

In the previous example the mass of hydrogen in each kilogramme of fuel is 0.11 kg, therefore there will be $9 \times 0.11 = 0.99$ kg of steam formed. The unavailable heat energy is therefore to be taken as $0.99 \times 2.442 = 2.418$.

Hence the lower calorific value of this fuel is:

$$44.46 - 2.418 = 42.042 \text{ MJ/kg}$$

The calorific values of solid and liquid fuels can be found experimentally by means of a bomb calorimeter.

COMPOSITION OF FLUE GASES

An estimate of the analysis of the flue gases can be calculated

from the composition of the fuel and mass of supply air as demonstrated in the following.

Example. An oil fuel is composed of 85% carbon, 12% hydrogen, 2% oxygen, and 1% incombustible solid matter. If the air supply is 50% in excess of the stoichiometric, find the mass of each product of combustion per kg of fuel burned and the percentage mass analysis of the flue gases.

$$\begin{aligned} \text{Stoichiometric air/kg fuel} &= \frac{100}{23} \left\{ 2\frac{2}{3} \text{ C} + 8 \left(\text{H}_2 - \frac{\text{O}_2}{8} \right) \right\} \\ &= \frac{100}{23} \left\{ 2\frac{2}{3} \text{ C} + 8 \text{ H}_2 - \text{O}_2 \right\} \\ &= \frac{100}{23} \left\{ 2\frac{2}{3} \times 0.85 + 8 \times 0.12 - 0.02 \right\} \\ &= \frac{100}{23} \times 3.207 = 13.94 \text{ kg air/kg fuel} \end{aligned}$$

$$\text{Excess air} = 0.5 \times 13.94 = 6.97 \text{ kg}$$

$$\text{Actual air supply} = 13.94 + 6.97 = 20.91 \text{ kg air/kg fuel}$$

Mass of each product of combustion, Ans. (i):

$$\text{Mass of CO}_2 \text{ formed} = 3\frac{2}{3} \times 0.85 = 3.117 \text{ kg}$$

$$\text{Mass of H}_2\text{O formed} = 9 \times 0.12 = 1.08 \text{ kg}$$

$$\begin{aligned} \text{Mass of O}_2 &= \text{excess of oxygen from the excess air} \\ &= 23\% \text{ of } 6.97 = 1.603 \text{ kg} \end{aligned}$$

$$\begin{aligned} \text{Mass of N}_2 &= \text{mass of nitrogen in all the air supply} \\ &= 77\% \text{ of } 20.91 = 16.1 \text{ kg} \end{aligned}$$

Total products of combustion

$$= 3.117 + 1.08 + 1.603 + 16.1 = 21.9 \text{ kg/kg fuel}$$

Alternatively, the total mass of products of combustion per kg fuel = mass of air supplied + (mass of fuel - incombustibles)

$$= 20.91 + (1 - 0.01)$$

$$= 20.91 + 0.99 = 21.9 \text{ kg/kg fuel}$$

Composition expressed as percentages, Ans. (ii):

$$\text{CO}_2 = \frac{3.117}{21.9} \times 100 = 14.23\%$$

$$\text{H}_2\text{O} = \frac{1.08}{21.9} \times 100 = 4.931\%$$

$$O_2 = \frac{1.603}{21.9} \times 100 = 7.319\%$$

$$N_2 = \frac{16.1}{21.9} \times 100 = 73.52\%$$

ORSAT APPARATUS

A volumetric analysis of the dry products of combustion can be made by the Orsat apparatus. This operates on the principle of passing a sample of the flue gases through a series of three bottles containing different solutions, each of which absorbs one of the constituents, and measuring each time the reduction in volume of the sample which is the volume of the constituent absorbed.

The solutions used for absorption are (i) caustic potash which can absorb CO_2 , (ii) pyrogallic acid which can absorb CO_2 and O_2 , (iii) cuprous chloride which can absorb CO_2 , O_2 and CO . Therefore the first bottle through which the sample is passed must be that containing the caustic potash solution so that only CO_2 is absorbed, the remainder of the sample then contains O_2 , CO and N_2 . The sample is next passed through the bottle containing the pyrogallic acid solution which takes out the O_2 , leaving CO and N_2 . Finally, the sample is passed through the cuprous chloride solution which absorbs the CO , and the remainder is assumed to be all N_2 . The temperature and pressure are maintained constant throughout the operation.

It is convenient to draw in a sample of exactly 100 ml to be tested so that the percentage volumetric analysis is simply the volumes in ml.

CONVERSION FROM VOLUMETRIC TO MASS ANALYSIS. The volumetric analysis obtained by the Orsat apparatus can now be converted into a mass analysis. By Avogadro's law, equal volumes of any gas, at the same temperature and pressure, contain the same number of molecules, therefore, although the *mass* of a molecule of one gas is different to the mass of a molecule of another gas, the *volume* of each molecule is the same at the same temperature and pressure.

Hence the ratio of the volumes of each gas in the mixture of flue gases (N mols) can be converted into a ratio of masses by multiplying by the molecular weight of the gas (M), and from this the percentage composition by mass (m%) can be calculated.

Example. In a test on a sample of funnel gases the percentage composition by volume was found to be: $CO_2 = 8.5\%$, $O_2 = 9.5\%$, $CO = 3\%$ and the remainder 79% was assumed to be N_2 . Convert these quantities into a mass analysis.

DFG	N	M	m	m%
CO_2	8.5	44	$8.5 \times 44 = 374$	$\frac{374}{2974} \times 100 = 12.58\%$
O_2	9.5	32	$9.5 \times 32 = 304$	$\frac{304}{2974} \times 100 = 10.22\%$
CO	3.0	28	$3 \times 28 = 84$	$\frac{84}{2974} \times 100 = 2.82\%$
N_2	79.0	28	$79 \times 28 = 2212$	$\frac{2212}{2974} \times 100 = 74.38\%$
			Total = 2974	Total = 100.00%

Example. The analysis of a sample of coal burned in the furnace of a boiler is 80% carbon, 5% hydrogen, 4% oxygen, and the remainder ash etc. Calculate (i) the stoichiometric air required per kg of coal, (ii) the actual mass of air if it is supplied with 70% excess, (iii) the percentage mass analysis of the products of combustion.

$$\begin{aligned} \text{Stoichiometric air} &= \frac{100}{23} \left\{ 2\frac{2}{3} C + 8 \left(H_2 - \frac{O_2}{8} \right) \right\} \\ &= \frac{100}{23} \left\{ 2\frac{2}{3} C + 8 H_2 - O_2 \right\} \\ &= \frac{100}{23} \left\{ 2\frac{2}{3} \times 0.8 + 8 \times 0.05 - 0.04 \right\} \\ &= \frac{100}{23} \times 2.493 = 10.84 \text{ kg air/kg fuel Ans. (i)} \end{aligned}$$

$$\text{Excess air} = 0.7 \times 10.84 = 7.59$$

$$\text{Actual air} = 10.84 + 7.59 = 18.43 \text{ kg air/kg fuel Ans. (ii)}$$

Mass products of combustion per kg of coal:

$$CO_2 = 3\frac{2}{3} C = 3\frac{2}{3} \times 0.8 = 2.933 \text{ kg}$$

$$9 H_2 = 9 \times 0.05 = 0.45$$

$$O_2 = 23\% \text{ of excess air} = 0.23 \times 7.59 = 1.746$$

$$N_2 = 77\% \text{ of all air} = 0.77 \times 18.43 = 14.19$$

$$\text{Total} = 19.319 \text{ kg}$$

% mass analysis Ans. (iii)

$$\text{CO}_2 = \frac{2.933}{19.319} \times 100 = 15.18\%$$

$$\text{H}_2\text{O} = \frac{0.45}{19.319} \times 100 = 2.33\%$$

$$\text{O}_2 = \frac{1.746}{19.319} \times 100 = 9.04\%$$

$$\text{N}_2 = \frac{14.19}{19.319} \times 100 = 73.45\%$$

Dry flue products are the total products less the amount of steam
 $= 19.319 - 0.45 = 18.869 \text{ kg}$

f CONVERSION FROM MASS TO VOLUMETRIC ANALYSIS. Is sometimes required and can be obtained by reversing the above procedure. To illustrate, determine the percentage analysis of the dry flue gases by volume from the previous worked example.

DFG	m%	M	N	N%
CO ₂	15.18	44	15.18 + 44 = 0.345	$\frac{0.345}{3.2505} \times 100 = 10.61\%$
O ₂	9.04	32	9.04 + 32 = 0.2825	$\frac{0.2825}{3.2505} \times 100 = 8.69\%$
N ₂	73.45	28	73.45 + 28 = 2.623	$\frac{2.623}{3.2505} \times 100 = 80.7\%$
			Total = 3.2505	Total = 100.00%

f HYDROCARBON FUELS C_xH_y. The group of fuels which have a chemical formula C_xH_y are called hydrocarbon fuels. Some examples are:

Methane	CH ₄
Propane	C ₃ H ₈
Butane	C ₄ H ₁₀

If we consider one kg of hydrocarbon fuel C_xH_y, it will be necessary to obtain the masses of carbon and hydrogen contained in the one kg in order to evaluate the air required for combustion.

$$\begin{aligned} \text{The mass of one mol of the fuel} &= 12 \times x + 1 + y \text{ kg} \\ M &= 12 \times x + 1 + y \end{aligned}$$

$$\begin{aligned} \text{The fraction of hydrogen by mass} &= \frac{1 \times y}{M} \text{ kg} \\ \text{and the fraction of carbon by mass} &= \frac{12 \times x}{M} \text{ kg} \end{aligned}$$

f Example. Determine the amount of air required for the complete combustion of 1 kg of propane, formula C₃H₈.

$$\begin{aligned} \text{Mass of 1 mol of the fuel} &= 12 \times 3 + 1 \times 8 = 44 \text{ kg} \\ \text{Fraction of hydrogen by mass} &= \frac{8}{44} = 0.1818 \\ \text{Fraction of carbon by mass} &= \frac{12 \times 3}{44} = 0.8182 \\ &= \underline{\underline{\Sigma 1.0000}} \end{aligned}$$

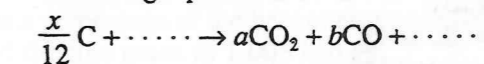
Hence 1 kg of C₃H₈ consists of 0.8182 kg of carbon and 0.1818 kg of hydrogen.

$$\begin{aligned} \text{Stoichiometric air} &= \frac{100}{23} [0.8182 \times 2\frac{2}{3} + 0.1818 \times 8] \\ &= 15.81 \text{ kg/kg of fuel.} \end{aligned}$$

Note: In 1 kg of a pure hydrocarbon fuel, if *x* is the mass of carbon then (1 - *x*) is the mass of hydrogen.

f INCOMPLETE COMBUSTION. This may occur if insufficient air, or unsatisfactory combustion, results. The exhaust gas analysis will then include carbon monoxide. Balancing of the fuel-gas combustion equation allows analysis of the constituents and dry gas and air amount.

Balancing equations:



$$\text{C balance is } \frac{x}{12} = a + b$$

Similarly for oxygen, hydrogen, etc. balances; these relations can be used to complete an otherwise unknown combustion equation; the mass of dry gases can be found from a C balance.

Relative mass of C in fuel

$$\begin{aligned} &= \frac{12}{44} \{ \text{Rel. mass (m) CO}_2 \} + \frac{12}{28} \{ \text{Rel. mass (m) CO} \} \text{ in gases} \\ &= \frac{12}{44} \{ \text{Rel. vol. (N) CO}_2 \times 44 \} + \frac{12}{28} \{ \text{Rel. vol. (N) CO} \times 28 \} \\ &= \{ \text{Rel. vol. (N) CO}_2 + \text{Rel. vol. (N) CO} \} \end{aligned}$$

f Example. The exhaust gas from an engine has a dry analysis by volume as follows:

CO₂ 8.8%, CO 1.25%, O₂ 6.9%, N₂ 83.05%.

The fuel supplied to the engine has a mass analysis as follows:

C 84%, H₂ 14%, O₂ 2%

Determine the mass of air supplied per kg of fuel burnt.

Note: Air contains 20.7% oxygen by volume and 23% oxygen by mass. Atomic mass relationships: hydrogen = 1, carbon = 12, nitrogen = 14, oxygen = 16.

DFG	N	M	m
CO ₂	8.8	44	387.2
CO	1.25	28	35
O ₂	6.9	32	220.8
N ₂	83.05	28	2325.4
			Total 2968.4

$$\text{Relative gas mass} = 2968.4$$

$$\begin{aligned} \text{Relative C mass} &= 12(8.8 + 1.25) \\ &= 120.6 \end{aligned}$$

i.e. 120.6 kg of C in 2968.4 kg of dry flue gases

$$0.84 \text{ kg of C in } \frac{2968.4 \times 0.84}{120.6} \text{ kg dry gases}$$

$$\text{Dry flue gases} = 20.68 \text{ kg}$$

$$\begin{aligned} \text{Water vapour} &= 9 \times 0.14 \\ &= 1.26 \text{ kg} \end{aligned}$$

$$\text{Total gases} = 21.94 \text{ kg}$$

$$\text{Mass of air supplied} = 20.94 \text{ kg/kg fuel Ans.}$$

TEST EXAMPLES 13

1. Steam at 30 bar, 375°C, is generated in a boiler at the rate of 30000 kg/h from feed water at 130°C. The fuel has a calorific value of 42 MJ/kg and the daily consumption is 53 tonne. Calculate (i) the boiler efficiency, (ii) the equivalent engine power if the overall efficiency of the plant is 0.13, (iii) the evaporative capacity of the boiler in kg/h from and at 100°C, (iv) the equivalent evaporation per kg of fuel from and at 100°C.

2. A boiler contains 3.5 tonne of water initially having 40 p.p.m. dissolved solids, and after 24 hours the dissolved solids in the water is 2500 p.p.m. If the feed rate is 875 kg/h, find the p.p.m. of dissolved solids contained in the feed water.

3. At its rated output a waste heat evaporator produces 10 tonne/day of fresh water containing 250 p.p.m. dissolved solids from sea water containing 31250 p.p.m. The dissolved solid content of the brine in the evaporator shell is maintained at 78125 p.p.m. Calculate the mass flow per day of:

- the sea water feed;
- the brine discharge.

4. The constituents of a fuel are 85% carbon, 13% hydrogen, and 2% oxygen. When burning this fuel in a boiler furnace the air supply is 50% in excess of the stoichiometric, inlet temperature of the air being 31°C and funnel temperature 280°C. Calculate (i) the calorific value of the fuel, (ii) mass of air supplied per kg of fuel, and (iii) the heat energy carried away to waste in the funnel gases expressed as a percentage of the heat energy supplied, taking the specific heat of the flue gases as 1.005 kJ/kg K.

5. A fuel consists of 84% carbon, 13% hydrogen, 2% oxygen, and the remainder incombustible solid matter. Calculate (i) the calorific value, (ii) the stoichiometric air required per kg of fuel, and (iii) an estimate of the mass analysis of the flue gases if 22 kg of air are supplied per kg of fuel burned.

6. The analysis of a fuel oil is 85.5% carbon, 11.9% hydrogen, 1.6% oxygen, and 1% impurities. Calculate the percentage CO₂ in the flue gases when (i) the quantity of air supplied is the stoichiometric, and when the excess air over the stoichiometric is (ii) 25%, (iii) 50%, and (iv) 75%.

f 7. The mass analysis of a fuel is 84% carbon, 14% hydrogen

and 2% ash. It is burnt with 20% excess air relative to stoichiometric requirement. If 100 kg/h of this fuel is burnt in a boiler determine (i) the volumetric analysis of the dry flue gas (ii) the volumetric flow rate of the gas if its temperature and pressure are 250°C and 1 bar respectively.

Note: Air contains 23% oxygen by mass. $R_o = 8.3143 \text{ kJ/kg molK}$ and atomic mass relationships are: oxygen 16, carbon 12, hydrogen 1.

f 8. The fuel oil supplied to a boiler has a mass analysis of 86% carbon, 12% hydrogen and 2% sulphur. The fuel is burned with an air/fuel ratio of 20:1.

Calculate:

- (a) the mass analysis of the wet flue gases;
- (b) the volumetric analysis of the wet flue gases.

f 9. Benzene fuel C_6H_6 is burnt in a boiler furnace under stoichiometric conditions of combustion. Calculate:

- (a) the mass of air required for one kilogram of benzene.
- (b) the mass analysis of the flue gases.
- (c) the volumetric analysis of the dry flue gas.

f 10. The volumetric analysis of a dry flue gas is: carbon dioxide 10.8%, oxygen 7.2%, carbon monoxide 0.8% and the remainder nitrogen.

Calculate:

- (a) the analysis of the dry flue gas by mass;
- (b) the mass percentage of carbon and hydrogen in the fuel assuming it to be a pure hydrocarbon.

CHAPTER 14

REFRIGERATION

The natural transfer of heat energy is from a hot body to a colder body. The function of a refrigeration plant is to act as a heat pump and reverse this process.

Refrigerating machines can be divided into two classes, (i) those which require a supply of mechanical work which is the vapour compression system, and (ii) those which require a heat supply and work on the absorption system. The latter is more suited to small domestic use and only the vapour compression system is described here.

The refrigerating agent used in the circuit is a substance which will evaporate at low temperatures. The boiling and condensation points of a liquid depend upon the pressure exerted upon it, for example, if water is under atmospheric pressure it will evaporate at 100°C, if the pressure is 7 bar the water will not change into steam until its temperature is 165°C, at 14 bar the boiling point is 195°C and so on. The refrigerant used must evaporate at very low temperatures. The boiling point of carbon dioxide at atmospheric pressure is about -78°C (note *minus 78*), by increasing the pressure the temperature at which liquid CO_2 will evaporate (or CO_2 vapour will condense) is raised accordingly so that any desired evaporation and condensation temperature can be attained, within certain limits, by subjecting it to the appropriate pressure.

Some of the agents employed as refrigerants and their more important characteristics are as follows:

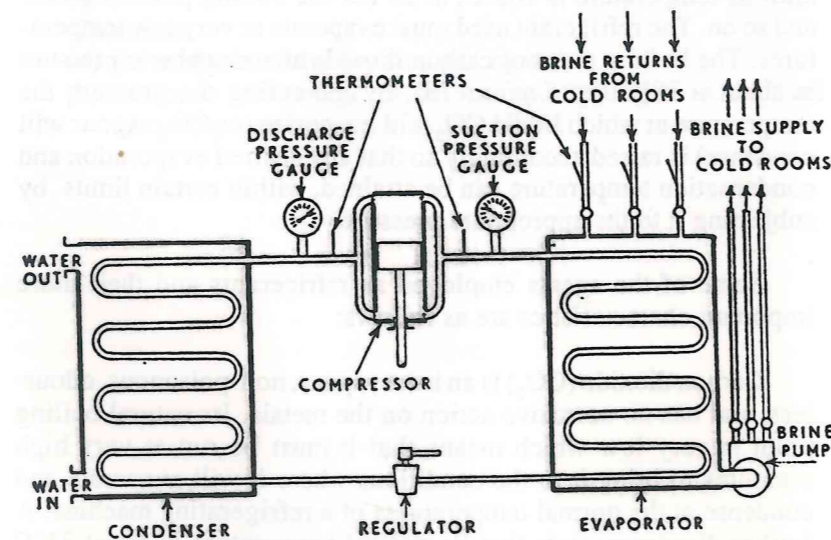
Carbon dioxide (CO_2) is an inert vapour, non-poisonous, odourless, and has no corrosive action on the metals. Its natural boiling point is very low which means that it must be run at very high pressures to bring it to the conditions where it will evaporate and condense at the normal temperatures of a refrigerating machine. A further disadvantage is that its critical temperature is about 31°C which falls within the range of coolant temperatures. At its critical temperature and above, it is impossible to liquefy the vapour no matter to what pressure it is subjected. Virtually outdated.

Ammonia (NH_3). Is a poisonous vapour and therefore an ammonia machine should have a compartment of its own so that it can be sealed off in the case of a serious leakage; water will absorb ammonia and therefore a water spray is a good combatant against a leakage. Ammonia will corrode copper and copper alloys and therefore parts in contact with it should be made of such metals as nickel steel and monel metal. Its natural boiling point is about -39°C therefore the pressures required are much lower than those required in a CO_2 machine. Largely outdated.

Freon-12 (CCl_2F_2). Requires low pressures. Is non-flammable, non-poisonous and non-corrosive. Physical properties make this a very desirable refrigerant - almost standard in use today. R12 is part of a large Freon group; others in use are R11, 22 and 502.

WORKING CYCLE

Fig. 68 illustrates diagrammatically the essential components of the vapour compression system, which consists of the Compressor driven by an electric motor, the Condenser which is circulated by the water, the Regulator (sometimes referred to as the



VAPOUR COMPRESSION REFRIGERATING MACHINE

Fig. 68

expansion valve), and the Evaporator which is circulated by brine. It is a completely enclosed circuit, the same quantity of refrigerant passes continually through the system and it only requires to be charged when there are losses due to leakage. Any of the above agents may be used as the refrigerant, the difference in their working being only the pressures throughout the system, therefore in the following description the words "refrigerant", "the liquid" and "the vapour", etc. will be used.

The refrigerant is drawn as a vapour at low pressure from the evaporator into the compressor where it is compressed to a high pressure and delivered into the coils of the condenser in the state of a superheated vapour. As it passes through the condenser coils, the vapour is cooled and condensed into a liquid at approximately coolant inlet water temperature, the latent heat energy given up being absorbed by the coolant surrounding the coils. The liquid, still at a high pressure, passes along to the regulator, this is a valve just partially open to limit the flow through it. The pipe-line from the discharge side of the compressor to the regulating valve is under high pressure but from the regulating valve to the suction side of the compressor is at low pressure due to the regulating valve being just a little way open. As the liquid passes through the regulator from a region of high pressure to a region of low pressure, throttling occurs, and some of the liquid automatically changes itself into a vapour absorbing the required amount of heat energy to do so from the remainder of the liquid and causing it to fall to a low temperature, this temperature being regulated by the pressure so that the refrigerant enters the evaporator at a temperature lower than that of the brine. The liquid (more correctly a mixture of liquid and vapour) now passes through the coils of the evaporator where it receives heat to evaporate it before being taken into the compressor to go through the cycle again. The required heat absorbed by the liquid refrigerant in the coils of the evaporator to cause evaporation is extracted from the brine surrounding the coils, resulting in the brine being cooled to a low temperature (or directly from the air).

THE CIRCUIT OF THE REFRIGERANT

It is necessary to have a clear picture in mind when performing calculations involving the changes that take place on the refrigerant as it flows through the components of the circuit, therefore a simple diagrammatic sketch such as Fig. 69 should be made with points in

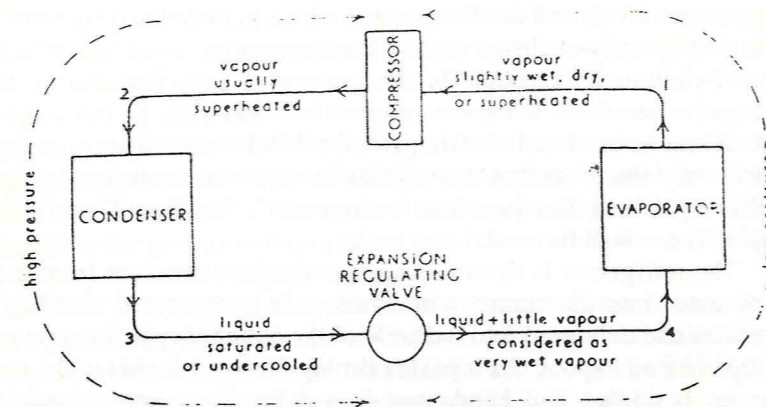


Fig. 69

the circuit suitably marked. Reference should be made to Fig. 69 when reading the following notes and examples.

Properties of some refrigerants are given in "Thermodynamic and Transport Properties of Fluids", SI Units, arranged by Y. R. Mayhew and G. F. C. Rogers, this booklet was referred to in previous Chapters dealing with properties of steam. In examples given here, extracts from these tables will be taken when appropriate, stating the page number.

Principles and calculations involving enthalpy of the refrigerant, dryness fraction, volume, effect of throttling, etc, follow the same pattern as for other vapours. However, whereas the enthalpy and entropy of steam are measured from the datum of water at 0°C , it is more convenient to choose a lower datum temperature for refrigerants to avoid negative quantities within the normal range of refrigeration. This datum is -40°C , thus the enthalpy and entropy of the saturated liquid refrigerant are taken as zero at this temperature.

Note that when the value of h_{fg} is not given in the tables, it is found by subtraction: $h_{fg} = h_g - h_f$. Similarly for the value of s_{fg} .

Referring to Fig. 69:

The work done on the vapour in the compressor increases the specific enthalpy of the refrigerant from h_1 to h_2 . Therefore the effective work done per second [kJ/s], which is the effective power [kW], is equal to the product of the mass flow [kg/s] of the refrigerant and its gain of specific enthalpy $(h_2 - h_1)$ [kJ/kg].

In the condenser, heat transfer takes place from the refrigerant to the condenser circulating water, causing condensation of the

refrigerant from a vapour to a liquid. The heat rejected by the refrigerant per kg is the change of specific enthalpy $h_2 - h_3$.

The effect of throttling the refrigerant through the expansion valve is to reduce the pressure of the saturated liquid with consequent reduction in temperature and the formation of a very small amount of vapour. During a throttling process there is no change of enthalpy, therefore h_4 is equal to h_3 .

In passing through the evaporator coils, heat transfer takes place from the brine (or other surrounds) to the refrigerant, which cools the brine and causes the refrigerant to evaporate. The gain in specific enthalpy of the refrigerant is $h_1 - h_4$ and this is the *refrigerating effect* or *cooling effect* per kg of the refrigerant flowing through the circuit.

Example. In an NH_3 refrigerator, the ammonia leaves the evaporator and enters the compressor as dry saturated vapour at 2.68 bar, it leaves the compressor and enters the condenser at 8.57 bar with 50°C of superheat. it is condensed at constant pressure and leaves the condenser as saturated liquid. If the rate of flow of the refrigerant through the circuit is 0.45 kg/min calculate (i) the compressor power, (ii) the heat rejected to the condenser cooling water in kJ/s, and (iii) the refrigerating effect in kJ/s.

From tables page 12, NH_3 :

$$2.68 \text{ bar, } h_g = 1430.5$$

$$8.57 \text{ bar, } h_f = 275.1 \quad h \text{ supht } 50^\circ = 1597.2$$

Mass flow of refrigerant

$$= \frac{0.45}{60} = 0.0075 \text{ kg/s}$$

Enthalpy gain per kg of refrigerant in compressor

$$= h_2 - h_1 = 1597.2 - 1430.5 = 166.7 \text{ kJ/kg}$$

Enthalpy gain per second

$$= \text{mass flow [kg/s]} \times 166.7 \text{ [kJ/kg]} = 0.0075 \times 166.7 = 1.25 \text{ kJ/s}$$

kJ/s = kW, therefore useful compressor power

$$= 1.25 \text{ kW Ans. (i)}$$

Enthalpy drop per kg of refrigerant through condenser

$$= h_2 - h_3 = 1597.2 - 275.1 = 1322.1 \text{ kJ/kg}$$

Heat rejected = 0.0075×1322.1

$$= 9.915 \text{ kJ/s Ans. (ii)}$$

$$\begin{aligned} \text{Enthalpy gain per kg of refrigerant through evaporator} \\ &= h_1 - h_4 \\ &= 1430.5 - 275.1 = 1155.4 \text{ kJ/kg} \end{aligned}$$

(Note that the value of h_4 is the same as h_3 because there is no change of enthalpy in the throttling process through the expansion valve between the condenser exit and the evaporator inlet).

$$\begin{aligned} \text{Refrigerating effect} &= 0.0075 \times 1155.4 \\ &= 8.666 \text{ kJ/s} \quad \text{Ans. (iii)} \end{aligned}$$

Example. In a Freon-12 refrigerator, the freon leaves the condenser as a saturated liquid at 20°C , and the evaporator temperature is -10°C and the freon leaves the evaporator as a vapour 0.97 dry. Calculate (i) the dryness fraction at the evaporator inlet, (ii) the cooling effect per kg of refrigerant, and (iii) the volume flow of refrigerant entering the compressor if the mass flow is 0.1 kg/s .

From tables page 13, Freon-12:

$$20^\circ\text{C}, \quad h_f = 54.87$$

$$-10^\circ\text{C}, \quad h_f = 26.87 \quad h_g = 183.19 \quad v_g = 0.0766$$

$$h_{fg} = h_g - h_f = 183.19 - 26.87 = 156.32$$

Throttling process through expansion valve:

$$\text{Enthalpy after throttling } (h_4) = \text{Enthalpy before } (h_3)$$

$$h_{fg} + x_4 h_{fg4} = h_3$$

$$26.87 + x_4 \times 156.32 = 54.87$$

$$x_4 \times 156.32 = 28$$

$$x_4 = 0.1792 \quad \text{Ans. (i)}$$

Enthalpy gain of refrigerant through evaporator

$$\begin{aligned} &= \text{cooling effect} = h_1 - h_4 \\ &= (26.87 + 0.97 \times 156.32) - (26.87 + 0.1792 \times 156.32) \\ &= (0.97 - 0.1792) \times 156.32 \\ &= 0.7908 \times 156.32 = 123.6 \text{ kJ/kg} \quad \text{Ans. (ii)} \end{aligned}$$

Alternatively, since $h_4 = h_3$:

$$\begin{aligned} \text{cooling effect} \\ &= (26.87 + 0.97 \times 156.32) - 54.87 \\ &= 123.6 \text{ kJ/kg} \end{aligned}$$

Specific volume of refrigerant leaving evaporator

$$= 0.97 \times 0.0766 \text{ m}^3/\text{kg}$$

Volume of refrigerant entering compressor per second [m^3/kg]

$$= 0.1 \times 0.97 \times 0.0766$$

$$= 7.43 \times 10^{-3} \text{ m}^3/\text{s}$$

or 7.43 litre/s Ans. (iii)

CAPACITY AND PERFORMANCE

The capacity of a refrigerating plant is usually expressed in terms of the tonnes of ice at 0°C that could be made in 24 hours from water at 0°C .

The *coefficient of performance* is the ratio of the refrigerating effect to the net work done on the refrigerant in the compressor:

$$\begin{aligned} \text{Coeff. of performance} &= \frac{\text{heat extracted by refrigerant}}{\text{work done on refrigerant}} \\ &= \frac{h_1 - h_4}{h_2 - h_1} \end{aligned}$$

f In Chapter 8, the reversed Carnot cycle was explained as the ideal cycle for a refrigerator and the coefficient of performance of this cycle given as:

$$\frac{T_2}{T_1 - T_2}$$

where T_1 and T_2 are the highest and lowest absolute temperatures in the cycle. This is referred to as the *Carnot coefficient of performance*. Note however, that the notation is different to that used in the practical circuit of Fig. 68, to avoid confusion write subscripts H for highest and L for lowest:

$$\frac{T_L}{T_H - T_L}$$

f Example. A Freon-12 refrigerating machine operates on the ideal vapour compression cycle between the limits -15°C and 25°C . The vapour is dry saturated at the end of isentropic compression and there is no undercooling of the condensate in the condenser. Calculate (i) the dryness fraction at the suction of the compressor, (ii) the refrigerating effect per kg of refrigerant, (iii) the coefficient of performance, (iv) the Carnot coefficient of performance.

Tables Freon-12, page 13:

$$25^\circ\text{C}, \quad h_f = 59.7 \quad h_g = 197.73 \quad s_g = 0.6869$$

$$-15^\circ\text{C}, \quad h_f = 22.33 \quad h_g = 180.97$$

$$h_{fg} = 180.97 - 22.33 = 158.64$$

$$s_f = 0.0906 \quad s_g = 0.7501$$

$$s_{fg} = 0.7051 - 0.0906 = 0.6145$$

Isentropic compression in compressor:

Entropy before (s_1) = Entropy after (s_2)

$$0.0906 + x_1 \times 0.6145 = 0.6869$$

$$x_1 = 0.9701 \quad \text{Ans. (i)}$$

$$h_1 = 22.33 + 0.9701 \times 158.64$$

$$= 153.9$$

$$\text{Refrigerating effect/kg} = h_1 - h_4$$

Since there is no undercooling, the condensate leaves the condenser as saturated liquid, hence $h_3 = 59.7$, and $h_4 = h_3$ because there is no change of enthalpy during the throttling process, therefore:

$$\text{Refrigerating effect} = 153.9 - 59.7$$

$$= 94.2 \text{ kJ/kg} \quad \text{Ans. (ii)}$$

$$\text{Work done in compressor/kg} = h_2 - h_1$$

$$= 197.73 - 153.9 = 43.83 \text{ kJ/kg}$$

$$\text{Coefficient of performance} = \frac{\text{refrigerating effect}}{\text{work transfer}}$$

$$= \frac{h_1 - h_4}{h_2 - h_1} = \frac{94.2}{43.83}$$

$$= 2.140 \quad \text{Ans. (iii)}$$

$$\text{Carnot coeff. of performance} = \frac{T_L}{T_H - T_L}$$

$$\text{where } T_H = 25 + 273 = 298 \text{ K}$$

$$T_L = -15 + 273 = 258 \text{ K}$$

$$\therefore \text{Carnot c.o.p.} = \frac{258}{298 - 258} = 6.45 \quad \text{Ans. (iv)}$$

$T-s$ and $p-h$ diagrams illustrate, and allow analysis of, the refrigeration circuit.

f Example. An ammonia refrigeration plant operates between temperature limits of -20°C and 32°C . The refrigerant leaves the evaporator as saturated vapour and enters the expansion valve as saturated liquid. Assuming the isentropic efficiency of the compressor is 0.85 determine the coefficient of performance of the plant and sketch the cycle on $p-h$ and $T-s$ diagrams.

From tables

$$h_2 = 1469.9 \text{ kJ/kg}$$

$$h_3 = h_4 = 332.8 \text{ kJ/kg}$$

$$\text{For isentropic compression } s_1 = s_2 = 4.962 \text{ kJ/kg K}$$

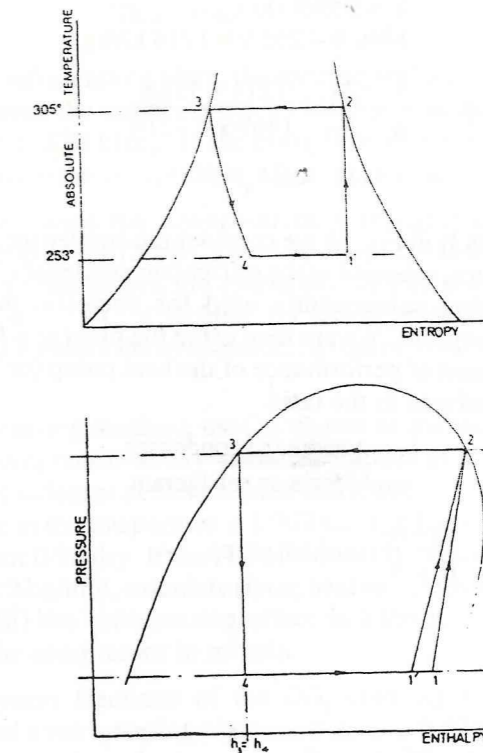


Fig. 70

at -20°C

$$s_1 = s_f + x(s_g - s_f)$$

$$4.962 = 0.368 + x(5.623 - 0.368)$$

$$x = 0.874 \text{ dry}$$

also at -20°C

$$h_1 = h_f + x(h_g - h_f)$$

$$= 89.8 + 0.874(1420 - 89.8)$$

$$= 1252.4 \text{ kJ/kg}$$

$$\text{Isentropic work done in compressor} = h_2 - h_1$$

$$= 1469.9 - 1252.4$$

$$= 217.5 \text{ kJ/kg}$$

Actual work done in compressor \times isentropic efficiency = isentropic work done in compressor

$$\therefore \text{Actual work done in compressor} = \frac{217.5}{0.85} = 255.9 \text{ kJ/kg}$$

$$\begin{aligned} \therefore h_1' &= h_2 - 255.9 \\ &= 1469.9 - 255.9 = 1214 \text{ kJ/kg} \end{aligned}$$

$$\begin{aligned} \text{c.o.p.} &= \frac{h_1' - h_4}{h_2 - h_1} = \frac{1214 - 332.8}{1469.9 - 1214} \\ &= 3.443 \text{ Ans.} \end{aligned}$$

HEAT PUMPS If in Fig. 69 we consider the condenser as a heater where air, or water, is heated as the refrigerant condenses, the heated air or water being subsequently used for domestic purposes in accommodation spaces. We are then using the plant as a *heat pump*.

The coefficient of performance of the heat pump (or heat pump coefficient) is defined as the ratio:

$$\frac{\text{heat transfer in condenser}}{\text{work done on refrigerant}}$$

$$\text{i.e. c.o.p.} = \frac{h_2 - h_3}{h_2 - h_1} \text{ for heat pump.}$$

TEST EXAMPLES 14

1. In a CO_2 refrigerating plant, the specific enthalpy of the refrigerant as it leaves the condenser is 135 kJ/kg, and as it leaves the evaporator it is 320 kJ/kg. If the mass flow of the refrigerant is 5 kg/min calculate the refrigerating effect per hour.
2. Freon-12 leaves the condenser of a refrigerating plant as a saturated liquid at 5.673 bar. The evaporator pressure is 1.509 bar and the refrigerant leaves the evaporator at this pressure and at a temperature of -5°C . Calculate (i) the dryness fraction of the refrigerant as it enters the evaporator, (ii) the refrigerating effect per kg.
3. A refrigerating machine uses ammonia as the working fluid. It leaves the compressor as dry saturated vapour at 8.57 bar, passes through the condenser at this pressure and leaves as saturated liquid. The pressure in the evaporator is 1.902 bar and the ammonia leaves the evaporator 0.96 dry. If the rate of flow of the refrigerant through the circuit is 2 kg/min, calculate (i) the heat rejected in the condenser in kJ/min, (ii) the refrigerating effect in kJ/min, (iii) the volume taken into the compressor in m^3/min .
4. The dryness fractions of the CO_2 entering and leaving the evaporator of a refrigerating plant are 0.28 and 0.92 respectively. If the specific enthalpy of evaporation (h_g) of CO_2 at the evaporator pressure is 290.7 kJ/kg, calculate (i) the refrigerating effect per kg, (ii) the mass of ice at -5°C that would theoretically be made per day from water at 14°C when the mass flow of CO_2 through the machine is 0.5 kg/s. Take the values:

$$\text{Specific heat of water} = 4.2 \text{ kJ/kg K}$$

$$\text{Specific heat of ice} = 2.04 \text{ kJ/kg K}$$

$$\text{Enthalpy of fusion of ice} = 335 \text{ kJ/kg}$$

5. A refrigerating machine is driven by a motor of output power 2.25 kW and 2.5 tonne of ice at -7°C is made per day from water at 18°C . Calculate the coefficient of performance of the machine and express its capacity in terms of tonnes of ice per 24 hours from and at 0°C , taking the following values:

$$\text{Specific heat of water} = 4.2 \text{ kJ/kg K}$$

$$\text{Specific heat of ice} = 2.04 \text{ kJ/kg K}$$

$$\text{Enthalpy of fusion of ice} = 335 \text{ kJ/kg}$$

6. The refrigerant leaves the compressor and enters the condenser of a Freon-12 refrigerating plant at 5.673 bar and 50°C, and leaves the condenser as saturated liquid at the same pressure. At compressor suction the pressure is 1.826 bar and temperature 0°C. Calculate the coefficient of performance.

f 7. In an ammonia refrigerating plant the discharge from the compressor is 14.7 bar 63°C, the refrigerant leaves the condenser at this pressure as liquid with no undercooling. The suction pressure at the compressor is 2.077 bar and compression is isentropic. Calculate, for a mass flow of refrigerant of 0.15 kg/s, (i) the refrigerating effect, (ii) the work transfer in the compressor, (iii) the coefficient of performance.

f 8. A refrigeration plant using Freon-12 operates between 4.914 bar and 1.004 bar. The refrigerant is dry saturated on entry to the compressor and no undercooling takes place in the condenser. If compression is isentropic and losses are neglected, determine:

(a) the c.o.p.

(b) power input to compressor if the flow rate of refrigerant entering the compressor is 0.15 m³/s

Sketch the cycle on temperature-entropy and pressure-enthalpy axes.

f 9. A heat pump using ammonia operates between saturation temperature limits of -16°C and 40°C. The ammonia leaves the compressor after isentropic compression as dry saturated vapour and is then used to heat air from ambient, 13°C to 20°C which is circulated in an insulated room of internal volume 300 m³. If the room requires 4 changes of air per hour determine the c.o.p. of the heat pump and the mass flow rate of ammonia.

Take for the air $c_p = 1005$ J/kg, $R = 287$ J/kgK and pressure 1.013 bar.

f 10. In a refrigeration plant Freon-12 leaves the condenser as a liquid at 25°C. The refrigerant leaves the evaporator as a dry saturated vapour at -15°C and leaves the compressor at 6.516 bar and 40°C. The cooling load is 73.3 kW.

Calculate:

(a) the mass flow rate of refrigerant;

(b) the power of the compressor

(c) the condition of the refrigerant after the expansion valve.

SOLUTIONS TO TEST EXAMPLES 1

- Mass of water lifted = 50×10^3 kg/h
 Weight of water lifted = $50 \times 10^3 \times 9.81$ N/h
 Power [W = J/s = N m/s]
 = weight lifted per second [N/s] \times height [m]
 = $\frac{50 \times 10^3 \times 9.81 \times 8}{3600}$
 = 1090 W = 1.09 kW Ans. (i)

Input power = $\frac{\text{output power}}{\text{efficiency}}$
 = $\frac{1.09}{0.69} = 1.58$ kW Ans. (ii)

Energy [kWh] = power [kW] \times time [h]
 = $1.58 \times 2 = 3.16$ kWh Ans. (iii)

Energy [MJ] = energy [kWh] \times 3.6 [MJ/kWh]
 = $3.16 \times 3.6 = 11.38$ MJ Ans. (iv)
- A column of water 1 mm high exerts a pressure of 9.81 N/m², therefore 20 mm water is equivalent to:
 $20 \times 9.81 = 196.2$ N/m² Ans. (ia)
 One mbar = 100 N/m²
 $\therefore 196.2 \text{ N/m}^2 = 1.962$ mbar Ans. (iia)

A column of mercury 1 mm high exerts a pressure of 133.3 N/m², therefore 750 mmHg is equivalent to:
 $750 \times 133.3 = 1 \times 10^5$ N/m²
 = 100 kN/m² Ans. (ib)
 = 1 bar Ans. (iib)
- Abs. press. = (barometer mmHg - vac. gauge mmHg) \times 133.3
 = $(757 - 715) \times 133.3$
 = 42×133.3
 = 5.6×10^3 N/m² = 5.6 kN/m²
 = 0.056 bar Ans.



$$\begin{aligned}
 4. \quad C &= (140 - 32) \times \frac{5}{9} & C &= (5 - 32) \times \frac{5}{9} \\
 &= 108 \times \frac{5}{9} & &= -27 \times \frac{5}{9} \\
 &= 60^\circ \text{ Ans. (i)} & &= -15^\circ\text{C Ans. (ii)} \\
 C &= (-31 - 32) \times \frac{5}{9} & C &= (-40 - 32) \times \frac{5}{9} \\
 &= -63 \times \frac{5}{9} & &= -72 \times \frac{5}{9} \\
 &= -35^\circ\text{C Ans. (iii)} & &= -40^\circ\text{C Ans. (iv)}
 \end{aligned}$$

$$\begin{aligned}
 5. \quad \text{Area of tube bore} &= 0.7854 \times 0.03^2 \text{ m}^2 \\
 \text{Area of 16 tubes} &= 0.7854 \times 0.03^2 \times 16 \text{ m}^2 \\
 \text{Volume flow [m}^3\text{/s]} &= \text{area [m}^2\text{]} \times \text{velocity [m/s]} \\
 &= 0.7854 \times 0.03^2 \times 16 \times 2 \\
 &= 0.02261 \text{ m}^3\text{/s} \\
 1 \text{ m}^3 &= 10^3 \text{ litre} \\
 \therefore \text{Volume flow [l/s]} &= 0.02261 \times 10^3 \\
 &= 22.61 \text{ l/s Ans. (i)} \\
 \text{density} &= 0.85 \text{ g/ml} = 0.85 \text{ kg/litre} \\
 \text{Mass flow [kg/min]} &= \text{vol. flow [litre/min]} \times \text{density [kg/litre]} \\
 &= 22.61 \times 60 \times 0.85 \\
 &= 1153 \text{ kg/min Ans. (ii)}
 \end{aligned}$$

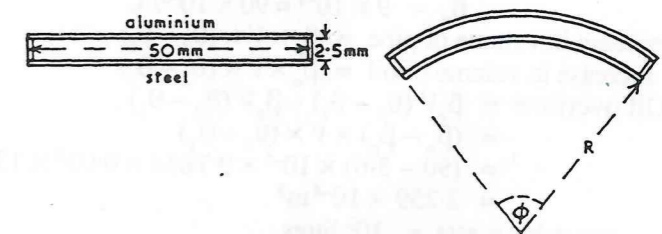
SOLUTIONS TO TEST EXAMPLES 2

- Mechanical energy converted into heat energy per minute:
 Energy [kJ] = power [kW = kJ/s] \times time [s]
 $= 70 \times 60$
 $= 4200 \text{ kJ} = 4.2 \text{ MJ}$ Ans. (i)
 All the heat energy being transferred to water:
 $Q \text{ [kJ]} = m \text{ [kg]} \times c \text{ [kJ/kg K]} \times (T_2 - T_1) \text{ [K]}$
 $4200 = m \times 4.2 \times 10$
 $m = 100 \text{ kg/min}$ Ans. (ii)
- Friction force at effective radius of pads
 $= \mu \times \text{force between surfaces}$
 $= 0.025 \times 240 = 6 \text{ kN}$
 Work to overcome friction in one revolution
 $= \text{friction force [kN]} \times \text{circumference [m]}$
 $= 6 \times 2\pi \times 0.23 \text{ kN m} = \text{kJ}$
 Power loss [kW = kJ/s] = work per rev [kJ] \times rev/s
 $= 6 \times 2\pi \times 0.23 \times \frac{93}{60}$
 $= 13.45 \text{ kW}$ Ans. (i)
 Mechanical energy converted into heat energy per hour:
 Energy [kWh] = power [kW] \times time [h]
 $= 13.45 \times 1 = 13.45 \text{ kWh}$
 $13.45 \text{ kWh} \times 3.6 = 48.42 \text{ MJ}$ Ans. (ii)
 Quantity of heat energy transferred to oil per hour:
 $Q \text{ [kJ]} = m \text{ [kg]} \times c \text{ [kJ/kg K]} \times (T_2 - T_1) \text{ [K]}$
 $48.42 \times 10^3 = m \times 2 \times 20$
 $m = 1210.5 \text{ kg/h}$ Ans. (iii)
- Let θ = initial temperature of the copper
 θ = temperature of the flue gases
 Heat lost by copper = Heat gained by water
 $m_c \times c_c \times \text{temp. fall}_c = m_w \times c_w \times \text{temp. rise}_w$
 $1.8 \times 0.395 \times (\theta - 37.2) = 2.27 \times 4.2 \times (37.2 - 20)$
 $0.711 \theta - 26.44 = 164$
 $0.711 \theta = 190.44$
 $\theta = 267.9^\circ\text{C}$ Ans.

4. Mass of 2.3 litre of water = 2.3 kg
 Let c [kJ/kg K] = specific heat of the iron
 Heat lost by iron = Heat gained by water and vessel
 $2.15 \times c \times (100 - 24.4) = (2.3 + 0.18) \times 4.2 \times (24.4 - 17)$
 $2.15 \times c \times 75.6 = 2.48 \times 4.2 \times 7.4$
 $c = \frac{2.48 \times 4.2 \times 7.4}{2.15 \times 75.6}$
 $= 0.4742 \text{ kJ/kg K}$ Ans.
5. Heat transferred to the ice to raise all of it in temperature from -5°C to 0°C
 $= 0.5 \times 2.04 \times 5 = 5.1 \text{ kJ}$
 Heat transferred from the water and vessel in cooling from 17°C to 0°C
 $= (1.8 + 0.148) \times 4.2 \times 17$
 $= 1.948 \times 4.2 \times 17 = 139.1 \text{ kJ}$
 Therefore, heat from water available to melt some of the ice
 $= 139.1 - 5.1 = 134 \text{ kJ}$
 Each kg of ice requires 335 kJ to melt it, therefore mass of ice melted by 134 kJ
 $= \frac{134}{335} = 0.4 \text{ kg}$
 Hence, $\left. \begin{array}{l} \text{Final mass of water} = 1.8 + 0.4 = 2.2 \text{ kg} \\ \text{Final mass of ice} = 0.5 - 0.4 = 0.1 \text{ kg} \end{array} \right\} \text{Ans.}$

SOLUTIONS TO TEST EXAMPLES 3

1. Free expansion = $\alpha \times l \times (\theta_2 - \theta_1)$
 $= 1.25 \times 10^{-5} \times 3.85 \times (260 - 18)$
 $= 0.01165 \text{ m}$
 $= 11.65 \text{ mm}$ Ans.
2. Vol. of sphere = $\frac{\pi}{6} d^3 = \frac{\pi}{6} \times 0.15^3 \text{ m}^3$
 Mass [kg] = volume [m^3] \times density kg/m^3
 $= \frac{\pi}{6} \times 0.15^3 \times 7.21 \times 10^3$
 $= 12.75 \text{ kg}$
 $Q[\text{kJ}] = m [\text{kg}] \times c [\text{kJ/kgK}] \times (T_2 - T_1) [\text{K}]$
 $2110 = 12.75 \times 0.54 \times (T_2 - T_1)$
 temp. rise = $\frac{2110}{12.75 \times 0.54} = 306.7 \text{ K}$
 $= 306.7^\circ\text{C}$
 Increase in diameter = $\alpha \times d \times (\theta_2 - \theta_1)$
 $= 1.12 \times 10^{-3} \times 150 \times 306.7$
 $= 0.5152 \text{ mm}$ Ans.
- 3.



- Free expansion = $\alpha \times l \times (\theta_2 - \theta_1)$
 Heated length = $l + \alpha l (\theta_2 - \theta_1) = l \{1 + \alpha (\theta_2 - \theta_1)\}$
 Heated length of aluminium strip
 $= 50 (1 + 2.5 \times 10^{-5} \times 200)$
 $= 50 \times 1.005 = 50.25 \text{ mm}$
 Heated length of steel strip
 $= 50 (1 + 1.2 \times 10^{-5} \times 200)$
 $= 50 \times 1.0024 = 50.12 \text{ mm}$
 Heated length of brass distance pieces
 $= 2.5 (1 + 2 \times 10^{-5} \times 200)$

$$= 2.5 \times 1.004 = 2.51 \text{ mm}$$

Let R = radius of curvature of steel

$$R + 2.51 = \dots \dots \dots \text{aluminium}$$

Subtended angle ϕ is common to both

$$\phi \text{ [rad]} = \frac{\text{arc length [mm]}}{\text{radius [mm]}}$$

$$\therefore \frac{50.25}{R + 2.51} = \frac{50.12}{R}$$

$$50.25R = 50.12(R + 2.51)$$

$$50.25R = 50.12R + 125.8$$

$$0.13R = 125.8$$

$$R = 967.7 \text{ mm}$$

$$\text{Radius of steel} = 967.7 \text{ mm} \quad \text{Ans.}$$

$$\text{Radius of aluminium} = 967.7 + 2.51$$

$$= 970.21 \text{ mm} \quad \text{Ans.}$$

$$4. \quad \text{Internal volume of pipe (= vol. of oil in pipe)} \\ = 0.7854 \times 0.03^2 \times 13.7 \text{ m}^3$$

Co-efficient of cubical expansion of steel pipe

$$\beta_p = 3 \times 1.2 \times 10^{-5} = 3.6 \times 10^{-5} / ^\circ\text{C}$$

Co-efficient of cubical expansion of oil

$$\beta_o = 9 \times 10^{-4} = 90 \times 10^{-5} / ^\circ\text{C}$$

$$\text{Increase in volume of pipe} = \beta_p \times V \times (\theta_2 - \theta_1)$$

$$\text{Increase in volume of oil} = \beta_o \times V \times (\theta_2 - \theta_1)$$

$$\text{Oil overflow} = \beta_o V (\theta_2 - \theta_1) - \beta_p V (\theta_2 - \theta_1)$$

$$= (\beta_o - \beta_p) \times V \times (\theta_2 - \theta_1)$$

$$= (90 - 3.6) \times 10^{-5} \times 0.7854 \times 0.03^2 \times 13.7 \times 27$$

$$= 2.259 \times 10^{-4} \text{ m}^3$$

$$\text{one cubic metre} = 10^3 \text{ litres}$$

$$\therefore 2.259 \times 10^{-4} \times 10^3 = 0.2259 \text{ litre} \quad \text{Ans.}$$

$$5. \quad \text{Free expansion} = \alpha \times l \times (\theta_2 - \theta_1)$$

If fully restricted,

$$\text{Strain} = \frac{\text{change of length}}{\text{original length}} = \frac{\alpha l (\theta_2 - \theta_1)}{l} = \alpha (\theta_2 - \theta_1)$$

$$\text{Stress} = \text{strain} \times E$$

$$= 1.12 \times 10^{-5} \times (220 - 17) \times 206 \times 10^9$$

$$= 1.12 \times 203 \times 206 \times 10^4$$

$$= 4.684 \times 10^8 \text{ N/m}^2 = 4684 \text{ MN/m}^2$$

This compressive stress is to be relieved by exerting an initial tensile stress so that the compressive stress does not exceed 350 MN/m²:

$$\text{Initial tensile stress} = 468.4 - 350 \\ = 118.4 \text{ MN/m}^2$$

SOLUTIONS TO TEST EXAMPLES 4

$$1. \quad Q [J] = \frac{k [W/m K] \times A [m^2] \times t [s] \times (T_1 - T_2) [K]}{S [m]}$$

$$= \frac{150 \times 0.7854 \times 0.127^2 \times 60 \times 5}{0.019}$$

$$= 3 \times 10^4 J = 30 kJ \quad \text{Ans.}$$

$$2. \quad \text{Total surface area of walls, ceiling and floor}$$

$$= 2(4.5 \times 2.5 + 4 \times 2.5 + 4.5 \times 4)$$

$$= 78.5 m^2$$

$$Q = \frac{kAt (T_1 - T_2)}{S}$$

$$= \frac{5.8 \times 10^{-2} \times 78.5 \times 3600 \times 20}{0.15}$$

$$= 2.185 \times 10^6 J$$

$$= 2.185 MJ \text{ or } 2185 kJ \quad \text{Ans.}$$

3. For each thickness:

$$Q = \frac{kAt \times \text{temp. diff.}}{S} \quad \therefore \text{temp. diff.} = \frac{QS}{kAt}$$

For the three thicknesses:

$$T_D = \frac{Q}{AT} \left\{ \frac{S_1}{k_1} + \frac{S_2}{k_2} + \frac{S_3}{k_3} \right\}$$

where $T_D = 20 - (-3) = 23^\circ C$ temp. diff. = 23 K

$$\Sigma \left\{ \frac{S}{k} \right\} = \frac{0.03}{0.2} + \frac{0.168}{0.042} + \frac{0.035}{0.2}$$

$$= 0.15 + 4 + 0.175$$

$$= 4.325$$

Heat transfer per second per metre² of area:

$$T_D = \frac{Q}{At} \Sigma \left\{ \frac{S}{k} \right\}$$

$$23 = \frac{Q}{1 \times 1} \times 4.325$$

$$Q = \frac{23}{4.325} = 5.318 J \quad \text{Ans. (i)}$$

Heat transfer per hour through whole side:

$$Q = 5.318 \times 6 \times 3.7 \times 3600$$

$$= 4.25 \times 10^5 J$$

$$= 425 kJ \quad \text{Ans. (ii)}$$

Temperature drop across outer wood wall:

$$= \frac{QS_1}{k_1 At}$$

$$= \frac{5.318 \times 0.03}{0.2}$$

$$= 0.7977 K \quad \text{say } 0.8^\circ C$$

Temperature at interface of cork and outer wood wall
= $20 - 0.8 = 19.2^\circ C$ Ans. (iiia)

Temperature drop across cork:

$$= \frac{QS_2}{k_2 At}$$

$$= \frac{5.318 \times 0.168}{0.042} = 21.27 K = 21.27^\circ C$$

Temperature at interface of cork and inner wood wall
= $19.2 - 21.27 = -2.07^\circ C$ Ans. (iiib)

$$4. \quad Q = 5.67 \times 10^{-11} At (T_1^4 - T_2^4)$$

$$T_1 = 488 K$$

$$T_2 = 318 K$$

$$T_1^4 - T_2^4 = 4.647 \times 10^{10}$$

$$Q = 5.67 \times 10^{-11} \times 0.7854 \times 0.5^2 \times 3600 \times 4.647 \times 10^{10}$$

$$= 1862 kJ \quad \text{Ans.}$$

$$5. \quad \frac{1}{U} = \frac{1}{h_G} + \frac{S_P}{k_P} + \frac{1}{h_A}$$

$$= \frac{1}{31.5} + \frac{0.01}{50} + \frac{1}{32}$$

$$= 0.03175 + 0.0002 + 0.03125$$

$$= 0.0632$$

$$U = 1/0.0632 = 15.82 W/m^2 K \quad \text{Ans. (i)}$$

$$Q = UA t T_D$$

$$Q = 15.82 \times 1 \times 60 \times (280 - 35) \\ = 2.327 \times 10^5 \text{ J} = 232.7 \text{ kJ} \quad \text{Ans. (ii)}$$

6. Heat flow from outside atmosphere to exposed surface of outer wood wall, and also from exposed surface of inner wood wall to room atmosphere:

$$Q = hA_t \times \text{temp. difference per m}^2 \text{ of surface area per second,} \\ 42 = 15 \times 1 \times 1 \times \text{temp. difference}$$

$$\text{temp. difference} = \frac{42}{15} = 2.8^\circ\text{C}$$

$$\text{Temperature of exposed surface of outer wood} \\ = 25 - 2.8 = 22.2^\circ\text{C} \quad \text{Ans. (ia)}$$

$$\text{Temperature of exposed surface of inner wood} \\ = -20 + 2.8 = -17.2^\circ\text{C} \quad \text{Ans. (ib)}$$

Heat flow through each wood wall:

$$Q = \frac{kA_t \times \text{temp. difference}}{0.03}$$

$$42 = \frac{0.2 \times 1 \times 1 \times \text{temp. difference}}{S}$$

$$\text{Temperature difference across each wood wall} \\ = \frac{42 \times 0.03}{0.2} = 6.3^\circ\text{C}$$

$$\text{Interface temperature of outer wood and cork} \\ = 22.2 - 6.3 = 15.9^\circ\text{C} \quad \text{Ans. (iia)}$$

$$\text{Interface temperature of inner wood and cork} \\ = -17.2 + 6.3 = -10.9^\circ\text{C} \quad \text{Ans. (iib)}$$

$$\text{Temperature difference across cork} \\ = 15.9 - (-10.9) = 26.8^\circ\text{C}$$

Heat flow through cork:

$$Q = \frac{kA_t \times \text{temp. difference}}{S}$$

$$S = \frac{0.05 \times 1 \times 1 \times 26.8}{42}$$

$$= 0.0319 \text{ m} = 31.9 \text{ mm} \quad \text{Ans. (iii)}$$

7. Let T_1 = abs. temp. of shell before lagging
= 503 K
 T_2 = abs. temp. of surrounds before lagging
= 324 K
 T_3 = abs. temp. of cleading after lagging
= 342 K

$$T_4 = \text{abs. temp. of surrounds after lagging} \\ = 300 \text{ K}$$

$$\text{Heat radiated from shell before lagging} \\ = 5.67 \times 10^{-11} (T_1^4 - T_2^4) \text{ kJ/m}^2 \text{ s}$$

$$\text{Heat radiated from cleading after lagging} \\ = 5.67 \times 10^{-11} (T_3^4 - T_4^4) \text{ kJ/m}^2$$

$$\text{Heat saved by lagging} \\ = 5.67 \times 10^{-11} \{ (T_1^4 - T_2^4) - (T_3^4 - T_4^4) \} \text{ kJ/m}^2 \text{ s} \\ = 4.74 \times 10^{10}$$

$$\text{Total surface area} = \text{area of ends} + \text{area of cylinder}$$

$$= \pi d^2 + \pi dl$$

$$= \pi d (d + l)$$

$$= \pi \times 1.22 (1.22 + 4.78)$$

$$= \pi \times 1.22 \times 6$$

$$\text{Lagged area} = 0.75 \times \pi \times 1.22 \times 6$$

$$= 17.26 \text{ m}^2$$

$$\text{Total heat saved by lagging per hour}$$

$$= 5.67 \times 10^{-11} A_t \{ (T_1^4 - T_2^4) - (T_3^4 - T_4^4) \}$$

$$= 5.67 \times 10^{-11} \times 17.26 \times 3600 \times 4.74 \times 10^{10}$$

$$= 1.67 \times 10^5 \text{ kJ}$$

$$= 167 \text{ MJ} \quad \text{Ans.}$$

$$8. \quad \text{For lagging } Q = \frac{2\pi k (T_1 - T_2)}{\ln(D/d)} \quad \text{W/m}$$

$$\text{For surface film } Q = \frac{hA (T_2 - T_3)}{L} \quad \text{W/m}$$

$$= h\pi D (T_2 - T_3)$$

$$Q \text{ for lagging} = Q \text{ for surface film}$$

$$\therefore \frac{2\pi k (T_1 - T_2)}{\ln(D/d)} = h\pi D (T_2 - T_3)$$

$$\text{i.e. } \frac{2\pi \times 0.05 (350 - T_2)}{\ln\left(\frac{340}{200}\right)} = 10 \times \pi \times \frac{340}{10^3} (T_2 - 15)$$

$$\text{From which } T_2 = 32.6^\circ\text{C} \quad \text{Ans. (i)}$$

$$\therefore Q = hA (T_2 - T_3)$$

$$= 10 \times \pi \times \frac{340}{10^3} \times 20 (32.6 - 15) / 10^3$$

$$= 3.76 \text{ kJ/s} \quad \text{Ans. (ii)}$$

9. Let T = temperature of the cooling water at outlet
 $\therefore \theta_1 = 410 - 10 \quad \theta_2 = 130 - T$
 $= 400$

$$\therefore \theta_m = \frac{400 - (130 - T)}{\ln \left(\frac{400}{130 - T} \right)} \quad (1)$$

For the gas $Q = 0.4 \times 1130 \times (410 - 130)/10^3$
 $= 126.56 \text{ kW}$

For the water $Q = 0.5 \times 4190 \times (T - 10)/10^3$
 $126.56 = 0.5 \times 4190 \times (T - 10)/10^3$

From which $T = 70.4^\circ\text{C}$ substitute in (1)

$$\therefore \theta_m = \frac{400 - (130 - 70.4)}{\ln \left(\frac{400}{130 - 70.4} \right)}$$

$$\theta_m = 178.8^\circ\text{C}$$

$$\text{also } Q = UA\theta_m$$

$$A = \frac{Q}{UA\theta_m}$$

$$= \frac{126.56 \times 10^3}{140 \times 1 \times 178.8}$$

$$= 5.06 \text{ m}^2 \text{ Ans.}$$

10. *Note:* This question illustrates one fluid being at constant temperature, i.e. the condensing steam

$$\text{Water mass flow} = 0.7854 \times 0.025^2 \times 0.6 \times 10^3$$

$$= 0.2945 \text{ kg/s}$$

$$Q = 0.2945 \times 4.18 \times 12$$

$$= 14.77 \text{ kW}$$

$$\theta_m = \frac{27 - 15}{\ln \left(\frac{27}{15} \right)}$$

$$= 20.41^\circ\text{C}$$

$$Q = UA\theta_m$$

$$U = \frac{14770}{\pi \times 0.025 \times 2.75 \times 1 \times 20.41}$$

Overall heat transfer coefficient

$$= 3350 \text{ W/m}^2 \text{ K Ans. (a)}$$

$$\frac{1}{U} = \frac{1}{h_o} + \frac{1}{h_i}$$

$$\frac{1}{3350} = \frac{1}{17000} + \frac{1}{h_i}$$

Inside surface heat transfer coefficient (i.e. between water and tube) is h_i :

$$= 4172 \text{ W/m}^2 \text{ K Ans. (b)}$$

SOLUTIONS TO TEST EXAMPLES 5

$$1. \quad \begin{aligned} p_1 V_1 &= p_2 V_2 \\ 120 \times 1 &= 960 \times V_2 \\ V_2 &= \frac{120 \times 1}{960} = 0.125 \text{ m}^3 \text{ Ans. (i)} \end{aligned}$$

$$\begin{aligned} p_1 V_1 &= p_2 V_2 \\ 1.05 \times V_1 &= 42 \times 5.6 \\ V_1 &= \frac{42 \times 5.6}{1.05} = 224 \text{ m}^3 \text{ Ans. (ii)} \end{aligned}$$

$$2. \quad \begin{aligned} \frac{p_1 V_1}{T_1} &= \frac{p_2 V_2}{T_2} \\ \frac{1350 \times 0.2}{450} &= \frac{250 \times 0.9}{T_2} \\ T_2 &= \frac{450 \times 250 \times 0.9}{1350 \times 0.2} = 375 \text{ K} \\ &= 102^\circ\text{C} \text{ Ans.} \end{aligned}$$

$$3. \quad \begin{aligned} pV &= mRT \\ m_1 &= \frac{pV}{RT} \\ &= \frac{10 \times 100 \times 2}{287 \times 293} \\ &= 0.02378 \text{ kg} \\ m_2 &= 0.92 \times 0.02378 \\ &= 0.02188 \text{ kg} \\ T &= \frac{pV}{mR} \\ &= \frac{20 \times 100 \times 2}{0.02188 \times 287} \\ &= 636.98 \text{ K} \\ \text{Temperature} &= 363.98^\circ\text{C} \text{ Ans.} \end{aligned}$$

4. By Charles' law, when volume is constant,

$$\frac{p_1}{p_2} = \frac{T_1}{T_2}$$

$$\begin{aligned} p_2 &= \frac{(3200 + 100) \times 308}{289} \\ &= \frac{3300 \times 308}{289} = 3518 \text{ kN/m}^2 \text{ abs.} \end{aligned}$$

or, $3518 - 100 = 3418 \text{ kN/m}^2 \text{ gauge Ans. (i)}$

$$\begin{aligned} Q \text{ [kJ]} &= m \text{ kJ} \times c \text{ [kJ/kg K]} \times (T_2 - T_1) \text{ [K]} \\ &= 20 \times 0.718 \times (35 - 16) \\ &= 272.9 \text{ kJ} \text{ Ans. (ii)} \end{aligned}$$

$$5. \quad \begin{aligned} \text{Volume of saloon} &= 12 \times 16.5 \times 4 = 792 \text{ m}^3 \\ \text{Volume of air to be supplied per hour} \\ &= 2 \times 792 = 1584 \text{ m}^3 \end{aligned}$$

Volume varies as the absolute temperature when the pressure is constant, therefore density varies inversely as the absolute temperature.

$$\begin{aligned} \text{Density at } 0^\circ\text{C and atmos. press.} &= 1.293 \text{ kg/m}^3 \\ \therefore \text{density at } 21^\circ\text{C and atmos. press.} \end{aligned}$$

$$= 1.293 \times \frac{273}{294} = 1.2 \text{ kg/m}^3$$

$$\begin{aligned} \text{Mass of air to be supplied per hour} \\ &= \text{volume [m}^3\text{/h]} \times \text{density [kg/m}^3\text{]} \\ &= 1584 \times 1.2 = 1901 \text{ kg/h} \end{aligned}$$

$$\begin{aligned} \text{Heat extracted per hour} \\ Q \text{ [kJ/h]} &= m \text{ [kg/h]} \times c_p \text{ [kJ/kg K]} \times (T_1 - T_2) \text{ [K]} \\ &= 1901 \times 1.005 \times (30 - 21) \\ &= 17190 \text{ kJ/h or } 17.19 \text{ MJ/h} \text{ Ans. (i)} \end{aligned}$$

$$\begin{aligned} \text{Power [kW]} &= \text{kilojoules per second} \\ &= \frac{17190}{3600} = 4.775 \text{ kW} \text{ Ans. (ii)} \end{aligned}$$

6. Negligible velocity, so kinetic energies are zero

Steady Flow Energy Equation:

$$h_1 + q = h_2 + w \text{ per unit mass}$$

$$w_{\text{OUT}} = h_1 - h_2 + q_{\text{IN}}$$

$$\begin{aligned} w_{\text{OUT}} &= 1.7 \times 0.9 \times 480 - 1.7 \times 10 \\ &= 717.4 \text{ kW} \text{ Ans.} \end{aligned}$$

7. At constant volume:

$$\begin{aligned} Q &= mc_v (T_2 - T_1) \\ &= 1.36 \times 0.718 \times (468 - 40) \\ &= 417.8 \text{ kJ} \quad \text{Ans. (a) (i)} \end{aligned}$$

External work done = nil Ans. (a) (ii)

From the energy equation,

$$\text{Heat energy supplied} = \text{Increase in internal energy} + \text{External work done}$$

$$417.8 = \text{Increase in internal energy} + 0$$

\therefore Increase in internal energy = 417.8 kJ Ans. (a) (iii)

At constant pressure:

$$\begin{aligned} Q &= mc_p (T_2 - T_1) \\ &= 1.36 \times 1.005 \times (468 - 40) \\ &= 584.8 \text{ kJ} \quad \text{Ans. (b) (i)} \end{aligned}$$

Internal energy depends only upon the temperature and is independent of changes in pressure and volume. Therefore the increase in internal energy is the same as before,

$$= 417.8 \text{ kJ} \quad \text{Ans. (b) (iii)}$$

$$\text{Heat energy supplied} = \text{Increase in internal energy} + \text{External work done}$$

$$584.8 = 417.8 + \text{work done}$$

$$\text{Work done} = 584.8 - 417.8$$

$$= 167 \text{ kJ} \quad \text{Ans. (b) (ii)}$$

Note also:

$$R = c_p - c_v = 1.005 - 0.718 = 0.287 \text{ kJ/kg K}$$

$$\begin{aligned} \text{Work done} &= mR (T_2 - T_1) \\ &= 1.36 \times 0.287 \times 428 \\ &= 167 \text{ kJ (as above)} \end{aligned}$$

8. Ratio of partial pressures is equal to the ratio of partial volumes.

$$\left. \begin{aligned} \text{Partial press. of CO}_2 &= 0.1 \times 1.015 = 0.1015 \text{ bar} \\ \text{Partial press. of O}_2 &= 0.08 \times 1.015 = 0.0812 \text{ bar} \\ \text{Partial press. of N}_2 &= 0.82 \times 1.015 = 0.8323 \text{ bar} \end{aligned} \right\} \text{Ans.}$$

$$pV = mRT \quad \therefore m = \frac{pV}{RT}$$

where p = partial pressure, and V = full volume
(or p = total pressure, and V = partial volume)

$$\begin{aligned} \text{Mass of CO}_2 &= \frac{0.1015 \times 10^2 \times 500 \times 10^{-6}}{0.189 \times 293} \\ &= 9.162 \times 10^{-5} \text{ kg} = 0.09162 \text{ g} \quad \text{Ans.} \end{aligned}$$

$$\begin{aligned} \text{Mass of O}_2 &= \frac{0.0812 \times 10^2 \times 500 \times 10^{-6}}{0.26 \times 293} \\ &= 5.33 \times 10^{-5} \text{ kg} = 0.0533 \text{ g} \quad \text{Ans.} \end{aligned}$$

$$\begin{aligned} \text{Mass of N}_2 &= \frac{0.8323 \times 10^2 \times 500 \times 10^{-6}}{0.297 \times 293} \\ &= 4.782 \times 10^{-4} \text{ kg} = 0.4782 \text{ g} \quad \text{Ans.} \end{aligned}$$

$$9. \quad pv = RT \text{ and } R = \frac{R_0}{M}$$

$$\text{hence } pv = \frac{R_0}{M} \times T$$

$$\text{also } h = u + pv$$

$$\therefore h = u + R_0 \frac{T}{M}$$

$$\text{Hence } h_1 = u_1 + R_0 \frac{T_1}{M} \text{ and } h_2 = u_2 + R_0 \frac{T_2}{M}$$

$$u_1 - u_2 = 80$$

$$\begin{aligned} \therefore h_1 - h_2 &= 80 + \frac{R_0}{M} (T_1 - T_2) \\ &= 80 + \frac{8.314}{30} (200 - 100) \\ &= 107.71 \text{ kJ/kg} \end{aligned}$$

Steady flow energy equation

$$h_1 + \frac{1}{2}c_1^2 + q = h_2 + \frac{1}{2}c_2^2 + w$$

$$\text{as } q \text{ and } w = 0$$

$$h_1 - h_2 = \frac{1}{2}(c_2^2 - c_1^2)$$

$$\text{hence } 107.71 \times 10^3 = \frac{1}{2}(c_2^2 - 50^2)$$

$$c_2 = \sqrt{107.71 \times 10^3 \times 2 + 50^2}$$

$$c_2 = 466.82 \text{ m/s} \quad \text{Ans.}$$

$$10. \quad 0.45 \text{ kg carbon monoxide } R = \frac{R_0}{M} = \frac{8.314}{28} = 0.2969$$

$$0.23 \text{ kg oxygen } \quad R = \frac{R_0}{M} = \frac{8.314}{32} = 0.2598$$

$$0.77 \text{ kg nitrogen } \quad R = \frac{R_0}{M} = \frac{8.314}{28} = 0.2969$$

$$1.45 R = 0.45 \times 0.2969 + 0.23 \times 0.2598 + 0.77 \times 0.2969$$

$$1.45 \text{ kg mixture } R = 0.2911$$

$$pV = mRT$$

$$p = \frac{mRT}{V}$$

$$= \frac{1.45 \times 0.2911 \times 288}{0.4}$$

$$= 303.9 \text{ kN/m}^2$$

$$\text{Total pressure} = 3.039 \text{ bar} \quad \text{Ans. (a)}$$

$$pV = mRT$$

$$p = \frac{mRT}{V}$$

$$= \frac{0.77 \times 0.2911 \times 288}{0.4}$$

$$= 164.6 \text{ kN/m}^2$$

$$\text{Partial pressure nitrogen} = 1.646 \text{ bar} \quad \text{Ans. (b)}$$

SOLUTIONS TO TEST EXAMPLES 6

1. For polytropic expansion

$$p_1 V_1^n = p_2 V_2^n$$

$$2550 \times V_1^{1.3} = 210 \times 0.75^{1.3}$$

$$V_1^{1.3} = \frac{210 \times 0.75^{1.3}}{2550}$$

$$V_1 = 0.75 \times \sqrt[1.3]{\frac{210}{2550}}$$

$$= 0.1099 \text{ m}^3 \quad \text{Ans.}$$

2. Ratio of compression = $\frac{8.6}{1} = \frac{\text{initial volume}}{\text{final volume}} = \frac{V_1}{V_2}$

$$p_1 V_1^n = p_2 V_2^n$$

$$98 \times 8.6^{1.36} = p_2 \times 1^{1.36}$$

$$p_2 = 98 \times 8.6^{1.36} = 1828 \text{ kN/m}^2 \quad \text{Ans. (i)}$$

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$$

$$\frac{98 \times 8.6}{301} = \frac{1828 \times 1}{T_2}$$

$$T_2 = \frac{301 \times 1828}{98 \times 8.6} = 653 \text{ K}$$

$$= 380^\circ\text{C} \quad \text{Ans. (ii)}$$

3. $p_1 V_1^n = p_2 V_2^n$

$$1750 \times 0.05^n = 122.5 \times 0.375^n$$

$$\frac{1750}{122.5} = \left\{ \frac{0.375}{0.05} \right\}^n$$

$$n = 1.32 \quad \text{Ans.}$$

4. $\frac{T_1}{T_2} = \left\{ \frac{p_1}{p_2} \right\}^{\frac{n-1}{n}}$

$$\frac{n-1}{n} = \frac{1.35-1}{1.35} = \frac{7}{27}$$

$$\frac{293}{T_2} = \left\{ \frac{101.3}{1420} \right\}^{\frac{7}{27}}$$

$$T_2 = 293 \times \left\{ \frac{1420}{101.3} \right\}^{\frac{7}{27}} = 581.1 \text{ K}$$

$$581.1 - 273 = 308.1^\circ\text{C} \text{ Ans.}$$

5. For adiabatic expansion

$$\frac{T_1}{T_2} = \left\{ \frac{V_2}{V_1} \right\}^{\gamma-1}$$

$$\text{where } \gamma = \frac{c_p}{c_v} = \frac{1.005}{0.718} = 1.4$$

$$\gamma - 1 = 1.4 - 1 = 0.4$$

$$\frac{339}{275} = \left\{ \frac{V_2}{0.014} \right\}^{0.4}$$

$$V_2 = 0.014 \times \sqrt[0.4]{\frac{339}{275}}$$

$$= 0.02362 \text{ m}^3 \text{ Ans.}$$

6.

$$\frac{T_1}{T_2} = \left\{ \frac{p_1}{p_2} \right\}^{\frac{n-1}{n}}$$

$$\frac{305}{773} = \left\{ \frac{117}{3655} \right\}^{\frac{n-1}{n}}$$

$$\text{or, } \frac{773}{305} = \left\{ \frac{3655}{117} \right\}^{\frac{n-1}{n}}$$

$$\log \left\{ \frac{773}{305} \right\} = \log \left\{ \frac{3655}{117} \right\} \times \left\{ \frac{n-1}{n} \right\}$$

$$0.4039 = 1.4947 \times \frac{n-1}{n}$$

$$1.4947 = 1.0908n$$

$$n = \frac{1.4947}{1.0908} = 1.37$$

∴ law of compression is:

$$pV^{1.37} = C \text{ Ans.}$$

7.

$$p_1V_1 = mRT_1$$

$$V_1 = \frac{1 \times 0.287 \times 473}{20 \times 100}$$

$$= 0.1142 \text{ m}^3$$

$$p_2V_2 = mRT_2$$

$$V_2 = \frac{1 \times 0.287 \times 398}{10 \times 100}$$

$$= 0.0679 \text{ m}^3$$

$$\text{Work transfer (expansion)} = \text{Area under process straight line of } pV \text{ diagram}$$

$$= \text{Mean pressure} \times \text{change of volume}$$

$$= 100 \times 15 (0.1142 - 0.0679)$$

$$= 69.45 \text{ kJ} \text{ Ans. (a)}$$

$$c_v = 1005 - 287 = 718 \text{ J/kg K}$$

$$\text{Change of internal energy} = mc_v(T_2 - T_1)$$

$$= 1 \times 0.718 (398 - 473)$$

$$= -53.85 \text{ kJ, i.e. decrease} \text{ Ans. (b)}$$

$$\text{Heat transfer} = -53.85 + 69.45$$

$$= 15.6 \text{ kJ} \text{ Ans. (c)}$$

$$\text{Change of enthalpy} = mc_p(T_2 - T_1)$$

$$= 1 \times 1.005 (398 - 473)$$

$$= -75.375 \text{ kJ, i.e. decrease} \text{ Ans. (d)}$$

8.

$$p_1V_1 = mRT_1$$

$$V_1 = \frac{1 \times 0.287 \times 486}{1 \times 100}$$

$$= 1.395 \text{ m}^3$$

$$p_2V_2 = m_2RT_2$$

$$m_2 = \frac{6 \times 100 \times 0.2}{0.287 \times 685}$$

$$= 0.6103 \text{ kg}$$

No heat transfer takes place (insulated)
 No external work is done
 The process is at constant internal energy

$$m_1 c_v T_1 + m_2 c_v T_2 = m_3 c_v T_3$$

$$1 \times 486 + 0.6103 \times 685 = 1.6103 \times T_3$$

$$T_3 = 561.4 \text{ K}$$

Final air temperature is 288.4°C Ans. (a)

$$p_3 V_3 = m_3 R T_3$$

$$p_3 = \frac{1.6103 \times 0.287 \times 561.4}{1.595}$$

$$= 162.7 \text{ kN/m}^2$$

Final air pressure is 1.627 bar Ans. (b)

9. $pV = mRT$

$$m = \frac{1 \times 100 \times 0.1}{0.287 \times 288}$$

$$= 0.1210 \text{ kg}$$

$$\frac{T_2}{T_1} = \frac{p_2}{p_1} \text{ as } V \text{ constant}$$

$$\frac{773}{288} = \frac{p_2}{1}$$

$$p_2 = 2.684 \text{ bar}$$

$$\frac{T_3}{T_2} = \left(\frac{p_3}{p_2} \right)^{\frac{n-1}{n}}$$

$$\frac{T_3}{773} = \left(\frac{1}{2.684} \right)^{\frac{0.5}{1.5}}$$

$$T_3 = 556.1 \text{ K}$$

Final temperature = 283.1°C Ans. (a)

$$\text{Work done} = \frac{mR(T_2 - T_3)}{n-1}$$

$$= \frac{0.121 \times 0.287(773 - 556.1)}{0.5}$$

$$= 15.05 \text{ kJ Ans. (b)}$$

$$\text{Heat supplied} = mc_v(T_2 - T_1) \text{ at constant volume}$$

$$= 0.121 \times 0.718(773 - 288)$$

$$= 42.14 \text{ kJ Ans. (c)}$$

$$\text{Heat transfer} = mc_v(T_3 - T_2)$$

$$= 0.121 \times 0.718(556.1 - 773)$$

$$= -18.84 + 15.05$$

$$= -3.79 \text{ kJ Ans. (c)}$$

10. $T_1 = 783 \text{ K}$
 $T_2 = 313 \text{ K}$

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$$

$$\frac{36 \times 0.125}{783} = \frac{p_2 \times 1.5}{313}$$

$$p_2 = \frac{36 \times 0.125 \times 313}{783 \times 1.5}$$

$$= 1.199 \text{ bar or } 119.9 \text{ kN/m}^2 \text{ Ans. (i)}$$

$$p_1 V_1^n = p_2 V_2^n \text{ or } \frac{T_1}{T_2} = \left(\frac{V_2}{V_1} \right)^{n-1}$$

$$36 \times 0.125^n = 1.199 \times 1.5^n$$

$$\frac{36}{1.199} = \left(\frac{1.5}{0.125} \right)^n$$

$$n = 1.37 \text{ Ans. (ii)}$$

$$p_1 V_1 = mRT_1$$

$$m = \frac{3600 \times 0.125}{0.284 \times 783} = 2.023 \text{ kg Ans. (iii)}$$

Increase in internal energy:

$$U_2 - U_1 = mc_v(T_2 - T_1)$$

$$= 2.023 \times 0.71 \times (313 - 783)$$

$$= 2.023 \times 0.71 \times (-470)$$

$$= -675.1 \text{ kJ}$$

The minus sign indicates a decrease in internal energy.

$$\text{Decrease in internal energy} = 675.1 \text{ kJ Ans. (iv)}$$

$$\begin{aligned} \text{Work done} &= \frac{p_1 V_1 - p_2 V_2}{n-1} \text{ or } \frac{mR(T_1 - T_2)}{n-1} \\ &= \frac{3600 \times 0.125 - 119.9 \times 1.5}{1.37 - 1} \\ &= \frac{450 - 179.9}{0.37} = 730.2 \text{ kJ Ans. (v)} \end{aligned}$$

$$\begin{aligned} \text{Heat supplied} &= \text{Increase in internal energy} + \text{External work done} \\ &= -675.1 + 730.2 \\ &= 55.1 \text{ kJ Ans. (vi)} \end{aligned}$$

SOLUTIONS TO TEST EXAMPLES 7

$$\begin{aligned} 1. \text{ Indicated mean effective pressure [kN/m}^2] &= \text{mean height of diagram [mm]} \times \text{spring scale [kN/m}^2\text{mm]} \\ &= \frac{\text{area of diagram}}{\text{length of diagram}} \times \text{spring scale} \\ &= \frac{390}{70} \times 1.6 \times 10^2 = 891.6 \text{ kN/m}^2 \\ \text{ip} &= p_m A L n \times 4 \text{ (for 4 cylinders)} \\ &= 891.6 \times 0.7854 \times 0.15^2 \times 0.2 \times \frac{5.5}{2} \times 4 \\ &= 34.66 \text{ kW Ans.} \end{aligned}$$

$$\begin{aligned} 2. \text{ ip} &= p_m A L n \times 8 \text{ (for 8 cylinders)} \\ &= 1172 \times 0.7854 \times 0.75^2 \times 1.125 \times \frac{110}{60 \times 2} \times 8 \\ &= 4272 \text{ kW Ans. (i)} \\ \text{bp} &= \text{ip} \times \text{mech. efficiency} \\ &= 4272 \times 0.86 \\ &= 3676 \text{ kW Ans. (ii)} \end{aligned}$$

$$3. \text{ ip} = \frac{\text{bp}}{\text{mech. eff.}} = \frac{2250}{0.84} = 2680 \text{ kW}$$

Let d = diameter of cylinders in metres
then L = $1.25d$ metres

$$\text{ip} = p_m A L n \times 6$$

$$2680 = 10 \times 10^2 \times 0.7854 \times d^2 \times 1.25d \times 2 \times 6$$

$$d = \sqrt[3]{\frac{2680}{1000 \times 0.7854 \times 1.25 \times 2 \times 6}}$$

$$\left. \begin{aligned} d &= 0.6103 \text{ m} = 610.3 \text{ mm} \\ L &= 1.25 \times 610.3 = 763 \text{ mm} \end{aligned} \right\} \text{ Ans.}$$

$$\begin{aligned} 4. \text{ Load on brake} &= 480 - 84 = 396 \text{ N} \\ \text{Effective radius} &= \frac{1}{2} (1220 + 24) = 622 \text{ mm} = 0.622 \text{ m} \\ \text{Brake power} &= T \omega \\ &= 396 \times 0.622 \times \frac{250 \times 2\pi}{60} \\ &= 6449 \text{ W} = 6.449 \text{ kW Ans. (i)} \end{aligned}$$

$$1 \text{ kWh} = 3.6 \text{ MJ}$$

$$\begin{aligned} \text{Heat carried away by water in one hour} \\ = 0.9 \times 6.449 \times 3.6 \times 10^3 \text{ kJ} \end{aligned}$$

$$\begin{aligned} \text{Heat received by water} \\ = \text{mass} \times \text{spec. heat} \times \text{temp. rise} \\ Q \text{ [kJ]} = m \text{ [kg]} \times c_p \text{ [kJ/kg K]} \times \theta \text{ [K]}, \\ = m \times 4.2 \times 18 \end{aligned}$$

$$\begin{aligned} m \times 4.2 \times 18 &= 0.9 \times 6.449 \times 3.6 \times 10^3 \\ m &= 276.4 \text{ kg} \end{aligned}$$

$$\begin{aligned} \text{Mass of one litre of fresh water} &= \text{one kg} \\ \text{Quantity in litres} &= 276.4 \text{ litres/h} \quad \text{Ans. (ii)} \end{aligned}$$

$$5. \text{ Brake power [W]} = \text{Torque [Nm]} \times \omega \text{ [rad/s]}$$

$$\text{When speed is } 24.5 \text{ rev/s}$$

$$\text{bp} = T \times 2\pi \times 24.5 = 153.9 T \text{ watts} = 0.1539 T \text{ kW}$$

$$\text{With all cylinders firing:}$$

$$\text{bp} = 0.1539 \times 193.8 = 29.84 \text{ kW} \quad \text{Ans. (i)}$$

$$\text{no. 1 cyl. cut out, bp} = 0.1539 \times 130.8 = 20.13 \text{ kW}$$

$$\text{ip of no. 1 cyl.} = 29.84 - 20.13 = 9.71 \text{ kW}$$

$$\text{no. 2 cyl. cut out, bp} = 0.1539 \times 130.2 = 20.04 \text{ kW}$$

$$\text{ip of no. 2 cyl.} = 29.84 - 20.04 = 9.8 \text{ kW}$$

$$\text{no. 3 cyl. cut out, bp} = 0.1539 \times 129.9 = 20.0 \text{ kW}$$

$$\text{ip of no. 3 cyl.} = 29.84 - 20 = 9.84 \text{ kW}$$

$$\text{no. 4 cyl. cut out, bp} = 0.1539 \times 131.1 = 20.18 \text{ kW}$$

$$\text{ip of no. 4 cyl.} = 29.84 - 20.18 = 9.66 \text{ kW}$$

$$\text{Total ip of engine}$$

$$= 9.71 + 9.8 + 9.84 + 9.66 = 39.01 \text{ kW} \quad \text{Ans. (ii)}$$

$$\text{Mech. effic.} = \frac{\text{brake power}}{\text{indicated power}}$$

$$= \frac{29.84}{39.01} = 0.7649 = 76.49\% \quad \text{Ans. (iii)}$$

$$6. \text{ Brake thermal effic.} = \frac{3.6 \text{ [MJ/kWh]}}{\text{kg fuel/brake kWh} \times \text{cal. value [MJ/kg]}}$$

$$= \frac{3.6}{0.255 \times 43.5} = 0.3245 \text{ or } 37.45\% \quad \text{Ans. (ii)}$$

$$\text{Indicated thermal efficiency} = \frac{\text{brake thermal efficiency}}{\text{mechanical efficiency}}$$

$$= \frac{0.3245}{0.86} = 0.3773 \text{ or } 37.74\% \quad \text{Ans. (i)}$$

$$\begin{aligned} \text{For each kg of fuel burned,} \\ \text{total mass of gases} &= 35 \text{ kg air} + 1 \text{ kg fuel} = 36 \text{ kg} \end{aligned}$$

$$\text{Heat carried away in exhaust gases,}$$

$$Q \text{ [kJ]} = m \text{ [kg]} \times c_p \text{ [kJ/kg K]} \times \theta \text{ [K]}$$

$$= 36 \times 1.005 \times (393 - 26)$$

$$= 1.327 \times 10^4 \text{ kJ}$$

$$= 13.27 \text{ MJ/kg fuel}$$

$$\begin{aligned} \text{Heat supplied} &= 43.5 \text{ MJ/kg fuel} \\ \therefore \text{percentage heat in exhaust gases} \end{aligned}$$

$$= \frac{13.27}{43.5} \times 100 = 30.52\% \quad \text{Ans. (iii)}$$

$$7. \text{ Ind. thermal effic.} = 100\% - (31.7 + 30.8) = 37.5\% \quad \text{Ans. (i)}$$

$$\text{Mechanical effic.} = \frac{\text{bp}}{\text{ip}} = \frac{4060}{4960}$$

$$= 0.8185 \text{ or } 81.85\% \quad \text{Ans. (ii)}$$

$$\begin{aligned} \text{Overall effic.} &= \text{ind. thermal effic.} \times \text{mech. effic.} \\ &= 0.375 \times 0.8185 \\ &= 0.307 \text{ or } 30.7\% \quad \text{Ans. (iii)} \end{aligned}$$

$$\text{Specific fuel consumption (indicated)}$$

$$= \frac{27 \times 10^3}{24 \times 4960} = 0.2269 \text{ kg/ind. kWh} \quad \text{Ans. (iv)}$$

$$\text{Indicated thermal effic.} = \frac{3.6 \text{ [MJ/kWh]}}{\text{kg fuel/ind. kWh} \times \text{cal. val. [MJ/kg]}}$$

$$0.375 = \frac{3.6}{0.2269 \times \text{cal. value}}$$

$$\text{Calorific value} = \frac{3.6}{0.375 \times 0.2269}$$

$$= 42.32 \text{ MJ/kg} \quad \text{Ans. (v)}$$

$$8. \text{ BEFORE ALTERATION:}$$

$$V_1 = 87.5 + 12.5 = 100$$

$$V_2 = 12.5$$

$$p_1 V_1^{1.35} = p_2 V_2^{1.35}$$

$$0.97 \times 100^{1.35} = p_2 \times 12.5^{1.35}$$

$$p_2 = 0.97 \times \left\{ \frac{100}{12.5} \right\}^{1.35}$$

$$= 0.97 \times 8^{1.35} = 16.07 \text{ bar} \quad \text{Ans. (i)}$$

AFTER ALTERATION:

$$V_1 = 87.5 + 10 = 97.5$$

$$V_2 = 10$$

$$p_1 V_1^{1.35} = p_2 V_2^{1.35}$$

$$0.97 \times 97.5^{1.35} = p_2 \times 10^{1.35}$$

$$p_2 = 0.97 \times 9.75^{1.35} = 20.99 \text{ bar} \quad \text{Ans. (ii)}$$

9. Due to the "hit and miss" governor cutting off the supply of gas, the number of power strokes per second is the number of explosions per second, which is $123 \div 60 = 2.05$

$$\begin{aligned} ip &= p_m ALn \\ &= 3.93 \times 10^2 \times 0.7854 \times 0.18^2 \times 0.3 \times 2.05 \\ &= 6.152 \text{ kW} \quad \text{Ans. (i)} \end{aligned}$$

$$\begin{aligned} \text{Mech. effic.} &= \frac{\text{brake power}}{\text{indicated power}} \\ &= \frac{4.33}{6.152} = 0.7039 = 70.39\% \quad \text{Ans. (ii)} \end{aligned}$$

Indicated thermal efficiency

$$\begin{aligned} &= \frac{\text{heat equivalent of work done in cyl. [kJ/h]}}{\text{heat supplied [kJ/h]}} \\ &= \frac{6.152 \times 3600}{3.1 \times 17.6 \times 10^3} \\ &= 0.4059 = 40.59\% \quad \text{Ans. (iii)} \end{aligned}$$

$$\begin{aligned} \text{Brake thermal effic.} &= \text{ind. thermal effic.} \times \text{mech. effic.} \\ &= 0.4059 \times 0.7039 \\ &= 0.2858 = 28.58\% \quad \text{Ans. (iv)} \end{aligned}$$

$$\begin{aligned} 10. \quad \text{Mean piston speed} &= 2Ln \\ 6 &= 2 \times 1.5 \times n \\ n &= 2 \text{ rev/s} \end{aligned}$$

$$\begin{aligned} \text{brake power} &= p_m ALn \\ &= 7 \times 100 \times 0.7854 \times 0.75^2 \times 1.5 \times 2 \end{aligned}$$

$$\text{brake power} = 927.8 \text{ kW}$$

$$\frac{\text{power 1}}{\text{swept volume 1}} = \frac{\text{power 2}}{\text{swept volume 2}}$$

$$\frac{370}{0.7854 \times D^2 \times 2.0D} = \frac{927.8}{0.7854 \times 0.75^2 \times 1.5}$$

$$D^3 = \frac{370 \times 0.5625 \times 1.5}{927.8 \times 2.0}$$

$$D = 0.552 \text{ m}$$

$$\text{cylinder bore} = 0.552 \text{ m} \quad \text{Ans. (a)}$$

$$\begin{aligned} \text{brake power} &= p_m ALn \\ 370 &= p_m \times 100 \times 0.7854 \times 0.552^2 \times 1.104 \times 2 \end{aligned}$$

$$p_m = 7.002 \text{ bar}$$

$$\text{brake mean effective pressure} = 7.002 \text{ bar} \quad \text{Ans. (b)}$$

SOLUTIONS TO TEST EXAMPLES 8

1. $p_1 V_1^{1.35} = p_2 V_2^{1.35}$
 $1 \times 8.5^{1.35} = p_2 \times 1^{1.35}$
 $p_2 = 8.5^{1.35} = 17.97 \text{ bar}$ Ans. (i)
- $$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$$
- $$\frac{1 \times 8.5}{313} = \frac{17.97 \times 1}{T_2}$$
- $$T_2 = \frac{313 \times 17.97}{8.5} = 661.8 \text{ K}$$
- $$= 388.8^\circ\text{C}$$
- Ans. (ii)
- $$\frac{T_3}{T_2} = \frac{p_3}{p_2}$$
- $$T_3 = \frac{661.8 \times 31}{17.97} = 1141 \text{ K}$$
- $$= 868^\circ\text{C}$$
- Ans. (iii)
2. $\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1}$ (Refer to Fig. 26, p - V diagram)
- $$T_2 = 323 \times 5^{0.4}$$
- $$= 615 \text{ K}$$
- Heat supplied = $c_v (T_3 - T_2)$
 $930 = 0.717 (T_3 - 615)$
 $T_3 = 1912 \text{ K}$
- Maximum temp. = 1639°C Ans. (a)
- $$\frac{T_3}{T_4} = \left(\frac{V_4}{V_3}\right)^{\gamma-1}$$
- $$T_4 = \frac{1912}{5^{0.4}}$$
- $$T_4 = 1004 \text{ K}$$
- Work done = heat supplied - heat rejected
 $= c_v [(T_3 - T_2) - (T_4 - T_1)]$
 $= 0.717 [(1912 - 615) - (1004 - 323)]$
 $= 441.7 \text{ kJ/kg of working fluid}$ Ans. (b)

$$\text{Ideal thermal efficiency of cycle} = \frac{441.7}{0.717 \times 1297} \times 100$$

$$= 47.5\% \text{ Ans. (c)}$$

3. $\gamma = \frac{c_p}{c_v}$
- $$= \frac{1005}{718}$$
- $$= 1.4$$
- $$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1}$$
- $$T_2 = 330 \times 8^{0.4}$$
- $$= 758 \text{ K}$$
- Energy added = $c_v (T_3 - T_2)$
 $1250 = 0.718 (T_3 - 758)$
 $T_3 = 2499 \text{ K}$
- Max. temperature = 2216°C Ans. (a) (i)
- $$p_1 V_1^\gamma = p_2 V_2^\gamma$$
- $$p_2 = 8^{1.4} \times 1$$
- $$= 18.37 \text{ bar}$$
- $$\frac{p_2}{T_2} = \frac{p_3}{T_3}$$
- $$p_3 = 2499 \times \frac{18.37}{758}$$
- Max. pressure = 60.56 bar Ans. (a)(ii)
- For pressure-volume diagram, refer to Fig. 26 Ans. (b)(i)
 For temperature-entropy diagram, refer to Fig. 49 Ans. (b)(ii)
4. Refer to Fig. 28 (p - V diagram)
- $$\gamma = \frac{c_p}{c_v}$$
- $$= \frac{1005}{718}$$
- $$= 1.4$$
- $$p_2 = 1 \times (16)^{1.4}$$

Max. pressure = 48.5 bar Ans. (a)

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1}$$

$$T_2 = 330 \times 16^{0.4}$$

$$= 1000 \text{ K}$$

Energy added = $c_p(T_3 - T_2)$

$$1250 = 1.005(T_3 - 1000)$$

$$T_3 = 2244 \text{ K}$$

Max. temperature = 1961°C Ans. (b)

$$\frac{V_3}{V_2} = \frac{T_3}{T_2}$$

$$V_3 = 1 \times \frac{2244}{1000}$$

$$= 2.244$$

$$\frac{T_4}{T_3} = \left(\frac{V_3}{V_4}\right)^{\gamma-1}$$

$$T_4 = 2244 \times \left(\frac{2.244}{16}\right)^{0.4}$$

$$T_4 = 1023 \text{ K}$$

$$\frac{p_4}{p_1} = \frac{T_4}{T_1}$$

$$p_4 = 1 \times \frac{1023}{330}$$

$$= 3.1 \text{ bar}$$

$$\text{Energy removed} = c_v(T_4 - T_1)$$

$$= 0.718(1023 - 330)$$

$$= 497.6 \text{ kJ/kg}$$

$$\text{Cycle efficiency (A.S.E.)} = \frac{1250 - 497.6}{1250} \times 100$$

$$= 60.19\%$$

$$\text{m.e.p.} = \frac{p_2(V_3 - V_2) + \frac{p_3V_3 - p_4V_4}{\gamma - 1} - \frac{p_2V_2 - p_1V_1}{\gamma - 1}}{V_1 - V_2}$$

$$= \frac{48.5(2.244 - 1) + \frac{48.5 \times 2.244 - 3.1 \times 16}{0.4} - \frac{48.5 \times 1 - 1 \times 16}{0.4}}{16 - 1}$$

$$= \frac{1}{15}(60.33 + 148.1 - 81.25)$$

$$= 8.479 \text{ bar Ans. (d)}$$

5. Referring to Fig. 30, data given:

$$p_1 = 1 \quad p_3 = p_4 = 41$$

$$V_1 = V_5 = 10.7 \quad V_2 = V_3 = 1$$

$$T_1 = 305$$

$$T_4 = 1866$$

$$\gamma = \frac{c_p}{c_v} = \frac{1.005}{0.718} = 1.4$$

COMPRESSION PERIOD:

$$p_1V_1^\gamma = p_2V_2^\gamma \quad p_2 = \frac{1 \times 10.7^{1.4}}{1^{1.4}}$$

$$= 27.62 \text{ bar}$$

$$\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2} \quad T_2 = \frac{305 \times 27.62 \times 1}{1 \times 10.7}$$

$$= 787.2 \text{ K}$$

$$= 514.2^\circ\text{C}$$

PART COMBUSTION AT CONSTANT VOLUME:

Absolute temperature varies as absolute pressure,

$$\therefore T_3 = T_2 \times \frac{p_3}{p_2} = \frac{787.2 \times 41}{27.62} = 1169 \text{ K}$$

$$= 896^\circ\text{C}$$

PART COMBUSTION AT CONSTANT PRESSURE:

Volume varies as absolute temperature,

$$\therefore V_4 = V_3 \times \frac{T_4}{T_3} = \frac{1 \times 1866}{1169} = 1.596$$

EXPANSION PERIOD:

$$p_4 V_4^{1.4} = p_5 V_5^{1.4}$$

$$p_5 = \frac{41 \times 1.596^{1.4}}{10.7^{1.4}} = 2.858 \text{ bar}$$

$$\text{Since } V_5 = V_1, \text{ then } \frac{T_5}{T_1} = \frac{p_5}{p_1}$$

$$T_5 = \frac{305 \times 2.858}{1} = 871.6 \text{ K}$$

$$= 598.6^\circ\text{C}$$

Hence, pressures and temperatures required:

$$\left. \begin{aligned} p_2 &= 27.62 \text{ bar}, \theta_2 = 514.2^\circ\text{C} \\ \theta_3 &= 896^\circ\text{C} \\ p_5 &= 2.858 \text{ bar}, \theta_5 = 598.6^\circ\text{C} \end{aligned} \right\} \text{Ans. (i)}$$

Ideal thermal efficiency

$$= 1 - \frac{\text{heat rejected}}{\text{heat supplied}}$$

$$= 1 - \frac{c_v (T_5 - T_1)}{c_v (T_3 - T_2) + c_p (T_4 - T_3)}$$

$$= 1 - \frac{(T_5 - T_1)}{(T_3 - T_2) + \gamma (T_4 - T_3)}$$

$$= 1 - \frac{871.6 - 305}{(1169 - 787.2) + 1.4 (1866 - 1169)}$$

$$= 1 - 0.4175$$

$$= 0.5825 = 58.25\% \text{ Ans. (ii)}$$

Note: Compression ratio and maximum pressures are low for this example compared to modern practice.

$$6. \text{ Cycle efficiency} = \frac{T_1 - T_2}{T_1} \times 100$$

$$= \frac{1300 - 300}{1300} \times 100$$

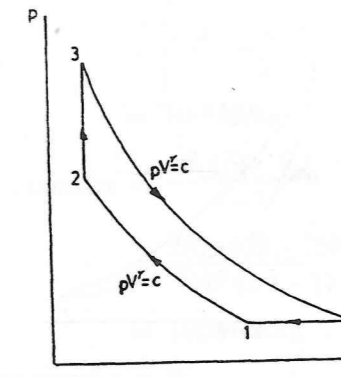
$$= 76.9\% \text{ Ans. (a)}$$

Refer to Fig. 48 (temperature-entropy) of Chapter 11.

Also the description of practical constraints to the Carnot cycle - leading on to Fig. 61 (temperature-entropy) for the vapour (steam) plant and the Rankine cycle (Chapter 12).

Difficulties with compressor for wet vapour Ans. (b)(i)
Substitution of feed pump to handle condensed liquid Ans. (b)(ii)

7.



$$\gamma = \frac{c_p}{c_v} = \frac{1000}{678} = 1.475$$

$$\frac{T_1}{T_2} = \left(\frac{p_1}{p_2} \right)^{\frac{\gamma-1}{\gamma}}$$

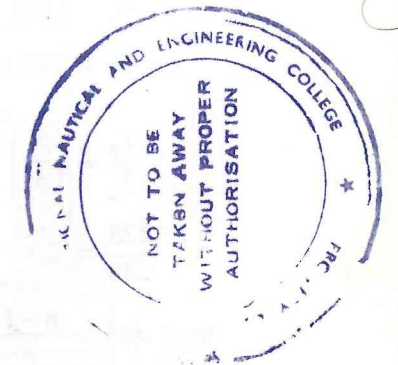
$$T_2 = 333 \times 4.5^{\frac{0.475}{1.475}} = 540.5 \text{ K}$$

$$\frac{p_3}{T_3} = \frac{p_2}{T_2}$$

$$T_3 = \frac{1.35}{1} \times 540.5 = 729 \text{ K}$$

$$\frac{T_3}{T_4} = \left(\frac{p_3}{p_4} \right)^{\frac{\gamma-1}{\gamma}}, \quad p_3 = 4.5 \times 1.35 = 6.075$$

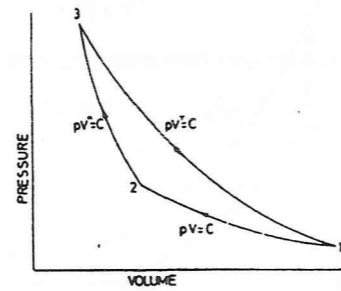
$$T_4 = 729 \left(\frac{1}{6.075} \right)^{\frac{0.475}{1.475}} = 407.8 \text{ K}$$



Thermal efficiency of the cycle

$$\begin{aligned}
 &= 1 - \frac{c_p (T_4 - T_1)}{c_v (T_3 - T_2)} \times 100\% \\
 &= 1 - 1.475 \frac{(407.8 - 333)}{(729 - 540.5)} \times 100\% \\
 &= 41.5\% \text{ Ans.}
 \end{aligned}$$

8.



$$\frac{T_3}{T_1} = \left(\frac{p_3}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$T_3 = 288 \times 4.5^{\frac{0.4}{1.4}} = 428 \text{ K}$$

$$\frac{T_3}{T_2} = \left(\frac{p_3}{p_2}\right)^{\frac{n-1}{n}} \text{ and } T_1 = T_2$$

$$\therefore \frac{428}{288} = 2^{\frac{n-1}{n}}$$

$$\log 1.486 = \frac{n-1}{n} \times \log 2$$

$$0.172 \times n = (n-1) \times 0.3010$$

$$n = 2.33$$

$$\begin{aligned}
 \text{From 1 to 2 Work transfer} &= -mRT_1 \ln r \\
 &= -287 \times 288 \times \ln 2 / 10^3 \\
 &= -57.29 \text{ kJ/kg}
 \end{aligned}$$

Ans. (a)(i)

$$\text{From 1 to 2 Heat transfer} = -57.29 \text{ kJ/kg} \quad \text{Ans. (b)(i)}$$

$$\begin{aligned}
 \text{From 2 to 3 Work transfer} &= -\frac{mR(T_3 - T_2)}{n-1} \\
 &= -\frac{287(428 - 288)}{10^3(2.33 - 1)} \\
 &= -30.2 \text{ kJ/kg}
 \end{aligned}$$

Ans. (a)(ii)

$$\begin{aligned}
 \text{From 2 to 3 Heat transfer} &= \frac{\gamma-n}{\gamma-1} \times (-30.2) \\
 &= \left(\frac{1.4-2.33}{1.4-1}\right) \times (-30.2) \\
 &= 70.3 \text{ kJ/kg}
 \end{aligned}$$

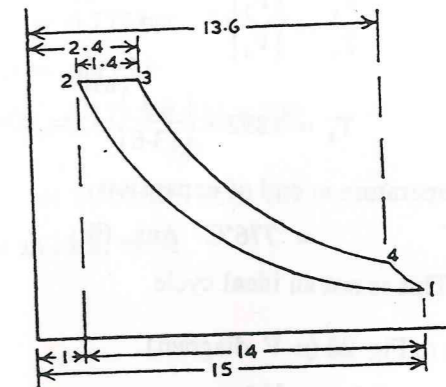
Ans. (b)(ii)

$$\begin{aligned}
 \text{From 3 to 1 Work transfer} &= \frac{mR(T_3 - T_1)}{\gamma-1} \\
 &= \frac{287(428 - 288)}{10^3(1.4 - 1)} \\
 &= 100.5 \text{ kJ/kg}
 \end{aligned}$$

Ans. (a)(iii)

$$\text{From 3 to 1 Heat transfer} = 0 \quad \text{Ans. (b)(iii)}$$

9.



$$\begin{aligned}
 \text{Let } V_1 &= 15, \quad \text{and } V_2 = 1 \\
 \text{then, stroke volume} &= 15 - 1 = 14 \\
 \text{Fuel burning period} &= \frac{1}{10} \times 14 = 1.4 \\
 \therefore V_3 &= 1.4 + 1 = 2.4
 \end{aligned}$$

$$V_4 = 1 + \frac{9}{10} \times 14 = 13.6$$

$$T_1 = 314 \text{ K}$$

COMPRESSION PERIOD:

$$\frac{T_2}{T_1} = \left\{ \frac{V_1}{V_2} \right\}^{n-1}$$

$$T_2 = 314 \times 15^{0.34} = 788.4 \text{ K}$$

$$\therefore \text{Temperature at end of compression} \\ = 515.4^\circ\text{C} \quad \text{Ans. (i)}$$

BURNING PERIOD:

$$\frac{T_3}{T_2} = \frac{V_3}{V_2}$$

$$T_3 = 788.4 \times 2.4 = 1892 \text{ K}$$

$$\therefore \text{Temperature at end of combustion} \\ = 1619^\circ\text{C} \quad \text{Ans. (ii)}$$

EXPANSION PERIOD:

$$\frac{T_4}{T_3} = \left\{ \frac{V_3}{V_4} \right\}^{n-1}$$

$$T_4 = 1892 \times \left\{ \frac{2.4}{13.6} \right\}^{0.34} = 1049 \text{ K}$$

$$\therefore \text{Temperature at end of expansion} \\ = 776^\circ\text{C} \quad \text{Ans. (iii)}$$

Note: This is not an ideal cycle10. Refer to Fig. 28 (p - V diagram)

$$p_1 V_1^n = p_2 V_2^n$$

$$p_2 = 1 \times 12^{1.36}$$

$$= 29.38 \text{ bar}$$

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2} \right)^{n-1}$$

$$T_2 = 353 \times 12^{0.36}$$

$$= 863.4 \text{ K}$$

$$\frac{T_3}{T_2} = \frac{V_3}{V_2}$$

$$V_3 = \frac{1923}{863.4}$$

$$= 2.227$$

$$p_3 V_3^n = p_4 V_4^n$$

$$p_4 = 29.38 \times \left(\frac{2.227}{12} \right)^{1.4}$$

$$= 2.780 \text{ bar}$$

$$\text{m.e.p.} = \frac{p_2(V_3 - V_2) + \frac{p_3 V_3 - p_4 V_4}{n-1} - \frac{p_2 V_2 - p_1 V_1}{n-1}}{V_1 - V_2}$$

$$= \frac{29.38(2.227 - 1) + \frac{29.38 \times 2.227 - 2.78 \times 12}{0.4} - \frac{29.38 \times 1 - 1 \times 12}{0.36}}{12 - 1}$$

$$= \frac{1}{11} (36.05 + 80.18 - 48.28)$$

$$= 6.177 \text{ bar} \quad \text{Ans. (a)}$$

$$\text{Power} = pm A L n$$

$$= \frac{6.177 \times 0.034 \times 200}{60}$$

$$= 70.0 \text{ kW} \quad \text{Ans. (b)}$$

Note: This is not an ideal cycle

SOLUTIONS TO TEST EXAMPLES 9

$$1. \text{ Clearance length [mm]} = \frac{\text{clearance volume [mm}^3\text{]}}{\text{area of cylinder [mm}^2\text{]}}$$

$$= \frac{900 \times 10^3}{0.7854 \times 250^2} = 18.33 \text{ mm}$$

$$V_1 = 350 + 18.33 = 368.33 \text{ mm}$$

$$p_1 V_1^{1.25} = p_2 V_2^{1.25}$$

$$0.986 \times 368.3^{1.25} = 4.1 \times V_2^{1.25}$$

$$V_2 = 368.3 \times \sqrt[1.25]{\frac{0.986}{4.1}}$$

$$= 117.8 \text{ mm}$$

$$\text{Compression period} = V_1 - V_2$$

$$= 368.3 - 117.8$$

$$= 250.5 \text{ mm Ans.}$$

2. Refer to Fig. 38 Ans. (a)

$$p_3 V_3^n = p_4 V_4^n$$

$$\left(\frac{p_3}{p_4}\right)^{\frac{1}{n}} = \frac{V_4}{V_3}$$

$$V_4 = 1.3 \sqrt[3]{8} \times 0.08$$

$$= 0.3961 \text{ m}^3$$

$$\text{Suction volume} = 1.08 - 0.3961$$

$$= 0.6839 \text{ m}^3$$

$$\text{Volumetric eff.} = \frac{0.6839}{1} \times 100$$

$$= 68.39\% \text{ Ans. (b)(i)}$$

$$\text{Free air delivery} = 0.7854 \times 0.152^2 \times 0.105 \times 0.6839 \times 12$$

$$= 0.01564 \text{ m}^3/\text{s Ans. (b)(ii)}$$

3. Refer to Fig. 38 Ans. (a)

$$pV = mRT$$

$$1 \times 100 \times V_1 = 0.0035 \times 0.287 \times 288$$

$$V_1 = 0.002893 \text{ m}^3$$

$$\text{Stroke volume} = V_1 - V_3$$

$$= 0.7854 \times 0.14^2 \times 0.18$$

$$= 0.002771 \text{ m}^3$$

$$\text{Clearance volume} = V_3$$

$$= 0.002893 - 0.002771$$

$$= 0.000122 \text{ m}^3 \text{ Ans. (b)(i)}$$

$$p_3 V_3^n = p_4 V_4^n$$

$$8.5 \times 0.000122^{1.32} = 1 \times V_4^{1.32}$$

$$V_4 = 0.0006166 \text{ m}^3$$

$$V_1 - V_4 = 0.002893 - 0.0006166$$

$$= 0.0022764 \text{ m}^3$$

$$\text{Volumetric eff.} = \frac{0.002276}{0.002771} \times 100$$

$$= 82.14\% \text{ Ans. (b)(ii)}$$

4. Refer to Fig. 36 Ans. (a)

$$\text{Stroke volume} = 0.7854 \times 0.1^2 \times 0.12$$

$$= 0.0009424 \text{ m}^3$$

$$\text{Let } N \text{ be rev/min}$$

$$\text{Suction volume/min} = 3 \times N \times 0.0009424$$

$$= \text{Free air delivery/min}$$

$$N = \frac{1.2}{3 \times 0.0009424}$$

$$\text{Operating speed} = 424.4 \text{ rev/min Ans. (b)(i)}$$

$$p_1 V_1^n = p_2 V_2^n$$

$$1 \times 0.0009424^{1.3} = 6.6 \times V_2^{1.3}$$

$$V_2 = 0.0002203 \text{ m}^3$$

$$\text{Work/cycle} = \frac{n}{n-1} (p_2 V_2 - p_1 V_1) \times 3$$

$$= \frac{3 \times 1.3 \times 10^2}{0.3} (6.6 \times 0.0002203 - 1 \times 0.0009424)$$

$$= 0.6651 \text{ kJ}$$

$$\text{Work/s} = 0.6651 \times \frac{424.4}{60}$$

$$= 4.704 \text{ kW}$$

$$\begin{aligned} \text{Input power} &= \frac{4.704}{0.9} \\ &= 5.227 \text{ kW Ans. (b) (ii)} \end{aligned}$$

5. Volume of free air (at 1.01 bar from the atmosphere) to make 22 m³ at 31.01 bar abs., at the same temperature

$$= 22 \times \frac{31.01}{1.01} \text{ m}^3$$

Volume of free air to make 22 m³ at 20.01 bar abs.

$$= 22 \times \frac{20.01}{1.01} \text{ m}^3$$

∴ Volume of free air to supply the difference

$$= \frac{22}{1.01} (31.01 - 20.01)$$

$$= \frac{22 \times 11}{1.01} = 239.6 \text{ m}^3 \dots (i)$$

Volume of free air dealt with by compressor per minute

$$\begin{aligned} &= 0.7854 (0.35^2 - 0.075^2) \times 0.3 \times 0.92 \times 170 \\ &= 4.306 \text{ m}^3/\text{min} \dots (ii) \end{aligned}$$

$$\therefore \text{Time} = \frac{239.6}{4.306} = 55.64 \text{ minutes Ans.}$$

6. Referring to Fig. 38

$$\begin{aligned} \text{Work/cycle} &= \frac{n}{n-1} \{ (p_2 V_2 - p_1 V_1) - (p_3 V_3 - p_4 V_4) \} \\ &= \frac{n}{n-1} \{ (p_2 V_2 - p_1 V_1) - (p_2 V_3 - p_1 V_4) \} \\ &= \frac{n}{n-1} \{ p_2 (V_2 - V_3) - p_1 (V_1 - V_4) \} \end{aligned}$$

Piston swept volume ($V_1 - V_3$)

$$= 0.7854 \times 0.2^2 \times 0.23 = 7.226 \times 10^{-3} \text{ m}^3$$

Working in litres of volume and bars of pressure:

$$V_3 = 364 \text{ cm}^3 = 0.364 \text{ litre}$$

$$V_1 = \text{piston swept vol.} + \text{clearance vol.}$$

$$= 7.226 + 0.364 = 7.59$$

$$\begin{aligned} p_1 V_1^n &= p_2 V_2^n \\ 1 \times 7.59^{1.28} &= 5 \times V_2^{1.28} \end{aligned}$$

$$V_2 = \frac{7.59}{1.28\sqrt[1.28]{5}} = 2.158$$

$$p_3 V_3^n = p_4 V_4^n$$

$$5 \times 0.364^{1.28} = 1 \times V_4^{1.28}$$

$$V_4 = 0.364 \times 1.28\sqrt[1.28]{5} = 1.28$$

Expressing pressure in kN/m² (1 bar = 10² kN/m²) and volumes in m³ (1 litre = 10⁻³ m³) to obtain work in kJ:

$$\text{Work/cycle} = \frac{n}{n-1} \{ p_2 (V_2 - V_3) - p_1 (V_1 - V_4) \}$$

$$= \frac{1.28}{0.28} \times 10^2 \times 10^{-3} \{ 5(2.158 - 0.363) - 1(7.59 - 1.28) \}$$

$$= 1.216 \text{ kJ}$$

$$\begin{aligned} \text{Indicated power [kW]} &= 1.216 \text{ [kJ/cycle]} \times 2 \text{ [cycle/s]} \\ &= 2.432 \text{ kW Ans. (i)} \end{aligned}$$

$$\text{Mean indicated press. [kN/m}^2] = \frac{\text{area of diagram [kJ]}}{\text{length of diagram [m}^3]}$$

$$= \frac{1.216}{7.226 \times 10^{-3}} = 168.3 \text{ kN/m}^2 = 1.683 \text{ bar Ans. (ii)}$$

$$\text{Vol. effic.} = \frac{\text{volume drawn in per stroke}}{\text{piston swept volume}}$$

$$= \frac{V_1 - V_4}{V_1 - V_3} = \frac{7.59 - 1.28}{7.226}$$

$$= 0.8732 \text{ or } 87.32\% \text{ Ans. (iii)}$$

7. Referring to Fig. 38

From $pV/T = \text{constant}$, volume rate of air induced at suction pressure and temperature

$$= \frac{5 \times 1.013 \times 297}{60 \times 0.98 \times 289} = 0.08853 \text{ m}^3/\text{s}$$

$$\text{Work/cycle} = \frac{n}{n-1} p_1 (V_1 - V_4) \left[\left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right]$$



The above expression is the work per cycle when $(V_1 - V_4)$ is the volume drawn in per cycle. Similarly, if $(V_1 - V_4)$ is taken as the volume drawn in per second, the expression will give work per second, which is power, thus:

$$\text{Power [kW]} = \text{work per second [kJ/s} = \text{kN m/s]}$$

$$\frac{n}{n-1} = \frac{1.25}{0.25} = 5 \quad \frac{n-1}{n} = \frac{1}{5}$$

$$V_1 - V_4 = 0.08853 \text{ m}^3/\text{s}$$

$$p_1 = 0.98 \times 10^2 \text{ kN/m}^2$$

$$\frac{p_2}{p_1} = \text{pressure ratio} = 4.55$$

$$\begin{aligned} \text{Power} &= \frac{n}{n-1} p_1 (V_1 - V_4) \left[\left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right] \\ &= 5 \times 0.98 \times 10^2 \times 0.08853 (4.55^{\frac{1}{5}} - 1) \\ &= 15.36 \text{ kW} \quad \text{Ans. (i)} \end{aligned}$$

Let stroke volume $(V_1 - V_3)$ on Fig. 38 be represented by unity, then $V_3 = 0.05$ and $V_1 = 1.05$

$$\text{From } p_3 V_3^n = p_4 V_4^n$$

$$V_4 = 0.05 \times 1.25 \sqrt[4]{4.55} = 0.1681$$

$$\begin{aligned} \text{Suction period} &= V_1 - V_4 \\ &= 1.05 - 0.1681 = 0.8819 \end{aligned}$$

$$\begin{aligned} \text{Vol. effic.} &= \frac{\text{vol. drawn in per stroke}}{\text{stroke volume}} \\ &= \frac{0.8819}{1} = 0.8819 \quad \text{Ans. (ii)} \end{aligned}$$

$$\text{Actual volume of air drawn in per stroke [m}^3]$$

$$= \frac{0.08853 \text{ [m}^3/\text{s]}}{2 \times 8 \text{ [strokes/s]}}$$

$$\text{Piston swept vol.} = \frac{\text{induced volume}}{\text{volumetric efficiency}}$$

$$= \frac{0.08853}{2 \times 8 \times 0.8819}$$

$$\text{Let } d \text{ [m]} = \text{cyl. diameter, stroke} = 1.2d$$

$$\text{Piston swept vol.} = 0.7854d^2 \times 1.2d$$

$$d = \sqrt[3]{\frac{0.08853}{0.7854 \times 1.2 \times 2 \times 8 \times 0.8819}}$$

$$= 0.1881 \text{ m}$$

$$\left. \begin{aligned} \text{Diameter of cylinder} &= 188.1 \text{ mm} \\ \text{Stroke} &= 1.2 \times 188.1 = 225.7 \text{ mm} \end{aligned} \right\} \text{Ans. (iii)}$$

$$\begin{aligned} 8. \quad \dot{m} &= \frac{p\dot{V}}{RT} \\ \dot{m} &= \frac{10^5 \times 17}{287 \times 306 \times 60} \\ \dot{m} &= 0.323 \text{ kg/s} \end{aligned}$$

$$\text{For minimum work } p_2 = \sqrt{p_3 p_1}$$

$$p_2 = \sqrt{16 \times 1}$$

$$p_2 = 4 \text{ bar}$$

$$\frac{T_1}{T_2} = \left(\frac{p_1}{p_2} \right)^{\frac{n-1}{n}}$$

$$\therefore T_2 = T_1 \left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} = 306 (4)^{\frac{0.3}{1.3}}$$

$$T_2 = 421.4 \text{ K or } 148.4^\circ\text{C}$$

$$\begin{aligned} \text{First stage power} &= \frac{n}{n-1} \dot{m} R (T_2 - T_1) \\ &= \frac{1.3}{0.3} \times 0.323 \times 287 (148.4 - 33) / 10^3 \\ &= 46.37 \text{ kW} \quad \text{Ans. (a)} \end{aligned}$$

$$\text{Heat rejected to intercooler}$$

$$\begin{aligned} &= \dot{m} c_p (T_2 - T_1) \\ &= 0.323 \times 1005 (148.4 - 33) \times 60 / 10^6 \\ &= 2.25 \text{ MJ/min} \quad \text{Ans. (b)} \end{aligned}$$

9. From $\frac{pV}{T} = \text{constant}$, volume rate of air at suction pressure and temperature

$$= \frac{1.013 \times 0.6083 \times 300}{0.97 \times 288} = 0.6617 \text{ m}^3/\text{s}$$

Let stroke volume $V_1 - V_3$ in Fig. 38 equal unity.

$$\text{Then } V_3 = 0.06, V_1 = 1.06$$

$$p_3 V_3^n = p_4 V_4^n$$

$$V_4 = 0.06 \times \sqrt[1.32]{\frac{4.85}{0.97}} = 0.2031$$

$$\begin{aligned} \text{Actual volume of air drawn in per stroke} &= \frac{0.6617}{2 \times 5} \left[\frac{\text{m}^3/\text{s}}{\text{stroke/s}} \right] \\ &= 0.06617 \text{ m}^3 \end{aligned}$$

$$\text{and } \frac{V_1 - V_4}{V_1 - V_3} = \frac{0.06617}{\text{piston swept volume}}$$

$$\therefore \text{Piston swept volume} = \frac{0.06617}{1.06 - 0.2031} = 0.07812 \text{ m}^3$$

$$\text{also, Piston swept volume} = 0.7854 d^2 \times d$$

d = cylinder diameter and stroke

$$\therefore 0.07812 = 0.7854 d^3$$

$$d = 0.463 \text{ m Ans. (a)}$$

Isothermal efficiency

$$= \frac{\ln r}{\frac{n}{n-1} \left[r^{\frac{n-1}{n}} - 1 \right]}$$

$$= \frac{\ln 5}{\frac{1.32}{0.32} \left[5^{\frac{0.32}{1.32}} - 1 \right]}$$

$$= 0.818 \text{ or } 81.8\% \text{ Ans. (b)}$$

10. Let r = stage pressure ratio for minimum work condition
Let s = number of stages of compression

$$\frac{p_2}{p_1} = \left(\frac{T_2}{T_1} \right)^{\frac{n}{n-1}}$$

$$r = \left(\frac{368}{308} \right)^{\frac{1.3}{0.3}}$$

$$r = 2.162$$

$$\text{now } p_2 = r p_1$$

$$p_3 = r p_2 = r^2 p_1$$

$$p_4 = r p_3 = r^3 p_1 \text{ etc.}$$

$$\therefore p_{s-1} = r^s p_1$$

$$\text{hence } 100 = 2.162^s \times 1$$

$$s = 5.97 \text{ say } 6 \text{ stages Ans. (a)}$$

$$\text{Power input per stage} = \frac{n}{n-1} \dot{m} R (T_2 - T_1)$$

$$= \frac{1.3}{0.3} \times 0.1 \times 0.287 (368 - 308)$$

$$= 7.45 \text{ kW}$$

$$\text{Compressor power input} = 7.45 \times 6 \text{ stages}$$

$$= 44.7 \text{ kW Ans. (b)}$$

SOLUTIONS TO TEST EXAMPLES 10

1. Tables page 2, water at 80°C, $h = 334.9$
 4, steam at 9 bar:

$$h = h_f + xh_{fg}$$

$$= 743 + 0.96 \times 2031 = 2693$$
 Heat energy transferred = change in enthalpy

$$= 2693 - 334.9$$

$$= 2358.1 \text{ kJ/kg} \quad \text{Ans.}$$

2. Tables page 7, 30 bar 350°C, $h = 3117$
 3, 0.06 bar dryness 0.88:

$$h = h_f + xh_{fg}$$

$$= 152 + 0.88 \times 2415 = 2277$$
 Enthalpy drop per kg = $3117 - 2277$

$$= 840 \text{ kJ/kg} \quad \text{Ans. (i)}$$

At 0.5 kg of steam per second, total change in enthalpy in the steam through the turbine per second

$$= 0.5 \times 840 = 420 \text{ kJ/s}$$
 kilojoules per second = kilowatts
 \therefore Power equivalent = 420 kW Ans. (ii)

3. Tables page 7, 20 bar 350°C, $h = 3138$, $v = 0.1386$
 4, 20 bar 0.98 dry:

$$h = h_f + xh_{fg}$$

$$= 909 + 0.98 \times 1890 = 2761 \text{ kJ/kg}$$

$$v = xv_g$$

$$= 0.98 \times 0.09957$$

$$= 0.09757 \text{ m}^3/\text{kg}$$
 Heat energy supplied to steam in superheaters

$$= 3138 - 2761 = 377 \text{ kJ/kg} \quad \text{Ans. (i)}$$
 % increase in specific volume

$$= \frac{0.1386 - 0.09757}{0.09757} \times 100$$

$$= 42.06\% \quad \text{Ans. (ii)}$$

4. Tables page 4, 4 bar, $v_g = 0.4623$
 8 bar, $v_g = 0.2403$
 For 8 bar, dryness 0.94,

$$v = xv_g = 0.94 \times 0.2403 = 0.2258 \text{ m}^3/\text{kg}$$

$$p_1 v_1^n = p_2 v_2^n$$

- $$8 \times 0.2258^{1.12} = 4 \times v_2^{1.12}$$
- $$v_2 = 0.2258 \times 1.12 \sqrt[1.12]{2} = 0.4194 \text{ m}^3/\text{kg}$$
- 0.4194 m³/kg is the specific volume of wet steam at 4 bar, let its dryness fraction = x :
- $$v = xv_g$$
- $$0.4194 = x \times 0.4623$$
- $$x = 0.9072 \quad \text{Ans.}$$
-
5. Tables page 4, steam 2.4 bar, $h_g = 2715$
 2, water 42°C, $h = 175.8$
 3, water 99.6°C, $h = 417$
 See Fig. 41. Consider one kg of steam from boiler, let x kg be tapped off low pressure turbine to heater, then $(1 - x)$ kg passes through condenser and as water to hotwell.

Enthalpy of heating steam and water entering heater	=	Enthalpy of water leaving heater
---	---	----------------------------------

$$x \times 2715 + (1 - x) \times 175.8 = 1 \times 417$$

$$2715x + 175.8 - 175.8x = 417$$

$$2539.2x = 241.2$$

$$x = 0.095$$
 \therefore % of steam tapped off = 9.5% Ans.

 6. Tables page 4, steam 16 bar, $h_f = 859$, $h_{fg} = 1935$
 steam 8 bar, $h_f = 721$, $h_{fg} = 2048$
 Enthalpy after throttling = Enthalpy before

$$721 + x \times 2048 = 859 + 0.98 \times 1935$$

$$x \times 2048 = 2034$$

$$x = 0.9931 \quad \text{Ans.}$$

 7. $t_s = 165^\circ\text{C}$
 $p = 7 \text{ bar}$
 $v_{g1} = 0.2728 \text{ m}^3/\text{kg}$ } tables, page 4
 $V_1 = 0.75 \times 0.2728 \times 0.2$

$$= 0.0409 \text{ m}^3$$

$$V_2 = 0.0818 \text{ m}^3$$

$$v_{g2} = \frac{0.0818}{0.2}$$

$$= 0.4090 \text{ m}^3/\text{kg}$$

$$t = 355^\circ\text{C}$$

i.e. using tables, page 7, interpolating at 7 bar with v_g 0.4090

Final temperature = 355°C Ans. (a)

$$\begin{aligned}\text{Work done} &= p(V_2 - V_1) \\ &= 7 \times 100(0.0818 - 0.0409) \\ &= 28.64 \text{ kJ} \quad \text{Ans. (b)}\end{aligned}$$

$$\begin{aligned}\text{Heat energy transfer} &= H_1 - H_2 \\ &= 0.2 [h_g - (h_f + xh_{fg})] \\ &= 0.2 [3175 - (697 + 0.75 \times 2067)] \\ &= 185.5 \text{ kJ} \quad \text{Ans. (c)}\end{aligned}$$

8. Tables page 4, 15 bar, $h_f = 845$, $h_{fg} = 1947$
1.1 bar, $t_s = 102.3$, $h_g = 2680$

Dryness fraction by separator,

$$x_1 = \frac{m_2}{m_2 + m_1} = \frac{10}{10.55} = 0.9479$$

Dryness fraction by throttling calorimeter:

$$\begin{aligned}\text{Enthalpy before throttling} &= \text{Enthalpy after} \\ 845 + x_2 \times 1947 &= 2680 + 2(111 - 102.3) \\ x_2 \times 1947 &= 1852.4 \\ x_2 &= 0.9513\end{aligned}$$

Dryness fraction of steam:

$$\begin{aligned}x &= x_1 \times x_2 \\ &= 0.9479 \times 0.9513 \\ &= 0.9018 \quad \text{Ans.}\end{aligned}$$

9. 1st Case: Absolute press. = 1.9 + 1 = 2.9 bar
Tables page 4, when temp. of steam is 130°C,
 $p = 2.7$ bar, $v_g = 0.6686 \text{ m}^3/\text{kg}$

$$\text{Mass of steam} = \frac{4.25}{0.6686} = 6.356 \text{ kg} \quad \text{Ans. (a)(i)}$$

$$\begin{aligned}\text{Air pressure} &= \text{total press.} - \text{steam press.} \\ &= 2.9 - 2.7 = 0.2 \text{ bar} = 20 \text{ kN/m}^2\end{aligned}$$

$$pV = mRT$$

$$\begin{aligned}\therefore m &= \frac{20 \times 4.25}{0.287 \times 403} \\ &= 0.7348 \text{ kg} \quad \text{Ans. (b)(i)}\end{aligned}$$

2nd Case: Absolute press. = 6.25 + 1 = 7.25 bar

Tables page 4, when temp. of steam is 165°C,
 $p = 7$ bar, $v_g = 0.2728 \text{ m}^3/\text{kg}$

$$\text{Mass of steam} = \frac{4.25}{0.2728} = 15.58 \text{ kg} \quad \text{Ans. (a)(ii)}$$

$$\begin{aligned}\text{Air pressure} &= \text{total press.} - \text{steam press.} \\ &= 7.25 - 7 = 0.25 \text{ bar} = 25 \text{ kN/m}^2\end{aligned}$$

$$pV = mRT$$

$$\begin{aligned}\therefore m &= \frac{25 \times 4.25}{0.287 \times 438} \\ &= 0.845 \text{ kg} \quad \text{Ans. (b)(ii)}\end{aligned}$$

10. $p_{ww} = 0.02337$ bar
Using tables, page 2, $t_s = 20^\circ\text{C}$

$$\begin{aligned}p_A &= 1 - 0.02337 \\ &= 0.97663 \text{ bar}\end{aligned}$$

$$\frac{p_N}{p_O} = \frac{V_N}{V_O} \quad \text{using partial volumes}$$

$$\frac{p_N}{0.97663 - p_N} = \frac{0.79}{0.21}$$

$$p_N = 3.674 - 3.762 p_N$$

$$p_N = 0.7715 \text{ bar}$$

$$p_O = 0.97663 - 0.7715$$

$$= 0.2051 \text{ bar}$$

$$\text{Partial pressures} = \left. \begin{array}{l} 0.2051 \text{ bar, oxygen} \\ 0.7715 \text{ bar, nitrogen} \\ 0.02337 \text{ bar, water vapour} \end{array} \right\} \text{Ans. (a)}$$

$$p_A v = R_A T$$

$$v = \frac{0.287 \times 293}{100 \times 0.97663}$$

$$= 0.861 \text{ m}^3/\text{kg of dry air}$$

$$v_g = 57.84 \text{ m}^3/\text{kg}$$

using tables, page 2, $t_s = 20^\circ\text{C}$

$$\text{kg of water vapour} = \frac{0.861}{57.84}$$

$$= 0.01488 \text{ per kg of dry air}$$

$$\% \text{ Absolute humidity} = 1.488 \quad \text{Ans. (b)}$$

SOLUTIONS TO TEST EXAMPLES 11

1. Tables page 4, 17 bar, $s_f = 2.372$ $s_{fg} = 4.028$
 $s = 2.372 + 0.95 \times 4.028$
 $= 6.198 \text{ kJ/kgK}$ Ans.
2. Tables page 4, 195°C (14 bar) $s_f = 2.284$ $s_{fg} = 4.185$
 $s = 2.284 + 0.9 \times 4.185$
 $= 6.05 \text{ kJ/kg}$ Ans.
3. Tables page 4, 5.5 bar, $s_g = 6.790$
 Tables page 3, 0.2 bar, $s_f = 0.832$, $s_{fg} = 7.075$
 Entropy after expansion = Entropy before
 $0.832 + x \times 7.075 = 6.79$
 $x \times 7.075 = 5.958$
 $x = 0.8422$ Ans.
4. Tables page 7, by interpolation:
 15 bar 350°C $s = 7.102$
 20 bar 350°C $s = 6.957$
 For 5 bar increase $s = 0.145$ decrease
 For 2 bar increase $s = \frac{2}{5} \times 0.145 = 0.058$ decrease
 \therefore 17 bar 350°C, $s = 7.102 - 0.058 = 7.044$
 Page 4, 1.7 bar, $s_f = 1.475$ $s_{fg} = 5.707$
 Entropy after expansion = Entropy before
 $1.475 + x \times 5.707 = 7.044$
 $x \times 5.707 = 5.569$
 $x = 0.9759$ Ans.
5. Tables page 4, 22 bar, $h_g = 2801$, $s_g = 6.305$
 7 bar, $h_g = 2764$
 1.4 bar, $s_f = 1.411$, $s_{fg} = 5.835$

THROTTLING PROCESS:

$$\text{Enthalpy after} = \text{Enthalpy before}$$

$$\therefore \text{Enthalpy at 7 bar} = 2801$$

Throttled steam is therefore superheated.

Tables page 7, interpolating:

$$\text{Enthalpy of superheat} = 2801 - 2764 = 37$$

$$\text{At 7 bar, } h = 2846 \text{ for } 200^\circ\text{C}$$

$$h = 2764 \text{ for } 165^\circ\text{C (sat. temp.)}$$

$$\text{increase } h = 82 \text{ for } 35^\circ\text{C increase}$$

$$\text{difference in temp. for } h = 37, = \frac{37}{82} \times 35 = 15.8^\circ\text{C}$$

\therefore Degree of superheat at 7 bar = 15.8°C Ans. (i)

Entropy of steam at 7 bar with 15.8°C of superheat:

$$7 \text{ bar } 200^\circ\text{C } s = 6.888$$

$$165^\circ\text{C } s = 6.709$$

$$\text{for increase } 35^\circ\text{C } s = 0.179 \text{ increase}$$

$$\dots \dots 15.8^\circ\text{C } s = \frac{15.8}{35} \times 0.179 = 0.0808$$

\therefore Entropy at 7 bar, 15.8°C superheat

$$= 6.709 + 0.0808 = 6.7898$$

$$\text{Increase in entropy} = 6.7898 - 6.305$$

$$= 0.4848 \text{ kJ/kgK}$$
 Ans. (ii)

ISENTROPIC EXPANSION from 7 to 1.4 bar:

$$\text{Entropy after} = \text{Entropy before}$$

$$1.411 + x \times 5.835 = 6.7898$$

$$x \times 5.835 = 5.3788$$

$$x = 0.9217$$
 Ans. (iii)

SOLUTIONS TO TEST EXAMPLES 12

1. Tables page 4, 8 bar, $h_g = 2769$
 5 bar, $h_f = 640$ $h_{fg} = 2109$
 $v_g = 0.3748$

$$\text{Enthalpy drop} = 2769 - (640 + 0.97 \times 2109)$$

$$= 83 \text{ kJ/kg}$$

$$\text{Velocity} = \sqrt{2 \times 83 \times 10^3} = 407.4 \text{ m/s Ans. (i)}$$

$$\text{Spec. vol. of steam at exit} = 0.97 \times 0.3748 \text{ m}^3/\text{kg}$$

$$\text{Mass flow [kg/s]} = \frac{\text{volume flow [m}^3/\text{s]}}{\text{spec. vol. [m}^3/\text{kg]}}$$

$$= \frac{\text{area [m}^2] \times \text{velocity [m/s]}}{\text{spec. vol. [m}^3/\text{kg]}}$$

$$= \frac{14.5 \times 10^{-4} \times 407.4}{0.97 \times 0.3748}$$

$$= 1.625 \text{ kg/s Ans. (ii)}$$

2. Referring to Fig. 57:

$$\text{Angle between } u \text{ and } v_{r1} = 180^\circ - \beta_1$$

$$= 180^\circ - 33^\circ = 147^\circ$$

$$\text{Angle between } v_1 \text{ and } v_{r1} = \beta_1 - \alpha_1$$

$$= 33^\circ - 20^\circ = 13^\circ$$

By sine rule:

$$\frac{u}{\sin 13^\circ} = \frac{v_1}{\sin 147^\circ}$$

$$u = \frac{450 \times \sin 13^\circ}{\sin 147^\circ}$$

$$\text{Rotational speed [rev/s]} = \frac{\text{linear velocity [m/s]}}{\text{circumference [m]}}$$

$$= \frac{185.9}{\pi \times 0.66} = 89.64 \text{ rev/s Ans. (ii)}$$

3. Referring to Fig. 60:

$$v_{a1} = v_1 \sin \alpha_1 = 243 \times \sin 23^\circ = 94.95 \text{ m/s}$$

$$v_{w1} = v_1 \cos \alpha_1 = 243 \times \cos 23^\circ = 223.7 \text{ m/s}$$

$$x = v_{w1} - u = 223.7 - 159 = 64.7 \text{ m/s}$$

$$\tan \beta_1 = \frac{v_{a1}}{x} = \frac{94.95}{64.7} = 1.468$$

$$\text{Blade inlet angle} = 55^\circ 44' \text{ Ans. (i)}$$

Since the combined vector diagram of inlet and exit velocities is symmetrical, $v_{w2} = x$,

$$v_w = v_{w1} + v_{w2} = 223.7 + 64.7 = 288.4 \text{ m/s}$$

$$\text{Force on blades} = \dot{m} v_w$$

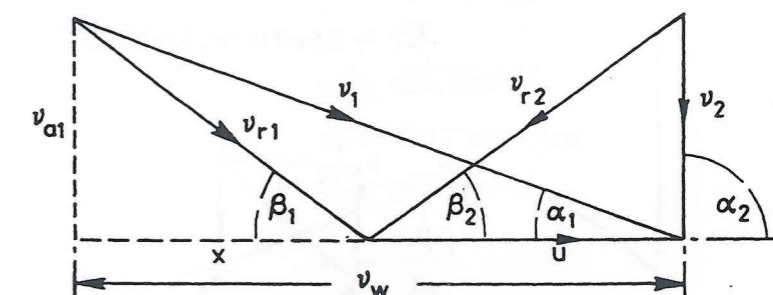
$$= 0.9 \times 288.4 = 259.5 \text{ N Ans. (ii)}$$

$$\text{Power [W]} = \text{force [N]} \times \text{velocity [m/s]}$$

$$= 259.5 \times 159$$

$$= 4.127 \times 10^4 \text{ W} = 41.27 \text{ kW Ans. (iii)}$$

4.



$$u = \pi \times 0.6 \times 100$$

$$= 188.5 \text{ m/s}$$

$$v_{r2} = \frac{188.5}{\cos 35^\circ}$$

$$= 230.1 \text{ m/s}$$

Let $v_{r1} = v_{r2}$ (see assumption following)

$$v_{a1} = 230.1 \times \sin 35^\circ$$

$$= 132 \text{ m/s}$$

$$v_w = 2 \times 188.5$$

$$= 377 \text{ m/s}$$