(i) *Sun at Position d*:
In fig. 35·3:

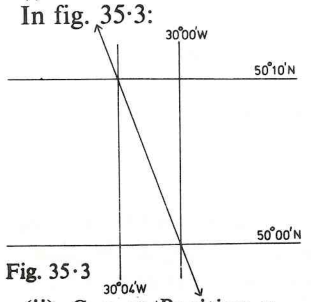


Fig. 35·3

Azimuth NZD = 077°
Position Line: Direction = 347°–167°
Using Latitude 50° 00′·0 N., the calculated Longitude = 30° 00′·0 W.

Using Latitude 50° 10′·0 N., the calculated Longitude = 30° 04′·0 W.

The error in Latitude, which is 10′·0, produces an error in Longitude of 2·5 miles, or 4′·0 W.

(ii) *Sun at Position e*:
In fig. 35·4:

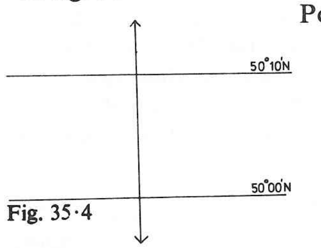


Fig. 35·4

Azimuth NZE = 090° (Sun on Prime Vertical Circle)
Position Line: Direction = 000°–180°
Using Latitude 50° 00′·0 N., the calculated Longitude = 30° 00′·0 W.

Using Latitude 50° 10′·0 N., the calculated Longitude = 30° 00′·0 W.

The error in Latitude, which is 10′·0, produces no error in the calculated Longitude.

(iii) *Sun at Position f*:
In fig. 35·5:

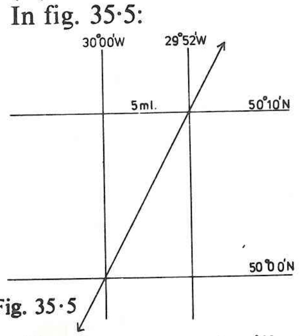


Fig. 35·5

Azimuth NZF = 118°
Position Line: Direction = 028°–208°
Using Latitude 50° 00′·0 N., the calculated Longitude = 30° 00′·0 W.

Using Latitude 50° 10′·0 N., the calculated Longitude = 29° 52′·0 W.

The error in Latitude, which is 10′·0, produces an error in the calculated Longitude of 5·0 miles, or 8′·0 E.

The above examples illustrate that if the Latitude used in the computation is not the True latitude of the vessel the calculated Longitude is NOT the vessel's actual Longitude, except in the special case when the observed body lies on the observer's prime vertical circle at the time of the observation.

2. Error in Longitude due to Error in Latitude

Error in Longitude due to Error in Latitude may be found as follows:

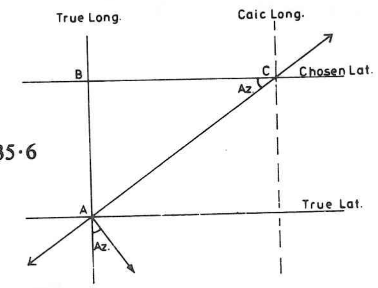


Fig. 35·6

Referring to fig. 35·6: In triangle ABC:

Error in Latitude = AB
Corresponding Error in Departure = BC
Azimuth = BCA

Now: $BC = AB \cot BCA$

That is: Error in Departure = Error in Latitude . cot Azimuth
But: Error in Departure = Error in Longitude . cot Latitude
Therfore: Error in Longitude = Error in Latitude . cot Azimuth . sec Latitude

From this formula it may be seen that, for any given Latitude, the error in Longitude consequent upon an error in Latitude varies as the cotangent of the azimuth. Thus, error in Longitude increases from zero when the azimuth is 90° to infinity when the azimuth is 0°. It is for this reason that when making an observation for Longitude, optimum conditions prevail when the observed body is on the observer's prime vertical circle; or, in other words, when the observed body bears 090° or 270°. It is for this reason that in days before the advent of astronomical position line navigation, navigators made a point of observing the Sun when on, or nearest to, the prime vertical circle.

It may also be seen from the formula that error in Longitude varies as the secant of the Latitude, so that the error increases with the Latitude; being least when the Latitude is 0° (secant 0° = 1), and infinity when the Latitude is 90° (secant 90° = ∞).

A table, given in Norie's and Burton's Nautical Tables, gives the error in Longitude consequent upon an error of one minute in Latitude, for all values of Latitude and Azimuth. This table is known as the Longitude Correction Table. The Longitude Correction may also be obtained from the ABC Table described in Chapter 38.

3. Effects of Errors in Altitude

Any error in an altitude used in calculating a position line affects the position through which to plot the position line. If the altitude is in error and too great, the position line will be displaced towards the direction of the observed body. If, on the other hand, the altitude is in error and too small, the position line will be displaced away from the direction of the observed body.

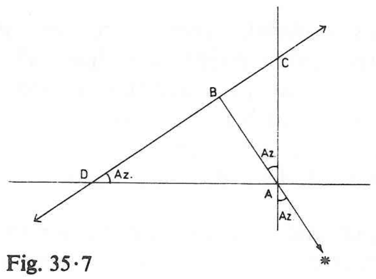


Fig. 35·7

Referring to fig. 35·7, suppose that the true position of a vessel is represented as being at A and that, due to an error in altitude amounting to AB, a false position line is drawn through B. The position line passes through the meridian of the vessel at C. Therefore, the error in Latitude due to an error of AB in the altitude is AC.

The position line passes through the parallel of the vessel's Latitude at D, so that the error in departure, due to the error AB in altitude, is AD.

In the triangles ABC and ABD which, if the distance AB is small, may be considered to be plane right-angled triangles, the angles BDA and BAC are each equal to the azimuth of the body at the time of the observation.

In triangle ABC:	AB =	$AC \cos CAB$
Thus:	Error in Altitude =	Error in Latitude . cos Azimuth
Or:	Error in Latitude =	Error in Altitude . sec Azimuth

These formulae indicate that Error in Latitude is greatest when secant Azimuth is greatest; that is to say, when the Azimuth is 90°. The Error is least when the Azimuth is 0°; that is, when the observed body is on the meridian. Now the secant of 0° is unity, so that, for a body at meridian passage, an error in altitude produces an equal error in latitude.

In triangle ABD	AD =	AB cosec ADB
Thus:	Error in Departure =	Error in Altitude . cosec Azimuth
Or:	Error in Altitude =	Error in Departure . sin Azimuth
But:	Error in Departure =	Error in Longitude . cos Latitude
Thus:	Error in Altitude =	Error in Longitude . cos Latitude . sin Azimuth
Or:	Error in Longitude =	Error in Altitude . sec Latitude . cosec Azimuth

This formula indicates that, for any error in altitude for a given latitude, the error in Longitude is greatest when the cosecant of the azimuth is greatest. This occurs when the azimuth is 90°, that is to say when the body is on the observer's prime vertical circle. The error decreases as the azimuth decreases from 90° to 0°.

It may also be seen from the formula that, for any given azimuth, error in Longitude due to error in altitude is greatest when secant latitude is greatest. This occurs when the latitude is 90°.

Error in altitude may arise from the sextant used in taking the altitude not being in correct adjustment. It may also result through faulty refraction and dip corrections.

Any error in altitude which affects all position lines equally may readily be eliminated or reduced in cases in which the position lines are obtained from observations of four stars one in each quadrant of the compass. If, for example, stars bearing 000°, 090°, 180° and 270°, are observed, the four false position lines will intersect to form a square; If the same error affects all four position lines the true position of the vessel is then at the centre of the square.

4. The Plotting Chart

The most convenient way of ascertaining a vessel's position from astronomical observations, is to plot the resulting position lines on a large-scale navigational chart. The Latitude and Longitude of the vessel may then be lifted from the chart provided that the scale of the chart is sufficiently large. The sea-chart is used at times when Nautical Astronomy is practised is usually a small-scale chart: it is, therefore, quite unsuitable for plotting astronomical position lines. In these circumstances resort is made to a Plotting Sheet.

A plotting sheet is simply a sheet of blank or squared paper, and is very commonly a page of the navigator's workbook. A convenient scale of distance is chosen—usually one inch to represent ten miles—and the position lines are drawn relative to a reference position on the plotting sheet. The D. Lat. and the Departure between the vessel's projected position and the reference position may readily be measured. Departure is then converted into D. Long., usually by means of the traverse table, and the vessel's position found by applying the D. Lat. and D. Long. to the reference position.

The method of finding a vessel's position by means of a plotting sheet is superior to those in which tabular methods are used. The latter, often used mechanically, tend to cause the user to lose sight of the principles of the problem. Moreover, the degree of liability of the observed position is better assessed from the plotting sheet on which tthe problem is clearly displayed, than is the case when a tabular method is employed.

The following examples illustrate the use of the plotting sheet.

Example 35·1—Using position Lat. 31° 23′·0 S., Long. 49° 43′·0 W. to reduce his sights, a navigator's observation of Canopus bearing 136° gave an intercept of 5′·0 TOWARDS, and that of α Pavonis bearing 220° gave an intercept of 8′·5 AWAY. Find the vessel's position by plotting.

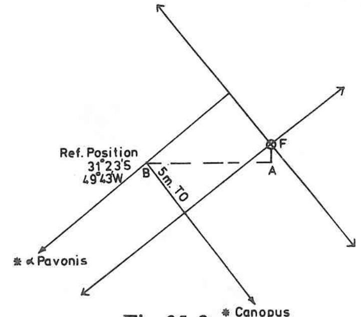


Fig. 35·8

In fig. 35·8:

Departure (AB) =	10′·0 E.		
D. Long. (from Trav. Tables) =	11′·7 E.		
Ref. Position: (B) Lat.	31° 23′·0 S.	Long.	49° 43′·0 W.
(AF) D. Lat.	2′·7 N.	D. Long	11′·7 E.
Vessel's Position: (F) Lat.	31° 20′·3 S.	Long.	49° 31′·3 W.

Answer—Vessel's Position: Lat. = 31° 20′·3 S., Long. = 49° 31′·3 W.

Example 35·2—An observation of the Sun bearing 120°, at 0930 hr. gave a calculated Longitude of 32° 10′·0 W., using Lat. 40° 00′·0 N., to reduce the sight. The vessel travelled on a course of 292° (T) at a speed of 12·0 knots until noon, when the Latitude by observation of the Sun on the meridian gave a Latitude of 40° 25′·0 N. Find the Longitude of the vessel at noon.

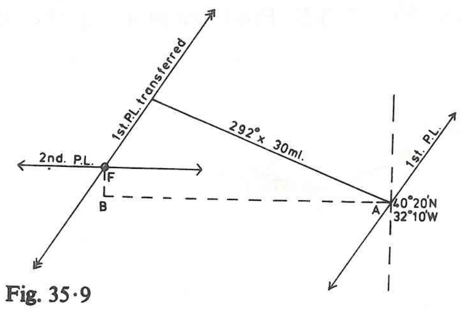


Fig. 35·9

Distance travelled = 2·5 × 12·0
= 30·0 miles

From fig. 35·9:

Departure (AB) =	31′·4
D. Long. =	41′·0 W.
Long. A =	32° 10′·0 W.
Long. F =	32° 51′·0 W.

Answer—Longitude at noon = 32° 51′·0 W.

Note—An alternative solution to Example 35·2 involves plotting the transferred position line through a position obtained by applying the D. Lat. and D. Long.—found from the Traverse Tables—corresponding to the course and distance made good (292° (T) × 30·0 miles) to the position through which the first position line is plotted in the above solution. By so doing the necessity of plotting the first position line and the run is obviated.

Example 35·3—Using Lat. 50° 45′·0 N., Long. 30° 10′·0 W., the intercept obtained from an observation of the Sun bearing 040° (T) was zero. The vessel then travelled on a course of 213° (T) for a distance of 40·0 miles, when a second observation of the Sun bearing 075° (T) gave an intercept of 5·0 miles TOWARDS. Find by plotting the vessel's position at the time of the second observation, given that the position used in reducing the second sight was Lat. 50° 00′·0 N., Long. 31° 00′·0 W.

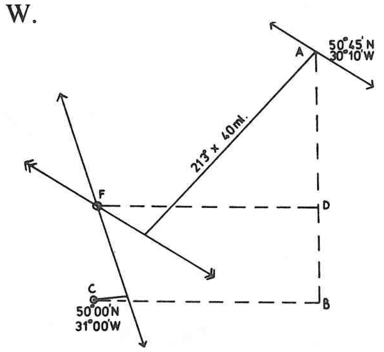


Fig. 35·10

D. Lat. (*AB*) =	45′ S.	Dep. (*AF*) =	32′·8
D. Long. (*AC*) =	50′ W.	D. Long. (*AF*) =	51′·0 W.
Dep. (*AC*) =	32·0 miles	D. Lat. (*AF*) =	25′·0
Lat. *A* =	50° 45′·0 N.	Long. *A* =	30° 10′·0 W.
D. Lat. (*AF*) =	25′·0 S.	D. Long. (*AF*) =	51′·0 W.
Lat. *F* =	50° 20′·0 N.	Long. *F* =	31° 01′·0 W.

Answer—Lat. = 50° 20′·0 N., Long. = 31° 01′·0 W.

Example 35·4—At 0800 hr., using Lat. 20° 00′·0 S., Long. 170° 00′·0 E., an observation of the Sun bearing 075° gave an intercept of 8·5 miles AWAY. The vessel travelled on a course of 275° (T) at a speed of 8·0 knots. At 1100 hr. the course was altered to 240° (T). At noon the Latitude by meridian altitude of the Sun was 20° 15′·0 S. Find, by plotting, the vessel's noon position.

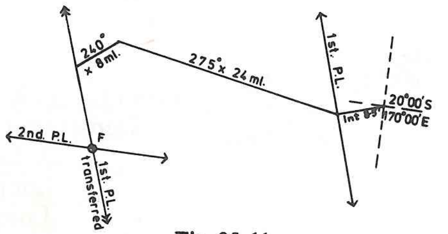


Fig. 35·11

From fig. 35·11:

Dep. *AF* =	37·0 miles
D. Long. *AF* =	39′·5 W.
Long. *A* =	170° 00′·0 E.
Long. *F* =	169° 20′·5 E.

Answer—Noon Position: Lat. = 20° 15′·0 S., Long. = 169 20′·5 E.

Example 35·5—At 0600 hr., using Lat. 46° 45′·0 N., Long. 51° 45′·0 W., an observation of a star bearing 045° gave an intercept of 5·0 miles AWAY. After travelling for 22·0 miles on a course of 250° (T) Cape Race (Lat. 46° 39′·0 N., Long. 53° 04′·0 W.) bore 032° (T). Find the vessel's position at the time at which the bearing of Cape Race was observed.

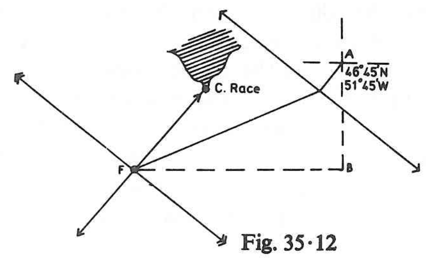


Fig. 35·12

From fig. 35·12:

D. Lat. *AF* =	17′·5 S.
Dep. *AF* =	18·5 miles
D. Long. *AF* =	26′·8 W.

Lat. *A* =	46° 45′·0 N.	Long. =	51° 45′·0 W.
D. Lat. *AF* =	17′·5 S.	D. Long. =	26′·8 W.
Lat. *F* =	46° 27′·5 N.	Long. =	52° 11′·8 W.

Answer—Lat. = 46° 27′·5 N., Long. = 52° 11′·8 W.

Exercises on Chapter 35

1. Explain why it is impossible to find a vessel's position when out of sight of land from a single astronomical observation.
2. Is it possible to find an observer's Longitude from a single observation? Explain your answer.
3. Discuss the factors which influence the error in Longitude consequent upon an error in Latitude when the Longitude Method is used for sight reduction.
4. Prove: Error in Long. = Error in Lat. . cot Az. . sec Lat.
5. Explain how the Longitude Correction Table is computed.
6. Explain clearly the effect on an astronomical position line due to an error in altitude.
7. Prove: Error in Lat. = Error in Alt. . sec Az.

8. Prove: Error in Long. = Error in Alt. . sec Lat. . cosec Az.
9. Describe the use of a simple plotting sheet. How does a plotting chart differ from a navigational chart when used for plotting astronomical position lines for fixing?
10. Using Lat. 40° 05′·0 N., for reducing an observation of the Sun bearing 285° by the Longitude Method, the calculated Longitude was 46° 15′·0 W. At the same time the Latitude by meridian altitude of Venus was found to be 40° 10′·0 N. Find the vessel's position.
11. Using Lat. 25° 10′·0 S., Long. 120° 33′·0 W., the intercept obtained from an observation of a star bearing 065° was 6·0 miles TOWARDS. At the same time an observation of a star bearing 134° gave an intercept of 4·5 miles TOWARDS. Find the vessel's position at the time of the observations.
12. Using Lat. 44° 10′·0 N., Long. 155° 30′·0 W., an observation of the Moon bearing 254° gave an intercept of 5·5 miles AWAY. At the same time an observation of a star bearing 105° gave an intercept of zero. Find the vessel's position at the time of the observations.
13. Using Lat. 60° 30′·0 S., Long. 30° 00′·0 W., simultaneous observations of stars bearing 158° and 020° gave intercepts respectively, of 4·5 miles AWAY and 3·0 miles AWAY. Find the vessel's position at the time of the observations.
14. The calculated Longitude obtained from an observation of the Sun which bore 085° was 43° 06′·0 W. The latitude used in reducing the sight was 50° 45′·0 N. The vessel travelled on a course of 280° (T) for a distance of 30·0 miles, when a second observation of the Sun which bore 165° (T) gave an intercept of 4·0 miles AWAY, the position used for reducing the sight being Lat. 51° 00′·0 N., Long. 43° 30′·0 W. Find the vessel's position at the time of the second observation.
15. Using Lat. 40° 30′·0 S., the calculated Longitude obtained from an observation of the Sun bearing 105° was 124° 20′·0 W. The vessel travelled on a course of 090° (T) for a distance of 20·0 miles, and then on a course of 140° (T) for a distance of 15·0 miles. At the end of this run the Latitude by meridian altitude of the Sun was 40° 30′·0 S. Find the Longitude of the vessel at noon.
16. At 1000 hr. when the log registered 46·0, the Sun was observed bearing 156° (T) and the intercept, using Lat. 40° 00′·0 S., Long. 00° 00′·0. to reduce the sight was 5·0 miles AWAY. At 1400 hr. when the log registered 86·0, an observation of the Sun bearing 210° gave an intercept of 4·0 miles TOWARDS, using Lat. 40° 00′·0 S., Long. 00° 30′·0 W. Find the vessel's position at 1400 hr. given that the course made good between the times of the observations had been 275° (T).
17. At 0300 hr. a point of land in Lat. 37° 00′·0 N., Long. 08° 54′·0 W., bore 040° (T). The vessel travelled for a distance of 25·0 miles on a course of 195° (T), during which time the current was estimated to have set 270° (T) for a distance of 5·0 miles. At the end of this run, an observation of a star bearing 040° gave an intercept of 5·0 miles away, using Lat. 36° 20′·0 N., Long. 09° 00′·0 W. to reduce the sight. Find the vessel's position at the time of the observation of the star.

CHAPTER 36

CELESTIAL BODIES NEAR THE MERIDIAN

1. General Remarks

It is sometimes useful to know what stars are within a given interval from their times of meridian passage.

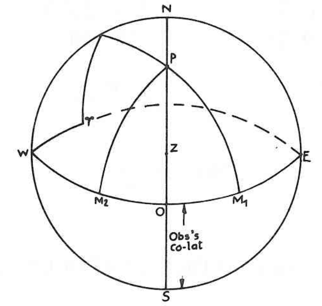


Fig. 36·1

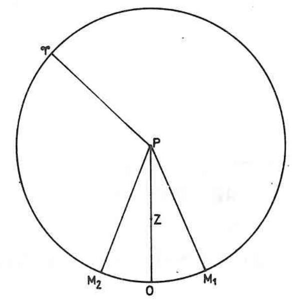


Fig. 36·2

In figs. 36·1 and 36·2, which are projections of the celestial sphere onto the planes of an observer's celestial horizon and celestial equator, respectively, *PO* represents the upper celestial meridian of the observer, and $p\gamma$ represents the celestial meridian of the First Point of Aries. The arc $O\gamma$ is the Local Hour Angle of γ. This may readily be found by applying the observer's Longitude to the G.H.A. of γ which is tabulated against G.M.T. in the *Nautical Almanac*. The L.H.A. of γ subtracted from 360° 00′ gives the Sidereal Hour Angle of the Observer's Upper Celestial Meridian.

Suppose that the celestial meridian PM_1 lies 15° to the east of the observer's upper celestial meridian, and that the celestial meridian PM_2 lies 15° to the West of the observer's celestial meridian. All celestial bodies which lie within these two celestial meridians have Sidereal Hour Angles within 15° of that of the observer's upper celestial meridian.

All celestial bodies which lie between the celestial meridian PM_1 and the observer's upper celestial meridian have S.H.A.s which are less than the S.H.A. of the observer's upper celestial meridian. On the other hand, all celestial bodies which lie between the observer's upper celestial meridian and the celestial meridian PM_2 have S.H.A.s which are more than the S.H.A. of the observer's upper celestial meridian.

It may be seen from fig. 36·1 that, for a celestial body to be above the horizon at the time of its upper transit, its declination, if of opposite name to that of the observer's Latitude, must be less than the co-Latitude of the observer. For a celestial body to be above the horizon at the time of its lower transit, its polar distance must be less than the observer's latitude. (Refer to Part 4, Chapter 28).

The following examples illustrate a method of finding what stars are within a given interval of time of meridian passage.

Example 36·1—What visible stars are less than 1 hour from the upper celestial meridian of an observer in Lat. 50° 00′ N., Long. 30° 00′ W. at 0736 G.M.T. on 22nd September.

G.H.A. γ at 07 hr. G.M.T. =	105° 38′·2	
Increment for 36 m. =	09° 01′·5	
G.H.A. γ at 0736 hr. G.M.T. =	114° 39′·7	
Longitude =	30° 00′·0 W.	
L.H.A. =	84° 39′·7	
	360° 00′·0	
S.H.A. Obs. Upper Mer. =	275° 20′·3	275° 20′·3
Interval 1 hour =	15° 00′·0	15° 00′·0
Limiting S.H.A.s =	290° 20′·3	260° 20′·3

The required stars have S.H.A.s between 260° 20′·3 and 290° 20′·3, and declinations North of 40° 00′·0 S.

From the Selected Star list in the *Nautical Almanac*, we find that, at 0736 hr. G.M.T.:

Betelgeuse (Mag. var.) is East of the meridian and will cross to the South of the observer.
Alnilam (Mag. 1·8) is West of the meridian and has crossed to the South of the observer.
Elnath (Mag. 1·8) is West of the meridian and has crossed to the South of the observer.
Bellatrix (Mag. 1·7) is West of the meridian and has crossed to the South of the observer.
Capella (Mag. 0·2) is West of the meridian and has crossed to the North of the observer.
Rigel (Mag. 0·3) is West of the meridian and has crossed to the North of the observer.

Example 36·2—What visible navigational stars are within 1½ hours of the lower celestial meridian of an observer in Lat. 50° 00′·0 S., Long. 34° 15′·0 W. at 2040 hr. G.M.T. on 30th December.

G.H.A. γ at 20 hr. G.M.T. =	38° 45′·0	
Increment for 40 m. =	10° 01′·6	
G.H.A. γ at 2040 hr. G.M.T. =	48° 46′·6	
Longitude =	34° 15′·0 W.	
L.H.A. =	14° 31′·6	
	360° 00′·0	
S.H.A. Observer's U. Mer. =	345° 28′·4	
	180° 00′·0	180° 00′·0
S.H.A. Observer's L. Mer. =	165° 28′·4	165° 28′·4
Interval 1½ hours =	+22° 30′·0	−22° 30′·0
Limiting S.H.A.s =	187° 54′·4	142° 58′·4

The required stars have S.H.A. between 187° 58′·4 and 142° 58′·4, and declinations South of 40° 00′·0 S.

From the Selected Star List, we find that at 2040 hr. G.M.T.:

Hadar (Mag. 0·9) is West of the Observer's lower meridian and will cross South of the observer.

Gacrux (Mag. 1·6) is East of the Observer's lower meridian and has crossed South of the observer.

Acrux (Mag. 1·1) is East of the Observer's lower meridian and has crossed South of the observer.

2. Position Line from Ex-Meridian Observation

The term Ex-Meridian applies to an altitude observation of a body which is relatively near to the observer's celestial meridian. Such an observation enables an observer to determine his Latitude provided that he knows his approximate Longitude. In practice, because an observer does not know his Longitude when making an ex-meridian observation, the Latitude he finds from his sight is not necessarily his actual Latitude. It is the Latitude of a point on an astronomical position line, the Longitude of the point being that used in the reduction of the ex-meridian sight.

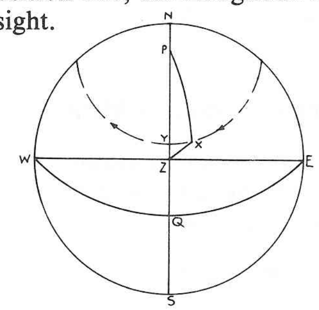


Fig. 36·3

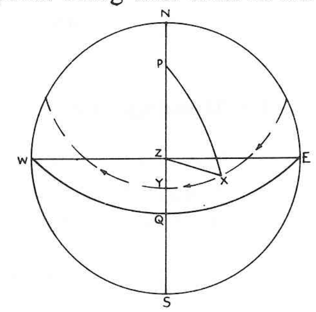


Fig. 36·4

Figs. 36·3 and 36·4 illustrate the celestial sphere, and depict typical *PZX*-triangles projected onto the plane of the celestial horizon of an observer whose zenith is projected at *Z*. Assume that the declination of the body *X* remains constant during the time it takes to move from its present position *X* to *Y* on the observer's celestial meridian. The arc *ZY* is the Meridian Zenith Distance (M.Z.D.) of the body; and, because *PY* and *PX* are equal, we have:

$$\text{M.Z.D.} = (PX \sim PZ)$$

The Spherical Haversine Formula applied to the *PZX*-triangle contains the term $(PX \sim PZ)$, thus:

$$\text{hav } P = \frac{\text{hav } ZX - \text{hav } (PX \sim PZ)}{\sin PX \sin PZ}$$

Transposing, we have:

$$\text{hav } (PX \sim PZ) = \text{hav } ZX - \text{hav } P \sin PX \sin PZ$$

Or:

$$\text{hav M.Z.D.} = \text{hav } ZX - \text{hav } P \sin PX \sin PZ$$
$$\text{hav M.Z.D.} = \text{hav } ZX - \text{hav } P \cos \text{Lat.} \cos \text{Dec.}$$

This formula is known as the Ex-Meridian Haversine Formula. It may be used for finding a position line provided that the Hour Angle (P), and the Latitude and declination are within certain limits.

The second term is the right-hand side of the ex-meridian formula is a small quantity if the angle P is a small angle. This term contains the factor cos. lat. Now the observer's Latitude is not known: the problem in hand being one in which a Latitude is to be computed. Provided that the Latitude used in the computation approximates to the actual but unknown Latitude of the vessel, there will only be a small error introduced into the term hav P cos. Lat. cos dec. It follows, therefore, that the M.Z.D. may be calculated; and, by applying to the M.Z.D. the declination of the observed body, a latitude may be found. The required position line passes through a point having this computed Latitude and a Longitude equal to that used for finding the angle P which figured in the computation. The direction of the position line is at right angles to the bearing of the observed body at the time of the observation.

The following practical rules shoud be observed when using the ex-meridian formula:

1. The numerical difference between the Latitude of the observer and the declination of the observed object should not exceed 4°.
2. The Hour Angle (Angle P) in minutes of time should not exceed the zenith distance of the observed body in degrees.

3. Ex-Meridian Observation Using the Intercept Method

We have shown in Chapter 34 that the zenith distance of an observed celestial body at a chosen position near to the actual but unknown position of the observer, may be computed by means of the Haversine Formula as follows:

$$\text{hav } ZX = \text{hav } P \sin PZ \sin PX + \text{hav } (PX \sim PZ)$$

or:

$$\text{hav M.Z.D.} = \text{hav H.A. cos lat. cos dec.} + \text{hav (Lat.} \pm \text{dec.)}$$

The Calculated Zenith Distance, when compared with the Observed Zenith Distance, yields an intercept. The observer's position lies on a position line which is drawn through the end of the intercept in a direction at right angles to the bearing of the body at the time of the observation.

The Azimuth is usually found by means of Azimuth Tables. When the azimuth is a small angle—and this is generally the case in ex-meridian observations—it may be ascertained by applying the Spherical Sine Formula to the PZX-triangle and modifying it as follows:

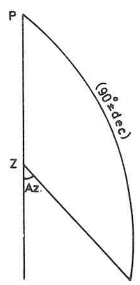


Fig. 36·5

Referring to fig. 36·5:

$$\frac{\sin Z}{\sin PX} = \frac{\sin P}{\sin ZX}$$

$$\sin Z = \sin P \sin (90° \pm \text{dec}) \text{ cosec } ZX$$

i.e.

$$\sin Z = \sin \text{H.A. cos dec. sec alt.}$$

This formula is general regardless of the value of Z, but when a celestial body is near meridian passage it bears almost due North or due South, and the angle Z is a small angle.

Therefore:

$$\sin Z = Z \text{ in radians}$$

Therefore:

$$Z^c = \text{H.A.}^c \text{ cos dec. sec alt.}$$

And

$$Z^o = \text{H.A.}^o \text{ cos dec. sec alt.}$$

Or:

$$Z^o = \frac{\text{H.A. in mins. of time}}{4} \text{ cos. dec. sec alt.}$$

In the case of the Sun, the cosine of whose declination is always nearly unity, its approximate azimuth may be found from the formula:

$$\text{Sun's Azimuth when near Meridian} = \frac{\text{H.A. in mins.}}{4} \text{ sec alt.}$$

The Traverse Table may be used to find the Sun's azimuth in these circumstances. Enter with (H.A. in mins./4) in the Distance column on the page corresponding to Altitude as a course angle. The azimuth is then lifted from the D. Lat. column.

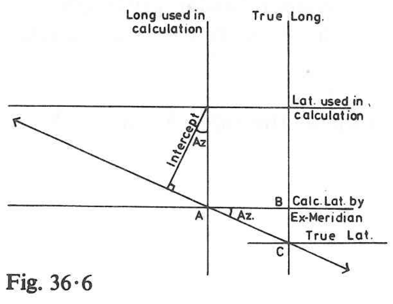


Fig. 36·6

Fig. 36·6 illustrates the relationship between the ex-meridian method and the intercept method of obtaining a position line.

The D. Lat. between the Latitude used in the ex-meridian formula and that used in the intercept formula is given by:

$$\text{D. Lat.} = \text{Intecept sec Azimuth.}$$

If the Longitude used in the ex-meridian formula is not the observer's actual Longitude at the time of the observation, the observer's actual latitude differs from the Latitude computed using the ex-meridian formula by an amount which depends upon the error in Longitude and the azimuth of the observed body. Because the position line lies nearly East–West, any such error is small.

Referring to fig. 36·6: if the error in Longitude corresponding to the error in Departure AB is e, then:

$$\begin{aligned} \text{Error in Latitude} &= BC \\ &= AB \text{ tan Azimuth} \\ &= e \text{ cos Lat. tan Azimuth} \end{aligned}$$

4. Ex-Meridian Observation Using Napier's Rules

The Ex-Meridian Haversine Formula is valid only for observations of celestial bodies near an observer's UPPER celestial meridian. When a celestial body is near LOWER meridian passage its M.Z.D. is the SUM of PX and PZ. Now the quantity ($PX + PZ$) does not appear in the Haversine Formula, so that the Ex-Meridian Haversine Formula cannot be used for solving ex-meridian problems involving bodies near the observer's lower celestial meridian.

The intercept method, in contrast, is valid for all ex-meridian sights. Another method which holds good for ex-meridian observations of bodies near lower as well as upper meridian passage, requires the division of the *PZX*-triangle into two right-angled spherical triangles and solving both using Napier's Rules.

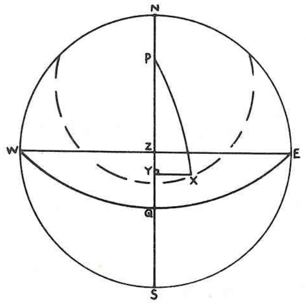


Fig. 36·7

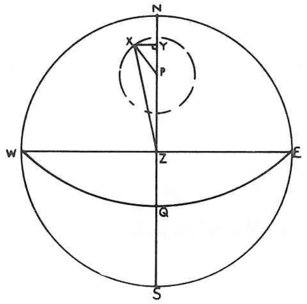


Fig. 36·8

Figs. 36·7 and 36·8 illustrate typical *PZX*-triangles. In fig. 36·7 the body *X* is near to the observer's upper celestial meridian. In fig. 36·8 the body *X* is near to the observer's lower meridian.

In solving the ex-meridian problem by Napier's Rules, a perpendicular great circle is dropped from the body onto the observer's celestial meridian to form two right-angled spherical triangles *PYX* and *ZYX*.

In triangle *PYX*: using side *PX* and angle *P* (or its supplement) the sides *XY* and *PY* are solved.

In triangle *ZYX*: using sides *XY* and *ZX* the side *ZY* is solved.

In triangle *PYX*:

$$\sin XY = \sin P \cos \text{dec.}$$
$$\tan PY = \cos P \cot \text{dec.}$$

In triangle *ZYX*:

$$\cos ZY = \cos ZX \sec XY$$

The Latitude, which is the complement of *PZ*, may then be found as follows:

$$\text{Lat.} = 90° - (PY - ZY)$$

Or:

$$\text{Lat.} = 90° - (ZY - PY)$$

5. Reduction to the Meridian

When a celestial body's Hour Angle is small, and Latitude and declination are within certain limits, the M.Z.D. may be found by applying a small correction to the ex-meridian zenith distance. This small correction is known as the Reduction to the Meridian. In practice, ex-meridian problems are invariably solved by means of Ex-Meridian Tables. There are numerous such tables, many of which give the reduction, against Latitude, declination and hour angle as arguments.

The ex-meridian tables given in Norie's and Burton's Nautical Tables appear to be the most popular in use amongst Merchant Naval officers. These tables are explained with reference to fig. 36·9.

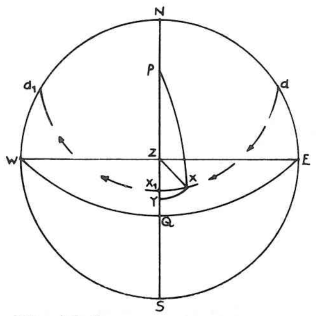


Fig. 36·9

Fig. 36·9 illustrates a typical *PZX*-triangle projected onto the plane of the celestial horizon of an observer whose zenith is *Z*. dd_1 represents the diurnal circle of a body which is at *X* when observed in order to find a position line using the ex-meridian method. The body is on the meridian at position X_1. Arc *ZY* is equal to arc *ZX*, so that the correction to apply to the ex-meridian zenith distance (*ZX*) to obtain the meridian zenith distance (ZX_1) is equal to the arc X_1Y. This correction is the reduction to the meridian denoted by *r*.

$$\begin{aligned} ZX &= ZX_1 + X_1Y \\ &= (PX_1 - PZ) + r \\ &= (\text{Lat.} - \text{dec.}) + r \\ &= (L - D) + r \end{aligned}$$

Applying the Spherical Cosine Formula to the *PZX*-triangle, we have:

$$\begin{aligned} \cos ZX &= \cos PZ \cos PX + \sin PZ \sin PX \cos P \\ \cos [(L - D) + r] &= \sin L \sin D + \cos L \cos D \cos P \\ &= \sin L \sin D + \cos L \cos D\,(1 - 2 \sin^2 P/2) \\ &= (\sin L \sin D + \cos L \cos D) - 2 \,.\, \cos L \,.\, \cos D \,.\, \sin^2 P/2 \\ &= \cos (L - D) - 2 \cos L \cos D \sin^2 P/2 \,.\,.\,.\,.\,.\,.\,.\,.\,.\,. \quad \text{(I)} \end{aligned}$$

Now:

$$\begin{aligned} \cos [(L - D) + r] &= \cos (L - D) \cos r - \sin (L - D) \sin r \\ &= \cos (L - D)\,[1 - 2 \sin^2 r/2] - \sin (L - D) \sin r \end{aligned}$$

If *r* is small,

then: $\sin r/2 = r/2$ radians

and: $\sin r = r$ radians

so that:

$$\cos [(L - D + r] = \cos (L - D)\,(1 - r^2/2) - \sin (L - D)\, r \,.\,.\,.\,.\,.\,.\,.\,.\,.\,. \quad \text{(II)}$$

Equating (I) and (II), we have:

$$\cos (L - D)\,(1 - r^2/2) - \sin (L - D)\, r = \cos (L - d) - 2 \cos L \cos D \sin^2 P/2$$

Thus:

$$\begin{aligned} r \sin (L - D) &= \cos (L - D)\,(1 - r^2/2) - \cos (L - D) + 2 \cos L \cos D \sin^2 P/2 \\ &= \cos (L - D)\,(1 - r^2/2 - 1) + 2 \cos L \cos D \sin^2 P/2 \\ &= -\cos (L - D)\, r^2/2 + 2 \cos L \cos D \text{ hav } P \end{aligned}$$

And,

$$r = \frac{2 \cos L \cos D \text{ hav } P}{\sin (L - D)} - \cot (L - D)\, r^2/2$$

The reduction to the meridian is, therefore, a combination of two parts. The second part, namely: $- \cot (L - D)\, r^2/2$, is a very small quantity if *P* is small. In this circumstance it may be ignored without introducing material error.

As a first approximation we may consider the reduction to the meridian as being:

$$r \text{ radians} = \frac{2 \cos L \cos D \text{ hav } P}{\sin (L - D)}$$

If we now express r in seconds of arc and consider P to be one minute of time:

$$r \text{ sec of arc} = \frac{2 \cos L \cos D \text{ hav } 1^m}{\sin (L - D)} \times 60 \times 3438$$

That is:

$$r'' = 1{\cdot}9635 \frac{\cos L \cos D}{\sin (L - D)}$$

This quantity is the change in a body's altitude in seconds of arc during one minute of time to or from the instant of meridian passage. The quantity is referred to as A in Norie's Ex-Meridian Tables, and as F in Burton's. So that:

$$A \text{ (or } F) = 1{\cdot}9635 \,.\, \frac{\cos L \cos D}{\sin (L - D)}$$

Values of A are given for all values of Latitude and declination in Ex-Meridian Table 1.

The motion in altitude during a short interval before or after the time of meridian passage is considered to be one of uniform acceleration. The rate of change of altitude at the instant of meridian passage is zero, so that if the change in altitude in one minute from or to the instant of meridian passage is A'', the rate of change of altitude one minute from the time of meridian passage must be $2{\cdot}A''$ per minute, and the average rate is A'' per minute. From the relationship in Dynamics, $s = \frac{1}{2}\, ft.^2$, we have:

$$\text{Change in altitude in } t \text{ mins.} = At^2$$

Ex-Meridian table 2 provides the result of multiplying A by t^2. The table is entered with A and the Hour Angle of the observed body. This is the required First Correction, which must be applied to the ex-meridian zenith distance to obtain the M.Z.D.

If t is large, then it becomes necessary to apply the second part of the reduction, namely: $- \cot (L - D)\, r^2/2$. This is referred to as the Second Correction. It is tabulated in Ex-Meridian Table 3 against Altitude and First Correction.

To obtain the position line by ex-meridian observation, the azimuth of the body for the time of observation must be found. *A B C* Tables or Davis' or Burdwood's Tables are used for finding the azimuth. The position line runs at right angles to the azimuth of the observed body.

6. Concluding Remarks on the Ex-Meridian Problem

The above discussion on the ex-meridian problem is largely of academic interest, although the problem is still considered by some navigators as being worth preserving for

practical use. Moreover, it still figures occasionally in examinations designed to test nautical astronomical knowledge and principles. But it must be admitted that the method is quite redundant: the Intercept Method of sight reduction is universal in its application and this method is preferable to all other methods of sight reduction.

Exercises on Chapter 36

1. Explain carefully how you would find the L.M.T. at which a given star is within a given interval of time of its upper meridian passage.
2. What navigational stars are above the horizon and within 1 hour East of the upper celestial meridian of an observer in Lat. 30° 00′·0 N., Long. 30° 00′·0 W. at 05 h. 36 m. on 30th December?
3. What navigational stars will be above the horizon of an observer in Lat. 40° 00′·0 S., Long. 60° 00′·0 E. and within 45 minutes of upper meridian passage and to the West of the observer's celestial meridian at 20 h. 43 m. on 1st January?
4. What navigational stars are above the horizon and within 1 hour of the upper celestial meridian of an observer in Lat. 35° 00′·0 N., Long. 45° 00′·0 W. at 05 h. 36 m. on 22nd September?
5. What navigational stars are within 1 hour of lower meridian passage and above the horizon of an observer in Lat. 70° 00′·0 S., Long. 60° 00′·0 W. at 21 h. 36 m. L.M.T. on 31st December?
6. What navigational stars are within half an hour of the lower meridian and are above the horizon of an observer in Lat. 60° 00′·0 N., Long. 60° 00′·0 W. at 05 h. 37 m. on 23rd September?
7. Describe how the Ex-Meridian Haversine Formula is derived from the Haversine Formula for finding P in the PZX-triangle.
8. Explain how an ex-meridian may be solved by using Napier's Rules.
9. Compare the Intercept Method with the Ex-Meridian Method for finding an astronomical position line.
10. Explain the First and Second Corrections given in the Ex-Meridian tables contained in Norie's and Burton's Tables.

CHAPTER 37

THE POLE STAR

1. Latitude from Observation of Polaris

In Chapter 28 it is demonstrated that:

Latitude of Observer = Altitude of Celestial Pole

Were a star located at the celestial pole its true altitude would equal the observer's Latitude. No star of declination 90° 00′·0 exists, but there is a bright star which is located very near to the North celestial pole. This star, of magnitude 2·2, is *α Ursa Minoris* or Polaris.

The declination of Polaris is about 89° and its Sidereal Hour Angle is about 330°. These values change comparatively rapidly on account of the precession of the equinoxes. For 1975, the mean values of the declination and S.H.A. of Polaris were, respectively, 89° 09′·2 N. and 327° 30′·0.

The diurnal circle of Polaris is a very small circle, centred at the North celestial pole, having a radius of about 1°, this being the approximate value of the Polar Distance of Polaris. It follows that the measure of the true altitude of Polaris is always within about 1° of that of the observer's Latitude.

By applying a correction to the True Altitude of Polaris, an observer readily may find his Latitude. This method of finding Latitude is one of the earliest astronomical methods used by seamen; and, as far back as the time of the Great Discoveries of the fifteenth century, the Portuguese and Spanish navigators were provided with information to enable them to find Latitude at sea from an observation of the Pole Star.

The Pole Star Tables given in the *Nautical Almanac* enable a navigator to find Latitude and azimuth of Polaris provided that the Latitude does not exceed about 65° N. Between the equator and the parallel of about 8° N., Polaris is too near the horizon to make it a suitable body for altitude observations.

In fig. 37·1, which is a projection of the celestial sphere onto the plane of the horizon of an observer whose zenith is projected at *Z*:

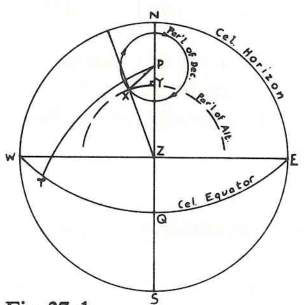


Fig. 37·1

N., E., S. and W., are the Cardinal Points of the Horizon.

P is the North Celestial Pole

X is polaris

Y is a point on the Observer's Celestial Meridian having the same altitude as that of Polaris

WQE is the celestial equator

γ is the First Point of Aries

YPX is the L.H.A. of Polaris.

Latitude of observer = Altitude of Celestial Pole
= *NP*
= NY − *PY*
= True Altitude of polaris − Correction

When the L.H.A. of Polaris is more than 18 h. 00 m. or less than 06 h. 00 m., the correction is to be subtracted from the True Altitude of Polaris to give the Latitude. This is the case illustrated in fig. 37·1. On the other hand, when the L.H.A. of Polaris is more than 06 h. 00 m. but less than 12 h. 00 m. the correction is to be added to the True Altitude of Polaris to give the Latitude.

Because arc *PY* is small, about 1°, the triangle *PXY* may be assumed to be a plane triangle right-angled at *Y*. On this assumption the correction to be applied to the True Altitude of Polaris (arc *NY*) to find the Latitude (arc *NP*) is given by:

$PY = PX \cos YPX$

That is: Correction = Polar Distance of polaris . cos L.H.A. of Polaris.

To allow for the fact that the triangle *PXY* is not a plane, the actual correction to be applied to the True Altitude is given by the formula:

$$\text{Correction} = p \,.\, \cos h + p/2 \,.\, \sin p \,.\, \sin^2 h \,.\, \tan l$$

where: p = Polar Distance of Polaris
$2h$ = L.H.A. of Polaris
and l = Latitude of Observer.

When the L.H.A. of Polaris is 000°, that is to say, when Polaris is at upper meridian passage, the L.H.A. γ is about 29°, which is (360° 00′ − S.H.A. of Polaris). In this circumstance the correction to apply to the True Altitude of Polaris to obtain the Latitude is equal to the Polar Distance of Polaris, and the correction is to be subtracted.

When the L.H.A. of Polaris is 180° 00′; that is to say, when Polaris is at lower meridian passage, the L.H.A. of γ is about 209°. In this circumstance the correction is equal to the Polar Distance of Polaris which is to be added to the True Altitude of Polaris to find the Latitude of the observer.

2. The Pole Star Altitude Tables

The Pole Star Tables for Latitude are in three parts. The quantities extracted from, the three parts are denoted by a_0, a_1 and a_2, for the first, second and third parts, respectively.

The quantity a_0 is tabulated against L.H.A. γ. This is used as an argument partly to obviate the necessity of finding the L.H.A. of Polaris, which latter quantity determines the value of a_0. The correction is computed from the formula given above, using a Latitude of 50° and mean values of declination and S.H.A. of Polaris for the year for which the tables apply. The computed value is adjusted by the addition of a constant so that it is always a positive quantity regardless of the L.H.A. of Polaris.

The quantity a_1, is tabulated against L.H.A. γ and Latitude. The excess (positive or negative) of the value of the second term of the formula, viz. $p/2.\ \sin p\ \sin^2 h\ \tan l$ over its mean value for Latitude 50°, is computed. This excess is increased by a constant to make a_1 always positive.

The quantity a_2 is tabulated against L.H.A. γ and the date. The correction to the first term of the formula, viz. $-\ p \cos h$, for the actual celestial position of Polaris from the mean position used in computing a_0 and a_1, is computed and increased by a constant to give a_2 which is always positive.

The sum of the three constants is exactly 1° 00′·0, so that the Latitude from an observation of Polaris is given by:

$$\text{Latitude} = \text{True Altitude of Polaris} - 1^\circ\ 00' + a_0 + a_1 + a_2$$

Example 37·1—13th June in approximate Long. 30° 00′·0 W., the observed altitude of Polaris was 46° 10′·0. The chronometer time of the observation was 07 h. 35 m. 40 s. Chronometer error 2 m. 25 s. slow on G.M.T. Height of eye 12 m. Find the observer's Latitude.

Chron. Time =	07 h. 35 m. 40 s.
Error =	+ 2 m. 25 s.
G.M.T. = (13)	07 h. 38 m. 05 s.
G.H.A. γ at 07 h. =	6° 05′·2
Increment =	9° 32′·8
G.H.A. γ =	15° 38′·0
Longitude W. =	30° 00′·0
L.H.A. γ =	345° 38′·0
Obs. Alt. =	46° 10′·0
Dip =	− 6′·1
Apparent Altitude =	46° 03′·9
Total corr. =	− 0′·9
True Altitude =	46° 03′·0
	− 1° 00′·0
	45° 03′·0
a_0 =	00° 18′·4
a_1 =	+ 0′·6
a_2 =	+ 0′·2
Latitude =	45° 22′·2 N.

Answer—Latitude Observer = 45° 22′·2 N.

When observing Polaris, the Latitude obtained from the Pole Star Tables is the Latitude of the observer only if the Longitude used in the finding L.H.A. γ is the observer's actual Longitude. In general, the exact Longitude of the observer at the time of an observation is not known, so that the result of a Pole Star Altitude observation is not a Latitude but a Position Line, the direction of which is at right angles to the bearing of Polaris at the time of the observation. The position line passes through a point the Latitude of which is that found from the tables, and the Longitude of which is that used in finding L.H.A. γ.

The azimuth of Polaris may readily be found from the Pole Star Azimuth Table.

3. The Pole Star Azimuth Table

The Pole Star Azimuth Table is entered with L.H.A. γ and Latitude as arguments. Azimuths are given to the nearest 0°·1, this being more than sufficiently accurate for all practical purposes.

Fig. 37·2 serves to illustrate the method used for computing Pole Star Azimuths.

In fig. 37·2:

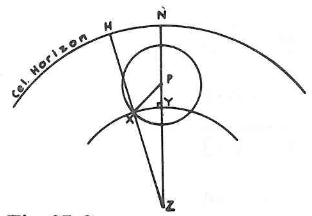


Fig. 37·2

Azimuth of Polaris = PZX
= arc NH

By the Parallel Sailing Formula:

$$NH = Xy \sec HX$$

But $XY = PX \sin P$

Therefore: Azimuth = $PX \sin P \sec HX$

Or: Azimuth = Polar Dist. sin P sec Alt.

Now Polar Distance = 1° (nearly)

and Altitude = Latitude (nearly)

Therefore: Azimuth = 1° sin L.H.A. Polaris sec Lat.

This formula indicates that the azimuth is zero when the L.H.A. of Polaris is 000° or 180°, and that it is maximum when the L.H.A. of Polaris is 090° or 270°. In other words the azimuth of Polaris is zero when the L.H.A. of γ is about 29° or 209°, and it is maximum when the L.H.A. of γ is 109° or 299°.

The Pole Star Azimuth Table provides an easy means for checking the compass error, especially in low Latitudes, in which circumstances Polaris has a small altitude and it is ideally placed, therefore, for azimuth observations.

Example 37·2—13th June in Lat. 40° 00′·0 N., Long. 150° 00′·0 W., at 11 h. 42 m. G.M.T., the Pole Star bore 004¼ (C). The variation was 6½° W. Find the deviation for the heading of the vessel at the time of the observation.

G.H.A., γ at 11 h. =	66° 15′·1
Increment =	10° 31′·7
G.H.A. γ =	76° 46′·8
Longitude =	150° 00′·0
L.H.A. γ =	286° 46′·8
True Azimuth =	001¼°
Obs. Azimuth =	004¼°
Error =	3° W.
Variation =	6½° W.
Deviation =	3½ E.

Answer—Deviation = 3½° E.

Example 37·3—22nd September in D.R. position Lat. 35° 20′·0 N., Long. 30° 00′·0 W., the sextant altitude of Polaris was 35° 20′·5. Index error 1′·8 off the arc. Height of eye 12 metres. Chronometer time 18 h. 38 m. 46 s. Chronometer error 2 m. 23 s. slow on G.M.T. Find the position line corresponding to the observation.

L.M.T. =	18 h. 42 m.	Sext. alt. =	35° 20′·5
Long. =	2 h. 00 m. W.	Index error =	+ 1′·8
G.M.T. =	20 h. 42 m.	Obs. Alt. =	35° 22′·3
G.H.A. γ 20 h. =	301° 10′·3	Dip =	− 6′·1
Increment =	10° 31′·7	Apparent Alt. =	35° 16′·2
G.H.A. γ =	311° 42′·0	Total corr. =	− 1′·4
Longitude =	30° 00′·0 W.	True Alt. =	35° 14′·8
L.H.A. γ =	281° 42′·0		− 1° 00′·0
Azimuth =	001·1°		34° 14′·8
		a_0 =	+ 1° 15′·7
		a_1 =	+ 0′·4
		a_2 =	+ 0′·9
		Latitude =	35° 31′·8 N.

Answer—Position Line 091°–271° through Lat. 35° 31′·8, Long 30° 00′·0 W.

Exercises on Chapter 37

1. Prove that the Latitude of an observer is equal to the altitude of the celestial pole.

2. Explain why the true altitude of Polaris is always within about 1° of the Latitude of an observer.
3. Describe carefully the construction of the Pole Star Tables.
4. 12th June in approx. Long. 29° 00′·0 W. the observed altitude of Polaris was 46° 14′·0. The chronometer time of the observation was 07 h. 40 m. 20 s. Chronometer error 3 m. 22 s. slow on G.M.T. Height of eye 9 m. Find the observer's Latitude.
5. 15th June in approx. Long. 175° 00′·0 W. the observed altitude of Polaris was 43° 25′·0. The chronometer time was 19 h. 38 m. 20 s. and the chronometer was 4 m. 13 s. slow on G.M.T. Height of eye 15·4 m. Find the observer's Latitude.
6. 23rd September in approx. Long. 33° 00′·W. the observed altitude of Polaris was 39° 25′·0. The chronometer time was 05 h. 47 m. 12 s. and the chronometer was 4 m. 04 s. fast on G.M.T. Height of eye was 12 m. Find the position line on which the observer was located at the time of the observation.
7. 24th September in approx. Long. 75° 00′·W. the observed altitude of Polaris was 28° 10′·5. The chronometer time was 11 h. 42 m. 20 s. and the chronometer error 1 m. 02 s. slow on G.M.T. Height of eye 6 m. Ascertain the position line on which the observer was located at the time of the observation.
8. 12th June in approx. position Lat. 50° 00′·0 N., Long. 30° 00′·0 W., the compass bearing of Polaris was 008°·5 when the G.M.T. was 07 h. 44 m. 00 s. Find the compass error for the heading of the vessel at the time of observation.
9. 23rd September in approx. position Lat. 42° 00′·0 N., Long. 150° 00′·0 E., the compass bearing of Polaris was 014°·0. when the chronometer time was 08 h. 43 m. 00 s. Chronometer error 1 m. 05 s. fast on G.M.T. Find the deviation for the present heading of the vessel given that the variation is 10°·0 W.
10. 14th June in Latitude 35° 00′·0 N., Long. 65° 00′·0 W., the compass bearing of Polaris was 015°·0 when the chronometer time was 08 h. 14 m. 20 s. Chronometer error 29 m. 04 s. slow on G.M.T. Find the deviation for the present heading of the vessel given that the variation is 12°·0 W.

CHAPTER 38

NAUTICAL ASTRONOMICAL TABLES

1. Introduction

The tables we shall describe in this chapter are some of these designed to facilitate the solution of spherical triangles—principally the *PZX*-triangles in sight reduction.

2. The Principles of *A B C* Tables

The *A B C* Tables found in Burton's, Norie's and other Nautical Table collections, may be used to give a direct solution to any one of any four adjacent parts of a spherical triangle, provided that the other three of the adjacent parts are known. The principle of the tables is based on the Four Parts Formula of spherical trigonometry. This formula is discussed in Chapter 6.

The navigational uses of the *A B C* Tables include:

1. Finding the azimuth of a celestial body.
2. Finding great circle courses.

In addition to these uses the tables are extensively used for finding the error in Longitude consequent upon an error in Latitude used in the Longitude method of sight reduction.

The arguments used in the *A B C* Tables are Hour Angle, Latitude and Declination. These, together with Azimuth, are involved in the principle type of problem which is solved by means of the tables.

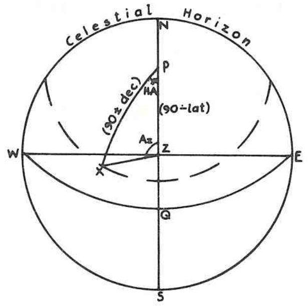


Fig. 38·1

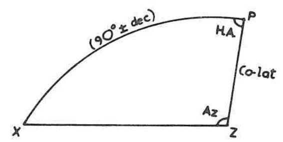


Fig. 38·2

Figs. 38·1 and 38·2 illustrate typical *PZX*-triangles with four adjacent parts marked.

By the Four Parts Formula:

$$\cos P \cos PZ = \sin PZ \cot PX - \sin P \cot Z$$

Dividing throughout by sin *P* sin *PZ*, we have:

$$\frac{\cos P \cos PZ}{\sin P \sin PZ} = \frac{\sin PZ \cot PX}{\sin P \sin PZ} - \frac{\sin P \cot Z}{\sin P \sin PZ}$$

From which:

$$\cot P \cot PZ = \operatorname{cosec} P \cot PX - \cot Z \operatorname{cosec} PZ$$

Or: $$\underbrace{\cot \text{H.A.} \tan \text{Lat.}}_{1} = \underbrace{\operatorname{cosec} \text{H.A.} \cot (90° \pm \text{Dec.})}_{2} - \underbrace{\cot \text{AZ.} \sec \text{Lat.}}_{3}$$

Expression 1 is tabulated for all combinations of H.A. and Latitude, as *A*-Correction.
Expression 2 is tabulated for all combinations of H.A. and Declintion, as *B*-Correction.
Expression 3 is tabulated for all combinations of Latitude and Azimuth as *C*-Correction.

It follows that:

$$A = B - C$$

and

$$C = B - A$$

(i) *When the H.A. is less than* 90° *and the Latitude and Declination have the Same name:*

B is a positive quantity because cot (90° − Dec.) and cosec H.A. are both positive quantities.
A is a positive quantity because cot H.A. and tan Lat. are both positive quantities.

In these circumstances, therefore:

$$C = +B - (+A)$$

That is: $$\underline{C = B - A}$$

(ii) *When the H.A. is less than* 90° *and the Latitude and Declination have the Opposite names:*
B is a negative quantity because cot (90° + Dec.) is negative and cosec H.A. is positive.
A is a positive quantity because cot H.A. and tan Lat. are both positive quantities.

In these circumstances, therefore:

$$C = -B - (+A)$$

Or: $$\underline{C = -B - A}$$

(iii) *When the H.A. is more than* 90° *and the Latitude and Declination have the Same name:*
B is a positive quantity because cot (90° − Dec.) and cosec H.A. are both positive quantities.
A is a negative quantity because cot H.A. is negative and tan Lat. is positive.

In these circumstances, therefore:

$$C = +B - (-A)$$

Or: $$\underline{C = +B + A}$$

(iv) *When the H.A. is more than* 90° *and the Latitude and Declination have Opposite names:*

B is a negative quantity because cosec H.A. is positive and cot (90° + Dec.) is negative. *A* is a negative quantity because cot H.A. and tan Lat. is positive.

In these circumstances, therefore:

$$C = -B - (-A)$$

Or:

$$C = -B + A$$

From (i), (ii), (iii) and (iv) it will be seen that, in general:

$$C = \pm (A + B)$$

When using *A B C* Tables care must be taken to interpolate carefully and also to apply the correct sign to each of the tabulated values of the *A, B* and *C,* Corrections.

3. Uses of the *A B C* Tables

(i) *To find Error in Longitude due to Error in Latitude:*

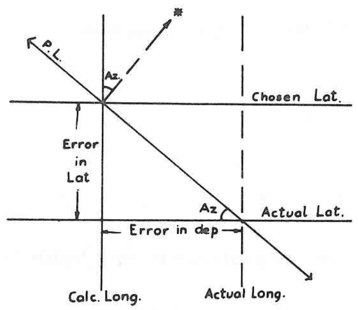


Fig. 38·3

Referring to fig. 38·3: it will be seen that

Error in Departure = Error in Latitude cot Azimuth

But: Departure = D. Long. cos lat.

Therefore:
Error in Long. cos Lat. = Error in Lat. cot Az.
And: Error in long. = Error in Lat. cot Az. sec Lat.

But: cot Az. sec Lat = *C*-correction

Therefore:
Error in Long. = Error in Lat. *C*-correction

It follows that the *C*-correction is numerically equal to the Error in Latitude consequent upon an Error of one minute in Latitude.

Because $(A \pm B) = C$; therefore, $(A \pm B)$ is also equal to the Error in Longitude consequent upon an Error of one minute in Latitude.

In general, Table *C* is entered with $(A \pm B)$ and Latitude, in order to find the Azimuth of the observed celestial body.

Example 38·1—A chosen Latitude of 50° 00′·0 N. was used to reduce a sight using the Longitude Method. The Sun's declination was 20° 00′·0 N., the calculated H.A. was 315° and the calculated Longitude was 30° 00′·0 W. The vessel travelled on a course of 240° for 32 ml., at the end of which the Sun was observed on the meridian and the Latitude by meridian altitude was found to be 49° 35′·0 N. Find the Longitude of the vessel at noon.

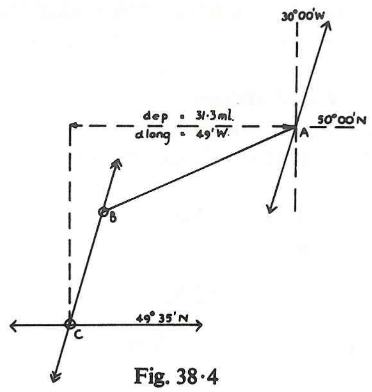


Fig. 38·4

The Longitude of the vessel at noon, found by scale drawing as illustrated in fig. 38·4, is 30° 49′·0 W.

The usual method employed at sea to find the noon Longitude using the *A B C* tables is as follows:

A =	1′·19 S.		
B =	0′·51 N.		
C =	0′·68 S.		
Azimuth =	113°·6		
Lat. (*A*) =	50° 00′·0 N.	Long. =	30° 00′·0 W.
D. Lat. =	16′·0 S.	D. Long. =	43′·0 W.
Lat. (*B*) =	49° 44′·0 N.	Long. =	30° 43′·0 W.
Lat. (*C*) =	49° 35′·0 N.	Error =	06′·0 W.
Error =	9′·0 S.	Long. (*C*) =	30° 49′·0 W.
C-correction =	× 0·68		
Error in Long. =	6′·0 W.		

Answer—Noon Longitude = 30° 49′·0 W.

(ii) *To find Courses in Great Circle Sailing:*

Suppose that it is necessary to find the initial course of the great circle track from *A* to *B* as illustrated in fig. 38·5:

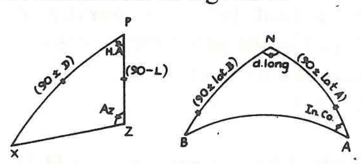


Fig. 38·5

Consider the initial Latitude as *Latitude* in Table *A*
Consider the final Latitude as *Declination* in Table *B*
Consider D. Long. between *A* and *B* as H.A. in Tables *A* and *B*
The *Azimuth*, found in Table *C*, is the required initial course.

Example 38·2—Find the initial course on a great circle path from a position in Lat. 51° 10′ N., Long. 10° 00′ W. to a position in Lat. 52° 00′·0 N., Long. 55° 00′·0 W.

Use 51°·2 as Latitude
Use 52°·0 as Declination
Use 45°·0, that is, D. Long. *AB*, as H.A.

A = 1′·24 S.
B = 1′·81 N.

C = 0′·57 N.

Initial Course = N. 70°·4 W.

Answer—Initial Course = 289°·6.

4. Inspection Tables

The only part of a *PZX*-triangle which can be obtained from direct measurement is the zenith distance. It is for this reason that there are two, and only two, general methods for finding an astronomical position line. These are the Longitude and Intercept methods, respectively.

The computation involved when reducing a sight using a direct method in which a *PZX*-triangle is solved by the common trigonometrical tables, is relatively complex and time-consuming. The chance, therefore, of the computer making clerical errors is considerable. For this and other reasons many attempts have been made to shorten the process of sight reduction, and numerous tables are available for this purpose.

The term Inspection Tables applies to tables from which certain unknown parts of the astronomical triangle corresponding to certain known parts, may be lifted. Apart from perhaps some interpolation, no computations are involved when solving a *PZX*-triangle using inspection tables.

The principal disadvantage of inspection tables used in Nautical Astronomy for solving Hour Angle or Altitude is the great bulk and the consequent expense of the tables.

The Altitude-Azimuth Tables by Burdwood in which azimuths are given against arguments Latitude, Declination and Altitude; and the Time-Azimuth Tables by Davis in which azimuths are given against arguments Latitude, Declination and Hour Angle; are examples of Inspection Tables in which only a relatively coarse degree of accuracy—to the nearest tenth of a degree— is required. These tables, as well as serving for azimuths are also useful for finding approximate star altitudes for setting the sextant prior to making an observation; and also for finding great circle courses in which problems great circle courses are analogous to azimuths. Because of their low degree of accuracy azimuth tables are not bulky. They are much used at sea because of their handiness.

A comprehensive set of altitude and azimuth tables is the United States publication H.O. 214. Each volume of H.O. 214 covers a range of 10 degrees. They are designed primarily for

marine navigation, and they are applicable to observations of all astronomical navigational bodies.

H.O. 214 were regrouped into six volumes, each covering a range of Latitude of 15 degrees, and published by the Hydrographic Department of the British Admiralty in 1951, as H.D. 486.

H.D. 486 is entered with Latitude, Declination and Local Hour Angle of the observed body as arguments, and the respondents are Altitude to the nearest 0′·1 and Azimuth to the nearest 0°·1. Each whole degree of Latitude covers 24 large pages—12 for cases in which Latitude and declination have the same name, and 12 for cases in which the Latitude and declination have different names.

Another very useful inspection table is that known as H.O. 249, published by the United States Hydrographic Office. These tables, similar to H.O. 214, provide pre-computed altitudes and azimuths, correct to the nearest minute and degree of arc respectively, for each of 38 stars. The arguments are L.H.A. and Latitude, each to an integral number of degrees. Although the accuracy of altitude and azimuth in H.O. 249 is not nearly so great as that in H.O. 214, they are valuable tables for star sight reduction.

A magnificent set of Inspection Tables, under the title *Sight Reduction Tables for Marine Navigation* was published by the U.S. Naval Oceanographical Office as Pub. No. 229 in 1970. In the following year the tables, in identical format, were published by the British Ministry of Defence as H.D. Publication N.P. 401. In all respects the new work is a veritable *tour de force* as far as inspection tables for marine use are concerned.

Each of the six volumes of N.P. 401 contains two eight-degree zones of Latitude, and there is an overlap of one degree between consecutive volumes. Within each of the 12 zones of eight degrees of Latitude, the main argument is Local Hour Angle (L.H.A.) between 0° and 360°. To each integral degree of L.H.A. (= P°) in the range 0°–90° there corresponds an opening of two facing pages. The left-hand page covers the case of Latitude and declination of the same name. The right-hand page contains on the upper portion, the tabulations for L.H.A. = P and L.H.A. = (36° − P), and declinations of contrary names; and on the lower portion the tabulations for supplementary L.H.A.s (180° − P), and (180° + P), and declinations of the same name as Latitude. The two portions on the right-hand page are separated in each column, by a horizontal rule which, together with the vertical lines separating the columns, form a stepped configuaration across the page. This separating line is called the "Contrary-Same" or "C-S" line. The horizontal segments of the C-S line indicate the degree of declination in which the horizon (alt. = 0°) occurs. Altitudes on one side of the line are positive, that is to say they are above the horizon; and on the other side they are negative or below the horizon. The advantage of this interesting arrangement is that the reduction of a sight taken within a particular zone of Latitude, in either hemisphere, is determined uniquely by the value of the L.H.A.

The respondents are altitude (denoted by H_c) to 0′·1; azimuth (denoted by Z) to 0°·1; and, in smaller type, the difference (denoted by d) between the tabular altitude for a given declination entry and that for a declination of one degree higher.

Rules are given at each opening for converting tabulated azimuths to true azimuths, and an interpolation table, in four pages occupying the insides of the covers and adjacent pages, facilitates finding the correction to the tabulated altitude for the odd minutes of declination. This table is in two parts: one giving the correction for the tens of minutes in the excess of the actual declination over the entering declination, and the other giving the correction for the units of declination. The interpolation table also provides for second-difference corrections and an indication of when a second-difference correction should be applied is provided by printing the "altitude-difference" d in *italics* followed by a dot (·) for easy recognition.

5. Short Method Tables

The PZX-triangle may be divided into two right-angled spherical triangles by dropping a perpendicular great circle from one of the vertices of the triangle onto the opposite side or side produced. This is the basis of the so-called Short Method Tables.

Short method tables, in respect of the time taken for sight reduction and bulk of tables, lie between inspection tables and the trigonometrical tables used in direct methods.

The principal disadvantage of short method tables is the relative complexity of the rules for using them. Before it is possible to gain full advantage from a short method table, considerable practice is necessary in the use of the table and in the handling of the rules. In other words, complete familiarity with the table is essential for optimum use.

6. Ogura's Table

S. Ogura of the Imperial Japanese Navy first published his ingeniously contrived short method table in 1920. Ogura's method is the basis of many short method tables, and it is primarily for this reason that we shall discuss the method in some detail.

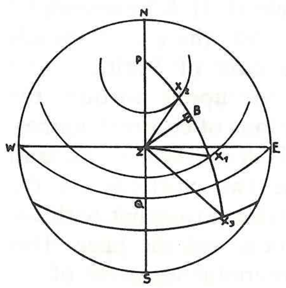


Fig. 38·6

The PZX-triangle is divided by dropping a perpendicular great circle from Z onto the side PX or PX produced. The right-angled spherical triangle PZB is known as the Time Triangle, and the right-angled spherical triangle BZX is known as the Altitude Triangle.

Fig. 38·6 illustrates the three cases, viz.:

Case 1—Latitude (φ) and Declination (d) of the same name, and $\varphi < d$ in which case: $BX_1 = d - K$, where K is the declination of the point B.

Case 2—Latitude and Declination of the same name but $\varphi > d$ in which case: $BX_2 = K - d$.

Case 3—Latitude and Declination having opposite names in which case: $BX_3 = K + d$.

In all cases:

$$BX = K \pm d \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Also, in all cases:

$$BX = (90° \pm d) - PB \quad \ldots\ldots\ldots\ldots (2)$$

Equating equations (1) and (2), we get:

$$K \pm d = 90° \pm d - PB$$
$$K = 90° - PB$$

In the Time Triangle (PZB), by Napier's Rules:

$$\sin \text{co} . P = \tan PB . \tan \text{co} . PZ$$

i.e. $\cos P = \tan PB . \cot PZ$

i.e. $\tan PB = \cos P . \tan PZ$

Inverting: $\cot PB = \sec P . \cot PZ$

$$\tan (90 - PB = \sec P . \cot PZ$$

i.e. $\tan K = \sec P . \tan \varphi \quad \ldots\ldots\ldots\ldots (3)$

In the Time Triangle (PZB), by the Spherical Sine Rule:

$$\sin BZ = \sin P . \sin PZ$$

i.e. $\sin BZ = \sin P . \cos \varphi$

Inverting $\text{cosec}\, BZ = \sin P . \cos \varphi \quad \ldots\ldots\ldots\ldots (4)$

In the Altitude Triangle (BZX), by Napier's Rules:

$$\sin \text{co}\, ZX = \cos BZ . \cos BX$$
$$\cos ZX = \cos BZ . \cos (K+d)$$

Inverting: $\sec ZX = \sec BZ . \sec (K+d) \quad \ldots\ldots\ldots\ldots (5)$

Values of K are pre-computed from equation (3) for integral degrees of Latitude (φ) and P (hour angle).

Values of BZ are computed from equation (4), and values of log sec BZ, denoted by A, are tabulated for integral degrees of Latitude (φ) and P (hour angle).

Ogura's tables are sometimes known as A and K Tables. The procedure for using them is as follows:

1. Choose a Latitude, an integral number of degrees nearest to the estimated Latitude of the ship.
2. Apply to the G.H.A. of the observed body for the time of the observation, a Longitude so as to make P an integral number of degrees.
3. Enter the table with φ and P and extract K.
4. Apply d to K to find ($K \pm d$).
   (+ when φ and d have different names,~ when φ and d have the same name).
5. Enter table with φ and P and extract A.
6. Enter the log secant table with ($K \pm d$). Add the log secant of ($K \pm d$) to A. This gives the log secant of the calculated zenith distance (from equation 5).
7. Compare the C.Z.D. with the O.Z.D. to find the intercept.
8. Use the $A\ B\ C$ table or Azimuth tables to find the azimuth of the observed body and hence the direction of the position line.

The A and K Tables afford a rapid means of finding the C.Z.D.—only three tabular entries being required.

The azimuth may be found by combining angle PZB in the Time Triangle with angle BZX in the Altitude Triangle.

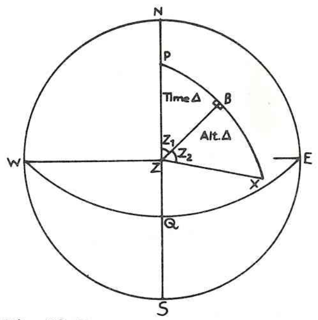


Fig. 38·7

Referring to fig. 38·7
In the Time Triangle:
By Napier's Rules:

$$\cos PZ = \cot PZB \,.\, \cot P$$

i.e. $\cot Z_1 = \cos PZ \,.\, \tan P$

Inverting: $\tan Z_1 = \text{cosec}\ \varphi \cot P$ . . . .(1)

In the Altitude Triangle:
By Napier's Rules:

$$\sin BZ = \cot BZX \,.\, \tan BX$$

i.e. $\cot Z_2 = \sin BZ \,.\, \cot BX$

Inverting: $\tan Z_2 = \text{cosec}\ BZ \,.\, \tan (K + d)$ . . . . .(2)

Z_1 and Z_2 may be computed from equation (1) and (2) respectively, and tabulated against φ and P. The azimuth of the observed body is then found by combining Z_1 and Z_2.

The Altitude and Azimuth table in Norie's Tables are based on Ogura's *A* and *K* tables, but they provide for azimuths as well.

Ogura's method of dividing the *PZX*-triangle into two right-angled spherical triangles has been used in tables by Dreisonstok; Smart and Shearm; Comrie (in Hughes' Sea and Air Navigation Tables); Gingrich; and Myerscough and Hamilton.

Exercises on Chapter 38

1. Show that, in solving an astronomical position line, an error in Latitude produces an error in Longitude proportional to the secant of the Latitude and the cotangent of the azimuth of the observed body.
2. Using the Four Parts Formula of spherical trigonometry, prove that:
   $\cot AZ \,.\, \sec \text{Lat.} = \text{cosec H.A.} \cot (90 \pm \text{dec.}) - \cot \text{H.A.} \tan \text{Lat.}$
3 What factors determine the signs (or names) of the *A, B* and *C* corrections? Show that, in all cases:
   $$C = \pm (A + B)$$
4. Explain how the *A B C* Tables are used to find:
   (i) the azimuth of a heavenly body
   (ii) a great circle course
   (iii) the Longitude correction factor.
5. Distinguish between Inspection- and Short Method-Navigation Tables.
6. Explain clearly the construction of Ogura's Short Method Table.

CHAPTER 39

RISING AND SETTING PHENOMENA

1. Introduction

When a celestial body is on an observer's celestial horizon its Local Hour Angle may readily be computed by Napier's Rules because the zenith distance of the body is 90° 00′.

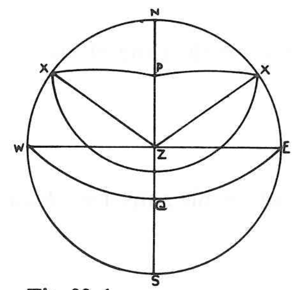


Fig. 39·1

Fig. 39·1 illustrates the visible celestial hemisphere of an observer whose zenith is projected at *Z*. *P* is the projection of the North celestial pole and *WQE* that of the celestial equator. *X* is a celestial body at rising or setting. Applying Napier's Rules to the spherical triangle *PZX* we have:

$$\sec P = -\cot \text{dec.} \cot \text{Lat.}$$

Example 39·1—Compute the L.H.A. of the planet Venus when it sets at a date when its declination is 20° 00′ N. given the Latitude of the observer as 42° 00′ N.

sec *P* =	− cot dec. cot Lat.	
dec. =	20° 00′ log cot	= 0·43893 (+)
Lat. =	42° 00′ log cot	= 0·04556 (+)
P =	log sec	= 0·48449 (−)

Therefore: *P* = 180° − 70° 52′
= 109° 08′

Answer—L.H.A. = 109° 08′

2. Sunrise and Sunset

If the L.H.A. of the Sun for the time of sunrise and sunset is known, the L.A.T. of sunrise or sunset or sunrise may be found from the relationship between Apparent Solar Time and Sun's Hour Angle, thus:

L.A.T. of Sunset = L.H.A. of the Sun at sunset + 12 hr.
L.A.T. of Sunrise = L.H.A. of the Sun at sunrise - 12 hr.

It should be noted that if the Latitude of the observer and the declination of the Sun have the same Name, the Sun rises before 0600 hr. L.A.T. and sets after 1800 hr. L.A.T. This is the case illustrated in fig. 39·1. On the other hand, if the Latitude of the observer and the Sun's declination have different names, the Sun rises after 0600 hr. L.A.T. and sets before 1800 hr. L.A.T.

The L.A.T. of sunset or sunrise may be found from Davis's or Burdwood's Azimuth Tables. The times of sunrise and sunset given in these tables are those at which the Sun's centre is on the celestial horizon of an observer. The terms Theoretical-Sunrise and -Sunset are used to denote these events. The terms Visible-Sunrise and -Sunset are the times at which the Sun's upper limb is on an observer's visible horizon.

Corresponding theoretical and visible sunsets or sunrises differ on account of dip, refraction, and semi-diameter. The observed altitude of the Sun's upper limb at the time of visible sunrise or sunset is 00° 00'·0, whereas the true altitude of the Sun's centre at the time of theoretical sunrise or sunset is 00° 00'·0.

It is an easy matter to compute the true altitude of the Sun's centre for the time of visible sunrise or sunset. The following example serves to illustrate this.

Example 39·2—Find the true altitude of the Sun's centre at the time of visible sunset on 13th June, if the observer's height of eye is 11 m.

Obs. Alt. of Sun's upper limb =	00° 00'·0
Dip =	− 05'·8
App. Alt. of Sun's upper limb =	− 00° 05'·8
Refraction =	− 33'·0
True Alt. of Sun's upper limb =	− 00° 38'·8
Semi-diameter =	− 15'·8
True Alt. of Sun's centre =	− 00° 54'·6

Answer—True Alt. = − 00° 54'·6.

In the circumstances applicable to example 39·2 the Sun's centre is nearly a whole degree, or double the Sun's angular diameter, below the observer's celestial horizon at the instant when its upper limb touches the observer's visible horizon. It follows that the Sun's centre is about 1° above the visible horizon when its centre is on the celestial horizon. It is important, when observing the amplitude of the Sun for checking the compass, to ensure that the observation is made at the instant when the Sun's centre is on the celestial horizon. Attention was drawn to this important matter in chapter 34, to which the reader is referred.

The G.M.T.s of sunrise and sunset at the Greenwich meridian for Latitudes at 10°-, 5°-, and 2°-intervals, are given on the right-hand daily pages of the *Nautical Almanac.* These times are approximately equal to the L.M.T.s of sunrise and sunset for all meridians other than the Greenwich meridian. To find the G.M.T. of sunrise or sunset the observer's Longitude must be applied to the time lifted from the *Nautical Almanac.* A table is provided in the *Nautical Almanac* to facilitate interpolating for Latitude.

3. Moonrise and Moonset

Theoretical Moonrise and Moonset occur when the Moon's centre is on the observer's celestial horizon. Visible Moonset and Moonrise occur when the Moon's upper limb is on the observer's visible horizon. Theoretical and visible moorise or moonset differ on account of dip, refraction, semi-diameter and horizontal parallax.

Example 39·3—Find the observed altitude of the Moon's upper limb at the time of theoretical moonrise in Lat. 40° 00' N., Long. 30° 00' W. on 24th September, if the height of the observer's eye is 18·3 m.

From *NauticaL Almanac* Extracts:

L.M.T. of Moonrise at Long. 0° =	16 h. 03 m.
Long. correction =	+ 03 m.
G.M.T. of Moonrise in Long. 30° W. =	16 h. 06 m.
H.P. =	− 55'·1
S.D. =	+ 15'·1
Ref. =	+ 33'·0
Dip. =	+ 7'·5
Total Correction =	+ 0'·5
True Alt. Moon's centre =	00° 00'·0
Total Correction =	+ 0'·5
Observed Alt. Moon's U.L. =	00° 00'·5

Answer—Obs. Alt. = 00° 00'·5.

In the circumstances applicable to example 39·3, the Moon's upper limb just touches the visible horizon at the instant it is on the observer's celestial horiizon. It follows that the correct conditions for observing the Moon' s amplitude are that its upper limb should be in contact with the observer's visible horizon.

The times of Moonrise and Moonset may be found from the tables given on the right-hand pages of the *Nautical Almanac.* The tabulated times are G.M.T.s of the phenomena at the Greenwich meridian. They are given for 10°-, 5°-, and 2°-intervals of Latitude, and tables are provided for interpolating for Latitude and Longitude.

4. Twilight

Owing to atmospheric refraction and the reflection of light from particles in the upper atmosphere, sunlight is received after the time of sunset or before the time of sunrise so long as the Sun is not more than about 18° below the observer's horizon. Sunlight received at a place in these circumstances is called Twilight.

Civil Twilight begins in the morning and ends in the evening when the Sun is 6° below the horizon. The light received from the Sun when it is between 6° and 12° below the horizon is known as Nautical Twilight. During the period of nautical twilight bright stars are visible and the horizon is clear enough for stellar observations. The light received from the Sun when it is beween 12° and 18° below the observer's horizon is known as Astronomical Twilight.

The times (G.M.T. at Greenwich meridian or L.M.T. at other meridians) of the beginning and end of Nautical- and Civil-Twilight are given on the right-hand daily pages of the *Nautical Almanac.*

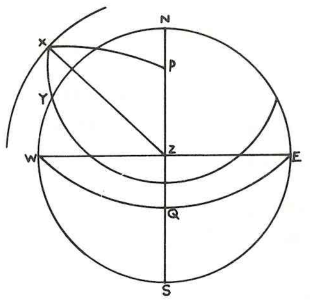


Fig. 39·2

In fig. 39·2 *Y* represents the Sun when it is on the celestial horizon of an observer whose zenith is projected at *Z*, and *X* represents the Sun when it is 18° below the observer's horizon. The time at which the period of astronomical twilight ends may be found by calculating the angle *XPZ* in the spherical triangle *PZX* in which *ZX* is equal to (90° + 18°), or 108°; *PZ* is the co-Latitude of the observer, and *PX* is the polar distance of the Sun.

If, in the calculation, the log haversine of *XPZ* is more than 10·0, no triangle can be formed, in which case twilight lasts all night.

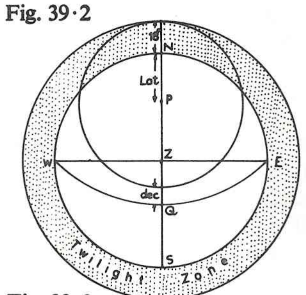


Fig. 39·3

Fig. 39·3 serves to illustrate the conditions in which twilight lasts all night. For twilight to last all night, as illustrated in fig. 39·3, the Sun's diurnal path must lie within the Twilight Zone, this being a spherical belt the limits of which are the observer's horizon and a small circle parallel to the horizon lying 18° below the observer's horizon.

For twilight to last all night (Lat. + 18°) must be less than (90° − dec.).

In the limiting case:

$$(90° - \text{dec.}) = (\text{Lat.} + 18°)$$

That is:

$$\text{Lat.} = (72° - \text{dec.})$$

or:

$$(\text{Lat.} + \text{dec.}) = 72°$$

Therefore, for twilight to last all night, the Latitude of the observer and the Sun's declination must be of the same name and the observer's Latitude must not be less than the sum of 72° and the Sun's declination. In other words (Lat. + dec.) must not be less than 72°.

The Sun's maximum declination is 23° 27′. Therefore the lowest Latitude at which twilight can last all night is (72° 00′ − 23° 27′), which is 48° 33′ N. or S.

In low latitudes, where the diurnal paths of celestial bodies cut the horizon at large angles, the Sun is in the twilight zone for a much shorter duration, than in high Latitudes.

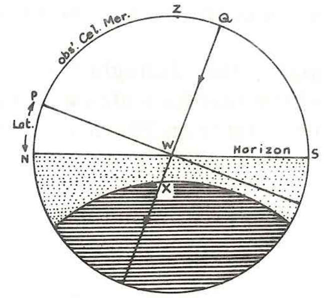


Fig. 39·4

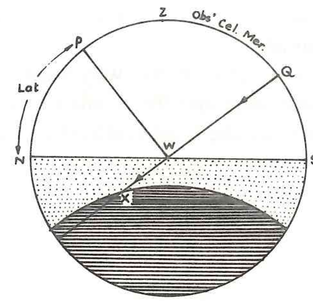


Fig. 39·5

Figs. 39·4 and 39·5 serve to illustrate that twilight is of shorter duration in the tropics than in higher Latitudes. In fig. 39·4 *WX* represents the path of the Sun when its declination is 00° 00′, at a place in latitude θ where θ is equal to arc *NP*. The angle *XPW* is a measure of the interval during which the Sun is in the twilight zone.

In fig. 39·5, the interval during which the Sun is in the twilight zone is angle *XPW*. Clearly this angle is greater than the corresponding angle in fig. 39·4, because the observer's Latitude is greater. Therefore, the Sun passes through the twilight zone more obliquely in the higher than in the lower latitude.

5. The Midnight Sun

If the Sun's diurnal circle does not cross the observer's horizon, the observer will experience daylight for the whole twenty-four hours of the day, and the Sun will be at lower meridian passage above the horizon. The limiting Latitude for the Sun to be visible at midnight on a given day of the year is (90° − dec.). At the limiting Latitude the Sun will graze the horizon at midnight. Thus, on 12th June, when the Sun's declination is 23° N., the Sun will graze the horizon, at midnight, of an observer in Lat. 67° N. In Latitudes higher than this the Sun's altitude at midnight on this day will be (Lat. − 67°). Thus, in Latitude 77° N. on 12th June, the Sun's lower meridian altitude will be 10° bearing North.

Exercises on Chapter 39

1. Define: Theoretical Sunrise; Visible Sunrise.
2. Derive a formula for finding the time of Sunrise in terms of the observer's Latitude and the Sun's declination.
3. In what circumstances will sunrise occur (*a*) before 0600 hr. L.A.T., (*b*) after 0600 hr. L.A.T.?
4. Explain why the Sun's lower limb is about a semi-diameter above the visible horizon when the Sun's centre is on the observer's celestial horizon.
5. Explain why the Moon's upper limb should be just in contact with the visible horizon when her amplitude is observed.
6. Define: Civil Twilight; Nautical Twilight; Astronomical Twilight.
7. In what circumstances will twilight be experienced all night in Latitude 60° 00′ N.?

8. Prove that for twilight to last all night:

$$(\text{Lat.} + 18°) \not> (90° - \text{dec. of Sun})$$

9. Explain the circumstances in which the Sun will be above the horizon of an observer at midnight.
10. Why is northern Norway sometimes called The Land of the Midnight Sun?
11. Explain why the duration of twilight on any day of the year increases with Latitude.
12. Show that the least duration of twilight occurs at the equator on 21st March of any year.

CHAPTER 40

RATE OF CHANGE OF HOUR ANGLE, AZIMUTH AND ALTITUDE

1. Rate of Change of Hour Angle

For a stationary observer the Hour Angle of the Mean Sun changes at the rate of 15° per hour. That is to say, at a rate of 900′ per hour or 15′ per minute of time.

The S.H.A. of the Mean Sun decreases uniformly at the rate of 360° per year; that is at a rate of 2·46′ per hour. Therefore, the Hour Angle of the First Point of Aries, and of any fixed star, changes at a rate of (900 + 2·46)′ per hour, or very nearly 902·5′ per hour.

The Hour Angle of the True Sun changes at an irregular rate. The average rate of change of the Hour Angle of the True Sun is, of course, equal to the uniform rate of change of the Mean Sun's Hour Angle. The variation from the mean rate of change of the Hour Angle of the True Sun is so small, that the hourly change in his Hour Angle may be taken as 900′.

The rate of change of the Hour Angle of the Moon or a planet, which may readily be found for any time from the corresponding daily page of the *Nautical Almanac*, is very irregular.

If h' per hour is the rate of decrease of the S.H.A. of a celestial body, the rate of change of its Hour Angle is $(902{\cdot}5 - h)'$ per hour.

Any motion of an observer over the Earth's surface, unless it be along a meridian, will tend to cause the rate of change of the Hour angle of a celestial body to increase if the observer's motion is easterly, and to decrease if his motion is westerly.

If x' per hour is the rate of change of an observer's Longitude towards the West, then:

$$\text{Rate of change of H.A. of any celestial body} = (902{\cdot}5 - h - x)' \text{ per hour.}$$

2. Rate of Change of Azimuth

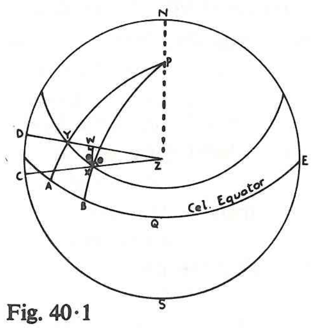


Fig. 40·1

Fig. 40·1 is a projection of the celestial sphere onto the plane of the celestial horizon of an observer whose zenith is projected at *Z*. N., E., S., and W., are the cardinal points of the horizon, and *WQE* is the projection of the celestial equator. *X* is a celestial body whose present altitude is *CX* and whose present azimuth is *NZX*.

Let the celestial body *X* in fig. 40·1 change its Hour Angle by 1 minute of time—this being represented by the arc *AB*. During this time the altitude of the body changes by an amount equal to *WY*, and its azimuth changes by an amount equal to the *C.D*. Because the change in Hour Angle is small, triangle *WXY* may be regarded as being a plane triangle right-angled at *W*.

Now, $WXZ = 90°$
and $WXP = 90°$
Now, $YXW = 90° - WXP$
and $PXZ = 90° - WXP$
Therefore: $YXW = PXZ$

The angles YXW and PXZ are denoted in fig. 40·1 by θ. θ is called the Parallactic Angle of the astronomical triangle-PZX.

$$\begin{aligned} \text{Rate of Change of Azimuth of } X &= CD' \text{ per minute of time} \\ &= XW \,.\, \sec CX'/\text{min.} \\ &= XY \,.\, \cos \theta \,.\, \sec CX'/\text{min.} \\ &= AB \,.\, \cos BX \,.\, \cos \theta \,.\, \sec CX'/\text{min.} \end{aligned}$$

Now arc AB = 1 minute of time = 15′ of arc
BX = Declination of X
Therefore: CX = Altitude of X

$$\text{Rate of Change of Azimuth} = 15 \,.\, \cos \text{dec.} \sec \text{Alt.} \cos \theta \,'/\text{min.}$$

From this formula it may readily be seen that for any given declination, the rate of change of a body's azimuth is greatest when its altitude is greatest. It may also be seen that the rate of change of the azimuth of a heavenly body is zero when its parallactic angle is 90°. When the parallactic angle is 0°, the body is on the observer's meridian and its rate of change of azimuth is greatest because its altitude is greatest.

The rate of change of azimuth of a celestial body may be expressed in terms of the body's Azimuth and Hour Angle as follows.

By applying the Spherical Sine Formula to the PZX-triangle, we have:

$$\sin PX = \sin \text{Az.} \operatorname{cosec} \text{H.A.} \sin ZX$$

or:

$$\cos \text{dec.} = \sin \text{Az.} \operatorname{cosec} \text{H.A.} \cos \text{Alt.}$$

By substituting for cos dec. in the formula derived above, we get:

$$\begin{aligned} \text{Rate of Change of Azimuth} &= 15 \,.\, \sin \text{Az.} \operatorname{cosec} \text{H.A.} \cos \text{Alt.} \sec \text{Alt.} \cos \theta \,'/\text{min.} \\ &= 15 \,.\, \sin \text{Az.} \operatorname{cosec} \text{H.A.} \cos \theta \,'/\text{min.} \end{aligned}$$

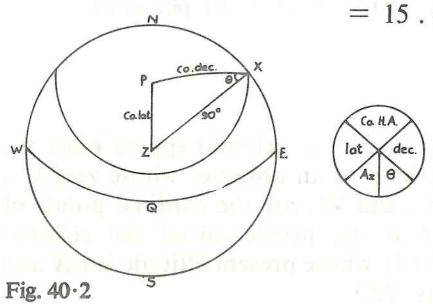


Fig. 40·2

When the altitude of a body is 0°, its rate of change of azimuth is proportional to the sine of the Latitude of the observer irrespective of the declination of the body. This may be proved with reference to fig. 40·2.

The secant of 0° is unity. Therefore, for a body on the horizon, we have, by substituting sec. 0° in the first formula derived:

$$\text{Rate of Change of Azimuth} = 15 \,.\, \cos \text{dec.} \cos \theta \,'/\text{min.}$$

By applying Napier's Rules to the PZX-triangle illustrated in fig. 40·2 we have:

$$\sin \text{Lat.} = \cos \text{dec.} \cos \theta$$

By substituting sin lat. for cos dec. cos θ we have:

$$\text{Rate of Change of Azimuth of a celestial body on the horizon} = 15 \sin \text{lat.} \,'/\text{min.}$$

This rate of change of azimuth is independent of the declination of the body. It follows that all celestial bodies at rising or setting change their azimuths at a rate which is the same for all bodies—this rate being proportional to the sine of the Latitude of the observer.

3. Rate of Change of Altitude

Referring to fig. 40·1 the change in the altitude of the body X during the interval of one minute is equivalent to WY. It follows that the rate of change of altitude is WY' per minute.

$$\begin{aligned} \text{Rate of Change of Altitude} &= WY\,'/\text{min.} \\ &= XY \,.\, \sin \theta \,'/\text{min.} \\ &= AB \,.\, \cos \text{dec.} \sin \theta \,'/\text{min.} \end{aligned}$$

By applying the Spherical Sine Formula to the triangle-PZX, we have:

$$\sin \theta = \frac{\sin PZ \,.\, \sin PZX}{\sin PX}$$

$$= \frac{\cos \text{Lat.} \sin \text{Az.}}{\cos \text{dec.}}$$

Substituting this value for sin θ in the above formula, we have:

$$\text{Rate of change of altitude} = AB \,.\, \cos \text{dec.} \frac{\cos \text{Lat.} \sin \text{Az.}}{\cos \text{dec.}}$$

$$= AB \,.\, \cos \text{lat.} \sin \text{Az.} \,'/\text{min.}$$

For a stationary observer the Mean Sun changes its altitude at the rate of 15 . cos lat. sin Az. ′/minute.

This rate is very nearly the same for the True Sun. In practice the same formula is used for finding the rate of change of a star's altitude, although the exact formula, which applies to any celestial; body is:

$$\text{Rate of change of Altitude} = \frac{(902{\cdot}5 - h - x)}{60} \,.\, \cos \text{Lat.} \sin \text{Az. in } '/\text{min.}$$

The rate of change of altitude of a heavenly body is zero when sin . Azimuth or cos . Latitude is zero. In other words the rate of change of altitude is zero when the body is at meridian passage or when the observer's Latitude is 90° North or South.

The maximum rate of change of altitude occurs for any given Latitude when the sine of the Azimuth is greatest. This occurs when the observed body has an azimuth of 90°; or, in cases when the body never crosses the prime vertical circle of the observer when it is at its limiting azimuth.

The formulae for rate of change of altitude, derived above, take no account of any declinational movement of the observed body, or meridianal movement of the observer. Changes of declination of the body or of Latitude of the observer are of particular importance when the body is at or near meridian passage.

When a celestial body is at meridian passage its rate of change of altitude due to its changing hour angle is zero. The body's altitude may change when the body is at meridian passage on account of changes in the body's declination or the observer's Latitude. Because of these factors the maximum daily altitude of a celestial body does not necessarily occur when the body is at meridian passage. Consequently the meridian altitude is not necessarily the maximum altitude. Maximum altitude and meridian altitude are the same only if the observer is not changing his Latitude at the time of the observation, and the body is not changing its declination.

When a body's geographical position and the observer's position are closing, the body's altitude at the time of meridian passage is increasing. In these circumstances maximum altitude occurs after meridian altitude.

When a body's geographical position and the observer's position are opening, the body's altitude at the time of meridian passsage is decreasing. In these circumstances maximum altitude occurs before meridian altitude.

The rate at which a body is changing its declination may be found from the appropriate daily page of the *Nautical Almanac*. The following examples show how a body's rate of change of altitude may be found for the time of the body's meridian passage.

Example 40·1—Find the rate of change of the Sun's altitude when it is on the observer's upper celestial meridian of an observer in Latitude 30° 00′ N. on board a vessel steaming 000° (T) at 20·0 knots. The date is 22nd September.

From *Nautical Almanac* Extracts:

Declination of Sun at Apparent Noon; that is, at 1153 L.M.T. or 1353 G.M.T. = 00° 22′·6 N.

The rate of change of the Sun's declination at this time is 01′·0 per hour towards South.

The ship's rate of change of Latitude is 20′·0 per hour towards North.

The Geographical position of the Sun and the observer's position are, therefore, opening at the rate of (20·0 + 1·0)′ per hour. That is 21′·0 per hour.

At apparent noon, therefore, the rate of change of the Sun's altitude is 21′·0 per hour, and the altitude is decreasing. Meridian altitude occurs, therefore, after maximum altitude.

Example 40·2—23rd September. Find the rate of change of the Moon's altitude when at upper meridian passage to an observer in Latitude 40° 00′ N. on board a vessel travelling 180° (T) at 15·0 knots. The observer's longitude is 30° 00′ W.

L.M.T. meridian passage at Long. 0°	=	(23)	20 h. 56 m.
Longitude correction 30/360 of 46 m.	=		+ 4 m.
L.M.T. meridian passage at 30° W.	=	(23)	21 h. 00 m.
Longitude in time	=		2 h. 00 m.
G.M.T. meridian passage at ship	=	(23)	23 h. 00 m.

Moon's declination = 11° 21′·8 S. Changing at the rate of 8′·0 per hour towards North. Rate of change of Latitude is 15′·0/hour towards South. Therefore, the geographical position of the Moon and the observer's position are closing at the rate of (8·0 + 15·0)′ per hour. That is, at 23′·0 per hour.

At the time of the Moon's meridian passage the rate of change of the Moon's altitude is 23′·0 per hour, and the altitude is increasing. Meridian altitude occurs, therefore, before maximum altitude.

4. Interval Between Maximum and Meridian Altitudes of the Sun

Let the combined rates of change of an observer's Latitude and observed body's declination be y' per hour. Maximum altitude occurs when y' per hour is equal numerically to the rate of change of altitude due to the body's changing hour angle.

Let the rate of change of the observer's longitude be x' per hour towards West. In other words x is positive or negative when the observer is travelling westwards or eastwards respectively.

$$\text{Rate of change of Sun's altitude} = (900 - x) \cos \text{lat.} \sin Az. \ '/\text{hour} \quad \ldots \quad (I)$$

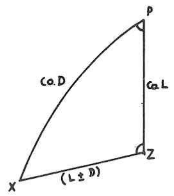


Fig. 40·3

Applying the Spherical Sine Formula to the PZX-triangle illustrated in fig. 40·3, we have:

$$\sin Z = \frac{\cos \text{dec.} \sin P}{\sin ZX}$$

Now when a celestial body is near meridian passage its zenith distance is very nearly equal to its M.Z.D.

When Latitude and declination have the same name:

$$\text{M.Z.D.} = (\text{Lat.} \sim \text{dec.})$$

When Latitude and declination have different names:

$$\text{M.Z.D.} = (\text{Lat.} + \text{dec.})$$

In general, therefore:

$$\text{M.Z.D.} = (\text{Lat.} \pm \text{dec.}), \text{ so that:}$$

$$\sin Z = \frac{\cos \text{dec.} \sin P}{\sin (\text{Lat.} \pm \text{dec.})}$$

Substituting for sin Z in formula (I), we have:

$$\text{Rate of change of Sun's altitude} = (900 - x) \cos \text{Lat.} \frac{\cos \text{dec.} \sin P}{\sin (\text{Lat.} \pm \text{dec.})} \ '/\text{hr.}$$

$$= \frac{(900 - x) \cos \text{Lat.} \cos \text{dec.} \sin P}{\sin (\text{Lat.} \pm \text{dec.})} \ '/\text{hr.}$$

Since P is a small angle:

$$\sin P = P \text{ radians}$$

Therefore:

$$\text{Rate} = \frac{(900 - x) \cos \text{Lat.} \cos \text{dec.}\ P^c}{\sin(\text{Lat.} \pm \text{dec.})}\ '/\text{hr.}$$

Let the interval between the times of maximum and meridian altitudes be t seconds of time. In other words, let the angle ZPX at the time of maximum altitude be t seconds or $t/3600$ hours. In this case:

$$P = \frac{t\ .\ (900 - x)}{3600\ .\ 3438} \text{ radians, so that:}$$

$$\text{Rate} = \frac{(900 - x) \cos \text{Lat.} \cos \text{dec.}}{\sin(\text{Lat.} \pm \text{dec.})} \frac{t\ .\ (900 - x)}{3600\ .\ 3438}\ '/\text{hr.}$$

$$= \frac{t\ .\ (900 - x)^2\ .\ \cos \text{Lat.} \cos \text{dec.})}{3600 \cdot 3438\ .\ \sin(\text{Lat.} \pm \text{dec.})}\ '/\text{hr.}$$

$$= \frac{t\ .\ (900^2(1 - x/900)^2\ .\ \cos \text{Lat.} \cos \text{dec.}}{3600 \cdot 3438\ .\ \sin(\text{Lat.} \pm \text{dec.})}\ '/\text{hr.}$$

Now, $(1 - x/900)^2 \approx (1 - x/450)$

and, $\sin(\text{Lat.} \pm \text{dec.}) = \sin \text{Lat.} \cos \text{dec.} + \cos \text{Lat.} \sin \text{dec.}$

Therefore:

$$\text{Rate} = \frac{t\ .\ (900^2(1 - x/450) \cos \text{Lat.} \cos \text{dec.}}{3600 \cdot 3438 \sin(\text{Lat.} \cos \text{dec} \pm \cos \text{Lat.} \sin \text{dec.})}\ '/\text{hr.}$$

$$= \frac{t(1 - x/450)}{15{\cdot}28\ (\tan \text{Lat.} \pm \tan \text{dec.})}\ '/\text{hr.}$$

At the time of maximum altitude the rate of change of the Sun's altitude is equal to y' per hour, so that:

$$y = \frac{t(1 - x/450)}{15{\cdot}28\ (\tan \text{Lat.} \pm \tan \text{dec.})}$$

and $$t = \frac{15{\cdot}28\ (\tan \text{Lat.} \pm \tan \text{dec.})}{(1 - x/450)}\ '/\text{hr.}$$

$$= 15{\cdot}28\ y\ (\tan \text{Lat.} \pm \tan \text{dec.})\ (1 - x/450)^{-1}$$

Now, $(1 - x/450)^{-1} \quad (1 + x/450)$,

so that: $t = 15{\cdot}28\ y\ (\tan \text{Lat.} \pm \tan \text{dec.})\ (1 + x/450)$ seconds

From this relatively simple formula, the interval t, between the times of the meridian and maximum altitudes may easily be found.

Example 40·3—Find the interval between the times of maximum and meridian altitudes of the Sun on 23rd September, to an observer in Lat. 50° 00′ N., Long. 45° 00′ W., on board a vessel travelling 030° (T) at the rate of 18·0 knots.

Rate of change of observer's Latitude = 15′·6 to the North
Rate of change of observer's Longitude = 9·0 miles/hr.
= 14′·0/hr. to the East

From *Nautical Almanac* Extracts:

Sun's Declination at 1153 L.M.T. or 1453 G.M.T. = 0° 01′·8 S.
Rate of change of Sun's Declination = 1′·0 towards the South
x = − 14′·0 per hour
y = (15·6 + 1·0)′ or 16′·6 per hour opening

Meridian altitude occurs AFTER maximum altitude.

$$t = 15{\cdot}28\ y\ (\tan \text{Lat.} + \tan \text{dec.})\ (1 - x/450)$$
$$= 15{\cdot}28\ .\ 16{\cdot}6\ (\tan 45° + \tan 0°\ 01'{\cdot}8)\ (1 - 14/450)$$
$$= 244 \text{ seconds or 4 minutes approx.}$$

Answer—Interval = 4 minutes approx.

Exercises on Chapter 40

1. Explain why the Rate of change of the Sun's Hour Angle is 900′ per hour, and that of a fixed star is about 2′·5 per hour faster.
2. Explain why westerly movement of the observer causes the rate of change of a celestial body's Hour Angle to be smaller than that for a stationary observer in the same Latitude.
3. What is the rate of change of a star's Hour Angle to an observer in Latitude 60° N. travelling due East at the rate of 20·0 knots?
4. Derive a formula for finding the rate of change of the Sun's azimuth in terms of the declination and altitude of the Sun and the parallactic angle.
5. Explain why, for any given declination of the Sun, the rate of change of the Sun's azimuth is greatest when its altitude is greatest.
6. Explain why an object's movement at the instant the parallactic angle is 90° is directly towards the observer's zenith.
7. Derive a formula for finding the rate or change of the Sun's azimuth in terms of the Sun's Azimuth, Hour Angle, and Parallactic Angle.
8. Show that the rate of change of azimuth of any celestial body at rising or setting is proportional to the sine of the Latitude of the observer.

9. Show that:
   Rate of change of Sun's Altitude to a stationary observer = 15 . cos Lat. sin Az. ′/min.
10. Explain why the rate of change of a celestial body's altitude is zero when it is at meridian passage.
11. Explain clearly why the times of maximum and meridian altitudes are not generally coincident to an observer travelling northwards or southwards.
12. Find the interval between maximum and meridian altitudes of the Sun on 24th September, to an observer in Lat. 42° 00′ N., Long. 60° 30′ W. travelling 165° (T) at the rate of 20·0 knots.
13. Find the interval between maximum and meridian altitudes of the Sun on 22nd September, for an observer travelling 340° (T) at 15·0 knots, given the observer's position: Lat. 20° 00′ N., Long. 15° 00′ W.

PART 6

THE INSTRUMENTS OF NAVIGATION

The present-day navigator is faced with a bewildering selection of instruments designed to assist him in bringing his vessel safely to harbour.

The principal instruments of navigation are chart, compass and log. Charts, and their complementary Sailing Directions, are discussed in Part 3 under the general heading of Coastal Navigation. In this part we shall describe the magnetic and gyroscopic compass; sextant and chronometer; sounding instruments and logs; radio direction finding; radar; hyperbolic aids to navigation; and, finally, satellite and inertial systems of navigation.

The usefulness of a tool is usually a function of the craftmanship of the user. For this reason a navigator should aim to be a complete master of every instrument designed for his specific use. There is much for the beginner to learn—but equally as much satisfaction, in being an expert, to be gained.

CHAPTER 41

THE MAGNETIC COMPASS

1. Magnetism

A magnet is a mass of iron, steel, or other magnetic material, which has the property of attracting like masses with a force known as Magnetic Force. Natural magnets are found in many regions of the Earth, but the strength of attraction or power of these is usually very meagre. Comparatively strong magnets may be made using the magnetic properties of an electric current. These are known as Artificial Magnets.

The strength of a magnet is concentrated at two points which are called the Poles of the magnet. In an artificial magnet, which is usually a bar or needle of steel, the poles are located near the ends. If a magnet is suspended at its centre of gravity, it will tend to align itself in a definite direction relative to the Earth's surface. This direction is roughly in the North-South plane. The end which tends to point northwards is known as the North-Seeking end. The other is known as the South-Seeking end.

The ends of magnets used on board in connection with the magnetic compass, are usually painted Red at the North-seeking end and Blue at the South-seeking end. For this reason seamen almost always refer to the poles of a magnet by the terms Red and Blue.

It will be found that if two artificial magnets are brought together a force of attraction exists between poles of the opposite colour, and a force of repulsion exists between poles of the same colour. A magnet's Field is the region around a magnet within which the force emanating from the magnet may influence magnetic material. The field of a magnet may be explored by means of a small Exploring Magnet. If the alignments of the needle of the exploring magnet at several points are linked, it will be found that Curves or Lines of Magnetic Force emerge from the Red end and enter the Blue end of the magnet whose field is being explored. The direction of a line of force is regarded conventionally as the direction towards which the Red end of the exploring magnet points. Thus we say that the lines of force of a magnet emanate from the Red end and enter the Blue end of a magnet.

The fundamental law of magnetism is:

Like poles repel each other, and
Unlike poles attract each other.

2. Terrestrial Magnetism

Observations of the alignment of a freely suspended magnet at many places on the Earth's surface indicate that the Earth itself has the properties of a huge natural magnet. The lines of force of the Earth-magnet cut the Earth's surface at different angles at different places. The angle at which the Earth's lines of force cut the Earth's surface at any place is called the angle of Dip for the place. The angle of dip may be found by means of a Dipping Needle which is simply a magnet suspended so that it is free to incline. At places where the Red end of a dipping needle tilts downwards Dip is named Positive. Where the Blue end dips downwards Dip is named negative.

At a position where the dip is $-90°$, the lines of force of the Earth-magnet leave the Earth's surface vertically. The position at which this occurs—a position known as the South Magnetic Pole—is in the vicinity of South Victoria Land in Antarctica.

At a position where the dip is $+90°$, the lines of force of the Earth-magnet enter the Earth vertically. The position of this point—the North Magnetic Pole—is in the region of Hudson Bay in North America.

Girdling the Earth in an approximate great circle path which is inclined at about 10° to the geographical equator is a line on which the dip at every point is 0°. This line is called the Magnetic Equator.

Lines on the Earth joining places of equal dip approximate to small circles which are parallel to the Magnetic Equator. These lines of equal dip are called Isoclinic Lines. The line of zero dip is called the Aclinic Line. This, of course, is the Magnetic Equator. All places at which the dip is negative are said to be in the South Magnetic Hemisphere and to have South Magnetic Latitude. All places at which the dip is positive are said to be in the North Magnetic Hemisphere and to have North Magnetic Latitude.

3. Variation

A freely suspended magnetic needle, when under the influence of the Earth's magnetic field alone, settles in a vertical plane which is known as that of the magnetic meridian. The horizontal direction towards the North Magnetic Pole is called Magnetic North, and the opposite direction is called Magnetic South. The horizontal angle between the directions of True North and Magnetic North at any place is called the Variation of the place. Variation is named, conventionally, according as the direction of Magnetic North lies to the right or left of that of True North. When the direction of Magnetic North lies to the right of that of True North, variation is named East. In other cases it is named West. The magnitude of variation may be anything between 0° and 180° E. and W.

Lines linking places at which the variation is the same are called Isogonic Lines or Isogonals. The variation changes not only from place to place but also from time to time. The value for any given epoch may be found from an isogonic chart. Information on the isogonic chart enables the navigator to bring the variation up to date.

4. The Magnetic Compass

There are two types of magnetic compasses known, respectively, as the Dry Card and the Liquid compasses.

The Dry Card Compass consists of a system of magnetised needles slung, by means of silk

threads, from a graduated Compass Card of about ten inches in diameter. The needles lie parallel to the plane of the North and South graduations on the card. At the centre of the card is a Cap in which is fitted a Jewel Bearing. The compass card rests on a hard metal sharply pointed Pivot. The point of support is above the centre of gravity of the system so that the card always remains in the horizontal plane and is unaffected by dip.

The card is made of very thin rice paper and is stiffened by a thin aluminium ring around its circumference, which provides the means for supporting the needle system. The concentration of mass at the edge of the card ensures that the period of the vibration of the card is large and that there will be little chance, therefore, of a state of synchronism existing between the oscillating card and the rolling of the ship.

The mass of the card and the needle system is very small—only a fraction of an ounce—hence the friction between the cap and pivot, the magnitude of which depends upon the resultant force acting, is very small. In a perfect magnetic compass the friction between cap and pivot is virtually nil, in which case the North-South axis of the compass card lies fixed in the plane of the magnetic meridian provided that the needle is under the influence of the Earth's magnetism alone.

Painted on the bowl which houses the compass card is a Lubber Line against which the compass course of the ship is read. The bowl is supported on Gymbals within the compass Binnacle.

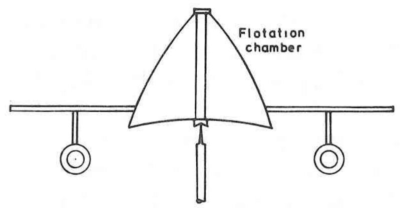


Fig. 41·1

In a Liquid Compass the card is housed in a bowl of liquid of low freezing point. The buoyancy of the compass card, which is provided by means of a copper Flotation Chamber, is adjusted so that the card almost floats. The weight of the card on the pivot, therefore, is very small, so that friction between the cap and the pivot is almost eliminated. The needles in a liquid compass are encased in brass to prevent them from rusting.

Fig. 41·1 shows a section through a liquid compass card.

5. Ship Magnetism

The magnetism of a piece of magnetic material may be Permanent or Temporary. The permanent magnetism of a mass of magnetic material is due to a magnetic property known as Hard Iron (H.I.). The temporary magnetism of a piece of magnetic material is due to its Soft Iron (S.I.) property. It should be noticed that H.I. and S.I. are names of Properties of magnetic material and that they have no direct connection with the physical properties of hardness of material.

A piece of unmagnetised magnetic material, when placed in a magnetic field, becomes magnetised by a phenomenon known as Induction. On removing the material from the inducing field only a proportion of the magnetism it acquired by induction remains: a proportion is always lost. That which is retained is due to the H.I. property of the material: that which is lost is due to its S.I. property.

A vessel is made of magnetic material, and every part of the structure of a vessel has both H.I. and S.I. properties simultaneously. The vessel acquires permanent magnetism during the time she is being constructed. This is due to the magnetising influence of the Earth. It must not be thought that a vessel's permanent magnetism is simply explained, especially in these times when prefabrication of vessel's plays an important part during the period of building. As a result of the vessel's permanent magnetism a permanent magnetic field acts at the position of the magnetic compass, and this field may cause deviation. This deviation is often regarded as being due to the existence of a Red and Blue pole within the vessel. This idea should be used reservedly as it may give rise to false ideas on magnetism of vessels.

In addition to the permanent magnetism which causes a permanent magnetic field to act at the compass position another magnetic field acts, this being due to the S.I., property of the vessel. This induced field changes in magnitude and direction relative to the fore-and-aft line of the vessel with every change in the vessel's course and geographical position. The permanent field, on the other hand, never varies in magnitude or direction relative to the vessel's fore-and-aft line.

6. Simple Ideas on Compass Adjustment

The term Compass Adjustment is something of a misnomer. The compass, as such, requires no adjustment: it is as near perfect as is humanly possible, when it is supplied to a vessel. The aim of a Compass Adjuster is to place correctors in the vicinity of the compass so that their combined magnetic effect neutralises the magnetic field of the vessel at the compass position. If he is successful the compass will be influenced by the Earth's magnetic field only and it will always indicate Magnetic North and no deviation will appear on any heading. In practice perfection is seldom attained, and the best that can be done is to reduce the magnitude of deviations to small proportions. When deviations appear there must be a magnetic force acting in addition to the force due to the Earth's magnetism on the magnetic compass. This force is the resultant of the vessel's magnetism and that due to the correctors.

The correcting devices used by the compass adjuster are of two types. Some are magnets having a large proportion of H.I. property. These are called Permanent Corrector magnets and they are used to neutralise the ship's permanent magnetism. The others have a large proportion of S.I. property. These are used to neutralise the effects of the ship's induced magnetism, and they are called the Soft Iron Correctors.

The Soft Iron Correctors are named after famous scientists who promoted the science of ship magnetism in the early days of its development. The spheres which are usually to be found placed athwart the compass on the binnacle are named after Lord Kelvin who first suggested their use. The Soft Iron Corrector placed in a brass case usually on the fore side of the binnacle is known as the Flinder's Bar after Captain Matthew Flinders the famous sailor scientist of the early nineteenth century.

For convenience of adjustment the permanent and induced magnetism of a vessel are resolved into fore-and-aft, athwartship, and keelward components.

The forces due to the vessel's permanent magnetism which act in the fore-and-aft, athwartship, and keelward directions at the Red end of the magnetic compass, are called respectively forces *P*, *Q*, and *R*.

Force *P* is eliminated by means of a permanent magnet placed fore and aft in the binnacle beneath the compass. Force *Q* is eliminated by means of an athwartship magnet placed in the binnacle beneath the compass. Force *R*, together with certain of the component forces of the Induced Magnetism of the vessel, is eliminated by means of a vertical magnet placed in the binnacle beneath the compass.

The resolution of the induced magnetism of a vessel is very much more complex than that of her permanent magnetism. Whereas three components are all that are necessary to deal with the permanent magnetism of a vessel, in the most complex cases her induced magnetism requires no less than nine components.

The purpose of Kelvin's spheres is to neutralise certain horizontal (fore-and-aft and athwartship) components of the vessel's induced magnetism. The purpose of the Flinders' Bar is to neutralise certain horizontal forces which result from induction in vertical components of the vessel's induced magnetism.

Although collectively the component forces of a vessel's magnetism are complex, the process of neutralising it at the compass position is relatively simple. As an illustration of the procedure let us suppose that the field due to the vessel's induced magnetism is properly neutralised by the Soft Iron correctors and that the permanent magnetism of the vessel acts in the fore-and-aft line towards the stern.

When the vessel is heading 000° or 180° by Compass, her permanent magnetism acts in line with the Earth's magnetic field, so that the vessel's magnetism does not produce a deviating force. On every other compass course the vessel's magnetic force acts across the magnetic meridian and it is, therefore, a deviating force. The maximum deviation occurs on 090° or 270° by compass.

When compensating any component of a vessel's magnetism a compass adjuster normally sets the course on which the deviation due to that particular component is maximum. Thus, in the case under consideration, the course would be set to 090° (C.) or 270° (C.).

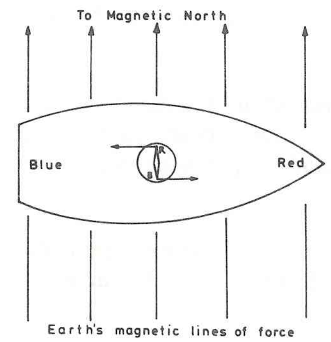


Fig. 41·2

Fig. 41·2 illustrates the magnetic couple due to the vessel's permanent magnetism which acts on the magnetic compass when the course is 090° (C.). On this course the maximum West deviation occurs. To eliminate the vessel's fore-and-aft permanent magnetism, the compass adjuster removes the West deviation, which he finds by comparing the Magnetic and Compass Bearings of a distant shore mark or of the Sun or other celestial body, by placing a fore-and-aft magnet (in the binnacle beneath the compass) with its Red end pointing aft.

To eliminate deviation due to the athwartship component of the vessel's permanent magnetism the course is set to 000° (C.) or 180° (C.), and the deviation appearing would be eliminated by means of an athwartship magnet placed in the binnacle beneath the compass.

7. The Azimuth Mirror

Compass Bearings of terrestrial or celestial bodies are observed by means of an Azimuth Mirror. This instrument which, when in use,. fits over the compass card, consists of a stand on which is mounted a glass prism. The prism may be rotated about a horizontal axis by means of a screw with a milled head on which is engraved an arrow. The stand of the azimuth mirror is provided with a Spirit Level and a Shadow Pin, and also with a Sighting Tube in which is housed a Magnifying Lens.

There are two methods of observing bearings with an azimuth mirror. They are known respectively as the Arrow Up and Arrow Down methods.

The Arrow Up method is usually employed when observing heavenly bodies. The prism is adjusted by means of the milled head screw until the light from the body is reflected to the observer's eye. This is denoted by *E* in fig. 41·3. The observer's eye is placed so that the compass card is seen through the magnifying lens in the sighting tube. A metal pointer *P* indicates the required bearing when it is in alignment with the image of the observed object.

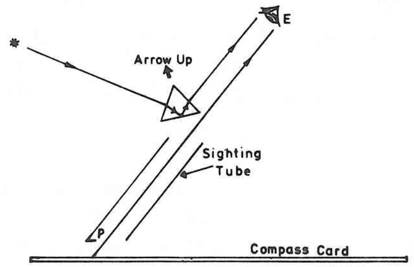


Fig. 41·3

Great care must be taken to ensure that the image of the observed body is in coincidence with the pointer: if they are not an error in the bearing will result. Such an error increases as the altitude of the body increases. For this reason it is advisable not to observe bodies whose altitudes are greater than about 40°, when bodies at smaller altitudes are available.

The Arrow Down method is used for observing terrestrial bodies or celestial bodies near the horizon. With this method the observer sights the object over the top of the prism, and adjusts the prism until the compass card is reflected to his eye. The Arrow Down method is illustrated in fig. 41·4.

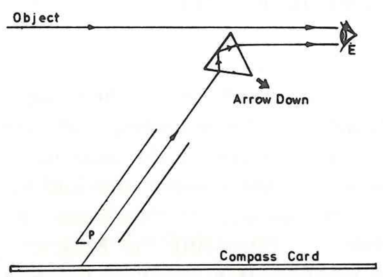


Fig. 41·4

The Arrow Down method may be used for observing celestial bodies which are well above the horizon, and it is preferred to the Arrow Up method when the body is indistinct.

Bearings using the two methods should agree unless the prism is out of adjustment. Rough bearings may be observed by means of the shadow pin with which the instrument is provided.

It is essential, when observing bearings, that the azimuth mirror is perfectly horizontal. The purpose of the spirit level fitted to the azimuth mirror stand, is to ensure that this is so.

Exercises on Chapter 41

1. What is an artificial magnet? Describe how an artificial magnet may be made.
2. What is the fundamental law of magnetism?

3. Describe a Dry Card Compass.
4. Describe a Liquid Compass such as that found in a life-boat.
5. Describe carefully the binnacle of a magnetic compass.
6. Write a short essay on Terrestrial Magnetism.
7. Define: Magnetic Poles; Magnetic Equator.
8. What are: (*a*) Isoclinic Lines, (*b*) Isogonic Lines?
9. Define: Magnetic Dip.
10. Describe how a vessel acquires her magnetism.
11. Distinguish between permanent magnetism and induced magnetism of a vessel.
12. Describe the permanent and Soft Iron correctors used for neutralising a vessel's magnetism at the position of the magnetic compass.
13. Describe an azimuth mirror.
14. Describe the two methods of using an azimuth mirror.
15. How may the accuracy of an azimuth mirror be checked?

CHAPTER 42

THE GYROSCOPIC COMPASS

1. Introduction

The magnetic compass, which has served the mariner for no less than half a millennium, and which has provided him in the past with his only sure guide of direction when out of sight of land, is now being superseded by the gyroscopic compass—a marvellous product of engineering skill in which navigators have complete confidence. An important feature of the gyro compass is that it indicates the direction of True North.

The ancestor of the complex gyro compass is a simple device, invented by Foucault of pendulum fame, to demonstrate the Earth's rotation. Foucault named the instrument a Gyroscope which means to "view a spin".

At the beginning of this century, when it became possible to spin a gyro wheel electrically, attention was directed to the possibility of developing a gyroscopic compass for use on board ship. First in the field was the German scientist Dr. Anschutz, who introduced his first successful gyro compass in 1908. Three years later, in 1911, the American scientist Dr. Elmer Sperry, patented his gyro compass. The British reply to the Anschutz and Sperry compasses was invented by Dr. S. G. Brown in 1916. Since then other makes of gyro compass have appeared, notably the Arma and Plath compasses.

The usefulness of a gyro compass is not confined to directional properties only: Steering and Bearing Repeater Compasses; Course Recorder, and Automatic Helmsman may be operated from a Master Gyro Compass.

The principal disadvantage of the gyro compass is that it requires an electrical supply, a failure in which requires the use of the stand-by magnetic compass.

2. The Principles of the Gyroscope

A device known as a mechanical or model gyroscope is used to demonstrate gyroscopic principles. It is a wheel mounted in such a way that its axle may be set in any desired direction relative to its housing. Fig. 42·1 illustrates a mechanical gyroscope.

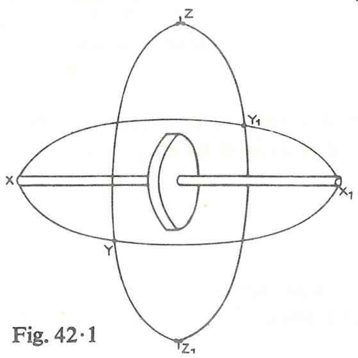


Fig. 42·1

A mechanical gyroscope is said to have three Axes or Planes of Freedom. Fig. 42·1 illustrates the three axes of freedom, which are:

XX_1—which allows freedom of the wheel to spin on its axle
YY_1—which allows freedom of the axle to incline to the horizontal
ZZ_1—which allows freedom of the axle to turn in azimuth.

The centre of gravity of a mechanical gyroscope should be at the centre of gravity of the wheel and friction at all bearings should be as small as possible.

A gyro wheel, in common with all spinning bodies, possesses the properties known as Gyroscopic Inertia and Precession.

Gyroscopic Inertia is the property of a spinning body by which it tends to maintain its axis or plane of spin. Precession is the seemingly paradoxical movement of the axis of a spinning body when a mechanical couple is applied to it.

To understand these properties it is necessary to understand elementary mechanics. The following brief notes should be familiar.

The Mass of a body is a measure of the quantity of matter in it, as indicated by the acceleration imparted to it by a given Force. The unit of mass is the Kilograms.

A body's Inertia is a measure of the tendency it has to maintain its present state of rest or uniform motion in a straight line. The amount of inertia a body possesses is dependent upon its mass and velocity. Newton's First Law of Motion, often referred to as the Law of Inertia, states that: "every body tends to maintain its present state of rest or uniform motion in a straight line and will do so if no force acts on it".

Anything that changes, or tends to change, a body's state of rest or uniform motion in a straight line, is known as a Force. A force is a vector quantity in that it has both magnitude and direction. A vector quantity is one that may be represented by a straight line, the direction of which represents the direction of the vector and the length of which represents its magnitude. Examples of kinds of force are:

(i) Gravitational Force which is the Force the Earth exerts on bodies on or near its surface. The gravitational force acting on a body is equivalent to the Weight of the body.
(ii) Frictional Force, which is the force of resistance to motion due to the contact of two surfaces moving relatively to one another.
(iii) Magnitude force, which is the force emanating from a pole of a magnet, which may change the motion of a mass of magnetic material.
(iv) Muscular Force.
(v) Electrical Force.

Velocity is a vector quantity and is defined as the rate of change of position in a given direction.

A body Accelerates when its velocity changes. Suppose a body starts from rest and moves with an average velocity of 1 m./second during the first second; 2 m./second during the second second; 3 m./second during the third; and so on. The body is said to accelerate at the rate of 1 m. per second, an acceleration written as 1m/sec./sec. or m/s^2. The unit of force is called the Newton (N). It is defined as the force which to a given mass of 1 kg produces an unit acceleration of $1 m/s^2$.

When a force is applied to a body which is free to move, the body's inertia is overcome, and it moves with ever-increasing velocity. As it accelerates it gathers Momentum. The momentum of a body is the quantity of motion it possesses, measured by the product of its mass and velocity. If equal forces are applied simultaneously to unequal masses, the momentum gathered by each mass is the same after any given interval of time. Hence the acceleration of the larger mass is less than that of the smaller.

Force is measured by rate of change of momentum.

If forces P and Q act simultaneously on equal masses m_1 and m_2, such that the velocity of m_1 is double that of m_2 after a given interval, then force P is double force Q.

If forces P and Q act simultaneously on masses m and $2m$ such that the velocity of each mass at any instant is the same, force P is half force Q.

The relation between force, mass and acceleration, is given in Newton's Second Law of Motion, which is:

"Rate of change of momentum is proportional to the applied force".

In other words:

$$\begin{aligned}\text{Applied force} &\propto \text{Rate of change of momentum}\\ &\propto \text{Rate of change of mass} \times \text{velocity}\\ &\propto \text{Rate of change of velocity} \times \text{mass}\\ &\propto \text{Acceleration} \times \text{mass.}\end{aligned}$$

The unit of force is chosen such that it is the force required to give unit acceleration to a unit mass.

Hence, if appropriate units are used:

$$\text{Applied Force} = \text{Acceleration} \times \text{Mass}$$

or:

$$F = a \,.\, m$$

A body spins, or tends to spin, when a Mechanical Couple is applied to it. A couple is formed when two equal opposite forces, acting in the same plane, are separated by a perpendicular distance known as the Arm of the Couple. The tendency of a body to spin when a couple is applied to it is dependent upon the magnitude of the forces and the arm of the couple. A measure of this tendency is called the Moment of the Couple or a torque. The moment of a couple is found by taking the product of one of the forces and the arm of the couple. This may readily be proved by the Principle of Moments. The Unit of torque is Newton metre (Nm).

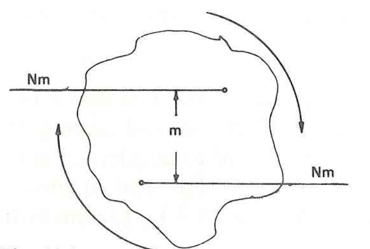


Fig. 42·2

Fig. 42·2 illustrates a couple the Moment of Newton metre (Nm) is acting on a body.

The quantity of motion possessed by a body moving in a circular path is called the body's Angular Momentum. The angular momentum of a spinning body is dependent upon its mass, its angular velocity, and the distribution of its mass.

When a body which is free to move is acted upon by a mechanical couple its angular velocity increases and it is said to move with Angular Acceleration.

A spinning body whose mass is concentrated near its axis of spin has less angular momentum than a spinning body of the same total mass and the same angular velocity but whose mass is concentrated away from the axis of spin.

Newton's Second Law of Motion applied to spinning bodies may be stated thus:

"The rate of change of angular momentum is proportional to the applied couple".

In other words:

$$\begin{aligned}\text{Applied Couple} &\propto \text{Rate of change of angular momentum}\\ &\propto \text{Rate of change of angular velocity} \times I\\ &\propto \text{angular acceleration} \times I\end{aligned}$$

If appropriate units are used:

$$\text{Applied Couple} = \text{Angular Acceleration} \times I$$
$$T = \propto I$$

The quantity I is known as the Moment of Inertia of the spinning body. It may be expressed in the form:

$$I = mr^2$$

where m is the mass of the body and r is a quantity called the radius of gyration of the body. The radius of gyration may be defined as the radius of motion of a particle having a mass equal to that of the spinning body.

If, through some internal cause, one particle of a revolving system changes its position and/or its angular velocity such that its contribution to the total momentum of the system changes, then one or more of the other particles of the revolving system will change position and/or angular velocity in such a way that the total angular momentum of the system remains the same.

This property of a revolving system of masses is implicit in a principle known as the Principle of the Conservation of Angular Momentum, which is:

"Every system of revolving masses, about a given axis, tends to maintain its angular momentum, and will do so if no external couple acts upon it".

Examples of revolving systems to which this principle applies are: the Solar System; the Earth-Moon system; the governor on an engine; and a pendulum.

The axis of revolution of a system or the spin axis of a rotating body, tends to maintain a fixed direction in space. If this axis changes its direction the angular momentum of the system or the body would change about the original axis, and this cannot be so if the principle of the conservation of angular momentum is to be obeyed. Newton's First Law of Motion applied to spinning bodies or revolving systems may, therefore, be stated thus:

"The axis of a spinning body, or of a revolving system, tends to maintain its present direction in space, and will do so if no external couple acts upon it".

An instructive illustration of the principle of the conservation of angular momentum is a piece of circus equipment known as a Performing Disc. This device serves also to explain the phenomenon of precession. A performing disc consists of a large platform of wood which is made to rotate about a vertical axis. A clown, by changing his position on the disc, is able to change the speed of the disc. When at the centre of the disc, the clown contributes no angular momentum to the system. When he moves from the centre towards the circumference of the disc he contributes an increasing amount of angular momentum to the system. The disc's contribution is, therefore, reduced, the reduction being affected by its slowing down. When the clown moves towards the centre of the disc, its angular velocity increases.

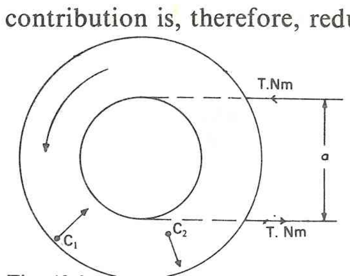


Fig. 42·3

Fig. 42·3 illustrates a performing disc the moment of the couple causing rotation is T. Nm. When the clown C_1 moves towards the centre of the disc, the speed of rotation of the disc increases. When clown C_2 moves towards the edge of the disc, the speed of rotation of the disc decreases.

From these facts the two following rules are deducted:

1. When a member of a revolving system moves towards the axis of revolution an apparent couple takes effect which acts *with* the couple causing the revolution of the system.

2. When a member of a revolving system moves away from the axis of revolution an apparent couple takes effect which acts *against* the couple causing the revolution of the system.

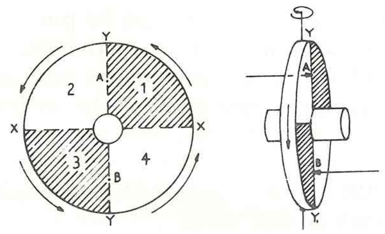


Fig. 42·4

The cause of precession may be explained thus: Imagine a couple acting at *A* and *B* on the spinning wheel illustrated in fig. 42·4. The couple acts about the axis *XX*. If the wheel were not spinning the couple would tend to rotate the wheel about the axis *XX*.

Every particle in quadrant 1 is receding from the axis of rotation of the couple and, therefore, an apparent couple acts such that every particle in this quadrant tends to move in a direction opposite to that in which force *A* acts.

Every particle in quadrant 2 is moving towards *XX* and, therefore, an apparent couple acts such that every particle in quadrant 2 tends to move in the direction in which force *A* acts.

Every particle in quadrant 3 is receding from *XX* and, therefore, an apparent couple acts such that every particle in this quadrant tends to resist force *B*.

Every particle in quadrant 4 is moving towards the axis *XX* and, therefore, an apparent couple acts such that every particle in quadrant 4 tends to act with the force *B*.

All particles in quadrants 1 and 4 tend to move in the same direction. That is in the direction of force *B*.

All particles in quadrants 2 and 3 tend to move in the same direction, which is the direction in which force *A* acts.

If the wheel is perfectly free to move it will rotate about the axis *YY*. This movement is called Precession. Note that the wheel tends to set its plane of spin in the plane of the applied couple.

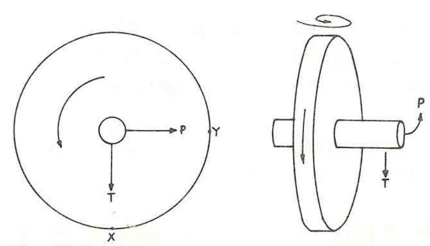


Fig. 42·5

Imagine the wheel illustrated in fig. 42·5 to be spinning in the plane of the paper. The rotary force *T* is one of the forces of a couple, known as a torque. This rotary force tends to cause *X* to move into the plane of the paper. Were the wheel not spinning, this is what *T* would do. But, because the wheel is spinning, the effect of *T* is for the point *Y* to move into the plane of the paper.

N.B. *Y* is 90° from *X* in the direction of spin of the wheel.

Examples of Gyroscopic Inertia

(i) *Deck Quoit*—The deck quoit must be given an initial spin so that it will remain horizontal during its flight.

(ii) *Rifling of a Gun Barrel*—The screw head in the gun barrel causes the shell to twist as it passes through the barrel. This ensures that the shell maintains its direction of flight.

(iii) *Spinning Top*—The axis of the top remains fixed in the vertical so long as it is spinning rapidly.

(iv) *The Earth*—Because it spins, the Earth tends to maintain its plane of spin and, therefore, the North Pole of the Earth tends to point to a fixed position on the celestial sphere. This position is the Celestial Pole.

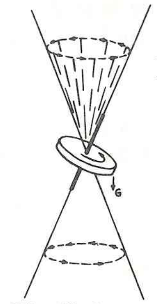


Fig. 42·6

Examples of Precession

(i) *The Spinning Top*—Because of the friction at the pivot of the top, the top tilts when its speed of rotation is insufficient to overcome this friction. As soon as this happens, the force of gravity tries to capsize it, but the effect of the force of gravity does not act at the point where it would act were the wheel not spinning: it acts 90° in advance of this position in the direction of spin of the wheel. The axis of the top, therefore, describes a conical movement the direction of which is the same as that of the spin of the wheel. This is illustrated in fig. 42·6.

(ii) *Precession of the Earth's Spin Axis*—The precession of the Earth's polar axis is due to a combination of the following contributory causes:
(*a*) The Earth's axial rotation.
(*b*) The Earth's orbital motion.
(*c*) The Solar attraction.
(*d*) The Earth's oblate shape.
(*e*) The obliquity of the ecliptic.

The attraction of the Sun on the Earth is such that it tends to force the Earth's axis perpendicular into the plane of its orbit around the Sun. This is due to the greater amount of matter in the Earth's equatorial plane than in its polar plane. Because the Earth rotates this force acts so that the extension of the Earth's polar axis on the celestial sphere sweeps out a conical path on the celestial sphere. This has a radius of 23½° and is centred at the pole of the ecliptic. The direction of this precessional motion is opposite to that of the Earth's axial spin.

The effect of the Earth's axial precession is to cause the equinoctial points to revolve in the ecliptic with a motion known as the Precession of the Equinoxes.

The First Points of Aries and Libra revolve in the ecliptic at an average rate of 50″ per year, taking 26,000 years to make the complete circuit. An effect of this is that the declination and S.H.A. of all fixed stars change gradually.

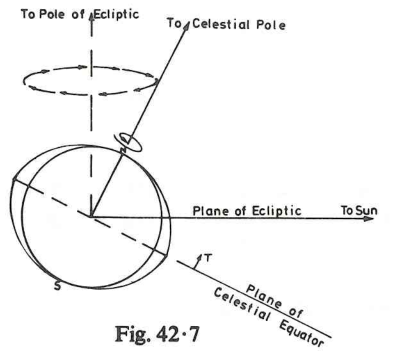


Fig. 42·7

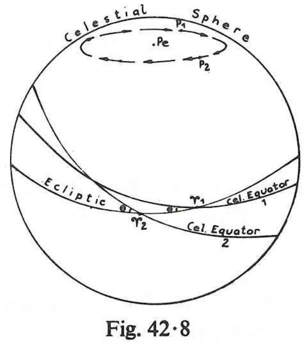


Fig. 42·8

Fig. 42·7 illustrates the precession of the Earth's axis. *T* represents the torque exerted by the Sun endeavouring to force the plane of the Earth's equator into the plane of the ecliptic.

Fig. 42·8 illustrates the precession of the equinoxes. When the celestial pole is at P_1, the equinoctial cuts the ecliptic at γ. As the celestial pole precesses from P_1 to P_2, the First Point of Aries, or the Spring Equinox, precesses from γ_1 to γ_2.

Another important effect of the precession of the equinoxes is that Polaris, which is less than 1° from the celestial pole at the present time will, in 13000 years, be about 46° from the celestial pole.

3. Tilt and Drift

The direction which a gyro axle endeavours to maintain is fixed with reference to space. Because of the Earth's rotation fixed directions in space change relative to the horizontal and vertical planes of an observer. All celestial bodies which are fixed in space appear to revolve around the Earth once in the time taken for the Earth to make one rotation on its axis.

Were the axle of a free gyro pointed to a fixed star, the axle would endeavour to maintain rigidly this direction because of the gyroscopic inertias of the gyroscope. It is for this reason that the property of gyroscopic inertia is sometimes referred to as Rigidity in Space.

Relative to the horizon and meridian of a terrestrial observer a star continually changes its altitude and azimuth. A gyro axle tends to do likewise. The component motions of a gyro axle corrrespond to changes in altitude and azimuth of a fixed star to which the axle is

pointing. The angle of inclination of a gyro axle to the horizontal plane corresponds to altitude: this angle is called Tilt. The angle which the vertical plane through the gyro axle makes with the plane of the meridian corresponds to azimuth: this angle is called Drift.

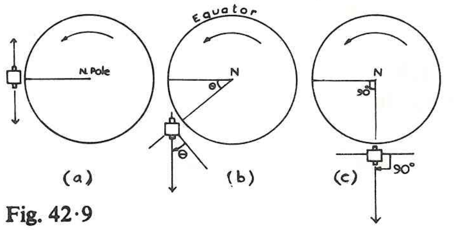


Fig. 42·9

Suppose a gyro wheel is set spinning on the equator with its axle horizontal and lying in a direction at right angles to the meridian. The effect of gyroscopic inertia is to cause that end of the axle which is pointing due East, to change its tilt upwards at the rate of 15° per hour. The gyro axle is simply illustrating the Earth's spin and, in one sidereal day it would perform a complete rotation about the horizontal N./S. axis.

The gyro in diagram (*a*) of fig. 42·9 has its axle horizontal. After the Earth has spun through an angle θ the gyro axle will have changed its tilt from 0° to θ°. This is illustrated in diagram (*b*) of fig. 42·9. After the Earth has spun through 90°, as illustrated in diagram (*c*) of fig. 42·9, the axle will have changed its tilt from 0° to 90°.

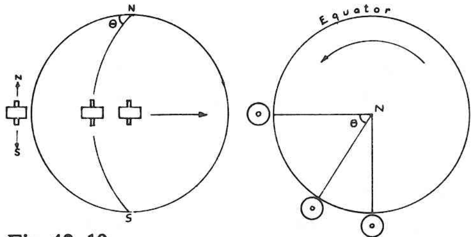


Fig. 42·10

Had the gyro axle been set horizontal and lying in the plane of the meridian it would have remained on the meridian. In this case the gyro axle would be pointing to the celestial pole.

Fig. 42·10 illustrates the gyro axle of a gyro set horizontal on the plane of the meridian at the equator, pointing continually to the celestial pole. In other words the tilt of the axle remains 0° continually.

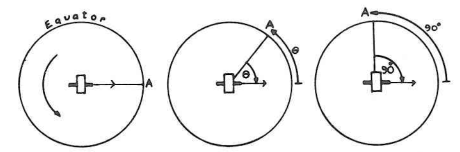


Fig. 42·11

Imagine a gyro axle set horizontally at the north pole of the Earth as illustrated in fig. 42·11. The gyro axle is at right angles to the Earth's axis of rotation and, because of gyroscopic inertia, it will remain so. The gyro axle will, therefore, remain horizontal, and its tilt will always be 0°. The gyro axle will, however, appear to rotate clockwise looking down on it. In so doing it is manifesting the Earth's real rotation in an anticlockwise direction. The apparent turntable movement of a gyro axle is analogous to change in azimuth of a fixed star: it is therefore, Change of the Drift of the gyro axle.

At either pole change of drift takes place at the rate of 15° per hour.

At either pole, provided that the gyro axle is not in the vertical direction, the apparent movement of the gyro axle is one of change of drift only.

At the equator, provided that the gyro axle is not in the plane of the meridian, the apparent movement of the horizontal gyro axle is change of tilt only.

In any position other than at either pole or at the equator, a horizontal gyro axle will change its tilt and drift simultaneously.

4. Tilting and Drifting

The rate at which the tilt of a gyro axle changes is called Tilting. The rate at which a gyro axle changes its drift is called Drifting.

$$\text{Tilting} = \text{rate of change of Tilt}$$
$$\text{Drifting} = \text{rate of change of Drift}$$

Tilting of a horizontal gyro axle at the equator when on the meridian
$= 0°$ per hour.

Tilting of a horizontal gyro axle at the equator when pointing due East/West
$= 15°$ per hour.

Therefore:

Tilting of a horizontal gyro axle at the equator $= 15 \, . \sin \text{Az.}$ per hr.
Tilting of a horizontal gyro axle at either pole $= 0°$ per hour.

Therefore:

Tilting of a horizontal gyro axle in Lat. θ Az. $\varphi = 15 \cos \theta \sin \varphi°/\text{hr.}$
$= 15 \cos \theta \sin \varphi\,'/\text{min.}$

Example 42·1—A gyro is spinning with its axle pointing 045° horizontally in Lat. 60° N. Find the change of tilt in 15 minutes assuming the change is uniform. Find the tilt after 15 minutes.

$$\text{Tilting} = 15 \, . \sin 45° \cos 60° \text{ degrees per hour}$$

$$= 15 \, . \frac{1}{\sqrt{2}} \, . \frac{1}{2} \text{ degrees per hour.}$$

Now, Change in Tilt

in any interval t = Rate of change of tilt $\times\, t$

Therefore:

$$\text{Change of tilt in 15 minutes} = 15 \, . \frac{1}{\sqrt{2}} \, . \frac{1}{2} \, . \frac{1}{4} \text{ degrees}$$

$= 1° \ 19\tfrac{1}{2}'$

Initial Tilt $= 0° \ 00'$

Final Tilt $= 1° \ 19\tfrac{1}{2}'$ End pointing eastwards inclined upwards.

Answer—Change in tilt $= 1° \ 19\tfrac{1}{2}'$
New Tilt $= 1° \ 19\tfrac{1}{2}'$ East end up.

The rate of change of Drift of a gyro axle is called Drifting.

Drifting of a horizontal gyro axle in Lat. $0° = 0°$ per hr.
Drifting of a horizontal gyro axle in Lat. $90° = 15°$ per hr.

Therefore:

Drifting of a horizontal gyro axle in Lat. $\theta = 15 \, . \sin \theta\,°/\text{hr.}$
$= 15 \, . \sin \theta\,'/\text{min.}$

Example 42·2—A gyro axle is set spinning with its axle horizontal and pointing 045° in Latitude 60° N. Assuming that the rate of change of drift is uniform, find the azimuth of the gyro axle after 15 minutes.

$$\text{Drifting} = 15 \, . \sin \text{Lat.} \, °/\text{hr.}$$
$$\text{Ch. of Drift in } \tfrac{1}{4} \text{ hr.} = 15 \, . \sin 60° \, . \tfrac{1}{4}$$

$$= 15 \, . \frac{\sqrt{3}}{2} \, . \frac{1}{4} \text{ degrees}$$

$= 3° \ 25'$ End pointing eastwards changes its drift towards the right in the northern hemisphere

Initial Azimuth $= 045°$

Final Azimuth $= 048° \ 25'$

Answer—Azimuth $= 048° \ 25'$.

5. The Gravity Controlled Gyroscope

The mechanical gyroscope described above is sometimes described as a Free Gyroscope. To make a free gyroscope into a Meridian-Seeking Gyroscope, it is necessary to control the apparent movements of the gyroscope axle due to the motion of the Earth on her axis. The Earth's force of gravity is used to effect this control. The harnessing of the Earth's force of gravity to control the apparent movement of the free gyro due to the Earth's rotation is achieved by fitting a device known as a Gravity Control.

There are several forms of gravity control. In the Anschutz and the Sperry Mark 20 compasses the gravity control consists of a mass fitted to the casing in which the gyro wheel is housed. This mass, sometimes known as a Bail Weight, moves out of the vertical when the gyro axle moves out of the horizontal, the force of gravity acting on it thereby providing the torque which precesses the axle. When the axle lies in the plane of the meridian, the tendency to change tilt is zero so that, provided the axle is horizontal, it remains on the meridian.

Another type of gravity control is the Liquid Gravity Control.

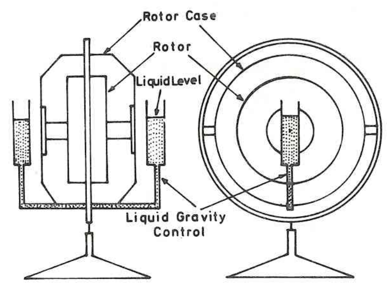


Fig. 42·12

6. The Liquid Gravity Control

Fig. 42·12 illustrates a liquid gravity control fitted to a mechanical gyroscope. The gravity control, which consists of two pots interconnected by a small bore tube lying in the vertical plane through the gyro axle, is secured to the base of the rotor case.

When the axle is horizontal the liquid in the system is such that there is an equal amount in each pot. When the axle is inclined to the horizontal the system is also inclined and more liquid is to be found in the lower than in the higher pot. When this is so, the force of gravity acting on the excess liquid in the lower pot provides a torque which causes the rotor axle to precess. The direction of precession is dependent upon the direction of the spin of the rotor.

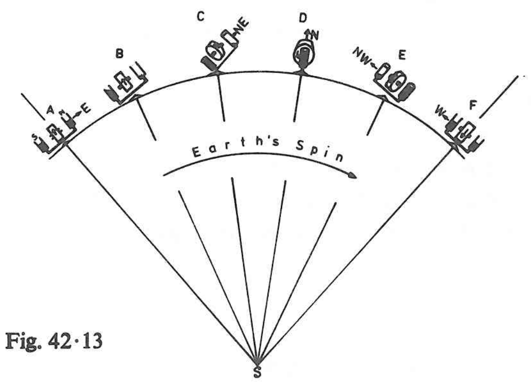


Fig. 42·13

Fig. 42·13 represents six successive positions of a liquid gravity controlled gyroscope, as it is carried around the Earth's axis due to the rotation of the Earth. The gyro wheel is supposed to be spinning anticlockwise when viewed from the South end, marked *S* in the diagrams, and the positions are supposed to be viewed from the South.

At position *A* the gyro axle is horizontal and at right angles to the meridian. The end marked *N* (for North) is pointing due east. When the gyro has been carried around to position *B*, the north end has inclined upwards and the liquid has flowed to the south pot. Torque due to gravity causes the north end of the gyro axle to precess towards the northwards. Note that the wheel is spinning anticlockwise when viewed from the south. At position *D* the axle is on the meridian, but the north end of the axle is inclined up with maximum tilt. At position *E*, the north end has precessed to the west of the meridian and the north end has begun to decline causing the liquid to level itself again. At position *F* the liquid is level and the north end is now pointing due west. The other end now begins to incline upwards, and the north end then precesses towards the meridian, with the axle inclining south end upwards.

7. The Movement of the Axle of a Gravity Controlled Gyroscope

(*a*) At the equator—Suppose a gyro axle is set horizontal with one end, say the north end, pointing N. θ E. At the equator the gyro axle tends to change its tilt but not its azimuth or drift. Were the movement of the north end of the axle projected onto a vertical screen lying in the east-west plane, the path traced out would be an ellipse. The diameter of the east-west axis of this ellipse is 2 . θ, and that of the vertical axis is dependent upon the magnitude of the gravity control. The vertical axis of the elliptical path traced out by the axle of an undamped gyro compass is never more than a couple of degrees.

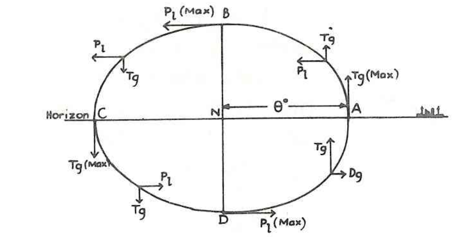


Fig. 42·14

Referring to fig. 42·14, the north end of the gyro axle is pointing to position *A*. Because of gyroscopic inertia, the north end will incline upwards. The rate of change of tilt is maximum at *A* and is represented by *Tg*. The liquid in the gravity control flows to the south pot and, as a consequence, the north end of the axle precesses towards north. Note that the direction of spin of the gyro wheel is clockwise when viewed from the north end.

As the azimuth decreases, tilting (rate of change of tilt) decreases, but the angle of tilt increases, until it reaches a maximum when the axle is on the meridian. At point *B*, the precession due to the liquid, which is represented by P_l is maximum. The north end of the axle crosses the meridian and begins to decline, because the south end is now east of the meridian and it will, therefore, incline upwards. The liquid now flows back to the north pot, the precession due to the liquid decreases and tilting increases until the north end of the axle is pointing N. θ W., when the axle will again be horizontal. At *C* the south end of the axle, which continues to incline upwards results in the liquid flowing to the north pot. Precession of the north end is now towards the meridian again, and it reaches the meridian with maximum tilt north end downwards. Excess of liquid in the north pot causes the north end to continue precessing towards the east, and it will point N. θ E. when the axle is again restored to the horizontal.

(*b*) *In Any North Latitude*—Suppose the axle is horizontal and the north end is pointing N. θ E. Because of gyroscopic inertia the north end inclines upwards and changes its azimuth towards the eastwards. As the axle increases its tilt liquid flows from the north to the south pot and a torque is introduced which tends to cause the north end to precess towards the meridian. The north end does move towards the meridian after the instant at which the precession due to the liquid exceeds drifting due to the the Earth's rotation, which latter is represented by *Dg* in fig. 42·15.

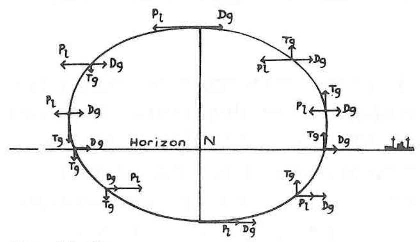


Fig. 42·15

Fig. 42·15 illustrates the path traced out by the north end of a liquid gravity controlled gyro in a northern Latitude. The centre of the elliptical path is vertically above the north point of the horizon.

(*c*) *In Any South Latitude*—In any south Latitude, where the change of azimuth or drift of the north end of a gyro axle is the opposite direction to that in the northern hemisphere, the path traced out by the north end of a liquid gravity controlled gyro is an ellipse, the centre of which is vertically below the north point of the horizon. This is illustrated in fig. 42·16.

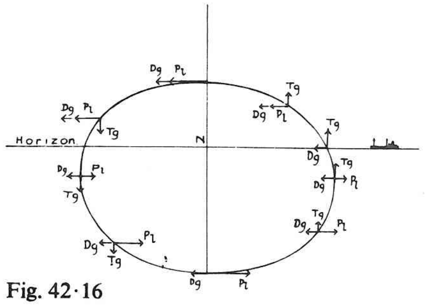


Fig. 42·16

8. Damping

The gravity controlled gyro described above is useless as a compass: the axle would perpetually oscillate about the meridian. In a gyro compass provision must be made to cause the oscillating axle ultimately to settle in the meridian. This is achieved by introducing a Damping Torque.

The elliptical path traced out by each end of the axle of a liquid gravity controlled gyro is made up of two component oscillations, one about the vertical, known as the Azimuthal Oscillation; and the other about the horizontal, known as the Tilt Oscillation.

In the Sperry Mark 14 compass a slight torque about the vertical axis of the gyro wheel is introduced. This torque causes precession which damps the tilt oscillations. In the Brown compass a torque about the horizontal axis causes precession which damps the azimuthal oscillations.

9. The Natural Errors of the Gyro Compass

A gyro compass on board ship may be affected by three Natural Errors. These are called: Latitude Error; Rolling Error; and Latitude/Course/Speed Error.

Latitude Error—This error arises when the damping torque acts about the vertical axis of the gyro wheel. Because of this, in every Latitude except the equator, the axle of such a gyro settles off the meridian with a slight tilt.

In the northern hemisphere the north end of the gyro axle settles east of the meridians with an up tilt. In southern Latitudes the north end of the axle settles west of the meridian with a slight down tilt. The horizontal angle between the vertical plane through the gyro axle and the true meridian due to this is known as Latitude Error.

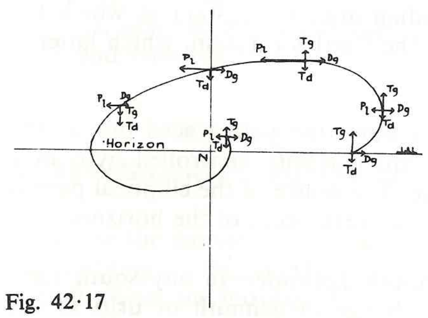


Fig. 42·17

Fig. 42·17 represents the path traced out by the projection of the north end of the gyro axle of a Sperry Mark 14 compass in a northern Latitude. In the diagram *d*, represents the effect of the damping torque.

If the north end of the gyro wheel is set horizontal and east of the meridian, tilting due to gravity causes the mercury in the control system to flow to the south pot. A torque due to the out-of-balance of mercury causes the north end of the axle to precess towards the meridian. Damping takes effect such that the north end reaches the meridian with a slight down tilt. The axle ultimately settles in a position at which the damping torque is equal and opposite to tilting due to gravity, and the precession due to the out-of-balance of mercury is equal and opposite to the drifting due to gravity.

Latitude error increases as the Latitude increases. In Latitude 50° N., the north end of the axle of a Sperry Mark 14 compass settles about 1½° to the east of the meridian with an up tilt of about 2′ of arc.

Because Latitude error is due to the method of damping it is sometimes known as Damping Error.

Rolling Error—When a ship rolls or pitches, the gyro is subjected to athwartship and fore-and-aft accelerations which may cause error in two way ways. The first is due to the asymmetry of the gyro, the mass of which in the east-west plane, that is in the plane of the wheel, is greater than that in the north-south plane.

Any mass which is caused to swing tends to align itself such that the plane of greatest moment of inertia lies in the plane of the swing. To compensate rolling error due to this cause, the gyro is fitted with Compensator Weights in north-south alignment so that the moment of inertia of the gyro in the plane of the axle is equal to that in the plane of the wheel.

Rolling error may also be caused in some compasses through the surging of the liquid in the gravity control when the ship rolls or pitches. When the ship rolls or pitches the liquid in the control system is subjected to accelerations resulting, in general, in an out-of-balance of liquid in the system. This provides a torque which causes error.

It may be shown that rolling error is maximum on intercardinal headings and nil on cardinal headings.

Latitude/Course/Speed Error—This error is sometimes referred to as Steaming Error. The tilting of a gyro axle is due to the Earth's rotation only when the gyro is placed on a stationary ship. Tilting, as we have seen, is maximum where the Earth's circumferential speed is maximum, that is at the equator.

The tilting of the gyro axle of a compass on board a moving ship is due to the resultant of the Earth's circumferential speed and the speed of the ship over the Earth's surface. In all cases, the gyro axle settles at right angles to the direction in which it is being carried through space.

On a ship moving due east or due west tilting is increased or decreased respectively, but the settling position will tend to be on the meridian. If however the gyro is carried in any direction other than east or west, the settling position will tend to lie off the meridian.

The axle of the gyro tends to settle in the plane of the Virtual Meridian, this lying at right angles to the resultant motion of the gyro through space. The horizontal angle between the vertical planes of the virtual and true meridians is called Latitude/Course/Speed Error.

Latitude/Course/Speed error is usually very small because the ship's speed over the ground is small relative to the Earth's circumferential speed.

Latitude/Course/Speed error depends not only upon the speed of the ship but also upon her course and the Latitude. It is the northerly or southerly component of the ship's speed which is responsible for this error. The error varies from zero when the course angle is 90°, to a maximum when the course angle is zero. It varies, therefore, as the cosine of the course angle. The Earth's circumferential speed varies with Latitude, it being 900 knots at the equator and zero at the pole. The speed therefore, varies as the cosine of the Latitude, the Latitude/Course/Speed error being least in Latitude 0° increasing as the Latitude increases.

Fig. 14·18 shows that Latitude/Course/Speed error may be computed for any combination of Latitude, course ad speed.

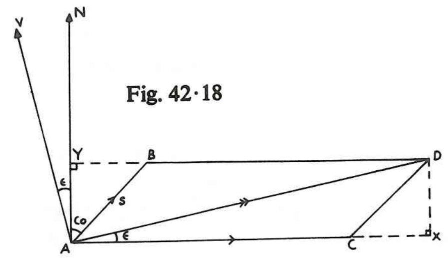


Fig. 42·18

In fig. 42·18:

AN represents the true meridian.

AB represents the ship's velocity, where *s* is the ship's speed.

AC represents the Earth's circumferential velocity.

By the parallelogram law, *AD* represents the velocity of the gyro through space. *VA* represents the virtual meridian which lies at right angles to *AD*.

$$NAV = DAC = \text{Latitude/Course/Speed Error} = \epsilon$$

$$\tan \epsilon = \frac{DX}{AX}$$

$$= \frac{AY}{AC + BY}$$

$$= \frac{s \,.\, \cos co}{900 \,.\, \cos Lat. + s \,.\, \sin co}$$

Had the course been westerly with the same course angle:

$$\tan \epsilon = \frac{s \,.\, \cos co}{900 \,.\, \cos Lat. - s \,.\, \sin co}$$

In general:

$$\tan \epsilon = \frac{s \,.\, \cos co}{900 \,.\, \cos Lat. \pm s \,.\, \sin co}$$

Because the error ε is a small angle:

$$\epsilon \text{ in radians} = \frac{s \,.\, \cos co}{900 \,.\, \cos Lat. \pm s \,.\, \sin co}$$

Now *s* . sin co is a small quantity and it may normally be ignored without introducing material error.

Therefore:

$$\epsilon \text{ in radians} = \frac{s \,.\, \cos co}{900 \,.\, \cos Lat.}$$

$$\epsilon \text{ in degrees} = \frac{s \,.\, \cos co}{900 \,.\, \cos Lat.} \,.\, \frac{180}{\pi}$$

$$\epsilon^\circ = \frac{s \,.\, \cos co}{5\pi \,.\, \cos Lat.}$$

For northerly courses error is West and gyro reads too high.
For southerly courses error is East and gyro reads too low.

Example 42·3—Find the Latitude/Course/Speed error in 60° N. when steaming at 30 knots due South.

$$\text{Speed Error} = \frac{s \,.\, \cos co}{5\pi \,.\, \cos Lat.}$$

$$= \frac{30}{5\pi \,.\, ½}$$

$$= \frac{12}{\pi}$$

$$= 4^\circ \text{ W. approx.}$$

Answer—Error = 4° W. or gyro reads too LOW.

10. Ballistic Deflection

The gyro compass is pendulously supported, so that when the ship accelerates, that is to say when she changes course and/or speed, the force causing the acceleration acts at the point of support of the gyro. The inertia of the gyro causes it to move out of the vertical plane, but the gyroscopic inertia of the wheel causes the plane of the spin to remain the same. The liquid in a gravity control that may be fitted to the compass, being mobile, surges from the pot in the direction of the acceleration, to the other pot. The excess of liquid in the pot lying in the direction opposite to that of the acceleration, provides a torque which causes the axle to precess. The amount of precession consequent upon an acceleration is known as Ballistic Deflection.

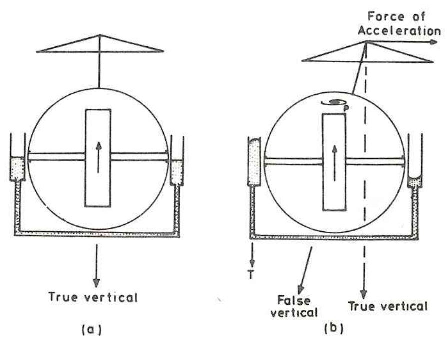


Fig. 42·19

Fig. 42·19(*a*) illustrates the gyro compass having a liquid control in a ship travelling due north at uniform velocity. Fig. 42·19(*b*) illustrates the same compass when the ship increases her speed towards the north.

When a ship accelerates the angle between the vertical planes of the virtual and true meridians changes and, therefore, the settling position of the gyro axle changes an amount equal to the change in Latitude/Course/Speed error consequent upon the acceleration. Referring to example 42·3, if the ship altered course to due south at 30 knots, the Latitude/Course/Speed error would change to the extent of 8°, that is from 4° W. to 4° E. This means that the virtual meridian would change its direction by the same amount, that is 8°. Now it is important that the gyro axle is always in the vertical plane of the virtual meridian. Therefore, when a ship accelerates, the gyro axle must be made to follow the virtual meridian. This is achieved by making the liquid gravity control of such a magnitude that the ballistic deflection is exactly equal to the change in course and/or speed error. The magnitude of the control influences the Period of the compass, this being the time taken for the axle to swing through one oscillation about the meridian before it settles on the meridian. The gravity control has the correct magnitude—the magnitude being influenced by the distance between the pots; the diameters of the pots and the interconnecting tube; and the density of the liquid used—when the period of the undamped gyro is about 85 minutes.

11. Ballistic Tilt

In the Sperry Mark 14 compass the liquid control is secured to the rotor case slightly to the east of the vertical through the centre of the wheel. This provides for damping. When the ship accelerates the surge of the liquid in the gravity control causes, not only a torque about the horizontal axis which causes precession about the vertical plane, but also a small torque about the vertical axis which causes precession about the horizontal plane. This latter precession is known as Ballistic Tilt. The ballistic tilt is very much smaller than the ballistic deflection for any given acceleration, because the offset of the point of attachment of the liquid control to the gyro case is very small.

Ballistic tilt causes the axle to wander slightly after an alteration of course and/or speed, because of the inclination of the axle. This wandering is referred to as Ballistic Tilt Effect.

Exercises on Chapter 42

1. Describe a mechanical gyroscope and the two properties of all spinning bodies for which the mechanical gyroscope may be used to demonstrate.
2. Give examples of gyroscopic inertia.
3. Explain the principle of the conservation of angular momentum, and thence explain why the axle of a spinning body tends to maintain a fixed direction in space.

4. Explain why a gyro axle precesses when a torque is applied at right angles to the axle at one of its ends.
5. State a rule for ascertaining the direction of precession for any given torque and direction of wheel spin.
6. Using your knowledge of the shape and motions of the Earth and that of elementary mechanics, explain the precession of the equinoxes.
7. Distinguish between a Free and Controlled gyroscope.
8. What is the function of a gravity control?
9. Describe a simple form of liquid gravity control.
10. Distinguish between tilt and tilting, and drift and drifting.
11. What is meant by damping? Explain how the oscillations of a liquid gravity controlled gyroscope are damped.
12. State the natural errors of a gyro compass.
13. Discuss latitude error and its cause.
14. Discuss rolling error and its causes and compensation.
15. What is Steaming error? Derive a formula for finding steaming error for any combination of Latitude, course, and speed.
16. Define Virtual meridian. Explain how a gyro compass maintains the virtual meridian when the ship on which the compass is fitted, changes her course and/or speed.
17. Describe ballistic deflection and ballistic tilt.
18. Compare the usefulness of a magnetic compass with that of a gyroscopic compass.

CHAPTER 43

THE SEXTANT AND CHRONOMETER

1. Description of Sextant

The sextant is an instrument used for measuring the angle between two distant objects. It may be used in Coastal Navigation for measuring horizontal angle between two conspicuous shore objects, or for measuring the vertical angle of a lighthouse or peak.

In the practice of Nautical Astronomy the sextant is used for measuring altitudes of celestial bodies. The angle, in this case, is the angle at the observer's eye between the direction of the celestial body and that of the visible horizon measured in the plane of the vertical circle on which the body lies.

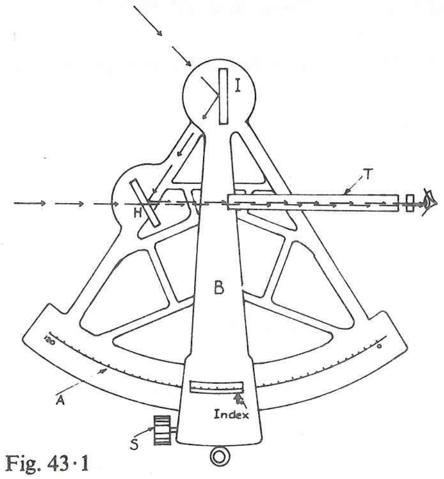


Fig. 43·1

The sextant consists of a Metal Frame on which is fitted a Graduated Arc denoted by *A* in fig. 43·1. At the centre of the circle of which the arc forms part is pivoted a bar *B* on which is a Pointer or Index to facilitate the reading on the arc. Fitted perpendicularly to the Index Bar is a metal frame in which is housed the Index Mirror indicated by *I* in fig. 43·1. Another metal frame houses the Horizon Glass indicated by *H*. The horizon glass is half silvered: the half which is nearer to the Plane of the Sextant, which is defined as the plane on which the arc lies, is silvered, the other half being clear glass. Both the index mirror and the horizon glass may be adjusted so that they may be set perpendicular to the plane of the sextant.

An Adjustable Collar is sometimes fitted to the sextant frame. The purpose of this is to house a telescope indicated by *T* in fig. 43·1. The collar is adjusted so that the axis of the telescope lies parallel to the plane of the sextant. The axis of the telescope passes through the horizon glass, and the telescope may on some instruments be adjusted so that the amount of the silvered portion of the horizon glass visible through the telescope may be altered to suit the observer and the observation. The device which facilitates the adjustment is known as the Rising Piece. Most modern sextants are not provided with this device.

The index bar may be clamped to any position along the arc of the sextant. For fine adjustment of the position of the index bar a Tangent Screw indicated by *S* in fig 43·1 is used; or, alternatively, the sextant may be fitted with a Micrometer.

The sextant is provided with several tinted glass Shades of different intensities, which are used to reduce the brilliance of the Sun, and sometimes the horizon, when observing the Sun's altitude. The shades at the index mirror are known as the Index Shades, and those at the horizon glass, the Horizon Shades.

2. The Sextant Telescopes

A good sextant outfit is usually provided with two telescopes. The telescope of higher magnification is an Astronomical or Inverting telescope, so called because the image appears inverted. The inverting telescope should be used for all observations of the Sun when the horizon is clear and the vessel reasonably steady. The high magnifying power of this telescope ensures an accurate grazing contact of the reflected image of the Sun's limb with the direct image of the horizon. The inverting telescope should also be used for measuring vertical angles of peaks and lighthouses, because, in this type of observation, great accuracy of the measured angle is essential to ensure a good position line.

The magnifying power of a telescope has an effect on the brightness of the image. The lower the magnifying power of a telescope, for a given object glass, the brighter is the image and the larger will be the field of view. Daylight observations, using the inverting telescope are not affected by the comparatively low degree of brilliance of the objects observed, nor by the small field of view. The inverting telescope, however, is useless for twilight observations of stars, hence the necessity for providing a second telescope.

The Erecting Telescope provided in the sextant outfit is bell-shaped. It has a large object glass and a low magnifying power. The large object glass provides for a large field of view, and the low magnifying power provides for a very bright image. These two properties are necessary for star observations, for which reason this telescope is known as the Star Telescope.

Many navigators use the star telescope for all observations. This is, no doubt, due to the slight difficulty in using the inverting telescope. And, in fact, because navigators in the past have tended to ignore the inverting telescope—using the star telescope for all observations—it is rare for a modern sextant outfit to include an inverting telescope. It is fair to add that the telescope provided in modern sextant outfits is generally a satisfactory all-purpose telescope.

3. The Principle of the Sextant

The principle of the sextant is based on the two simple laws of optics. The first, which is illustrated in fig. 43·2, is:

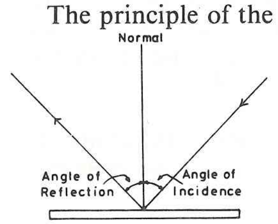


Fig. 43·2

"When a ray of light strikes a plane mirror, the angle incidence is equal to the angle of reflection".

The second law of optics is:

"The incident ray, the normal, and the reflected ray all lie in the same plane".

It follows that if a ray of light is doubly reflected by two plane mirrors, the angle between the first incident ray and second reflected ray is twice the angle between the mirrors. This is illustrated in fig. 43·3.

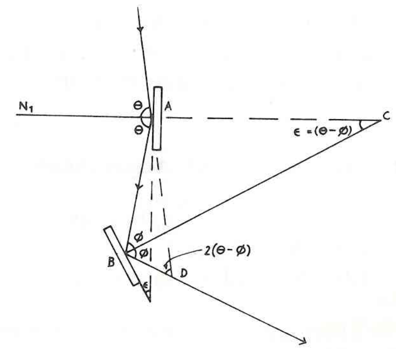


Fig. 43·3

Let A and B in fig. 42·3 be two mirrors. Let the first angle of incidence be θ, and the second angle of reflection be φ.

Angle between the mirrors = Angle between their normals

$$= \epsilon$$

In triangle ABC: $ACB = (\theta - \varphi)$

In triangle ABD: $ADB = 2\theta - 2\varphi$

$$= 2\,(\theta - \varphi)$$

Therefore: $ADB = 2\,.\,ACB$

Hence the angle between the first incident ray and second reflected ray is twice the angle between the mirrors.

Fig. 43·4 serves to illustrate that when a plane mirror is rotated through any given angle the angle which the reflected ray turns is equal to twice the angle through which the mirror has been rotated.

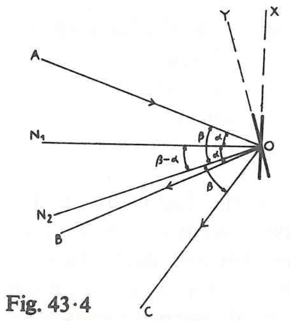


Fig. 43·4

Suppose the plane mirror illustrated in fig. 43·4 is rotated through an angle XOY.

Let the first angle of incidence be α
Let the second angle of incidence be β
Angle through which mirror rotates $= N_1ON_2$
$= (\beta - \alpha)$
Angle through which reflected ray rotates $= BOX$
$= 2\beta - 2\alpha$
$= 2\,(\beta - \alpha)$

Therefore, the angle through which the reflected ray rotates is twice the angle through which the mirror rotates.

For this reason the arc of a sextant is graduated to indicate twice the angle through which the index bar—and hence the index mirror—rotates from the zero mark on the scale. Although the sextant arc extends to only about 60%, the instrument may be used to measure angles up to about 120°.

Angles measured with the sextant may be determined with an accuracy of 10″ of arc. This is achieved by fitting either a Vernier or Micrometer to the index bar. Most modern sextants are fitted with popular micrometer.

4. The Errors and Adjustments of the Sextant

The errors of a sextant are classified under two headings:
(1) Adjustable Errors.
(2) Non-adjustable Errors.

There are four adjustable errors: These are error of perpendicularity; side error; index error; and collimation error.

Error of Perpendicularity is due to the index mirror not being perpendicular to the plane

of the sextant. To ascertain if a sextant has error of perpendicularity, hold the sextant face upwards with the arc away. Move the index bar along the arc until the true arc and the reflected arc from the index mirror may be seen simultaneously. If the true and reflected images of the arc in alignment the sexant is free from this error. If they are not, the adjusting screw at the back of the frame, which holds the mirror, must be turned until they are in line so as to adjust the sextant for this error.

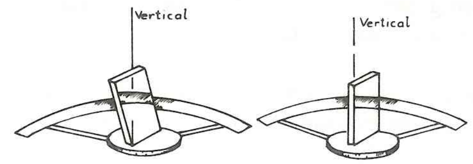


Fig. 43·5

Fig. 43·5 (*a*) indicates the appearance of the true and reflected images of the arc in a sextant which has error of perpendicularity.

Fig. 45 (*b*) depicts the same sextant when the error has been removed.

If the horizon glass is not perpendicular to the plane of the sextant, Side Error exists. To detect the presence of this error, the index bar should be adjusted until the true image of the horizon, as seen in the unsilvered part of the horizon glass, is in alignment with the reflected image as seen in the silvered part of the horizon glass. If the true and reflected images remain in alignment when the sextant is rotated about the line of sight, the horizon glass is perpendicular to the plane of the instrument and the sextant is free from side error. If the images are not in alignment when the sextant is rotated, the adjusting screw at the back of the horizon glass frame must be turned until they are in alignment, so as to adjust the Side Error.

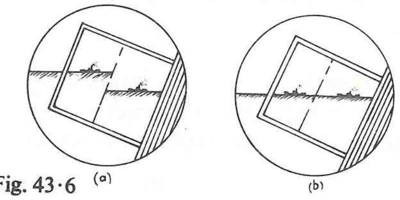


Fig. 43·6

Fig (43·6 (*a*) illustrates the appearance of the true and reflected images of the horizon in a sextant which has side eror. Fig 43·6 (*b*) shows the same sextant when side error has been removed.

The index mirror and the horizon glass should be parallel to one another when the index of the index bar is at the zero mark of the scale: if they are not, Index Error exists. To ascertain if the sextant is free from index error, the sextant is held vertically, and the index bar adjusted until the true and reflected images of the horizon appear in alignment. When this is so, the index on the index bar coincides with the zero mark on the scale. If the index is not at the zero mark, the reading is referred to as the Index Error of the Sextant.

Index error may be removed by means of the adjusting screw at the back of the horizon glass frame. Note that there are two screws at the back of the horizon glass frame: one at the top of the frame and the other at the bottom. The top one is for adjusting the horizon glass so as to remove side error. The lower screw is used to eliminate index error.

An alternative method for ascertaining index error is by means of a star. If the true and reflected images of a star are in exact coincidence when the index bar is at zero, no index error exists. When using this method the reflected image of the star is observed at the very edge of the mirror of the horizon glass; that is to say on the line which divides the silvered from the unsilvered part of the horizon glass.

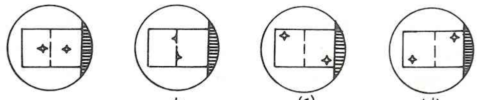

(a) (b) (c) (d)

Fig. 43·7

Fig. 43·7 illustrates the process of removing index error and side error using a star.

When adjusting a sextant it will be found that an adjustment for side error affects the adjustment for index error, and *vice versa*. Therefore, it is the correct practice to make these two adjustments simultaneously, by first removing the initial side error and the initial index error. When this has been done half the remaining side error is removed and half the remaining index error is removed—and so on until both errors are eliminated.

Using the star method for adjusting side and index error, if side error but no index error exists, the true and reflected images of the star appear as in diagram (*a*) of fig. 43·7.

If index error but no side error exists, the images appear as in diagram (*b*).

If both errors exist, the images appear as in diagrams (*c*) or (*d*).

Collimation Error exists when the axis of the telescope is not parallel to the plane of the sextant. Collimation error may be detected by the method outlined as follows, if the sextant outfit includes an inverting telescope.

Ship the inverting telescope and set the eyepiece so that one pair of cross wires is parallel to the plane of the sextant. Select two stars which are at least 90° apart, and adjust the index bar until the true image of one and the reflected image of the other are in perfect contact on one of the cross wires. Tilt the sextant until the images are on the other cross wire. If they close or separate, the telescope is not parallel to the plane of the sextant. It should be made so by adjusting the two screws of the collar of the telescope.

In many modern sextants the telescope collar is permanently set and no adjustment is possible. It is important when using the sextant to ensure that the observed objects occupy the centre of the field of view of the telescope. If they do not collimation error will result even if the telescope axis is correctly set.

Most modern sextants are manufactured by highly skilled craftsmen, who have at their disposal tools of the highest precision. Modern sextants, therefore, leave the hands of the makers in a state as near to perfection as is possible. However, it is not unlikely that many old sextants do possess certain errors which cannot be adjusted. Most important of these non-adjustable errors is Centring Error. This error is due to the axis of the index bar not coinciding with the centre of the circle of which the arc forms part. It is very likely that an old well-used sextant possesses centring error on account of the wear of the bush on which the index bar pivots.

Other non-adjustable errors are:

(1) Shade Error due, to the surfaces of the coloured shades not being parallel to each other.

(2) Prismatic Error, due to the surfaces of the index mirror or horizon glass not being parallel to each other.

(3) Graduation Error, due to inaccuracy in the dividing of the vernier or micrometer and/or the arc.

5. Using the Sextant

(i) *Observing Horizontal Angles*—The star telescope is used for horizontal angle observations. The sextant is held horizontally face up. The index bar is set to the zero mark on the scale. The right-hand object is observed. As the index bar is rotated to higher readings the sextant is turned about a vertical axis, so that the reflected image of the right-hand object is kept in the silvered portion of the horizon glass. When the left-hand object is seen through the unsilvered part of the horizon glass the index bar is clamped and the tangent screw or micrometer is used to make an accurate contact between the reflected image of the right-hand and the direct image of the left-hand object.

If the right-hand object is indistinct, or if it is a flashing light, and the left-hand object is a fixed light, it is better to hold the sextant face downwards, and bring the reflected image of the left-hand object into coincidence with the direct image of the right-hand object.

(ii) *Observing Vertical Angles*—The inverting telescope should be used for observing vertical angles. It is a good plan to observe the vertical angle both On and Off the Arc, using the Arc of Excess to measure the angle off the arc. The arc of excess is the part of the graduated arc which lies to the right of the zero mark on the scale. The average of the two readings should then be taken as the vertical angle.

(iii) *Observing a Star's Altitude*—The star telescope is shipped and focused and the sextant held vertically with the index set at zero. The star is observed, and the true and reflected images distinguished. The index bar is now rotated and the sextant tilted about a horizontal axis simultaneously, so that the reflected image of the star is kept in view in the silvered part of the horizon glass, until the horizon is seen through the unsilvered part. The index bar is then clamped. To ensure that the arc of a vertical circle is being measured, the sextant is rocked gently to and fro, so that the reflected image of the star appears to describe an arc of a circle. The tangent screw is adjusted so that this arc just grazes the horizon, as the sextant is rocked.

If the horizon is indistinct it is better to hold the sextant vertically with the arc uppermost, and to bring the reflected image of the horizon up to the true image of the star. When this has been accomplished, the sextant is held right way up and the tangent screw used for the accurate measurement.

(iv) *Observing the Sun's Altitude*—To observe the Sun, the inverting telescope is shipped and focused. Appropriate glass shades are turned down into position, and the sextant held with its plane vertically. The observer faces the direction of the Sun, and the index bar is gently rotated to and fro in the vicinity of the arc which reads the approximate altitude of the Sun until the reflected image of the Sun is observed through the silvered portion of the horizon glass, at the same time as the direct image of the horizon is visible through the unsilvered portion. The tangent screw is then used to make the fine adjustment.

6. Care of the Sextant

At all times the sextant should be handled with the greatest of care. The slightest knock may derange the adjustments or permanently damage the instrument.

The silvering of the mirrors may be impaired if moisture is allowed to remain on the mirrors so that after using the sextant, the surfaces of the index mirror and the horizon glass should be wiped carefully and lightly with a piece of soft chamois, which is kept specially for the purpose. The working parts of the sextant should occasionally be lightly smeared with high grade lubricating oil to reduce the rate of wear of working parts.

It may be thought that a modern sextant remains in perfect adjustment for an indefinite period. Experience shows that this is not so. Navigators are advised, therefore, to check the adjustments of their sextants frequently. Rather than meddle with the adjusting screws to the extent that the threads were causing the screws to become loose, it is better to measure the extent of the error and to apply it to the sextant reading.

7. The Chronometer and its Care

A chronometer is an accurate timekeeper suitable for keeping time, for nautical astronomical purposes, on a lively vessel at sea and which is capable of maintaining a uniform and small rate of gaining or losing.

We have seen in chapter 29 that time and Longitude are intimately related, and that the essence of Longitude-finding at sea by nautical astronomical methods is the comparing of L.M.T. with G.M.T. The L.M.T. is found from astronomical observation and the chronometer provides G.M.T. if its error is known.

In recent years precision chronometers, capable of maintaining an accuracy of 0·1 seconds per week, have become available: these employing quartz crystal oscillators.

A quartz chronometer provides a digital read-out and operates from the mains supply. Should this fail a built-in battery circuit automatically takes over. The instrument requires minimal maintenance and attention. The traditional chronometer, on the other hand, is a mechanical time-piece which depends on the energy of a wound mainspring which, through a train of gears, is transmitted to the hands which register the time on a dial of the usual clockface style. The important feature of this type of chronometer is the provision of compensation for changes in temperature—without which the going of the time-piece would be so erratic as to render the instrument entirely unfit for nautical astronomical purposes.

A mechanical chronometer is usually designed to run for about 56 hours without rewinding. Such an instrument is known as a Two-Day Chronometer. To ensure a systematic routine for winding, so as to ensure that the daily rate is kept as steady as possible, the chronometer should be wound at the same time each day, and preferably by the same officer.

The chronometer is mounted on gymbals and housed in a locker in the chartroom which should be dust- and draught-free, and insulated to offset the effects on the chronometer of temperature changes. To wind the chronometer the instrument is turned on its side, within its gymbals, and the metal guard covering the keyhole slid back. The key is then inserted and the chronometer wound until the key butts—this normally requiring about seven half-turns. The winding key, known as a Tipsy Key, is designed so that it cannot be turned in the wrong direction. The tipsy key is also used for resetting the hands of the chronometer should this become necessary; and, in no circumstances, should the hands be altered except by means of the tipsy key. A small dial on the face of the chronometer indicates the state of winding.

A chronometer should be cleaned and re-oiled at intervals of about two years. This should be done by the manufacturer or by an approved instrument maker. When it is

necessary to transport a chronometer the balance wheel should be wedged by means of two thin cork wedges cut for the purpose. To do this it is necessary to remove the instrument from its box and to lift out the working mechanism from its brass case. This requires the greatest of care: the glass front is first unscrewed and the brass case is inverted, the key being used if necessary to ease the mechanism from the case.

8. Use of Chronometer

In general an altitude observation must be timed by the chronometer. Ideally an assistant should be employed to record the time of a sight; although, with practice at counting seconds, the observer himself is able to time his own sights with accuracy.

When recording a chronometer time the three hands should be read in descending order of the rapidity of their motions. This means that the seconds should be read first; then the minutes; and finally the hours. It is customary to record the chart-room clock time as well, so that a future check may be made if necessary.

The error of the chronometer which, together with its daily rate, is recorded in the Chronometer Journal kept by the navigating officer, is to be applied to the chronometer reading to obtain the corresponding G.M.T.

The error of the chronometer should be checked frequently—at least once daily when the vessel is at sea—by means of Radio Time Signals, full particulars of which are to be found in the *Admiralty List of Radio Signals, Volume 5.*

Exercises on Chapter 43

1. Describe the angles which a sextant is designed to measure.
2. Describe the construction of a sextant.
3. Discuss sextant telescopes and their uses.
4. Define: Rising Piece; Plane of the Sextant.
5. State the laws of optics on which the principle of the sextant is based.
6. Prove that when a plane mirror is rotated the angle through which a reflected ray turns is twice the angle through which the mirror is rotated.
7. Explain why the sextant arc, which is an arc of 60°, is graduated from 0° to 120°.
8. Enumerate the errors of the sextant.
9. Explain carefully how you would detect and eliminate Side Error.
10. Explain how you would detect and eliminate Error of Perpendicularity.
11. Explain why Side Error and Index Error are eliminated simultaneously. Explain how you would eliminate these errors using a star.
12. What is Collimation Error: How would you detect collimation error in a sextant, and how would you remove it?
13. Describe the non-adjustable errors of a sextant.
14. Explain how a sextant is used for measuring the altitude of a star.
15. Explain how a Sun-sight is made with a sextant. Why should an inverting telescope be used for Sun observations: assuming that one is available?
16. Write an essay on the Care of Sextants.
17. Describe, in detail, the process of timing a sight.

18. Discuss the chronometer and the care it should receive by the navigating officer.
19. Explain how a chronometer should be prepared for transport from a vessel to the shore.
20. Examine the *Admiralty List of Radio Signals*, Volume 5, and describe the "English" and the "ONOGO" systems of radio time signal transmissions.

CHAPTER 44

SOUNDING INSTRUMENTS AND LOGS

1. The Lead Line and Mechanical Sounding Machine

When navigating in coastal waters in thick weather, a knowledge of the depth of the water under the keel is often of great value in affording the means of checking a vessel's position. The earliest method of ascertaining the depth of water was by means of the Lead Line. The Hand Lead Line was used for measuring shallow water depths of up to about 20 fathoms, and the Deep Sea Lead Line was used for measuring depths of up to about 100 fathoms. The cumbersome method of sounding by means of a lead line has been superseded by the Kelvin Sounding Machine and the Kelvin Sounding Tube; and, more recently, by the Echo-sounder.

2. The Sounding Machine and Sounding Tube

The sounding machine consists of a metal frame in which is housed a drum controlled by a suitable brake. Around the drum is wound about 300 fathoms of fine guage galvanised steel wire. A brass case, in which is fitted the sounding tube, is secured to the outboard end of the sounding wire. To the end of the brass case attachment is secured a 28 lb. sinker. The drum is fitted with crank handles or an electric motor so that the wire may be wound in after a cast has been taken. The Kelvin sounding tube is a glass tube of small bore sealed at one end. The inside of the tube is coated wth a chemical composition which changes colour when acted upon by salt water. The sounding tube is placed in the brass container with its open end downwards. The sounding wire is led through a suitable fairlead, and the sinker and container hung over the ship's side. When ready to cast the brake of the sounding machine is released, whereupon the sinker carries the sounding tube to the sea bed. With increasing depth the water pressure causes the air within the sounding tube to be compressed into an increasingly smaller volume. The sea water which enters the tube discolours the chemical coating. The depth of water corresponding to the length of discolouration is obtained from a graduated boxwood scale.

The principle of the sounding tube is based upon Boyle's law, which is:

> "The volume of a given mass of gas at constant temperature varies directly as the pressure".

In other words, the product of the volume and the pressure of a given mass of gas at a uniform temperature is a constant amount.

A useful rule, the proof of which illustrates Boyle's Law as it applies to the sounding tube, is stated and derived as follows:

> The air pressure at sea level is equal to the pressure of a head of sea water about 33 feet or 5·5 fathoms.

Suppose the length of discolouration of a sounding tube is W (wet), and the length of the remaining part D (dry). Let the sounding be S fathoms.

Then: by Boyle's law:

$$\text{Pressure} \times \text{Volume} = \text{constant}$$

so that:

$$A \,.\, D(5{\cdot}5 + S) = A(W + D) \,.\, 5{\cdot}5$$

where A is the cross sectionasl area of the sounding tube.

Therefore:

$$5{\cdot}5 \,.\, D + D \,.\, S = 5{\cdot}5 \,.\, W + 5{\cdot}5 \,.\, D$$

From which:

$$S = \frac{W}{D} \,.\, 5{\cdot}5$$

Example 44·1—A sounding tube 20 inches long is discoloured for 16 inches. Find the approximate depth to which it had descended.

$$\text{Depth} = \frac{16}{4} \times 5{\cdot}5 \text{ fathoms}$$
$$= 22 \text{ fathoms}$$

Answer—Depth = 22 fathoms.

Example 44·2—A sounding tube is found to be discoloured for exactly half its length. Find the depth to which it had descended.

$$\text{Depth} = \frac{W}{D} \times 5{\cdot}5$$

$$= \frac{W}{W} \times 5{\cdot}5 \quad \text{because } W = D$$

$$= 5{\cdot}5 \text{ fathoms}$$

Answer—Depth = 5·5 fathoms.

Other types of sounding tube appeared on the market after the advent of Lord Kelvin's chemical tube. Notable amongst these is the Wigzell tube, which is a plastic tube sealed at one end, and having a cap fitted with a non-return valve at the other end. When the tube is lowered into the sea, water is driven into the tube owing to the increase of water pressure. As the tube is being hove up, the valve prevents the entrapped water from escaping from the tube. The length of the entrapped water column is read against a suitable boxwood scale on which depths are marked.

3. The Echo-sounder

The principle of echo sounding is simple, althought the sounding instrument is a very complex and delicate piece of equipment. If the time interval between the instant of transmission of a sound pulse and the receipt of the echo pulse from the sea-bed is measured, the depth of water may be found by a simple calculation, the speed of sound in sea water

being known. Althought the speed of sound in sea water varies with the temperature and salinity of the water, the average speed of 800 fathoms per second is sufficiently accurate for sounding purposes in almost all cases.

The echo sounder consists of three basic parts, these being a Transmitter, a Receiver, and a Recorder. An electrical impulse causes a pulse of sound energy of high frequency, to be sent out from the transmitter which is fitted to the bottom of the hull of the vessel. The sound energy is reflected from the sea-bed and is returned to the vessel where it is received, as a low energy pulse, by the receiver. The receiver, on receipt of the echo pulse, sends a faint electrical impulse to a valve amplifier, which converts the weak signal into a comparatively strong electrical current.

Fitted in the recorder is a roll of sensitive paper saturated with an iodine solution. The paper is drawn by means of an electric motor, across the Platen, which is merely a metal plate. A stylus is caused to sweep across the paper starting its traverse each time a pulse of sound energy is transmitted. The incoming impulse causes an electric current to pass through the paper from the stylus to the platen. The current passing through the recording paper causes the paper to be marked, through electrolytic action, at the point occupied by the stylus at the instant the incoming pulse is received. The paper moves across a graduated scale against which the depth of water may be read.

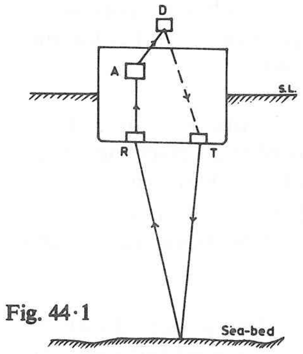


Fig. 44·1

Fig. 44·1 illustrates diagrammatically the essential parts of an echo-sounder. T represents the transmitter, R the receiver, A the valve amplifier, and D the recorder.

4. Logs

The earliest device for measuring the speed of a vessel through the water consisted of a piece of wood in the form of a quadrant, weighted on its curved edge so that it floated more or less vertically. This was secured by means of a three-legged bridle, to the Log-Line. Made fast to the log-line at equally spaced intervals were pieces of knotted cord. The knotted cord nearest to the Log-Ship—the name given to the quadrant of wood—had one knot; the next, two knots; the next three, and so on. The log-line was wound around a wooden reel.

When measuring the speed of the vessel the log-reel, log-line, and log-ship, were assembled aft. The log-ship was hove into the wake and the log-line allowed to run freely from the reel as the vessel sailed away from the log-ship. After a certain amount of line had been rendered—sufficient for the log-ship to have cleared the confused water of the wake—the hand in charge of the operation called out "turn", whereupon a sand glass would be turned. When the sand had ran out—usually in 14 or 28 seconds—the order "hold" was given, and the log-line held. The vessels's speed through the water was then found from the knotted cord nearest to the hand of the seaman who held the line.

The distance between successive knotted cords and the running time of the glass were proportional, respectively, to the length of a nautical mile and an hour. Thus, if say four knotted chords passed over the stern during the running time of the glass, the speed of the vessel would have been four knots.

It is from the old-fashioned knotted log-line that we get the name Knot, which denotes the navigational unit of speed. From the wooden log-ship we get the name Log which, nowadays, is used to describe any device that measures speed or distance through the water.

The mechanical log commonly in use at the present time registers not speed but distance travelled through the water since the log-clock was set to zero. The log-clock dial is usually graduated from 0 to 100 with secondary dials graduated from 0·0 to 10·0 and from 100 to 1000 miles.

Fastened to the log-clock, which fits into a shoe on the taffrail or at the end of a log-boom, is the log-line at the end of which is a brass rotator in the form of a screw.

The inboard end of the log-line is fitted with a wheel which, acting as a governor, ensures the smooth running of the clockwork mechanism of the log-clock.

To stream the log, the inboard end is first clipped onto the log-clock, and the rotator is clipped onto the outboard end. The rotator is lowered into the water, care being taken to prevent it knocking the vessel's hull and damaging the fins of the rotator, and allowed to stream astern until the line is taut.

To house or ship the log, the inboard end is disconnected from the governor and, as the rotator is hove in, the inboard end is allowed to run out over the stern. This is necessary in order that the turns in the line due to the rotator turning as the log is hove in, will come out; and the coiling of the line, starting at the rotor end, is thus facilitated.

The accuracy of a mechanical log is affected by the length of the log-line and the distance of the taffrail from the sea-surface. In general, the greater the speed of the vessel the longer should be the log-line. A line too short, especially if the taffrail is high above the sea, will cause the rotator to lie so near to the surface that it will be affected by waves; and, in rough weather, it will sometimes be dragged out of the water. For correct working the rotator should be well below the sea surface.

As a rough guide, for a speed of about 10 knots the log-line should be about 40 fathoms long; for 15 knots about 60 fathoms; and for 20 knots it should be about 80 fathoms. The correct length is best found by trial and error, comparing distances by log with those actually made through the water as found from observations.

The Taffrail Log, as the mechanical log described above is sometimes called, may incorporate a small dynamo, and the registering mechanism, in this case, conveniently may be fitted in the chart-room.

The Pitometer- and Chernikeeff-logs are sophisticated devices for measuring the ship's speed through the water.

The pitometer log depends for its action on a specially designed Pitot Tube which projects below the hull of the vessel. The orifice of the tube points ahead, and the pressure of water which enters the tube, and which thrusts on a float which is geared to the registering mechanism, is a function of the speed of the ship through water.

The Chernikeeff log consists of a small propeller which projects below the hull of the vessel, the speed of rotation of which is a function of the vessel's speed.

Exercises on Chapter 44

1. Describe a sounding machine, and explain clearly how a cast is made using a sounding machine.
2. Describe a sounding tube and the principle on which it is based.
3. Derive as simple formula, in terms of the lengths of the discoloured and un-discoloured lengths of a Kelvin sounding tube, for finding the depth to which the tube descended in fathoms.
4. Show that if the discoloured length of a Kelvin sounding tube is a quarter of the length of the tube, the tube has descended to about 11 feet.
5. Describe an echo-sounder of the type fitted on your ship.
6. Explain carefully how a taffrail log should be streamed and housed.

CHAPTER 45

RADIO DIRECTION FINDING

1. The Simple Detection Finder

The simplest form of a radio direction finder comprises a Loop Aerial, an Amplifying Unit, and a Headphone. The loop aerial consists of one or more turns of wire mounted in such a way that it may be rotated about a vertical axis. Radio signals from a transmitting station may be received by the loop aerial. The ends of the aerial are connected to the headphone by way of the amplifying unit.

The strength of the signal received is dependent upon the angle between the directions of the transmitting station and the plane of the loop aerial. When the plane of the loop aerial lies in the direction of the transmitting station the e.m.f. induced in the aerial, by the electro-magnetic energy which emanates from the transmitter, is maximum, and the signal strength is greatest. When the plane of the aerial lies at right angles to the direction of the transmitter, no e.m.f. is induced in the loop aerial and, consequently, the signal strength is zero.

The signal strength varies as the cosine of the angle which the plane of the loop aerial makes with the direction of the transmitter.

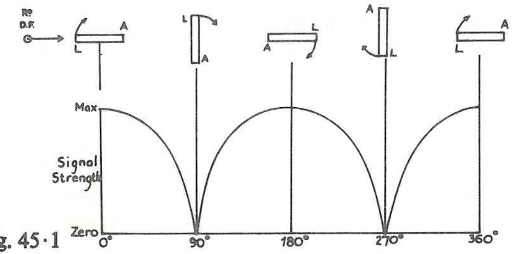


Fig. 45·1

Fig. 45·1 serves to illustrate graphically the relationship between the signal strength and the angle contained between the plane of the loop aerial and the direction of the transmitter. Imagine the loop aerial, represented in plan by *LA*, to be rotated clockwise as indicated in fig. 45·1. It will be noticed that the maximum signal strength remains more or less the same for an appreciable angle in the vicinity of 0° and 180° on the scale; and that the zero strength is approached very sharply when the angle between the plane of the loop aerial and the direction of the transmitter is 90° and 270°. The angle which the plane of the loop aerial makes with the vessel's fore-and-aft line is indicated by a Bearing Pointer which can be rotated within a graduated dial.

Because it is easier to discriminate the zero signal than the maximum signal, the bearing pointer is fixed at right angles to the directions indicated by the plane of the loop aerial, such that when the plane of the aerial is at right angles to the direction of the transmitter the signal strength is a minimum or zero. In other words, when the signal strength is zero the bearing pointer indicates the direction of the transmitter or its reciprocal direction.

In order to resolve the ambiguity—the 180°-Ambiguity, as it is called—a Sense-Finding Unit is fitted to the direction finder.

The sense-finding unit is provided with a vertical aerial which is separately connected to the amplifying unit, as is the loop aerial. The strength of the signal received by the sense aerial is adjusted so that it is the same as that received by the loop aerial when the latter lies in the direction of the transmitter. In other words, the strength of the signal received by the sense aerial is equal to the maximum received by the loop aerial.

Fitted to the graduated dial, in addition to the bearing pointer, is a Sense Pointer which is set at right angles to the bearing pointer.

When the sense aerial switch is made, the sense pointer indicates the direction of the transmitter (or its reciprocal direction) when the plane of the loop aerial is in alignment with the direction of the transmitter. If the sense pointer is then turned through 180° the strength of the signal, although a maximum, will have a different value from the maximum signal received when the sense pointer was set to its original direction.

The sense pointer is fitted relative to the bearing pointer such that when the former indicates the weaker of the two maximum signals, it also indicates the direction of the transmitter.

The principle of sensing is as follows: Depending upon whether the transmitter lies in a certain direction or the opposite direction, the e.m.f. induced in the loop aerial is altered in phase by 180°. The phase of the e.m.f. induced in the vertical aerial, on the other hand, is not affected by the direction of the transmitter. When the sense aerial switch is made the signals from both loop and vertical aerials are received. The e.m.f. in the loop aerial is a maximum when the plane of the aerial lies in the direction of the transmitter. When the loop aerial is rotated 180° from this direction, the strength is again maximum but the phase is different and it may be described as negative maximum.

Thus, if the signals from the loop aerial and the sense aerial are received simultaneously they will oppose each other when the sense pointer indicates the bearing of the transmitter, and reinforce each other when the sense pointer indicates the reciprocal bearing of the transmitter.

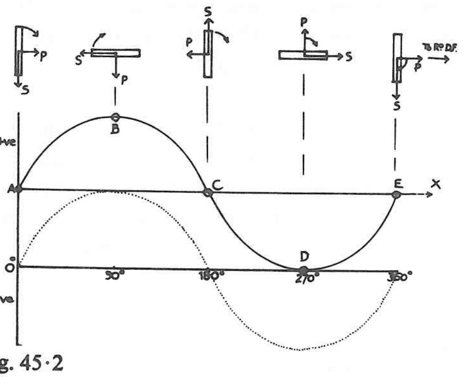


Fig. 45·2

Referring to fig. 45·2, *P* represents the bearing pointer and the *S* the sense pointer. The straight line *AX* represents the graph of the constant e.m.f. received by the sense aerial, and the dotted curved line represents the graph of the variable e.m.f. received by the loop aerial. The combined effect produces the curve *ABCDE*.

The simultaneous reception of the signals received by the loop and the sense aerial produces a single minimum and a single maximum for 360° rotation of the loop aerial. The single minimum occurs at a position on the graduated dial which is the bearing of the transmitter indicated by the sense pointer. The single maximum occurs at the opposite position.

2. The Bellini-Tosi Direction Finder

The Bellini-Tosi direction finder consists of a fixed "cross-loop" aerial instead of a rotating single loop aerial as in the simplest direction finder described in Paragraph 1. The ends of the loop aerials are led to two coils which are mounted so that their planes are at right angles to each other in an instrument known as a Goniometer. Within the two crossed coils of the geniometer is another coil—a small coil known as a Search Coil. The search coil may be rotated, and the e.m.f. induced in it is dependent upon the e.m.f.s induced in the crossed coils of the geniometer; and these e.m.f.s, in turn, depend upon the direction of the transmitter relative to the planes of the two cross-loop aerials.

The bearing and the sense pointers are fitted to the spindle which carries the search coil, and the method of ascertaining the bearing of a transmitter is similar to the method used with the simple direction finder.

3. Errors in Radio Direction Finding

(i) *Night Effect*—Hitherto, it has been assumed that the radio energy received by the aerial of a direction finder has travelled from the transmitter along a path which coincides with the great circle arc connecting transmitter and receiver. The reception, however, is sometimes due, at least in part, to energy that has been reflected from ionized layers high up in the Earth's atmosphere. This radiation, not being horizontal, induces an e.m.f. in the loop different from what it would be had the radiation been direct- or Ground Radiation. The resulting error is said to be due to Night Effect.

During the hours of darkness, the indirect radiation affects reception of radio energy to a greater extent than during the daylight. This is especially the case when the distance between transmitter and receiver is great; for, in this circumstance, ground signals are weak and sky signals, as radiations reflected from ionized layers are called, become important for reception.

During the dark hours the reliable range for radio direction finding is reckoned to be within about 25 miles. At and near the times of sunrise and sunset, when the ionized layers of the atmosphere are particularly agitated, reliable bearings cannot be obtained.

(ii) *Land Effect*—The direction of a ray of radio energy over the Earth is influenced by the nature of the surface over which it travels. A radio ray which travels partly over land and partly over sea may be refracted at the coast. This may result in an error in an observed radio bearing which is said to be due to Land Effect. Before using a radio bearing for the purpose of fixing a vessel it is prudent to ascertain, either from the chart or from the *Admiralty List of Radio Signals*, if land is likely to be present.

(iii) *Quadrantal Error*—Electro-magnetic energy emanating from a transmitter may, on arriving at a vessel, induce currents in the metal parts of the vessel and these, in turn, may radiate energy which may be received by the direction finder aerials, and which may result in error in an observed radio bearing. The effect of this tends to be least when the transmitter is dead ahead, astern or on either beam. The effect is greatest for transmitters which lie 45° on the low or quarter, for this reason the error is known as Quadrantal Error.

The direction finder should be calibrated from simultaneous radio and visual bearings, and a table or curve of errors prepared, from which the correction to an observed bearing may be lifted as occasion demands.

(iv) *Half Convergency*—Radio energy tends to travel from transmitter to receiver along a great circle arc joining transmitter and receiver. The observed bearing of a transmitter is, therefore, a great circle bearing. Before laying down a position line from a radio observation, it is necessary to apply a correction to the great circle bearing to obtain a rhumb-line bearing. The correction is equal to the Half-convergency of the meridians at the transmitting and receiving stations. Convergency is discussed in detail in Chapter 23.

(v) *Semi Circular Effect*—Radiation from vertical parts of ship's structure acts as vertical aerials. The radiated signals are 90° out of phase with the direct signals resulting in a blurring of the signal. Semi-circular effect is eliminated by using sense aerial in antiphase thereby cancelling the unwanted signals leaving the required signals uncontaminated.

Exercises on Chapter 45

1. Describe a simple radio direction finder.
2. Explain sensing, and indicate how the 180°-ambiguity in radio direction finding is resolved.
3. Explain the Bellini-Tosi direction finder.
4. Discuss the errors in Radio Direction Finding due to Night Effect and Land Effect.
5. Describe quadrantal error in radio direction finding.
6. Explain how a radio direction finder is calibrated.
7. Discuss the causes and effects of semi-circular effect and explain why semi-circular error should be eliminated during the calibration of the direction finder.

CHAPTER 46

HYPERBOLIC NAVIGATION

1. Introduction

A number of navigational instruments employing radio techniques have, since about 1940, come into general use. By means of this type of instrument a navigator may fix his vessel at the intersection of two position lines which are the projections on the chart of spherical hyperbolae on the Earth. The techniques in which these instruments are employed form a branch of navigation which has become known as Hyperbolic Navigation.

2. The Hyperbola

A hyperbola is one of the conic sections; but for our purposes, we shall define it in terms of one of its important geometrical properties. It is a curve in a plane such that the difference between the distances of any point on it from two fixed points, is a constant amount.

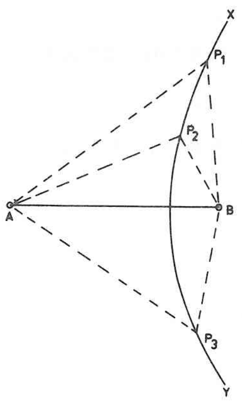


Fig. 46·1

Fig. 46·1 illustrates a typical hyperobla. The two fixed points denoted in fig. 46·1 by *A* and *B*, are known as the focal points, or foci, of the hyperbola *XY*.

At all points *P* on the hyperbola *XY* the difference between *AP* and *BP* is constant. Thus:

$$AP_1 - BP_1 = AP_2 - BP_2 = \mathrm{AP}_3 - \mathrm{BP}_3$$

It is for this reason that we may define a hyperbola as a locus such that the difference between the distances from any point on it to each of two fixed points, called the foci of the hypberbola, is constant.

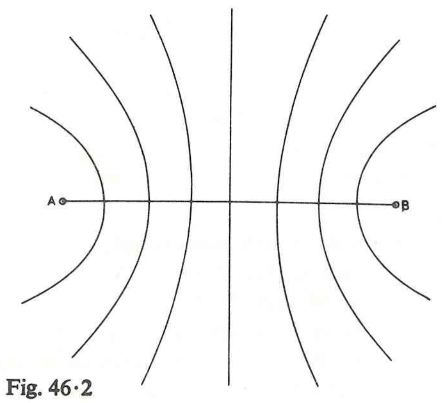


Fig. 46·2

Fig. 46·2 illustrates a series of hyperbolae all having the same focal points *A* and *B*. Such a series is known as a Family of Con-focal Hyperbolae.

Spherical hyperbolae are loci on the surface of a sphere such that the difference between the great circle distances from any point on such a spherical hyperbola to fixed points on the sphere is constant. Families of terrestrial spherical hyperbolae, when projected on a chart, give rise to a complicated network of lines. Such a chart is called a Lattice Chart, and such charts are important parts of any system of hyperbolic navigation.

Principles of Hyperbolic Navigation

Hyperbolic Navigation is based on the accurate measurement of the difference in times taken by signals transmitted from each of two fixed radio stations to reach an observer. If the velocity of radio energy is assumed to be constant, it follows that distances travelled are proportional to travel times. Thus, a hyperbolic navigation system may be dependent upon the accurate measurement of differences of "Distance" instead of differences of "Time". For this reason some hyperbolic navigation systems are known as "Distance-Difference Systems", and others are known as "Time-Difference Systems".

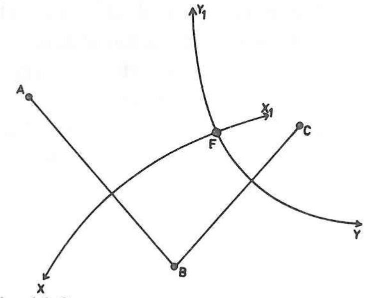


Fig. 46·3

Referring to fig. 43·6: let us suppose that an observer on board a vessel received radio signals, respectively, from radio stations *A* and *B*, from which he could measure the intervals of time taken for the radio energy to travel from *A* and from *B*, to his vessel. Knowing that radio energy travels at the rate of 300×10^6 metres per second, he could translate the diference of time intervals into a corresponding distance difference and, accordingly, plot the hyperbola XX_1, which has for its foci the positions of the respective radio stations *A* and *B*. The hyperbola XX_1 is a locus of constant distance-difference relative to *A* and *B*; so that, having plotted it on the chart, the navigator is able to say with confidence that his vessel may be fixed on XX_1, which line therefore, is a position line.

By repeating the procedure, but this time using radio stations *B* and *C*, the navigator is able to determine a second hyperbola YY_1, which has for its foci the stations *B* and *C*. When plotted on the chart this provides him with a second position line which intersects the first at *F*, which is a fix obtained from two hyperbolic position lines.

In practice, of course, it is not necessary for the navigator to plot hyperbolic position lines. This tedious task is rendered unnecessary by the availability of the appropriate lattice charts.

4. The Nature of Hyperbolic Position Lines

In a hyperbolic navigation system the transmitters are located at the foci of families of hyperbolae.

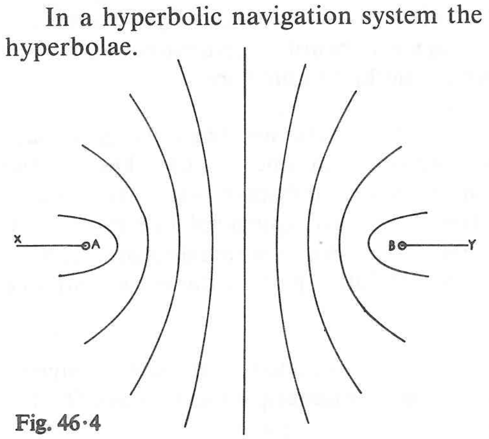


Fig. 46·4

Referring to fig. 46·4, *A* and *B* denote two transmitters located at the foci of the family of hyperbolae illustrated. The straight line or, more strictly, the great circle arc are joining *A* and *B* is known as the Base Line, and the arcs, *AX* and *BY* as the Base Line Extensions. The perpendicular bisector of *AB* is the locus of zero-time or zero-distance difference in respect of *A* and *B*, and this is a great circle arc which cuts the base line at 90°. The curvature of the remaining hyperbolae of a family varies with distance from the base line.

Accuracy of a hyperbolic position line is greatest along the base line where the members of the family of hyperbolae are most closely spaced. As any two adjacent hyperbolae separate more and more as distance from the base line increases, position line accuracy falls off. But the rate of decrease of accuracy is greatest along the base line extension: and, for this reason, in many hyperbolic systems the areas near the base line extensions are of no use for navigational purposes.

Any given member of a family of hyperbolae, other than the base line and the base line extensions, has its greatest curvature at the point where it intersects the base line. As distance from the base line increases the curvature diminishes until a point is reached at which the hyperbola may, for practical purposes, be considered to coincide with the great circle arc. The length of the base line determines the distance from the base line at which the hyperbola and great circle arc are considered to be coincident. In some systems the transmitters are closely spaced and these systems are more "directional" than "hyperbolic". In others the base line may be of many hundreds of miles, and nowhere may the hyperbola be considered to blend with a great circle arc.

5. Consol

Consol is a long-range medium-frequency hyperbolic system in which the transmitters are closely spaced; so that it is, essentially, a directional navigation system in which a navigator may determine the great circle bearing of the Consol station with a relatively high degree of accuracy for ranges of 1,000 miles or more in favourable conditions. The principle of Consol is illustrated in fig. 46·5.

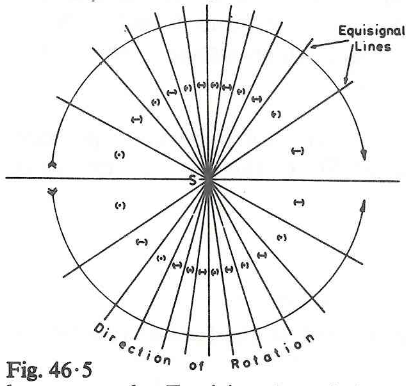


Fig. 46·5

At the Consol transmitting station, denoted by S in fig. 46·5, three aerials radiate energy which produce a "dot and dash" system for audible signals which may be received by means of an ordinary radio receiver on board. By counting the numbers of dots and dashes during a transmission cycle, the bearing of the consol station may be found from a suitable table or from a Consol lattice chart. The intersecting lines illustrated in fig. 46·5 are the boundaries of sectors within which dots or dashes are received at the beginnings of the transmission cycles. On the boundary lines themselves, a continuous signal is heard. The continuous note is known as the Equisignal, and the boundary is known as the Equisignal Line.

The transmission cycle consists of signals each of 60 dots and dashes. The dots and dashes have the same period, and the cycle is usually completed in one minute. During the transmission of the 60 dots and dashes, the pattern of the dot and dash system is made to rotate at a uniform rate such that at the end of the cycle each equisignal line has swung through one sector. This means that if a vessel is located in a sector in which dots are heard at the beginning of the cycle, dashes will be heard during the latter part of the cycle, and *vice versa*.

At the end of the cycle of transmission the station sends a coded identification signal, during which the equisignal lines are brought back to their original positions ready for the next cycle to commence.

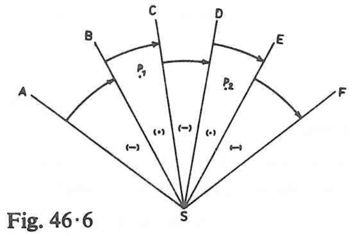


Fig. 46·6

Consider an observer at P_1 in fig. 46·6. At the beginning of the transmission cycle he will receive dash signals but, because of the rotation of the pattern, the signal will presently change to dots. The relative numbers of dots and dashes received during the cycle will assist him in establishing his position relative to the equisignal line BS.

It is evident that ambiguity may arise; an observer cannot determine if he is within the sector BSC or the sector DSE from the dot and dash count alone, the count being the same at P_2 as it is at P_1. His estimated position may assist him in resolving the ambiguity that may exist; but, failing this, the radio direction finder may be used to obtain a rough bearing of the station. The coded identification signal, which is transmitted between successive dot and dash cycles, will assist in this process.

The advantages of Consol are; first, simplicity of operation; and, second, no special receiving apparatus is needed on board. At the present time Consol stations provide facilities for position finding in the North Atlantic Ocean and the adjacent seas.

The operation of Consol is performed by tuning a radio receiver to the frequency of the Consol station and counting the number of dots and dashes in the keying cycle. The total number of dot and dash characters should be 60; but, because the change from dots to dashes, or from dashes to dots, is masked by the width of the equisignal, the number counted is normally less than 60. The count, therefore, had to be corrected for the lost characters. This involves subtracting the total count from 60 and adding half the difference to each of the dot and dash counts.

Example 46·1—Find the correct count if the observed count was 18 dots and 38 dashes.

$$\text{Total Count} = 18+38 = 56$$
$$\text{Number of lost characters} = 60 - 56$$
$$= 4$$
$$\text{Correction necessary} = 2$$

Corrected Count: 20 dots, 40 dashes.

Answer—20 dots, 40 Dashes.

By entering a table provided in the *Admiralty List of Radio Signals* with the corrected count, the true bearing of the vessel from the Consol station may be found. It must be remembered that the true bearing found from a Consol table is a great circle bearing and it may be necessary to apply a half-convergency correction to it before laying it down on a chart as a position line.

On a Consol lattice chart, the lines of the lattice are marked with the count numbers so that a position line is readily found. There will be ambiguity, as explained earlier, unless the bearing of the station is known to an accuracy of about 10°.

6. Loran

The name "Loran" is derived from **L**ong **R**ange **N**avigation. It is a long-wave hyperbolic system by means of which an observer may determine his vessel's position by measuring the time interval—using suitable equipment to do so—between the instants of receipt of synchronized signals from each of two Loran transmitting stations.

Loran stations are usually located many hundreds of miles apart, groups of stations forming Loran Chains. Within the coverage area, which at present is most of the North Atlantic and North Pacific Oceans, signals may be received from at least two pairs of stations of a chain. Three Loran stations working as two pairs, are sufficient to give two hyperbolic position lines which, if they intersect at a good angle of cut, give a reliable fix of relatively low accuracy.

Long-wave energy of the type used in Loran is reflected from the ionized regions of the atmosphere, and such refflected energy forms the so-called "sky waves". The ionized region of the atmosphere rises to higher levels during night-time, and this has the effect of increasing the range of sky-wave reception during the hours of darkness.

In general, ground-wave reception is possible for receivers within about 700 miles of the transmitter, but sky-wave reception, especially at night, increases the range to about double this distance.

Signals received at a Loran receiver on board a vessel activate an indicator in the form of a Cathode Ray Tube. On the face of the C.R.T. "traces of the signals from each of a pair of Loran transmitters have to be matched, after which the required time-difference is obtained. Families of Loran hyperbolic position lines are over-printed on the appropriate navigation chart, and such a Loran Chart facilitates fixing from Loran observations.

Loran is a useful aid, especially for aircraft flying over oceans, in which an approximate position is sufficient for the navigator's purpose. The accuracy of a Loran fix is related to the manner in which the radiated energy travels between transmitter and receiver, and on whether ground- or sky-waves are used for measuring the required time differences. It also depends upon the degree of skill of the observer in identifiying the signals as they appear on the C.R.T. indicator.

7. Decca Navigator

The Decca Navigation System employs a chain of transmitters formed by a Master and three Slave stations, the Slave stations being designated Red, Green and Purple, respectively. The distance between a Master station, which is located centrally in the chain, from a Slave station is about 70 miles, and the system operates on medium frequency, continuous wave (C.W.) transmissions.

Each station of a chain transmits C.W.s at a specified frequency which is related harmonically to the frequencies used by other stations. The four frequencies used are in the ratio 5, 6, 8 and 9.

On board the vessel is fitted a multi-channel receiver tuned to receive the four frequencies used by the station of the chain. The Master frequency is combined with each of the frequencies of the Slaves, in turn, to form a Comparison Frequency which is the L.C.M. of the Master and appropriate Slave frequencies.

The C.W. transmissions of Master and each Slave, in turn, are phased-locked, so that the

signals which combine to make the Comparison Frequency have the same phase on lines on the Earth's surface which are spherical hyperbolae having the Master and Slave stations at their foci.

Decca Lattice Charts provide the projections of the three families of hyperbolae appropriate to a given chain; the families of hyperbolae being coloured Red, Green and Purple, for each of the Master and Slave combinations.

The exact phase relationship between C.W. signals from Master and Slave is obtained by means of a phase-measuring instrument known as a Decometer. There are three Decometers in the indicating unit, these being coloured Red, Green and Purple, and which give details of the particular Red, Green and Purple hyperbolae on which the vessel is located.

In moving across a Decca coverage area from one point to another during which the phase difference of the signals from Master and Slave changes form 0° to 360°, the vessel is said to traverse a Decca Lane. A Decca Lane is merely the space between two hyperbolic position lines on both of which the phase difference of the signals is 0°; or, in other words, the signals are "in phase".

The 360°-phase difference resulting from traversing one lane is divided into hundredths of a lane width, and the corresponding fraction of a low-width from the boundary of a lane, together with the lane "number" and "zone", are indicated on each of the three Decometers. It is an easy matter, therefore, to transfer the Decometer readings to the Decca Lattice Chart, and hence to fix a vessel.

The lane-width on the base line depends upon the comparison frequency used, but its maximum value is under 2,000 feet. It follows, therefore, that a Decometer reading given to the nearest hundredth of a lane-width permits the navigator to fix his vessel to a theoretical accuracy of better than 20 feet on the base line. Of course the lane-width varies according to position in the coverage area, and propagation and other errors may affect the fix; but there can be no doubt that a Decca fix is highly accurate, and the Decca Navigation System is a remarkable radio aid to coastal navigation.

The reliable maximum range of Decca is about 250 miles from the Master station of a given chain. Details of propagation and other errors are promulgated by the Decca Navigation Company in the form of Data Sheets which are issued periodically as needed. The *Admiralty List of Radio Signals* also gives information about Decca and other hyperbolic navigation systems.

A large number of Decca Chains have been established in many parts of the globe where shipping activity warrants the use of an accurate position fixing system.

8. Omega

The Omega is a very low frequency (VLF) Radio Navigation system operating in the band between 10 KHz and 14 KHz. Worldwide coverage system of Omega started in 1968 and since 1971, Omega Navigation Systems Operation Details (ONSOD) of U.S. Coastguard unit took responsibility for operations and engineering for the transmitting stations.

Omega makes use of the long groundwave propagation to create a hyperbolic navigation system with baseline distance of up to five thousand miles. Such long base-lines gives the worldwide coverage with only eight transmitting stations with a system accuracy of two to four nautical miles. The time difference measurement in the Omega system is measured as difference in phase of two received signals.

Omega tables are available from which a hyperbolic position line may be laid down on a navigation chart. Omega lattice charts have, however, been provided for certain ocean areas. A position fix is obtained from a minimum of two line positions (LOPs) which can be derived from either three stations by making one common, or four stations. The propagation characteristics are such that up to six stations can be received at any location. The eight stations are designated alphabetically with letters A to H are as follows:—

Station letter designation	Location	Latitude	Longitude
A	Aldra, Norway	66° 25′ N.	13° 08′ E.
B	Monrovia, Liberia	6° 18′ N.	10° 40′ W.
C	Haiku, Hawaii	21° 24′ N.	157° 50′ W.
D	La Moure, North Dakota	46° 21′ N.	98° 20′ W.
E	Le Reunion	20° 58′ S.	55° 17′ E.
F	Golfo Nuevo, Argentina	43° 03′ S.	65° 11′ W.
G	Australia	38° 29′ S.	146° 56′ E.
(G)	Trinidad (temporary)	10° 42′ N.	61° 38′ W.
H	Tsushima, Japan	34° 37′ N.	129° 27′ E.

The basic Omega frequency of 10·2 KHz. is common to all eight transmitting stations and to avoid problems of mutual interference, each station transmits for approximately one second, with 0·2 second break between each transmission.

At the receiving end the signal is identified by a process known as synchronization and the phase comparisons is achieved by using reference oscillations and phase stores within the Omega receiver.

More sophisticated methods of fully automatic synchronization continuously co-relates the commutator with the received signal while adjusting the turning of the commutator until synchronization is achieved.

Similarly, precise synchronization of all transmissions within the Omega system is maintained in order to ensure the phases of each station are accurately related. Transmissions are controlled by cesium beam standards.

The accuracy of positions by Omega depends to some extent upon the performance of the transmitting and receiving equipments. However, the errors caused by various propagational effects are far more significant. Although propagation correction tables are available where changes in earth's conductivity, as the signal pass over different strata, water and geographical anomalies are accounted for in the correction tables, yet, errors due to sunrise/sunset effect, polar cap absorbtion and ionospheric disturbance cannot be predicted. Because of the uncertainties of skywave reception, at times, the accuracy is somewhat less than expected.

Excercises on Chapter 46

1. Define a hyperbola in terms of its important geometrical property of value to position line navigation.
2. Describe carefully the relationship between the curvature of hyperbolae of a given family, and the distance between the foci of the hyperbolae.
3. Describe a lattice chart of the type used in hyperbolic navigaton.
4. Describe the principles of Consol, and explain how to correct a Consol Count.
5. Describe how to resolve ambiguity of sector when using Consol.
6. Explain why Consol is described as being a "Directional" hyperbolic system.
7. Describe the principles of Loran.
8. What errors may affect the position obtained from a Loran observation?
9. Describe the Decca Navigation System.
10. Compare the Loran System of Navigation with that of Decca.
11. What is meant by "sky-wave reception"?
12. Discuss the Omega System of Navigation.

CHAPTER 47

RADAR NAVIGATION

1. Principles of Radar

Radar is an instrument by which the bearing and range of a distant object may be found provided that the object is within the so-called Radar Horizon. The name Radar is derived from **Ra**dio **D**irection **a**nd **R**ange.

The principle of radar is similar to that of echo-sounding. In radar, pulses of radio energy of very high frequency—known as radar frequency—are transmitted, and corresponding echoes received, just as in echo-sounding. If the speed at which a radar pulse travels is known, and the interval between the instants of transmission of the pulse and the receipt of its echo can be measured, the range of the object responsible for returning the echo is immediately determined. Moreover, as in echo-sounding, radar pulses are transmitted through the atmosphere in a narrow beam so that the direction of the object, as well as its range, is determined.

Radar energy, like light, is refracted as it passes through the Earth's atmosphere; but, because of the difference of frequencies of radar energy and light, the Radar Horizon has a range of about 15% more than that of the Visible Horizon. The range R of the Radar Horizon, for an aerial height of H feet above sea level is given by the formula:

$$R = 1{\cdot}22\sqrt{H} \text{ or } R = 2{\cdot}2\sqrt{H} \text{ where } H \text{ is in metres.}$$

The radar equipment comprises a transmitter which generates the pulses or signals at a rate, known as the Pulse Repetition Frequency (P.R.F.), in the order of about 1000 per second. The signals are passed to a specially-designed aerial from which they are transmitted horizontally, and the same aerial is used to receive the echoes of signals. The interval between the transmission of a pulse and the receipt of its corresponding echo is timed, and the range of the object responsible for the echo is indicated in the form of a light-spot on the face of a cathode ray tube which forms the display.

The normal display is in a form known as a P.P.I., or Plan Position Indicator, on which bearings, as well as ranges, are indicated.

The principal use of radar is as an anti-collision aid. But radar is also capable of providing navigational information of particular value when coasting in thick weather when visual observations are not possible.

2. The Use of Primary Radar in Navigation

The term Primary Radar applies to the radar equipment of a vessel by means of which ranges and bearings of objects such as other vessels, land, ice-bergs, may be found.

Provided that an identifiable object appears on a radar display, a single observation gives the object's range and bearing; and hence the vessel may be fixed on the navigational chart at the intersection of a position line (obtained from the observed bearing) and a position circle (obtained from the observed range). But the bearing discrimination of radar is not as good as that of visual observations, although the range discrimination is excellent. For this reason, when using radar as a navigational aid, a vessel is best fixed by radar ranges of two suitably-placed identifiable objects. In other words, a fix by cross-ranges, or two position circles, is to be preferred to a cross-bearing fix from radar observations.

The radar response of different objects varies considerably, and a good deal of skill and experience is necessary if radar is to be used to best advantage for navigating coastwise. A variety of Chart Comparison Units are available by means of which the radar display may be matched with the charted information.

The use of primary radar in navigation is hampered by the presence on the display of so-called unwanted, or false, echoes. These may arise from returns from wave fronts—a form of unwanted echo known as sea clutter which is troublesome in rough seas; multiple and indirect reflections; and echoes due to rain.

On the normal P.P.I. display the range of an object which appears as a light-spot is proportional to the distance of the light-spot from the centre of the display. The navigator has the choice of a "North-up" or "Head-up" display. In the former true bearings of objects are obtained, whereas with the head-up display relative bearings are obtained. The navigator may also choose between a "Relative" or "True-Motion" display. In the relative display the centre of the display denotes the observer's position at all times, whereas in the true-motion display the centre of the display denotes a given geographical position, and on this display light-spots representing all moving objects detectable by radar, as well as the observer's own vessel, move in their real directions across the display at speeds proportional to those of the objects they represent.

Provision is often made for offsetting the centre of a P.P.I. display to allow an extended period of observation of distant objects lying in particular directions.

3. Radar Beacons

The term Secondary Radar applies to radar equipment located ashore or on a light vessel, which is triggered by pulses from the primary radar with which a vessel in the vicinity is equipped. Secondary radar equipment is usually in the form of a Radar Beacon known as a Racon.

A racon, a name derived from the term **Ra**dar Bea**con**, consists of a transponder which transmits a signal only when the beacon has been interrogated by the original transmitted signal made by the vessel's radar. The re-transmitted signal made by the beacon is in the form of a coded group which manifests itself on the display by a line of dots and/or dashes radially aligned in the direction of the beacon from the vesssel. Not only is direction of the racon obtained but so also is its range.

Another type of radar beacon is known as Ramark, a name derived from the term **Ra**dar **Mark**. This form of beacon transmits continuously in all directions. The bearing of a ramark is indicated on the P.P.I. of a vessel in the vicinity as a radial line which indicates the bearing of the radar beacon.

Exercise on Chapter 47

1. Explain the principle of radar and state the more important parts of radar equipment.
2. Distinguish between primary and secondary radar.
3. Discuss the use of radar for navigating coastwise when the visibility is poor.
4. Explain why cross-ranges are to be preferred to cross-bearings when fixing by radar.
5. Describe the unwanted echoes that may hamper the use of radar as an aid to navigation.
6. What is meant by Radar Horizon? Explain why the range of the radar horizon exceeds that of the visible horizon.
7. Describe a Remark and a Racon.
8. Discuss the advantages and disadvantages of
   (*a*) True Motion and Relative Motion Display.
   (*b*) North-up and Head-up Displays.

CHAPTER 48

NAVIGATIONAL SATELLITES AND INERTIAL NAVIGATION

1. Introduction

It is interesting to reflect in the closing chapter of this book that the essential processes in navigation are finding and setting course, and determining the distance to travel to reach one's destination. The basic instruments of navigation are, therefore, the compass and the log. Had these instruments been capable of providing the navigator with completely reliable information, systems of position-finding at sea (except perhaps, for those related to finding position in respect of the depth of water under a vessel's keel) would have been unnecessary. In other words, had D.R. navigation been perfect Nautical Astronomy and the whole range of navigational equipment based on modern technology need never have been invented.

Of course, in the earliest days of ocean navigation, the mariner realized the insufficiency of his compass and log (and even of his chart, as well); and he was quick to enlist the support of land-based scholars who have, down the ages, devoted untiring attention to the improvement of navigational instruments and techniques. The importance of safe voyaging, especially at times when maritime trade has grown rapidly, has resulted in numerous of the world's leading philosophers, including men of the intellectual stature of Flamsteed, Newton, Halley, Lalande, Euler, Mayer and Lord Kelvin, to name but few, giving serious attention to the improvement of navigation and nautical astronomy. This applies particularly to the present when the most advanced technology finds its application in the field of navigation.

The concluding chapter of this book is devoted to navigational satellites and inertial navigation. These navigationl systems will be but briefly described.

Implicit in the following descriptions is a concept of which every practising navigator ought to be aware. This is the concept of "perpetual change", which applies to Nature in general and to human activities—ideas, science and technology—in particular.

Navigators of the past often were censured for their staunch and dogged resistance to change. The accusations, in large measure, were justified; for many a navigator, refusing to accept new ideas, clung to archaic methods long after these had outlived their usefulness. Young navigators of the present, however, cannot be but keenly aware of the rapid advances currently being made in science and technology. These men (and some women too) realize, most surely, that their noble art is continually in a state of evolution.

2. Navigational Satellites

Soon after the first artificial Earth satellite was launched in 1957, physicists of the Applied Physics Laboratory of Johns Hopkins University, who had set up equipment for receiving radio signals from such satellites, were struck by the change in frequency of the signals, familiarly known as the Doppler Shift, which results when there is a change of relative motion between transmitter and receiver.

The Doppler Shift, Δf is given by the formula:

$$\Delta f = -\frac{f}{c} \cdot r$$

where f is the transmitted frequency; c the velocity of radio energy, viz. 300×10^6 metres per second; and r is the rate of change of distance between transmitter and receiver. If f and c are known, a measure of the Doppler Shift (Δf) is equivalent to that of the rate at which the range of the transmitter is changing.

Because the motion of an artificial satellite is completely predictable—excepting small errors due to atmospheric drag and those due to imprecise knowledge of the Earth's gravitational field—it is possible to determine the details of the orbit of the satellite; that is to say, the "parameters" of the orbit, from observations of the Doppler Shift.

It was realized that if it was possible to determine the orbit of an artificial satellite from Doppler Shift observations, Doppler information received on board a vessel could be used to fix the vessel's position. It was this realization that led to the satellite navigational system now known as TRANSIT.

The transit satellite system is a refinement of celestial navigation using artificial Satellite and advanced technique of computer usage. It is operative in all weather conditions and at any time of the day or night.

The satellite contains a memory store, receiver and transmitter powered by solar energy. Satellite are in polar orbiting passing above the poles at approximately 600 nautical miles from the earth crossing the equator. The period of satellite orbit is about 108 minutes. During one orbit of satellite, the earth revolves about 27° of longitude beneath the plane of such a satellite.

There are at present five satellites, four in orbit each separated by 45° at the poles and the fifth is orbiting at 180° declination from the first, i.e. when the first satellite is on top of the north pole, the fifth satellite is on the same orbital plane, on top of the south pole.

Velocity of satellite is about 7,000 metres/sec. which is determined by the formula:—

$$\text{Satellite velocity} = \sqrt{\frac{r^2g}{(r+h)}}$$

where:
r = radius of earth = 3,444 miles
= 981 cms/sec^2
h = height of orbit

Length of orbital path = $2\pi(r+h)$

$$\text{Time of orbit} = \frac{2\pi(r+h)}{\sqrt{\frac{r^2g}{(r+h)}}}$$

e.g. when h = 600 miles, time = 108 minutes, the velocity works out to be approximately 7000 metres/sec.

Each satellite contains a receiver equipment to accept injection data and operational commands from the ground station. The position of the satellite in terms of height, declination and celestial longitude of next sixteen hours are transmitted from shore station every twelve hours intervals and the information is stored in satellite memory and instantaneous values of these informations are transmitted every two minutes as a code. In short, each satellite is telling the users which satellite it is, what time it is according to satellite clock and where the satellite is right now. On board the ship the receiver accepts the coded information and a computer then calculates the distance of the satallite from the ship's D.R. position, every two minutes interval and finds the receiver's position in relation to the satellite using the doppler shift method.

The doppler shift method may be explained further to state that if the source and the observer are drawing closer together, then the received frequency is increased; stable oscillator frequencies radiating from a satellite coming towards the receiver are first received higher than transmitted due to velocity of approaching satellite but as the satellite comes to the moment of closest approach the cycle of the received frequency exactly matches with the generated frequency. As the satellite passes beyond this point and travels away from the receiver then the received frequency count drops below the generated frequency in the ratio of the widening distance and the speed of the receeding satellite.

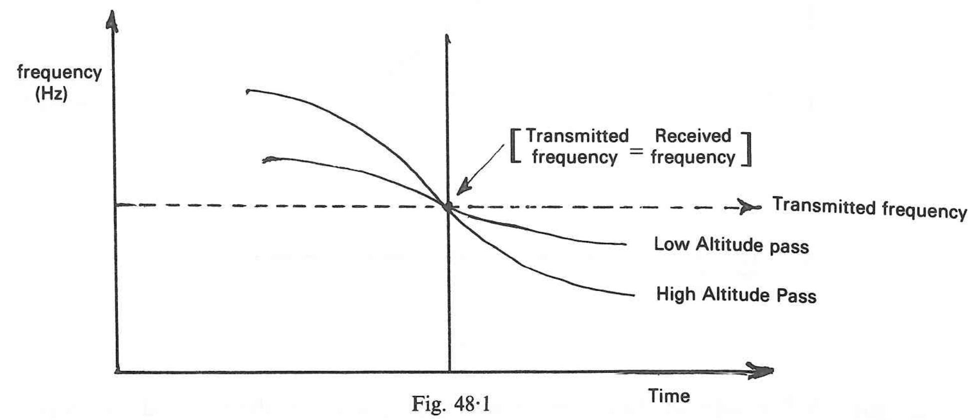


Fig. 48·1

In figure 48·1, the high altitude pass curve indicates that the d'long between the satellite and the ship is small. Similarly, the low altitude pass curve means that the d'long between the satellite and the ship is large. The instant when the received frequency is the same as the transmitted frequency is the instant when the latitude of the ship and the satellite are the same. Longitude of the ship is therefore a function of the steepness of the curve where as latitude of the ship is a function of the time of satellite pass. In practice, number of 'doppler counts' are made during the pass because it is possible to very accurately take cycle counts of radio frequencies and the amount of doppler shift can be measured by a computer.

The observed doppler shift is caused by three sources of relative velocity:
a) orbital velocity of the satellite
b) velocity of the observer (ship)
c) rotation of the earth about its axis

Accurate measurement of the instantaneous received frequency meets with difficulties of a technical nature and so another method is used. The received frequency f (obs.) is mixed with a frequency (local) generated in the receiver. The frequency f (local) is higher value that f (obs.) can obtain and equals 400 MHz. The number of beats per second originated from this mixing is given by:-

$$f\,(\text{beat}) = f\,(\text{local}) - f\,(\text{obs.})$$

and is, therefore, always positive. The number of beats during an interval of two minutes are marked by time signals from the satellite transmitters at time t_1 and t_2. In figure 48·2, the areas N_1, N_2 etc., are proportional to the number of beats obtained. These beats are found by mixing the received frequency f (obs.) with the generated frequency f (local). From this information the 'doppler count', a locus of the ship's position can be obtained.

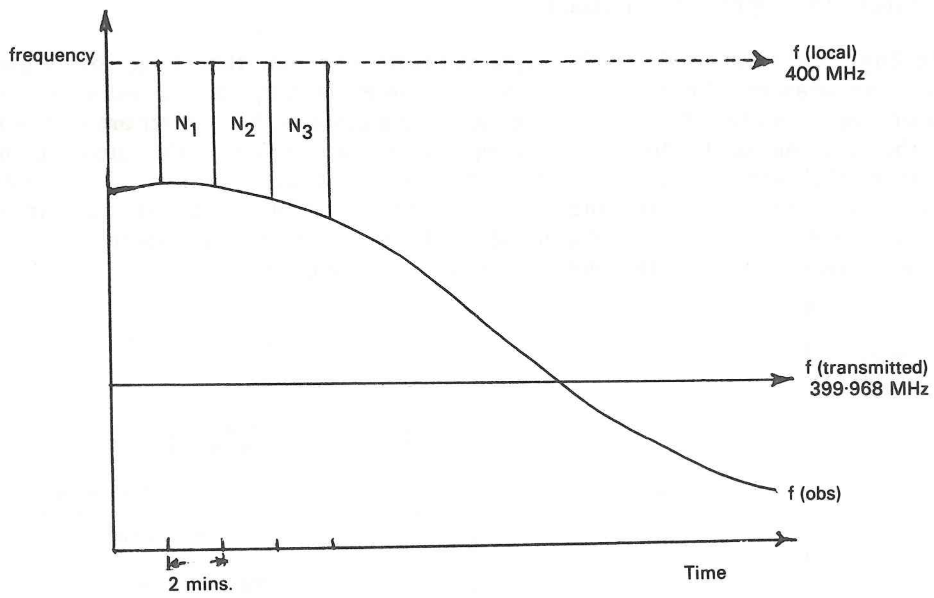


Fig. 48·2

In figure 48·2, a doppler curve has been sketched. Each of the hatched area has a variable height equal to the instantaneous beat frequency f (local) $-$ f (obs.) and a width equal to a time of two minutes. By multiplying the frequencies m cycles/second by the time in seconds, the number of cycles N_1, N_2 etc., is obtained.

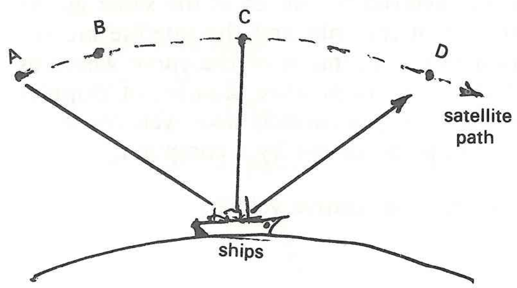


Fig. 48·3 (a)

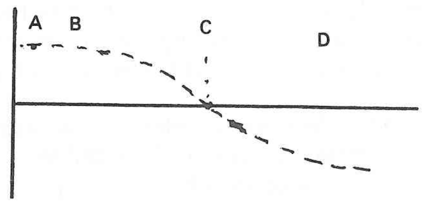


Fig. 48·3 (b)

The frequency received therefore, depends upon the velocity of the transmitting source. Figure 48·3 (a) illustrates a satellite moving from point A to point D in two minutes. If the transmitter is assumed to be stationary, transmitting a signal at a frequency of ft cycles per second and if the speed of propagation is C metres per second producing a wavelength of λ metres, then these waves will extend to a distance of C metres in one second.

$$\text{i.e. } C = \lambda\,\text{ft}. \;.\;.\;.\;.\;.\;.\;.\; (1)$$

If the transmitter is moving at a speed of V metres per second towards the observer then in one second there will be a ft waves contained in a distance of (C $-$ V) metres and then the apparent wavelength λ, will be

$$\lambda_1 = \frac{C - V}{\text{ft}}\,.$$

substituting the value of ft from the equation (1), we get:

$$\frac{C - V}{C}\lambda\,.$$

Therefore received frequency—

$$\text{fr} = \frac{C}{\lambda_1} \quad \text{or} \quad \frac{C}{\dfrac{C - V}{\text{ft}}}$$

$$= \frac{C}{(C - V)}\,.\,\text{ft}$$

and shift in frequency—

$$(\text{ft} - \text{fr}) = \text{ft} - \text{ft}\left(\frac{C}{C - V}\right)$$

$$= \text{ft}\left(1 - \frac{C}{C - V}\right).$$

When a satellite is transmitting in an orbit at a constant speed then the received frequency at the ship depends on the velocity component towards the ship. Referring to figure 48·3 (b) the velocity towards the ship decreases from A to become zero at C and then becoming negative as the satellite passes and moves away to D. The received frequency will therefore be maximum at A becoming the same as the transmitted frequency at C and falling below the transmitted frequency as the satellite moves away.

The doppler shift is then ascertained by comparing the frequency of the signal received from the satellite with a signal of an appropriate frequency of 400 MHz. generated by a local oscillator. It is thus measured by counting the number of cycles of a beat frequency over an extended time interval of two minutes, during which the satellite transmits the appropriate ephemeral data from its memory. Since the doppler shift is a measure of slant range difference, each range difference is associated with hyperboloid (a three dimensional figure swept out by rotating a hyperbola about an axis on which its foci lie) having the satellite at one of its focal point. Such a hyperbola intertsects the surface of the earth along a spherical hyperbola. Two observations of ephemeral data and doppler shift provide two spherical hyperbolae and at the intersection of which the observer is located.

The calculations necessary to find the vessel's position from satellite observations is facilitated by means of a purpose designed computer. This is fed with ephemeral data received from the satellite; the measured doppler shift; the latitude and the longitude of the vessel by estimation; the vessel's course and speed; and the height of the satellite above a reference surface corresponding to the earth's surface.

The computer calculation is repeated a few times, starting each time with latitude and longitude found by the previous calculations. The difference between calculated and observed doppler counts is called the 'residual'. If the new D.R. is nearer to the real position, the residual is reduced and further calculation is carried out while the computer converges on the real position. If the residuals diverge, the position must be suspect since some of the data is unsatisfactory. Usually no more than three or four iterations are necessary to find a fix.

However, the accuracy of the fix depends on the following factors, these are:-

a) An error in the ships course and speed provided in the computer
b) Anomalies of signal propagation
c) An error in the antenna height provided to the computer
d) Inaccuracy of apparatus on board the ship
e) Error in the predicted orbit deviations.

The actual course and speed of own ship during the period of taking the observations reduces the accuracy. This happens partly due to the small frequency shift generated by own ship's velocity, and partly due to the need to introduce comparatively crude course and speed data to adjust the running fix.

Height of aerial is an another important data to complete the calculation. Since the earth is not a perfect sphere, the aerial height above the earth's surface varies as a distance from the earth's centre. A correction is therefore necessary to be applied to the aerial height. A geoid map is supplied, giving corrections for the sea level as difference from a perfect ellipsoid.

Errors in propagation path due to refraction in passing through ionosphere is resolved by transmitting and receiving in two separated frequencies namely 400 MHz and 150 MHz. Since the amount of refraction depends upon frequency, the different shifts may be assessed and a correction is applied to the readings.

Received frequency may drift over a period of time as long as the drift during the period of taking the counts is negligible, this may be disregarded.

The errors due to orbit deviations are caused because of:—

1. Irregularities of earth's gravity field, of an extent not exactly known.
2. The air drag experienced by the satellite (even though at the height above 11,000 kilometres the atmosphere is extremely rarified).
3. The pressure exerted on the satellite by solar radiation.
4. The attraction of other celestial bodies (for instance, the moon).

Of these causes, the first creates the greatest difficulty. The field of gravity shows irregularities because the mass of the earth is not equally distributed. Nevertheless, it has been possible to determine these irregularities to some extent in the following way.

If one knows exactly a satellite's deviation from its normal orbit at any given time, one can determine one's position on earth by making doppler frequency shift measurements. This is what the navigator needs to do for position fixing. Conversely, therefore, one can calculate the satellite's deviations as a function of time by making doppler measurements from a number of known positions on earth. By extrapolation the deviations from subsequent orbits can also be determined. Scientists have succeeded in calculating a gravity model of the earth by observing the doppler shifts of the Navy navigation satellite system satellites and also by tracking other satellites (by means other than doppler observations) from a global net of tracking stations at known positions. Terrestrial information about gravity anomalies was also used. As further research is carried out and more information becomes available, the gravity model is continually improved.

It is apparent that the accuracy of out-put depends to some extent on the accuracy of input data and occasionally up to about six hours this system is not suitable for observation in some areas because the orbital path of satellite goes beyond the visible range.

Despite several inaccuracies, it is still highly accurate system of position fixing. It has no geographical limitations and can be used twenty four hours in all weather conditions. Digital readout of latitude and longitude makes it easy to note the position without plotting. This system has no saturation hence many users can use the system simultaneously.

3. Global Positioning System (NAVSTAR)

A new global positioning system expected to be operational in the foreseeable future has now being installed which will provide highly accurate position fixes for the world's commercial shipping using Transit and Nova Satellite. The system is called the Navy Navigation Satellite System (NNSS). However, there has been a further attempt to provide the ultimate system, to replace NNSS. The U.S. armed forces under the direction of joint services programme is developing a completely new satellite navigation system known as GPS (Global Positioning System). The GPS will be totally non-compatible with the existing system. Although GPS is in its second phase of development, yet it is not expected to be operational for commercial shipping before 1990. When operational the GPS will provide with precise three dimensional position information using eighteen satellites twelve hour orbits at a distance of approximately 20,183 kilometres (10,900 nautical miles) above the earth. Three orbital planes of each satellite will have an inclination of 53° to the equatorial plane and each plane off set from one another by 120° longitude. There will be six satellites in each orbital plane, transmitting information on two frequencies namely 1227 MHz. and 1575 MHz. thus permitting a correction to be made for propagation delays in the ionosphere. The system is so designed that at any point on earth at least six satellites will always be in view. The injection data and control of information will be monitored from the earth by one Master Control Station; an upload station and four monitor stations.

Unlike existing transit system, the Nav Star system will provide fixes by simultaneous determination of range of the user from several satellites. Although a user on the earth need only two range measurements to define a position in terms of latitude and longitude but a third satellite will provide the height above the mean sea level making it a three dimensional position information. It will also provide a reliable estimate of the receiver's velocity.

The message transmitted by each satellite will provide information relating to satellite clock correction, propagation delay correction, its position in space and also the information leading to the most suitable satellite to use for any given area.

The receiver set on earth will comprise of antenna, receiver, computer and data display segment. The receiver will be so designed that it will automatically select at any instant of time the optimum satellites for use in terms of their angular separation and good elevation.

4. Inertial Navigation

The system of navigation designated "inertial" is based on Newton's First Law of Motion in which it is stated that every body tends to maintain its present velocity and that it will do so if no resultant force acts on it. Velocity, which is a vector quantity, changes whenever the speed or direction of a moving body changes; and the rate at which velocity changes is known as acceleration. Newton's Second Law relates the force f, which, acting on a body of mass m, causes it to accelerate at acceleration a. If appropriate units of mass, force and acceleration, are used Newton's Second Law is expressed as:

$$f = ma$$

It is possible to detect or "sense" an acceleration of any moving body by means of a pendulum which, when fitted to an accelerating body on the Earth's surface, takes up a "false" vertical; and the angle which the false vertical makes with the true vertical is a measure of the acceleration of the body at the instant.

From the well-known equations of motion, viz: $v = at$, and $s = vt$, it is clear that, by integrating acceleration a with respect to time it is possible to find velocity v; and that by integrating velocity v with respect to time t, distance s is determined.

A device which is capable of sensing accelerations is known as an Accelerometer, and such devices are basic to inertial naviation.

Inertial navigation is, essentially, a sophisticated Dead Reckoning System in which the motion of a vessel, given its initial velocity, is sensed, without compass or log, so that the vessel's position relative to its starting point is at all times known.

Two accelerometers are needed in an inertial system suitable for surface vehicles, each to measure horizontal accelerations in mutually perpendicular planes. The accelerometers are fitted to a device known as a Stable Platform. This has three planes of freedom and it maintains a fixed orientation in space irrespective of the motion of the vessel. The platform is stabilized by means of gyroscopes which sense the rotation of the platform relative to space.

The accuracy of an inertial navigation system is closely related to the degree of precision of the gyroscopes and accelerometers which are vital to its performance; and it was not until relatively recently that the state of technology made it possible to manufacture these devices to the necessary precision required.

An important feature of inertial navigation for surface vessels is the so-called Schuler tuning* by which the platform on which the accelerometers are mounted is maintained in a horizontal plane. The merest angle of tilt of the platform out of the horizontal results in substantial error. Error in an inertial navigation system, like that of D.R. by compass and log, tends to be proportional to time, so that the importance of precision gyroscopes is paramount.

In addition to the accelerometers and gyroscopes, the third requirement of an inertial navigational system is a computer designed to deal with the complex integration problems associated with the accelerations of the vessel.

Errors in inertial navigation systems are primarily due to gyro drift; and such errors, which, for a gyro drift of 1° per hour amounts to about 6 miles per hour, are cumulative and increase with time. It become necessary, therefore, to update the inertial system by other position finding methods.

The inertial system used for surface vessels is known as SINS, which stands for Ship's Inertial Navigation System. Its important feature is that it is a self-contained system which functions independently of weather conditions which can hamper nautical astronomy; and of radio energy which may suffer from interference—man-made as well as natural. For this latter reason SINS is of importance for naval vessels, particularly submarines. At present its high cost, coupled with the fact that alternative navigational systems are available, does not warrant the use of the inertial navigation system in commercial vessels. But who can foresee the future?

*It is Schuler tuning that is used in gyro compasses to ensure that change in course and speed error is equal to ballistic deflection.

Extracts from

Admiralty Tide Tables

and

The Nautical Almanac

TABLE Ia

MULTIPLICATION TABLE for use with Tables I and II

RANGE AT STANDARD PORT

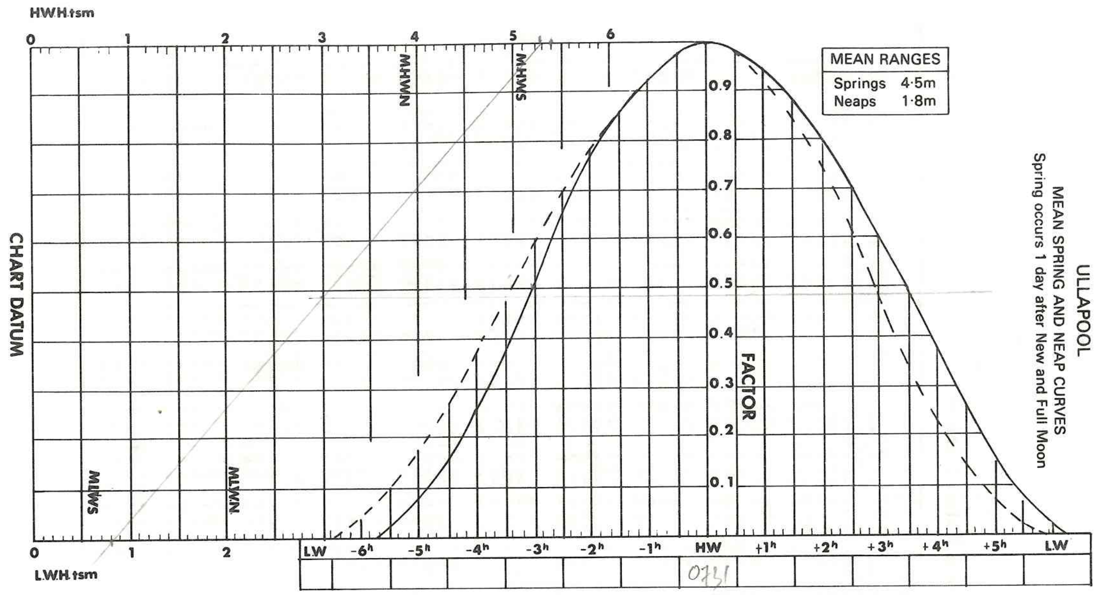
FACTOR 1·00	3·0	4·0	5·0	6·0	7·0	8·0	9·0	10·0	11·0	12·0	13·0	14·0	15·0	16·0	17·0	18·0	19·0	20·0	21·0	22·0	23·0	24·0	25·0	26·0	1·00 FACTOR
·98	2·9	3·9	4·9	5·9	6·9	7·8	8·8	9·8	10·8	11·8	12·7	13·7	14·7	15·7	16·7	17·6	18·6	19·6	20·6	21·6	22·5	23·5	24·5	25·5	·98
·96	2·9	3·8	4·8	5·8	6·7	7·7	8·6	9·6	10·6	11·5	12·5	13·4	14·4	15·4	16·3	17·3	18·2	19·2	20·2	21·1	22·1	23·0	24·0	25·0	·96
·94	2·8	3·8	4·7	5·6	6·6	7·5	8·5	9·4	10·3	11·3	12·2	13·2	14·1	15·0	16·0	16·9	17·9	18·8	19·7	20·7	21·6	22·6	23·5	24·4	·94
·92	2·8	3·7	4·6	5·5	6·4	7·4	8·3	9·2	10·1	11·0	12·0	12·9	13·8	14·7	15·6	16·6	17·5	18·4	19·3	20·2	21·2	21·1	23·0	23·9	·92
·90	2·7	3·6	4·5	5·4	6·3	7·2	8·1	9·0	9·9	10·8	11·7	12·6	13·5	14·4	15·3	16·2	17·1	18·0	18·9	19·8	20·7	21·6	22·5	23·4	·90
·88	2·6	3·5	4·4	5·3	6·2	7·0	7·9	8·8	9·7	10·6	11·4	12·3	13·2	14·1	15·9	15·8	16·7	17·6	18·5	19·4	20·2	21·1	22·0	22·9	·88
·86	2·6	3·4	4·3	5·2	6·0	6·9	7·7	8·6	9·5	10·3	11·2	12·0	12·9	13·8	14·6	15·5	16·3	17·2	18·1	18·9	19·8	20·6	21·5	22·4	·86
·84	2·5	3·4	4·2	5·0	5·9	6·7	7·6	8·4	9·2	10·1	10·9	11·8	12·6	13·4	14·3	15·1	16·0	16·8	17·6	18·5	19·3	20·2	21·0	21·8	·84
·82	2·5	3·3	4·1	4·9	5·7	6·6	7·4	8·2	9·0	9·8	10·7	11·5	12·3	13·1	13·9	14·8	15·6	16·4	17·2	18·0	18·9	19·7	20·5	21·3	·82
·80	2·4	3·2	4·0	4·8	5·6	6·4	7·2	8·0	8·8	9·6	10·4	11·2	12·0	12·8	13·6	14·4	15·2	16·0	16·8	17·6	18·4	19·2	20·0	20·8	·80
·78	2·3	3·1	3·9	4·7	5·5	6·2	7·0	7·8	8·6	9·4	10·1	10·9	11·7	12·5	13·3	14·0	14·8	15·6	16·4	17·2	17·9	18·7	19·5	20·3	·78
·76	2·3	3·0	3·8	4·6	5·3	6·1	6·8	7·6	8·4	9·1	9·9	10·6	11·4	12·2	12·9	13·7	14·4	15·2	16·0	16·7	17·5	18·2	19·0	19·8	·76
·74	2·2	3·0	3·7	4·4	5·2	5·9	6·7	7·4	8·1	8·9	9·6	10·4	11·1	11·8	12·6	13·3	14·1	14·8	15·5	16·3	17·0	17·8	18·5	19·2	·74
·72	2·2	2·9	3·6	4·3	5·0	5·8	6·5	7·2	7·9	8·6	9·4	10·1	10·8	11·5	12·2	13·0	13·7	14·4	15·1	15·8	16·6	17·3	18·0	18·7	·72
·70	2·1	2·8	3·5	4·2	4·9	5·6	6·3	7·0	7·7	8·4	9·1	9·8	10·5	11·2	11·9	12·6	13·3	14·0	14·7	15·4	16·1	16·8	17·5	18·2	·70
·68	2·0	2·7	3·4	4·1	4·8	5·4	6·1	6·8	7·5	8·2	8·8	9·5	10·2	10·9	11·6	12·2	12·9	13·6	14·3	15·0	15·6	16·3	17·0	17·7	·68
·66	2·0	2·6	3·3	4·0	4·6	5·3	5·9	6·6	7·3	7·9	8·6	9·2	9·9	10·6	11·2	11·9	12·5	13·2	13·9	14·5	15·2	15·8	16·5	17·2	·66
·64	1·9	2·6	3·2	3·8	4·5	5·1	5·8	6·4	7·0	7·7	8·3	9·0	9·6	10·2	10·9	11·5	12·2	12·8	13·4	14·1	14·7	15·4	16·0	16·6	·64
·62	1·9	2·5	3·1	3·7	4·3	5·0	5·6	6·2	6·8	7·4	8·1	8·7	9·3	9·9	10·5	11·2	11·8	12·4	13·0	13·6	14·3	14·9	15·5	16·1	·62
·60	1·8	2·4	3·0	3·6	4·2	4·8	5·4	6·0	6·6	7·2	7·8	8·4	9·0	9·6	10·2	10·8	11·4	12·0	12·6	13·2	13·8	14·4	15·0	15·6	·60
·58	1·7	2·3	2·9	3·5	4·1	4·6	5·2	5·8	6·4	7·0	7·5	8·1	8·7	9·3	9·9	10·4	11·0	11·6	12·2	12·8	13·3	13·9	14·5	15·1	·58
·56	1·7	2·2	2·8	3·4	3·9	4·5	5·0	5·6	6·2	6·7	7·3	7·8	8·4	9·0	9·5	10·1	10·6	11·2	11·8	12·3	12·9	13·4	14·0	14·6	·56
·54	1·6	2·2	2·7	3·2	3·8	4·3	4·9	5·4	5·9	6·5	7·0	7·6	8·1	8·6	9·2	9·7	10·3	10·8	11·3	11·9	12·4	13·0	13·5	14·0	·54
·52	1·6	2·1	2·6	3·1	3·6	4·2	4·7	5·2	5·7	6·2	6·8	7·3	7·8	8·3	8·8	9·4	9·9	10·4	10·9	11·4	12·0	12·5	13·0	13·5	·52
·50	1·5	2·0	2·5	3·0	3·5	4·0	4·5	5·0	5·5	6·0	6·5	7·0	7·5	8·0	8·5	9·0	9·5	10·0	10·5	11·0	11·5	12·0	12·5	13·0	·50
·48	1·4	1·9	2·4	2·9	3·4	3·8	4·3	4·8	5·3	5·8	6·2	6·7	7·2	7·7	8·2	8·6	9·1	9·6	10·1	10·6	11·0	11·5	12·0	12·5	·48
·46	1·4	1·8	2·3	2·8	3·2	3·7	4·1	4·6	5·1	5·5	6·0	6·4	6·9	7·4	7·8	8·3	8·7	9·2	9·7	10·1	10·6	11·0	11·5	12·0	·46
·44	1·3	1·8	2·2	2·6	3·1	3·5	4·0	4·4	4·8	5·3	5·7	6·2	6·6	7·0	7·5	7·9	8·4	8·8	9·2	9·7	10·1	10·6	11·0	11·4	·44
·42	1·3	1·7	2·1	2·5	2·9	3·4	3·8	4·2	4·6	5·0	5·5	5·9	6·3	6·7	7·1	7·6	8·0	8·4	8·8	9·2	9·7	10·1	10·5	10·9	·42
·40	1·2	1·6	2·0	2·4	2·8	3·2	3·6	4·0	4·4	4·8	5·2	5·6	6·0	6·4	6·8	7·2	7·6	8·0	8·4	8·8	9·2	9·6	10·0	10·4	·40
·38	1·1	1·5	1·9	2·3	2·7	3·0	3·4	3·8	4·2	4·6	4·9	5·3	5·7	6·1	6·5	6·8	7·2	7·6	8·0	8·4	8·7	9·1	9·5	9·9	·38
·36	1·1	1·4	1·8	2·2	2·5	2·9	3·2	3·6	4·0	4·3	4·7	5·0	5·4	5·8	6·1	6·5	6·8	7·2	7·6	7·9	8·3	8·6	9·0	9·4	·36
·34	1·0	1·4	1·7	2·0	2·4	2·7	3·1	3·4	3·7	4·1	4·4	4·8	5·1	5·4	5·8	6·1	6·5	6·8	7·1	7·5	7·8	8·2	8·5	8·8	·34
·32	1·0	1·3	1·6	1·9	2·2	2·6	2·9	3·2	3·5	3·8	4·2	4·5	4·8	5·1	5·4	5·8	6·1	6·4	6·7	7·0	7·4	7·7	8·0	8·3	·32
·30	0·9	1·2	1·5	1·8	2·1	2·4	2·7	3·0	3·3	3·6	3·9	4·2	4·5	4·8	5·1	5·4	5·7	6·0	6·3	6·6	6·9	7·2	7·5	7·8	·30
·28	0·8	1·1	1·4	1·7	2·0	2·2	2·5	2·8	3·1	3·4	3·6	3·9	4·2	4·5	4·8	5·0	5·3	5·6	5·9	6·2	6·4	6·7	7·0	7·3	·28
·26	0·8	1·0	1·3	1·6	1·8	2·1	2·3	2·6	2·9	3·1	3·4	3·6	3·9	4·2	4·4	4·7	4·9	5·2	5·5	5·7	6·0	6·2	6·5	6·8	·26
·24	0·7	1·0	1·2	1·4	1·7	1·9	2·2	2·4	2·6	2·9	3·1	3·4	3·6	3·8	4·1	4·3	4·6	4·8	5·0	5·3	5·5	5·8	6·0	6·2	·24
·22	0·7	0·9	1·1	1·3	1·5	1·8	2·0	2·2	2·4	2·6	2·9	3·1	3·3	3·5	3·7	4·9	4·2	4·4	4·6	4·8	5·1	5·3	5·5	5·7	·22
·20	0·6	0·8	1·0	1·2	1·4	1·6	1·8	2·0	2·2	2·4	2·6	2·8	3·0	3·2	3·4	3·6	3·8	4·0	4·2	4·4	4·6	4·8	5·0	5·2	·20
·18	0·5	0·7	0·9	1·1	1·3	1·4	1·6	1·8	2·0	2·2	2·3	2·5	2·7	2·9	3·1	3·2	3·4	3·6	3·8	4·0	4·1	4·3	4·5	4·7	·18
·16	0·5	0·6	0·8	1·0	1·1	1·3	1·4	1·6	1·8	1·9	2·1	2·2	2·4	2·6	2·7	2·9	3·0	3·2	3·4	3·5	3·7	3·8	4·0	4·2	·16
·14	0·4	0·6	0·7	0·8	1·0	1·1	1·3	1·4	1·5	1·7	1·8	2·0	2·1	2·2	2·4	2·5	2·7	2·8	2·9	3·1	3·2	3·4	3·5	3·6	·14
·12	0·4	0·5	0·6	0·7	0·8	1·0	1·1	1·2	1·3	1·4	1·6	1·7	1·8	1·9	2·0	2·2	2·3	2·4	2·5	2·6	2·8	2·9	3·0	3·1	·12
·10	0·3	0·4	0·5	0·6	0·7	0·8	0·9	1·0	1·1	1·2	1·3	1·4	1·5	1·6	1·7	1·8	1·9	2·0	2·1	2·2	2·3	2·4	2·5	2·6	·10
·08	0·2	0·3	0·4	0·5	0·6	0·6	0·7	0·8	0·9	1·0	1·0	1·1	1·2	1·3	1·4	1·4	1·5	1·6	1·7	1·8	1·8	1·9	2·0	2·1	·08
·06	0·2	0·2	0·3	0·4	0·4	0·5	0·5	0·6	0·7	0·7	0·8	0·8	0·9	1·0	1·0	1·1	1·1	1·2	1·3	1·3	1·4	1·4	1·5	1·6	·06
·04	0·1	0·2	0·2	0·2	0·3	0·3	0·4	0·4	0·4	0·5	0·5	0·6	0·6	0·6	0·7	0·7	0·8	0·8	0·8	0·9	0·9	1·0	1·0	1·0	·04
·02	0·1	0·1	0·1	0·1	0·1	0·2	0·2	0·2	0·2	0·2	0·3	0·3	0·3	0·3	0·3	0·4	0·4	0·4	0·4	0·4	0·5	0·5	0·5	0·5	·02

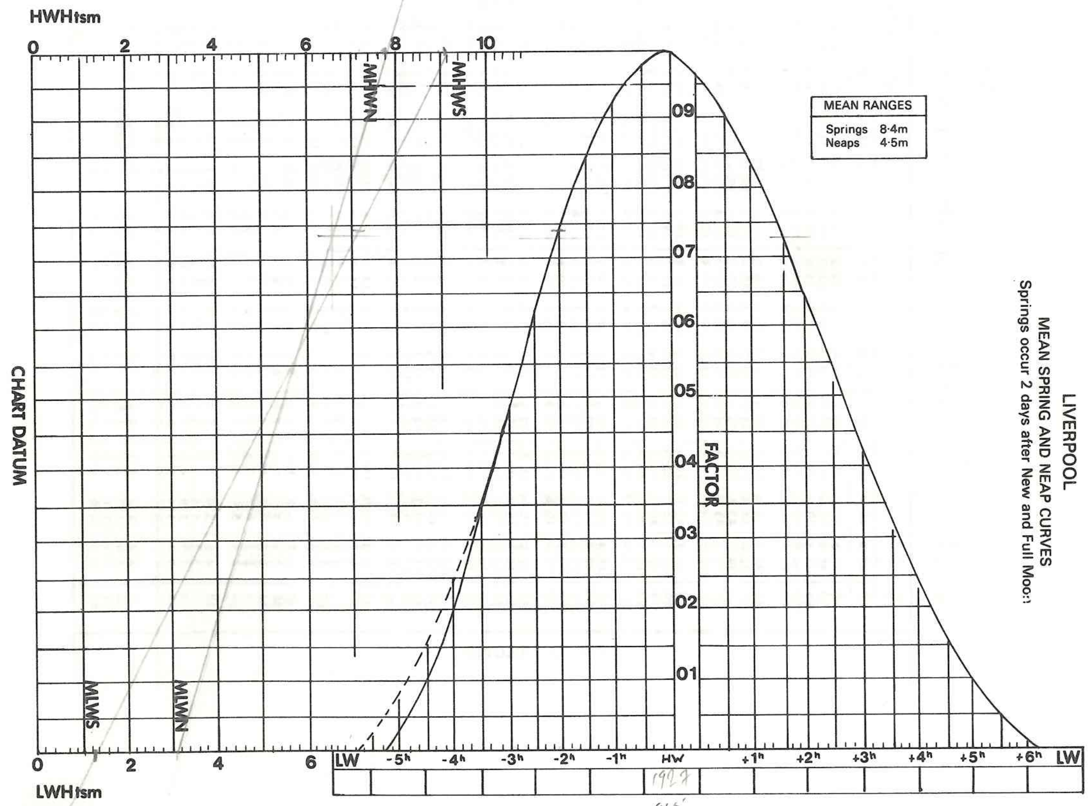
TABLE Ia (*cont.*)

MULTIPLICATION TABLE for use with Tables I and II

RANGE AT STANDARD PORT

FACTOR 1·00	27·0	28·0	29·0	30·0	31·0	32·0	33·0	34·0	35·0	36·0	37·0	38·0	39·0	40·0	41·0	42·0	43·0	44·0	45·0	46·0	47·0	48·0	49·0	50·0	1·00 FACTOR
·98	26·5	27·4	28·4	29·4	30·4	31·4	32·3	33·3	34·3	35·3	36·3	37·2	38·2	39·2	40·2	41·2	42·1	43·1	44·1	45·1	46·1	47·0	48·0	49·0	·98
·96	25·9	26·9	27·8	28·8	29·8	30·7	31·7	32·6	33·6	34·6	35·5	36·5	37·4	38·4	39·4	40·3	41·3	42·2	43·2	44·2	45·1	46·1	47·0	48·0	·96
·94	25·4	26·3	27·3	28·2	29·1	30·1	31·0	32·0	32·9	33·8	34·8	35·7	36·7	37·6	38·5	39·5	40·4	41·4	42·3	43·2	44·2	45·1	46·1	47·0	·94
·92	24·8	25·8	26·7	27·6	28·5	29·4	30·4	31·3	32·2	33·1	34·0	35·0	35·9	36·8	37·7	38·6	39·6	40·5	41·4	42·3	43·2	44·2	45·1	46·0	·92
·90	24·3	25·2	26·1	27·0	27·9	28·8	29·7	30·6	31·5	32·4	33·3	34·2	35·1	36·0	36·9	37·8	38·7	39·6	40·5	41·4	42·3	43·2	44·1	45·0	·90
·88	23·8	24·6	25·5	26·4	27·3	28·2	29·0	29·9	30·8	31·7	32·6	33·4	34·3	35·2	36·1	37·0	37·8	38·7	39·6	40·5	41·4	42·2	43·1	44·0	·88
·86	23·2	24·1	24·9	25·8	26·7	27·5	28·4	29·2	30·1	31·0	31·8	32·7	33·5	34·4	35·3	36·1	37·0	37·8	38·7	39·6	40·4	41·3	42·1	43·0	·86
·84	22·7	23·5	24·4	25·2	26·0	26·9	27·7	28·6	29·4	30·2	31·1	31·9	32·8	33·6	34·4	35·3	36·1	37·0	37·8	38·6	39·5	40·3	41·2	42·0	·84
·82	22·1	23·0	23·8	24·6	25·4	26·2	27·1	27·9	28·7	29·5	30·3	31·2	32·0	32·8	33·6	34·4	35·3	36·1	36·9	37·7	38·5	39·4	40·2	41·0	·82
·80	21·6	22·4	23·2	24·0	24·8	25·6	26·4	27·2	28·0	28·8	29·6	30·4	31·2	32·0	32·8	33·6	34·4	35·2	36·0	36·8	37·6	38·4	39·2	40·0	·80
·78	21·1	21·8	22·6	23·4	24·2	25·0	25·7	26·5	27·3	28·1	28·9	29·6	30·4	31·2	32·0	32·8	33·5	34·3	35·1	35·9	36·7	37·4	38·2	39·0	·78
·76	20·5	21·3	22·0	22·8	23·6	24·3	25·1	25·8	26·6	27·4	28·1	28·9	29·6	30·4	31·2	31·9	32·7	33·4	34·2	35·0	35·7	36·5	37·2	38·0	·76
·74	20·0	20·7	21·5	22·2	22·9	23·7	24·4	25·2	25·9	26·6	27·4	28·1	28·9	29·6	30·3	31·1	31·8	32·6	33·3	34·0	34·8	35·5	36·3	37·0	·74
·72	19·4	20·2	20·9	21·6	22·3	23·0	23·8	24·5	25·2	25·9	26·6	27·4	28·1	28·8	29·5	30·2	31·0	31·7	32·4	33·1	33·8	34·6	35·3	36·0	·72
·70	18·9	19·6	20·3	21·0	21·7	22·4	23·1	23·8	24·5	25·2	25·9	26·6	27·3	28·0	28·7	29·4	30·1	30·8	31·5	32·2	32·9	33·6	34·3	35·0	·70
·68	18·4	19·0	19·7	20·4	21·1	21·8	22·4	23·1	23·8	24·5	25·2	25·8	26·5	27·2	27·9	28·6	29·2	29·9	30·6	31·3	32·0	32·6	33·3	34·0	·68
·66	17·8	18·5	19·1	19·8	20·5	21·1	21·8	22·4	23·1	23·8	24·4	25·1	25·7	26·4	27·1	27·7	28·4	29·0	29·7	30·4	31·0	31·7	32·3	33·0	·66
·64	17·3	17·9	18·6	19·2	19·8	20·5	21·1	21·8	22·4	23·0	23·7	24·3	25·0	25·6	26·2	26·9	27·5	28·2	28·8	29·4	30·1	30·7	31·4	32·0	·64
·62	16·7	17·4	18·0	18·6	19·2	19·8	20·5	21·1	21·7	22·3	22·9	23·6	24·2	24·8	25·4	26·0	26·7	27·3	27·9	28·5	29·1	29·8	30·4	31·0	·62
·60	16·2	16·8	17·4	18·0	18·6	19·2	19·8	20·4	21·0	21·6	22·2	22·8	23·4	24·0	24·6	25·2	25·8	26·4	27·0	27·6	28·2	28·8	29·4	30·0	·60
·58	15·7	16·2	16·8	17·4	18·0	18·6	19·1	19·7	20·3	20·9	21·5	22·0	22·6	23·2	23·8	24·4	24·9	25·5	26·1	26·7	27·3	27·8	28·4	29·0	·58
·56	15·1	15·7	16·2	16·8	17·4	17·9	18·5	19·0	19·6	20·2	20·7	21·3	21·8	22·4	23·0	23·5	24·1	24·6	25·2	25·8	26·3	26·9	27·4	28·0	·56
·54	14·6	15·1	15·7	16·2	16·7	17·3	17·8	18·4	18·9	19·4	20·0	20·5	21·1	21·6	22·1	22·7	23·2	23·8	24·3	24·8	25·4	25·9	26·5	27·0	·54
·52	14·0	14·6	15·1	15·6	16·1	16·6	17·2	17·7	18·2	18·7	19·2	19·8	20·3	20·8	21·3	21·8	22·4	22·9	23·4	23·9	24·4	25·0	25·5	26·0	·52
·50	13·5	14·0	14·5	15·0	15·5	16·0	16·5	17·0	17·5	18·0	18·5	19·0	19·5	20·0	20·5	21·0	21·5	22·0	22·5	23·0	23·5	24·0	24·5	25·0	·50
·48	13·0	13·4	13·9	14·4	14·9	15·4	15·8	16·3	16·8	17·3	17·8	18·2	18·7	19·2	19·7	20·2	20·6	21·1	21·6	22·1	22·6	23·0	23·5	24·0	·48
·46	12·4	12·9	13·3	13·8	14·3	14·7	15·2	15·6	16·1	16·6	17·0	17·5	17·9	18·4	18·9	19·3	19·8	20·2	20·7	21·2	21·6	22·1	22·5	23·0	·46
·44	11·9	12·3	12·8	13·2	13·6	14·1	14·5	15·0	15·4	15·8	16·3	16·7	17·2	17·6	18·0	18·5	18·9	19·4	19·8	20·2	20·7	21·1	21·6	22·0	·44
·42	11·3	11·8	12·2	12·6	13·0	13·4	13·9	14·3	14·7	15·1	15·5	16·0	16·4	16·8	17·2	17·6	18·1	18·5	18·9	19·3	19·7	20·2	20·6	21·0	·42
·40	10·8	11·2	11·6	12·0	12·4	12·8	13·2	13·6	14·0	14·4	14·8	15·2	15·6	16·0	16·4	16·8	17·2	17·6	18·0	18·4	18·8	19·2	19·6	20·0	·40
·38	10·3	10·6	11·0	11·4	11·8	12·2	12·5	12·9	13·3	13·7	14·1	14·4	14·8	15·2	15·6	16·0	16·3	16·7	17·1	17·5	17·9	18·2	18·6	19·0	·38
·36	9·7	10·1	10·4	10·8	11·2	11·5	11·9	12·2	12·6	13·0	13·3	13·7	14·0	14·4	14·8	15·1	15·5	15·8	16·2	16·6	16·9	17·3	17·6	18·0	·36
·34	9·2	9·5	9·9	10·2	10·5	10·9	11·2	11·6	11·9	12·2	12·6	12·9	13·3	13·6	13·9	14·3	14·6	15·0	15·3	15·6	16·0	16·3	16·7	17·0	·34
·32	8·6	9·0	9·3	9·6	9·9	10·2	10·6	10·9	11·2	11·5	11·8	12·2	12·5	12·8	13·1	13·4	13·8	14·1	14·4	14·7	15·0	15·4	15·7	16·0	·32
·30	8·1	8·4	8·7	9·0	9·3	9·6	9·9	10·2	10·5	10·8	11·1	11·4	11·7	12·0	12·3	12·6	12·9	13·2	13·5	13·8	14·1	14·4	14·7	15·0	·30
·28	7·6	7·8	8·1	8·4	8·7	9·0	9·2	9·5	9·8	10·1	10·4	10·6	10·9	11·2	11·5	11·8	12·0	12·3	12·6	12·9	13·2	13·4	13·7	14·0	·28
·26	7·0	7·3	7·5	7·8	8·1	8·3	8·6	8·8	9·1	9·4	9·6	9·9	10·1	10·4	10·7	10·9	11·2	11·4	11·7	12·0	12·2	12·5	12·7	13·0	·26
·24	6·5	6·7	7·0	7·2	7·4	7·7	7·9	8·2	8·4	8·6	8·9	9·1	9·4	9·6	9·8	10·1	10·3	10·6	10·8	11·0	11·3	11·5	11·8	12·0	·24
·22	5·9	6·2	6·4	6·6	6·8	7·0	7·3	7·5	7·7	7·9	8·1	8·4	8·6	8·8	9·0	9·2	9·5	9·7	9·9	10·1	10·3	10·6	10·8	11·0	·22
·20	5·4	5·6	5·8	6·0	6·2	6·4	6·6	6·8	7·0	7·2	7·4	7·6	7·8	8·0	8·2	8·4	8·6	8·8	9·0	9·2	9·4	9·6	9·8	10·0	·20
·18	4·9	5·0	5·2	5·4	5·6	5·8	5·9	6·1	6·3	6·5	6·7	6·8	7·0	7·2	7·4	7·6	7·7	7·9	8·1	8·3	8·5	8·6	8·8	9·0	·18
·16	4·3	4·5	4·6	4·8	5·0	5·1	5·3	5·4	5·6	5·8	5·9	6·1	6·2	6·4	6·6	6·7	6·9	7·0	7·2	7·4	7·5	7·7	7·8	8·0	·16
·14	3·8	3·9	4·1	4·2	4·3	4·5	4·6	4·8	4·9	5·0	5·2	5·3	5·5	5·6	5·7	5·9	6·0	6·2	6·3	6·4	6·6	6·7	6·9	7·0	·14
·12	3·2	3·4	3·5	3·6	3·7	3·8	4·0	4·1	4·2	4·3	4·4	4·6	4·7	4·8	4·9	5·0	5·2	5·3	5·4	5·5	5·6	5·8	5·9	6·0	·12
·10	2·7	2·8	2·9	3·0	3·1	3·2	3·3	3·4	3·5	3·6	3·7	3·8	3·9	4·0	4·1	4·2	4·3	4·4	4·5	4·6	4·7	4·8	4·9	5·0	·10
·08	2·2	2·2	2·3	2·4	2·5	2·6	2·6	2·7	2·8	2·9	3·0	3·0	3·1	3·2	3·3	3·4	3·4	3·5	3·6	3·7	3·8	3·8	3·9	4·0	·08
·06	1·6	1·7	1·7	1·8	1·9	1·9	2·0	2·0	2·1	2·2	2·2	2·3	2·3	2·4	2·5	2·5	2·6	2·6	2·7	2·8	2·8	2·9	2·9	3·0	·06
·04	1·1	1·1	1·2	1·2	1·2	1·3	1·3	1·4	1·4	1·4	1·5	1·5	1·6	1·6	1·6	1·7	1·7	1·8	1·8	1·8	1·9	1·9	2·0	2·0	·04
·02	0·5	0·6	0·6	0·6	0·6	0·6	0·7	0·7	0·7	0·7	0·7	0·8	0·8	0·8	0·8	0·8	0·9	0·9	0·9	0·9	0·9	1·0	1·0	1·0	·02





SCOTLAND, WEST COAST - ULLAPOOL

LAT 57°54'N LONG 5°10'W

TIME ZONE GMT — TIMES AND HEIGHTS OF HIGH AND LOW WATERS — YEAR 1987

JANUARY

	TIME	M		TIME	M
1 TH	0132 0731 1400 2002	0.8 5.3 0.6 5.1	16 F	0140 0741 1417 2002	1.4 4.9 1.3 4.7
2 F	0218 0819 1448 2053	0.8 5.3 0.5 4.9	17 SA	0213 0810 1450 2035	1.4 5.0 1.2 4.7
3 SA	0304 0908 1535 2145	0.9 5.3 0.6 4.7	18 SU	0246 0842 1524 2109	1.4 5.0 1.3 4.6
4 SU	0350 1000 1623 2241	1.0 5.1 0.8 4.5	19 M	0321 0915 1558 2146	1.4 4.9 1.3 4.5
5 M	0437 1055 1713 2340	1.3 4.9 1.1 4.3	20 TU	0357 0952 1634 2228	1.5 4.8 1.4 4.4
6 TU	0528 1152 1805	1.6 4.6 1.4	21 W	0438 1033 1715 2318	1.6 4.7 1.6 4.3
7 W	0043 0625 1252 1904	4.1 1.9 4.4 1.7	22 TH	0523 1122 1803	1.8 4.5 1.7
8 TH	0150 0730 1356 2009	4.0 2.1 4.2 1.9	23 F	0019 0618 1222 1904	4.2 2.0 4.3 1.9
9 F	0257 0842 1503 2117	3.9 2.2 4.1 2.0	24 SA	0133 0726 1343 2021	4.1 2.1 4.2 1.9
10 SA	0359 0951 1606 2218	4.0 2.2 4.1 2.0	25 SU	0252 0848 1513 2142	4.2 2.1 4.2 1.9
11 SU	0451 1052 1700 2309	4.2 2.1 4.2 1.9	26 M	0402 1011 1629 2252	4.3 1.9 4.3 1.6
12 M	0533 1144 1744 2353	4.4 1.9 4.3 1.8	27 TU	0501 1120 1730 2350	4.6 1.6 4.6 1.3
13 TU	0609 1228 1823	4.5 1.7 4.5	28 W	0552 1218 1822	4.9 1.2 4.9
14 W	0031 0641 1307 1857	1.7 4.7 1.5 4.6	29 TH	0040 0639 1308 ● 1909	1.0 5.2 0.8 5.1
15 TH	0107 0711 1343 O 1930	1.5 4.8 1.4 4.7	30 F	0125 0723 1353 1953	0.8 5.4 0.5 5.2
			31 SA	0207 0806 1436 2036	0.6 5.6 0.3 5.1

FEBRUARY

	TIME	M		TIME	M
1 SU	0248 0849 1517 2118	0.6 5.5 0.4 5.0	16 M	0227 0818 1458 2041	1.1 5.2 1.0 5.0
2 M	0329 0932 1557 2200	0.8 5.4 0.6 4.8	17 TU	0259 0849 1530 2114	1.1 5.2 1.0 4.9
3 TU	0409 1015 1637 2244	1.0 5.1 1.0 4.5	18 W	0334 0923 1604 2151	1.2 5.1 1.2 4.8
4 W	0450 1100 1717 2333	1.4 4.7 1.4 4.2	19 TH	0412 1001 1641 2235	1.3 4.9 1.4 4.5
5 TH	0535 1150 1801	1.8 4.3 1.8	20 F	0454 1045 1724 2331	1.6 4.6 1.6 4.3
6 F	0036 0627 1253 1856	3.9 2.1 4.0 2.2	21 SA	0542 1144 1818	1.8 4.2 1.9
7 SA	0200 0739 1417 2014	3.7 2.4 3.8 2.4	22 SU	0051 0646 1320 1940	4.0 2.1 3.9 2.1
8 SU	0322 0916 1540 2148	3.8 2.5 3.8 2.4	23 M	0228 0823 1510 2130	4.0 2.2 3.9 2.1
9 M	0428 1041 1645 2256	3.9 2.3 3.9 2.2	24 TU	0349 1009 1630 2250	4.1 2.0 4.2 1.8
10 TU	0517 1138 1732 2343	4.2 2.0 4.2 2.0	25 W	0452 1121 1729 2345	4.5 1.5 4.5 1.4
11 W	0555 1220 1810	4.4 1.7 4.4	26 TH	0542 1213 1815	4.9 1.0 4.8
12 TH	0020 0626 1255 1843	1.7 4.7 1.5 4.6	27 F	0030 0626 1256 1856	1.0 5.2 0.6 5.1
13 F	0053 0655 1327 O 1912	1.5 4.9 1.2 4.8	28 SA	0110 0707 1336 ● 1934	0.7 5.5 0.3 5.3
14 SA	0125 0722 1358 1941	1.3 5.0 1.1 4.9			
15 SU	0155 0749 1428 2010	1.2 5.2 1.0 4.9			

MARCH

	TIME	M		TIME	M
1 SU	0149 0745 1414 2009	0.5 5.6 0.2 5.3	16 M	0131 0721 1359 1942	1.0 5.2 0.8 5.2
2 M	0226 0822 1450 2044	0.5 5.6 0.3 5.1	17 TU	0202 0750 1429 2012	0.9 5.3 0.8 5.2
3 TU	0302 0859 1525 2117	0.6 5.4 0.6 4.9	18 W	0236 0822 1501 2045	0.9 5.3 0.8 5.1
4 W	0338 0935 1558 2150	0.9 5.1 1.0 4.6	19 TH	0311 0857 1536 2122	0.9 5.1 1.0 4.9
5 TH	0414 1011 1632 2225	1.3 4.7 1.4 4.3	20 F	0349 0937 1613 2205	1.1 4.8 1.3 4.6
6 F	0452 1052 1708 2308	1.7 4.3 1.9 3.9	21 SA	0431 1024 1656 2302	1.4 4.4 1.6 4.2
7 SA	0537 1151 1751	2.1 3.9 2.3	22 SU	0521 1134 1751	1.7 4.0 2.0
8 SU	0039 0640 1333 1900	3.7 2.4 3.6 2.6	23 M	0034 0630 1333 1923	3.9 2.0 3.7 2.3
9 M	0236 0842 1512 2117	3.6 2.6 3.6 2.6	24 TU	0217 0826 1515 2129	3.9 2.1 3.8 2.1
10 TU	0355 1027 1622 2240	3.8 2.3 3.8 2.3	25 W	0336 1011 1626 2241	4.1 1.7 4.1 1.7
11 W	0449 1120 1711 2324	4.0 2.0 4.1 2.0	26 TH	0437 1111 1718 2330	4.4 1.2 4.5 1.3
12 TH	0528 1158 1747 2359	4.3 1.6 4.4 1.8	27 F	0525 1156 1759	4.8 0.8 4.8
13 F	0600 1230 1818	4.6 1.4 4.6	28 SA	0011 0606 1235 1835	0.9 5.2 0.5 5.1
14 SA	0030 0627 1300 1846	1.5 4.8 1.1 4.8	29 SU	0048 0644 1311 ● 1909	0.7 5.4 0.3 5.2
15 SU	0100 0654 1329 O 1913	1.2 5.1 0.9 5.0	30 M	0124 0719 1346 1940	0.5 5.5 0.3 5.2
			31 TU	0200 0753 1419 2010	0.5 5.4 0.5 5.1

APRIL

	TIME	M		TIME	M
1 W	0234 0826 1451 2039	0.7 5.2 0.8 4.9	16 TH	0213 0801 1436 2023	0.7 5.2 0.7 5.1
2 TH	0309 0859 1523 2108	1.0 4.9 1.1 4.6	17 F	0252 0840 1514 2103	0.8 5.0 0.9 4.9
3 F	0344 0934 1554 2139	1.3 4.5 1.5 4.3	18 SA	0333 0926 1555 2152	1.0 4.6 1.2 4.6
4 SA	0421 1015 1628 2219	1.6 4.1 1.9 4.0	19 SU	0420 1026 1642 2300	1.2 4.2 1.6 4.2
5 SU	0504 1115 1708 2331	2.0 3.7 2.2 3.7	20 M	0516 1157 1744	1.5 3.8 2.0
6 M	0604 1256 1808	2.3 3.5 2.5	21 TU	0034 0634 1341 1925	4.0 1.8 3.7 2.2
7 TU	0138 0803 1433 2022	3.6 2.4 3.5 2.6	22 W	0202 0828 1505 2112	4.0 1.7 3.8 2.0
8 W	0303 0950 1544 2200	3.7 2.2 3.7 2.4	23 TH	0314 0951 1608 2216	4.2 1.4 4.1 1.6
9 TH	0403 1043 1635 2247	3.9 1.9 4.0 2.1	24 F	0412 1045 1657 2303	4.4 1.1 4.4 1.3
10 F	0447 1121 1713 2323	4.2 1.6 4.3 1.8	25 SA	0500 1129 1736 2344	4.7 0.8 4.7 1.0
11 SA	0521 1154 1745 2356	4.5 1.3 4.6 1.5	26 SU	0542 1207 1811	4.9 0.7 4.9
12 SU	0551 1225 1814	4.7 1.0 4.9	27 M	0022 0619 1243 1843	0.8 5.1 0.6 5.0
13 M	0029 0621 1256 1844	1.2 5.0 0.8 5.1	28 TU	0059 0654 1317 ● 1913	0.8 5.1 0.7 5.0
14 TU	0102 0652 1328 O 1914	1.0 5.2 0.7 5.2	29 W	0135 0727 1349 1942	0.8 5.0 0.8 4.9
15 W	0137 0725 1401 1947	0.8 5.3 0.7 5.2	30 TH	0210 0801 1421 2011	0.9 4.9 1.0 4.8

ENGLAND, WEST COAST - LIVERPOOL

LAT 53°25'N LONG 3°00'W

TIME ZONE GMT | TIMES AND HEIGHTS OF HIGH AND LOW WATERS | YEAR 1987

SEPTEMBER

Day	TIME	M	Day	TIME	M
1	0328	8.1	16	0455	6.9
	1013	2.8		1126	3.8
TU	1603	7.7	W	1736	6.8
	2242	3.0			
2	0433	7.6	17	0046	3.7
	1125	3.2		0641	6.6
W	1723	7.4	TH	1319	3.9
				1926	7.0
3	0011	3.1	18	0220	3.3
	0611	7.3		0812	7.1
TH	1307	3.2	F	1442	3.5
	1902	7.5		2034	7.5
4	0151	2.8	19	0322	2.8
	0748	7.6		0905	7.6
F	1436	2.7	SA	1534	2.9
	2022	8.1		2121	8.1
5	0314	2.1	20	0407	2.2
	0903	8.2		0946	8.2
SA	1545	2.1	SU	1614	2.4
	2125	8.8		2159	8.6
6	0419	1.4	21	0444	1.8
	0959	8.9		1020	8.6
SU	1641	1.4	M	1648	2.0
	2216	9.4		2233	8.9
7	0513	0.7	22	0518	1.4
	1047	9.4		1051	8.9
M	1729	0.9	TU	1720	1.6
	○ 2302	9.9		2304	9.2
8	0600	0.3	23	0550	1.2
	1129	9.6		1120	9.1
TU	1812	0.6	W	1753	1.4
	2344	10.1		● 2333	9.3
9	0642	0.1	24	0621	1.1
	1210	9.7		1150	9.2
W	1853	0.5	TH	1825	1.2
10	0025	10.1	25	0003	9.4
	0720	0.2		0652	1.1
TH	1248	9.6	F	1219	9.2
	1930	0.7		1857	1.3
11	0103	9.9	26	0034	9.4
	0755	0.6		0723	1.2
F	1324	9.3	SA	1252	9.2
	2006	1.0		1931	1.4
12	0141	9.5	27	0104	9.2
	0829	1.2		0754	1.5
SA	1359	8.9	SU	1324	9.0
	2042	1.6		2004	1.7
13	0218	8.9	28	0140	8.9
	0901	1.8		0825	1.9
SU	1434	8.4	M	1401	8.7
	2118	2.2		2040	2.1
14	0257	8.3	29	0219	8.5
	0935	2.6		0901	2.4
M	1517	7.8	TU	1446	8.3
	2200	2.9		2127	2.5
15	0346	7.5	30	0311	8.0
	1016	3.3		0952	2.9
TU	1610	7.2	W	1545	7.8
	2304	3.5		2231	3.0

OCTOBER

Day	TIME	M	Day	TIME	M
1	0426	7.4	16	0001	3.7
	1111	3.4		0556	6.6
TH	1713	7.4	F	1227	4.1
				1834	6.9
2	0007	3.1	17	0133	3.4
	0611	7.2		0728	6.9
F	1259	3.3	SA	1355	3.7
	1850	7.6		1949	7.4
3	0147	2.6	18	0236	2.9
	0744	7.7		0826	7.5
SA	1425	2.7	SU	1451	3.1
	2009	8.3		2040	7.9
4	0303	1.9	19	0324	2.3
	0851	8,4		0907	8.1
SU	1529	2.0	M	1534	2.5
	2108	9.0		2119	8.4
5	0403	1.2	20	0403	1.9
	0942	9.0		0942	8.5
M	1621	1.4	TU	1612	2.1
	2156	9.5		2155	8.8
6	0452	0.7	21	0440	1.5
	1026	9.4		1014	8.9
TU	1706	1.0	W	1647	1.7
	2240	9.9		2227	9.1
7	0534	0.5	22	0515	1.3
	1105	9.6		1045	9.1
W	1747	0.7	TH	1722	1.4
	○ 2319	10.0		● 2259	9.3
8	0612	0.4	23	0549	1.1
	1143	9.6		1118	9.3
TH	1825	0.7	F	1758	1.3
	2358	9.9		2332	9.4
9	0648	0.6	24	0624	1.1
	1218	9.5		1151	9.4
F	1902	0.9	SA	1835	1.3
10	0035	9.7	25	[illegible]07	9.4
	0721	1.0		[illegible]657	1.3
SA	1252	9.2	SU	1227	9.4
	1937	1.3		1912	1.4
11	0110	9.2	26	0045	9.2
	0752	1.5		0731	1.5
SU	1326	8.9	M	1304	9.2
	2011	1.8		1949	1.6
12	0147	8.7	27	0124	8.9
	0822	2.1		0808	1.9
M	1401	8.4	TU	1347	8.8
	2046	2.4		2032	2.0
13	0225	8.1	28	0212	8.5
	0853	2.8		0850	2.4
TU	1440	7.9	W	1437	8.4
	2125	3.0		2125	2.4
14	0311	7.4	29	0311	7.9
	0931	3.4		0948	2.9
W	1531	7.3	TH	1543	7.9
	2224	3.5		2235	2.7
15	0417	6.8	30	0430	7.5
	1033	3.9		1109	3.2
TH	1649	6.9	F	1706	7.7
			31	0005	2.7
				0603	7.4
			SA	1245	3.1
				1832	7.9

NOVEMBER

Day	TIME	M	Day	TIME	M
1	0128	2.3	16	0131	3.0
	0724	7.8		0724	7.3
SU	1401	2.6	M	1348	3.3
	1944	8.4		1940	7.7
2	0239	1.8	17	0226	2.6
	0826	8.4		0813	7.8
M	1503	2.0	TU	1442	2.8
	2042	8.9		2027	8.1
3	0335	1.3	18	0314	2.1
	0917	8.8		0856	8.2
TU	1555	1.6	W	1527	2.3
	2131	9.3		2110	8.5
4	0423	1.0	19	0357	1.8
	0959	9.1		0934	8.7
W	1640	1.3	TH	1610	1.9
	2214	9.5		2148	8.9
5	0505	0.9	20	0438	1.5
	1038	9.3		1012	9.0
TH	1720	1.1	F	1652	1.6
	○ 2254	9.6		2227	9.2
6	0543	1.0	21	0519	1.3
	1116	9.4		1049	9.3
F	1800	1.1	SA	1734	1.4
	2333	9.5		● 2308	9.3
7	0617	1.2	22	0558	1.3
	1151	9.3		1129	9.4
SA	1836	1.3	SU	1817	1.3
				2349	9.4
8	0010	9.3	23	0638	1.3
	0649	1.5		1210	9.4
SU	1225	9.1	M	1859	1.3
	1913	1.6			
9	0046	8.9	24	0034	9.2
	0721	1.9		0719	1.5
M	1300	8.8	TU	1255	9.3
	1948	2.0		1944	1.4
10	0123	8.5	25	0120	9.0
	0751	2.3		0801	1.8
TU	1335	8.5	W	1342	9.0
	2023	2.4		2034	1.7
11	0201	8.0	26	0213	8.6
	0823	2.8		0850	2.2
W	1415	8.1	TH	1437	8.7
	2103	2.9		2129	1.9
12	0246	7.5	27	0312	8.2
	0901	3.2		0948	2.5
TH	1501	7.6	F	1539	8.4
	2153	3.2		2235	2.1
13	0342	7.1	28	0421	7.9
	0953	3.7		1058	2.8
F	1604	7.2	SA	1648	8.2
	2304	3.4		2346	2.2
14	0455	6.8	29	0536	7.7
	1112	3.9		1214	2.8
SA	1723	7.1	SU	1800	8.2
15	0025	3.3	30	0056	2.1
	0617	6.9		0649	7.9
SU	1242	3.7	M	1326	2.5
	1839	7.3		1909	8.4

DECEMBER

Day	TIME	M	Day	TIME	M
1	0202	1.9	16	0121	2.8
	0752	8.1		0707	7.5
TU	1429	2.2	W	1341	3.1
	2009	8.6		1927	7.8
2	0301	1.7	17	0222	2.5
	0846	8.4		0804	7.9
W	1525	2.0	TH	1442	2.7
	2103	8.8		2023	8.1
3	0352	1.6	18	0317	2.2
	0934	8.7		0854	8.3
TH	1614	1.8	F	1536	2.2
	2150	8.9		2114	8.5
4	0437	1.6	19	0407	1.8
	1016	8.9		0942	8.8
F	1659	1.6	SA	1627	1.8
	2234	9.0		2203	8.9
5	0516	1.6	20	0454	1.6
	1055	9.0		1028	9.1
SA	1740	1.6	SU	1716	1.5
	○ 2315	9.0		● 2251	9.1
6	0551	1.7	21	0540	1.4
	1133	9.0		1115	9.4
SU	1819	1.7	M	1805	1.2
	2353	8.8		2340	9.3
7	0625	1.8	22	0627	1.2
	1208	8.9		1201	9.5
M	1856	1.8	TU	1855	1.0
8	0031	8.7	23	0029	9.3
	0657	2.0		0713	1.3
TU	1243	8.8	W	1250	9.6
	1933	2.0		1945	1.0
9	0106	8.4	24	0119	9.2
	0730	2.2		0759	1.4
W	1320	8.6	TH	1340	9.5
	2008	2.2		2034	1.0
10	0144	8.2	25	0209	8.9
	0804	2.5		0847	1.6
TH	1357	8.3	F	1430	9.3
	2044	2.5		2125	1.2
11	0223	7.9	26	0303	8.6
	0840	2.8		0938	1.9
F	1439	8.0	SA	1524	9.0
	2125	2.7		2217	1.5
12	0308	7.6	27	0357	8.3
	0925	3.1		1033	2.2
SA	1525	7.7	SU	1619	8.7
	2214	2.9		2313	1.8
13	0400	7.3	28	0458	7.9
	1019	3.3		1134	2.5
SU	1619	7.5	M	1720	8.4
	2312	3.0			
14	0459	7.2	29	0015	2.1
	1122	3.4		0603	7.7
M	1720	7.4	TU	1241	2.6
				1827	8.1
15	0017	3.0	30	0120	2.3
	0604	7.2		0712	7.7
TU	1234	3.3	W	1351	2.6
	1825	7.5		1935	8.0
			31	0226	2.4
				0816	7.9
			TH	1458	2.5
				2039	8.1

TABLE V (*cont.*)

TIDAL LEVELS IN METRES AT STANDARD PORTS

(with data concerning predictions, etc.)

Standard Port	L.A.T. (*b*)	M.L.W.S. (*c*)	M.L.W.N. (*d*)	M.T.L. (*f*)	M.H.W.N. (*d*)	M.H.W.S. (*c*)	H.A.T. (*b*)	Authority for (g): Observations	Constants	Predictions	Method of Predicting (*h*)	Years of Observations (*j*)
Plymouth (Devonport)	0·0	+0·8	+2·2	+3·2	+4·4	+5·5	+6·1	Hyd.	I.O.S.	Hyd.	H.	1961-62
Portland	−0·1	+0·2	+0·7	+1·1	+1·4	+2·1	+2·7	Hyd.	I.O.S.	Hyd.	H.C.	1961-62
Southampton	0·0	+0·5	+1·8	+2·6	+3·7	+4·5	+4·9	H.A.	I.O.S.	I.O.S.	H.C.	1957
Portsmouth	0·0	+0·6	+1·8	+2·7	+3·8	+4·7	+5·2	Hyd.	I.O.S.	Hyd.	H.C.	1961-62
Shoreham	0·0	+0·7	+1·9	+3·5	+5·0	+6·2	+6·7	H.A.	I.O.S.	I.O.S.	H.	1959
Dover	0·0	+0·8	+2·0	+3·7	+5·3	+6·7	+7·3	H.A.	I.O.S.	I.O.S.	H.C.	1975
Margate	−0·1	+0·5	+1·4	+2·6	+3·9	+4·8	+5·2	H.A.	I.O.S.	I.O.S.	H.	1969-70
Sheerness	−0·1	+0·6	+1·5	+3·1	+4·8	+5·7	+6·4	H.A.	I.O.S.	I.O.S.	H.	1969-70
London Bridge	−0·2	+0·5	+1·6	+3·7	+5·8	+7·1	+7·9	H.A.	I.O.S.	I.O.S.	H.C.	1969-70
Walton-on-the-Naze	−0·1	+0·4	+1·1	+2·3	+3·4	+4·2	+4·7	H.A.	I.O.S.	I.O.S.	H.	1969-70
Harwich	−0·1	+0·4	+1·1	+2·2	+3·4	+4·0	+4·4	H.A.	I.O.S.	I.O.S.	H.C.	1961
Lowestoft	0·0	+0·5	+1·0	+1·5	+2·1	+2·4	+2·8	H.A.	I.O.S.	I.O.S.	H.	1959
Immingham	−0·1	+0·9	+2·6	+4·1	+5·8	+7·3	+8·0	H.A.	I.O.S.	I.O.S.	H.C.	1977
River Tees	0·0	+0·9	+2·0	+3·2	+4·3	+5·5	+6·1	H.A.	I.O.S.	I.O.S.	H.	1948
River Tyne	0·0	+0·7	+1·8	+2·9	+3·9	+5·0	+5·7	H.A.	I.O.S.	I.O.S.	H.	1962
Leith	0·0	+0·8	+2·1	+3·3	+4·5	+5·6	+6·1	H.A.	I.O.S.	I.O.S.	H.C.	1956
Rosyth	0·0	+0·8	+2·2	+3·4	+4·7	+5·8	+6·4	Hyd.	I.O.S.	Hyd.	H.C.	1966
Aberdeen	0·0	+0·6	+1·6	+2·5	+3·4	+4·3	+4·8	H.A.	I.O.S.	I.O.S.	H.	1964
Lerwick	0·0	+0·5	+0·9	+1·3	+1·6	+2·2	+2·5	H.A.	I.O.S.	Hyd.	H.	1967-71
Ullapool	0·0	+0·7	+2·1	+3·0	+3·9	+5·2	+5·8	H.A.	I.O.S.	Hyd.	H.	1966
Oban	0·0	+0·7	+1·8	+2·3	+2·9	+4·0	+4·5	Hyd.	I.O.S.	Hyd.	H.	1971-72
Greenock	−0·1	+0·4	+1·0	+1·9	+2·9	+3·4	+4·1	H.A.	I.O.S.	Hyd.	H.C.	1948
Liverpool	−0·2	+0·9	+2·9	+5·2	+7·4	+9·3	+10·3	H.A.	I.O.S.	I.O.S.	H.C.	1970
Holyhead	0·0	+0·7	+2·0	+3·2	+4·5	+5·7	+6·5	H.A.	I.O.S.	I.O.S.	H.	1959-60
Milford Haven	−0·1	+0·7	+2·5	+3·8	+5·2	+7·0	+8·0	Hyd.	I.O.S.	I.O.S.	H.	1961
Swansea	0·0	+1·0	+3·2	+5·3	+7·3	+9·6	+10·5	H.A.	I.O.S.	I.O.S.	H.C.	1961
Port of Bristol (Avonmouth)	−0·0	+0·9	+3·5	+6·9	+10·0	+13·2	+14·6	H.A.	I.O.S.	I.O.S.	H.C.	1974-75
Dublin	0·0	+0·5	+1·5	+2·4	+3·4	+4·1	+4·7	H.A.	I.O.S.	I.O.S.	H.	1948-49
Belfast	0·0	+0·4	+1·1	+2·0	+3·0	+3·5	+3·9	H.A.	I.O.S.	I.O.S.	H.	1947-48
Londonderry	0·0	+0·4	+1·0	+1·5	+2·0	+2·7	+3·2	H.A.	I.O.S.	I.O.S.	H.	1935-36
Galway	−0·1	+0·6	+2·0	+2·9	+3·9	+5·1	+5·7	H.A.	I.O.S.	I.O.S.	H.	1965
Tarbert Island	0·0	+0·5	+1·7	+2·8	+3·8	+5·0	+5·5	H.A.	I.O.S.	I.O.S.	H.	1966
Cobh (Ringaskiddy)	−0·1	+0·5	+1·4	+2·3	+3·3	+4·2	+4·6	H.A.	Hyd.	I.O.S.	H.	1983-84
Reykjavik	−0·4	+0·2	+1·3	+2·1	+2·9	+4·0	+4·6	I.	I.	Hyd.	H.	1963
Port of Kem'	+0·1	+0·4	+0·6	+1·1	+1·6	+1·9	+2·0	R.	R.	Hyd.	H.	1910
Ostrov Yekaterininskaya	−0·1	+0·5	+1·3	+2·1	+3·0	+3·7	+4·2	R.	G.	G.	H.	1906-07
Narvik	+0·1	+0·5	+1·2	+1·8	+2·4	+3·2	+3·7	—	—	Nor.	—	—
Bergen	−0·1	+0·2	+0·5	+0·8	+1·1	+1·4	+1·6	—	—	Nor.	—	—
Esbjerg	−0·4	−0·1	+0·2	+0·8	+1·4	+1·6	+1·9	D.	D.	D.	H.	—
Helgoland	−0·5	0·0	+0·4	+1·4	+2·3	+2·7	+3·0	—	—	G.	—	—
Cuxhaven	−0·5	0·0	+0·4	+1·7	+3·0	+3·4	+3·7	—	—	G.	—	—
Wilhelmshaven	−0·5	0·0	+0·6	+2·0	+3·8	+4·3	+4·6	—	—	G.	—	—
Hoek van Holland	−0·2	+0·4	+0·3	+1·2	+1·8	+2·3	+2·6	—	—	N.	—	—
Vlissingen (Flushing)	−0·4	+0·5	+1·0	+2·6	+4·0	+4·9	+5·3	—	—	N.	—	—
Antwerp (Prosperpolder)	−0·6	0·0	+1·0	+2·7	+4·2	+5·8	+6·1	B.	I.O.S.	I.O.S.	H.C.	1981
Dunkerque	+0·2	+0·6	+1·5	+3·1	+4·8	+5·8	+6·2	—	—	F.	—	—
Calais	+0·3	+0·9	+2·1	+4·0	+5·9	+7·1	+7·5	—	—	F.	—	—
Boulogne	+0·2	+1·1	+2·8	+5·0	+7·2	+8·9	+9·5	—	—	F.	—	—
Dieppe	+0·2	+0·7	+2·6	+4·9	+7·2	+9·3	+10·0	—	—	F.	—	—
Le Havre	+0·3	+1·2	+3·0	+4·7	+6·6	+7·9	+8·3	—	—	F.	—	—
Cherbourg	+0·5	+1·1	+2·5	+3·5	+5·0	+6·3	+7·0	—	—	F.	—	—
St. Helier	0·0	+1·3	+4·1	+6·2	+8·1	+11·1	+12·3	H.A.	I.O.S.	I.O.S.	H.	1952
Brest	+0·5	+1·4	+3·0	+4·4	+5·9	+7·5	+8·3	—	—	F.	—	—
Pointe de Grave	+0·5	+1·0	+2·1	+3·1	+4·3	+5·3	+6·0	—	—	F.	—	—
Lisbon	0·0	+0·5	+1·4	+2·2	+3·0	+3·8	+4·2	—	—	Hyd.	—	—
Gibraltar	−0·1	+0·1	+0·3	+0·5	+0·7	+1·0	+1·1	Hyd.	I.O.S.	Hyd.	H.	1961-62
Venezia (Venice)	−0·1	+0·1	+0·4	+0·5	+0·6	+0·8	+1·0	—	—	Hyd.	—	—

TABLE VI

FORTNIGHTLY SHALLOW WATER CORRECTION

Investigations have shown that at certain ports with large Shallow Water Effects the tidal constituent with a period of one cycle per fortnight (MSf) can be significant. The normal time and height differences allow for this effect but at certain ports predictions obtained by the use of the Simplified Harmonic Method of Tidal Prediction N.P.159 can be improved by a simple additional adjustment. Corrected values of M.L. are given below, for the ports concerned, for the day of springs and for each day before or after springs. The date of springs occurs (x) days (see table below) after New and Full Moon. The corrected value should be entered as M.L. on N.P.159. The seasonal correction is then applied to this value as usual.

Port No.	(x)	Average M.L.	At Springs	Corrected M.L. Days before or after springs						
				1	2	3	4	5	6	7
86a	2	1·97	2·13	2·11	2·07	2·01	1·93	1·87	1·83	1·81
112	3	3·67	3·74	3·73	3·71	3·69	3·65	3·63	3·61	3·60
113	3	3·70	3·81	3·80	3·78	3·74	3·70	3·66	3·64	3·63
124	2	2·7	2·87	2·85	2·81	2·74	2·66	2·59	2·55	2·53
162	2	3·56	3·73	3·71	3·67	3·60	3·52	3·45	3·41	3·39
166	2	3·31	3·54	3·52	3·45	3·36	3·26	3·17	3·10	3·08
174a	2	1·97	2·15	2·13	2·08	2·01	1·93	1·86	1·81	1·79
176	2	2·41	2·69	2·66	2·59	2·47	2·35	2·23	2·16	2·13
371b	2	1·2	1·26	1·25	1·24	1·21	1·19	1·16	1·15	1·14
453	2	5·3	5·45	5·43	5·39	5·33	5·27	5·21	5·17	5·15
472	2	4·22	4·32	4·31	4·28	4·24	4·20	4·16	4·13	4·12
513a	2	6·2	6·27	6·26	6·24	6·22	6·18	6·16	6·14	6·13
515	2	6·04	6·14	6·13	6·10	6·06	6·02	5·98	5·95	5·94
518	3	6·43	6·60	6·58	6·54	6·47	6·39	6·32	6·28	6·26
519	3	5·66	5·96	5·93	5·85	5·73	5·59	5·47	5·39	5·36
521	3	3·93	4·31	4·27	4·17	4·02	3·84	3·69	3·59	3·55
522	3	3·43	3·97	3·92	3·77	3·55	3·31	3·09	2·94	2·89
522b	2	3·26	3·89	3·83	3·66	3·40	3·12	2·86	2·69	2·63
523	2	6·92	7·02	7·01	6·98	6·94	6·90	6·86	6·83	6·82
547	2	3·45	3·51	3·50	3·49	3·46	3·44	3·41	3·40	3·39
1528	3	1·84	1·89	1·89	1·87	1·85	1·83	1·81	1·79	1·79
1530	3	2·16	2·22	2·21	2·20	2·17	2·15	2·12	2·11	2·10
1534	2	2·45	2·49	2·49	2·47	2·46	2·44	2·43	2·41	2·41
1536	2	2·63	2·69	2·68	2·67	2·64	2·62	2·59	2·58	2·57
1537	3	2·71	2·77	2·76	2·75	2·72	2·70	2·67	2·66	2·65
1538	3	2·72	2·80	2·79	2·77	2·74	2·70	2·67	2·65	2·64
1539	3	2·66	2·75	2·74	2·72	2·68	2·64	2·60	2·58	2·57
1666	2	3·32	3·48	3·47	3·42	3·36	3·28	3·22	3·17	3·16
1667	2	3·74	3·97	3·94	3·88	3·79	3·69	3·60	3·54	3·51

A2 ALTITUDE CORRECTION TABLES 10°-90°—SUN, STARS, PLANETS

SUN

OCT.—MAR. App. Alt.	Lower Limb	Upper Limb	APR.—SEPT. App. Alt.	Lower Limb	Upper Limb
° ′	′	′	° ′	′	′
9 34			9 39		
	+10·8	−21·5		+10·6	−21·2
9 45			9 51		
	+10·9	−21·4		+10·7	−21·1
9 56			10 03		
	+11·0	−21·3		+10·8	−21·0
10 08			10 15		
	+11·1	−21·2		+10·9	−20·9
10 21			10 27		
	+11·2	−21·1		+11·0	−20·8
10 34			10 40		
	+11·3	−21·0		+11·1	−20·7
10 47			10 54		
	+11·4	−20·9		+11·2	−20·6
11 01			11 08		
	+11·5	−20·8		+11·3	−20·5
11 15			11 23		
	+11·6	−20·7		+11·4	−20·4
11 30			11 38		
	+11·7	−20·6		+11·5	−20·3
11 46			11 54		
	+11·8	−20·5		+11·6	−20·2
12 02			12 10		
	+11·9	−20·4		+11·7	−20·1
12 19			12 28		
	+12·0	−20·3		+11·8	−20·0
12 37			12 46		
	+12·1	−20·2		+11·9	−19·9
12 55			13 05		
	+12·2	−20·1		+12·0	−19·8
13 14			13 24		
	+12·3	−20·0		+12·1	−19·7
13 35			13 45		
	+12·4	−19·9		+12·2	−19·6
13 56			14 07		
	+12·5	−19·8		+12·3	−19·5
14 18			14 30		
	+12·6	−19·7		+12·4	−19·4
14 42			14 54		
	+12·7	−19·6		+12·5	−19·3
15 06			15 19		
	+12·8	−19·5		+12·6	−19·2
15 32			15 46		
	+12·9	−19·4		+12·7	−19·1
15 59			16 14		
	+13·0	−19·3		+12·8	−19·0
16 28			16 44		
	+13·1	−19·2		+12·9	−18·9
16 59			17 15		
	+13·2	−19·1		+13·0	−18·8
17 32			17 48		
	+13·3	−19·0		+13·1	−18·7
18 06			18 24		
	+13·4	−18·9		+13·2	−18·6
18 42			19 01		
	+13·5	−18·8		+13·3	−18·5
19 21			19 42		
	+13·6	−18·7		+13·4	−18·4
20 03			20 25		
	+13·7	−18·6		+13·5	−18·3
20 48			21 11		
	+13·8	−18·5		+13·6	−18·2
21 35			22 00		
	+13·9	−18·4		+13·7	−18·1
22 26			22 54		
	+14·0	−18·3		+13·8	−18·0
23 22			23 51		
	+14·1	−18·2		+13·9	−17·9
24 21			24 53		
	+14·2	−18·1		+14·0	−17·8
25 26			26 00		
	+14·3	−18·0		+14·1	−17·7
26 36			27 13		
	+14·4	−17·9		+14·2	−17·6
27 52			28 33		
	+14·5	−17·8		+14·3	−17·5
29 15			30 00		
	+14·6	−17·7		+14·4	−17·4
30 46			31 35		
	+14·7	−17·6		+14·5	−17·3
32 26			33 20		
	+14·8	−17·5		+14·6	−17·2
34 17			35 17		
	+14·9	−17·4		+14·7	−17·1
36 20			37 26		
	+15·0	−17·3		+14·8	−17·0
38 36			39 50		
	+15·1	−17·2		+14·9	−16·9
41 08			42 31		
	+15·2	−17·1		+15·0	−16·8
43 59			45 31		
	+15·3	−17·0		+15·1	−16·7
47 10			48 55		
	+15·4	−16·9		+15·2	−16·6
50 46			52 44		
	+15·5	−16·8		+15·3	−16·5
54 49			57 02		
	+15·6	−16·7		+15·4	−16·4
59 23			61 51		
	+15·7	−16·6		+15·5	−16·3
64 30			67 17		
	+15·8	−16·5		+15·6	−16·2
70 12			73 16		
	+15·9	−16·4		+15·7	−16·1
76 26			79 43		
	+16·0	−16·3		+15·8	−16·0
83 05			86 32		
	+16·1	−16·2		+15·9	−15·9
90 00			90 00		

STARS AND PLANETS

App. Alt.	Corrⁿ
° ′	′
9 56	
	−5·3
10 08	
	−5·2
10 20	
	−5·1
10 33	
	−5·0
10 46	
	−4·9
11 00	
	−4·8
11 14	
	−4·7
11 29	
	−4·6
11 45	
	−4·5
12 01	
	−4·4
12 18	
	−4·3
12 35	
	−4·2
12 54	
	−4·1
13 13	
	−4·0
13 33	
	−3·9
13 54	
	−3·8
14 16	
	−3·7
14 40	
	−3·6
15 04	
	−3·5
15 30	
	−3·4
15 57	
	−3·3
16 26	
	−3·2
16 56	
	−3·1
17 28	
	−3·0
18 02	
	−2·9
18 38	
	−2·8
19 17	
	−2·7
19 58	
	−2·6
20 42	
	−2·5
21 28	
	−2·4
22 19	
	−2·3
23 13	
	−2·2
24 11	
	−2·1
25 14	
	−2·0
26 22	
	−1·9
27 36	
	−1·8
28 56	
	−1·7
30 24	
	−1·6
32 00	
	−1·5
33 45	
	−1·4
35 40	
	−1·3
37 48	
	−1·2
40 08	
	−1·1
42 44	
	−1·0
45 36	
	−0·9
48 47	
	−0·8
52 18	
	−0·7
56 11	
	−0·6
60 28	
	−0·5
65 08	
	−0·4
70 11	
	−0·3
75 34	
	−0·2
81 13	
	−0·1
87 03	
	0·0
90 00	

App. Alt.	Additional Corrⁿ
1977	
VENUS	
Jan. 1-Jan. 29	
°	
0	
47	+ 0·2′
Jan. 30-Feb. 26	
0	
46	+ 0·3′
Feb. 27-Mar. 14	
0	
11	+ 0·4′
41	+ 0·5
Mar. 15-Mar. 23	
0	
6	+ 0·5′
20	+ 0·6
31	+ 0·7
Mar. 24-Apr. 19	
0	
4	+ 0·6′
12	+ 0·7
22	+ 0·8
Apr. 20-Apr. 28	
0	
6	+ 0·5′
20	+ 0·6
31	+ 0·7
Apr. 29-May 13	
0	
11	+ 0·4′
41	+ 0·5
May 14-June 8	
0	
46	+ 0·3′
June 9-July 23	
0	
47	+ 0·2′
July 24-Dec. 31	
0	
42	+ 0·1′
MARS	
Jan. 1-Nov. 12	
0	
60	+ 0·1′
Nov. 13-Dec. 31	
0	
41	+ 0·2′
75	+ 0·1

DIP

Ht. of Eye	Corrⁿ	Ht. of Eye
m	′	ft.
2·4		8·0
	−2·8	
2·6		8·6
	−2·9	
2·8		9·2
	−3·0	
3·0		9·8
	−3·1	
3·2		10·5
	−3·2	
3·4		11·2
	−3·3	
3·6		11·9
	−3·4	
3·8		12·6
	−3·5	
4·0		13·3
	−3·6	
4·3		14·1
	−3·7	
4·5		14·9
	−3·8	
4·7		15·7
	−3·9	
5·0		16·5
	−4·0	
5·2		17·4
	−4·1	
5·5		18·3
	−4·2	
5·8		19·1
	−4·3	
6·1		20·1
	−4·4	
6·3		21·0
	−4·5	
6·6		22·0
	−4·6	
6·9		22·9
	−4·7	
7·2		23·9
	−4·8	
7·5		24·9
	−4·9	
7·9		26·0
	−5·0	
8·2		27·1
	−5·1	
8·5		28·1
	−5·2	
8·8		29·2
	−5·3	
9·2		30·4
	−5·4	
9·5		31·5
	−5·5	
9·9		32·7
	−5·6	
10·3		33·9
	−5·7	
10·6		35·1
	−5·8	
11·0		36·3
	−5·9	
11·4		37·6
	−6·0	
11·8		38·9
	−6·1	
12·2		40·1
	−6·2	
12·6		41·5
	−6·3	
13·0		42·8
	−6·4	
13·4		44·2
	−6·5	
13·8		45·5
	−6·6	
14·2		46·9
	−6·7	
14·7		48·4
	−6·8	
15·1		49·8
	−6·9	
15·5		51·3
	−7·0	
16·0		52·8
	−7·1	
16·5		54·3
	−7·2	
16·9		55·8
	−7·3	
17·4		57·4
	−7·4	
17·9		58·9
	−7·5	
18·4		60·5
	−7·6	
18·8		62·1
	−7·7	
19·3		63·8
	−7·8	
19·8		65·4
	−7·9	
20·4		67·1
	−8·0	
20·9		68·8
	−8·1	
21·4		70·5

Ht. of Eye	Corrⁿ
m	′
1·0	− 1·8
1·5	− 2·2
2·0	− 2·5
2·5	− 2·8
3·0	− 3·0
See table ←	
m	′
20	− 7·9
22	− 8·3
24	− 8·6
26	− 9·0
28	− 9·3
30	− 9·6
32	−10·0
34	−10·3
36	−10·6
38	−10·8
40	−11·1
42	−11·4
44	−11·7
46	−11·9
48	−12·2
ft.	′
2	− 1·4
4	− 1·9
6	− 2·4
8	− 2·7
10	− 3·1
See table ←	
ft.	′
70	− 8·1
75	− 8·4
80	− 8·7
85	− 8·9
90	− 9·2
95	− 9·5
100	− 9·7
105	− 9·9
110	−10·2
115	−10·4
120	−10·6
125	−10·8
130	−11·1
135	−11·3
140	−11·5
145	−11·7
150	−11·9
155	−12·1

App. Alt. = Apparent altitude = Sextant altitude corrected for index error and dip.
For daylight observations of Venus, see page 260.

ALTITUDE CORRECTION TABLES 0°–10°—SUN, STARS, PLANETS A3

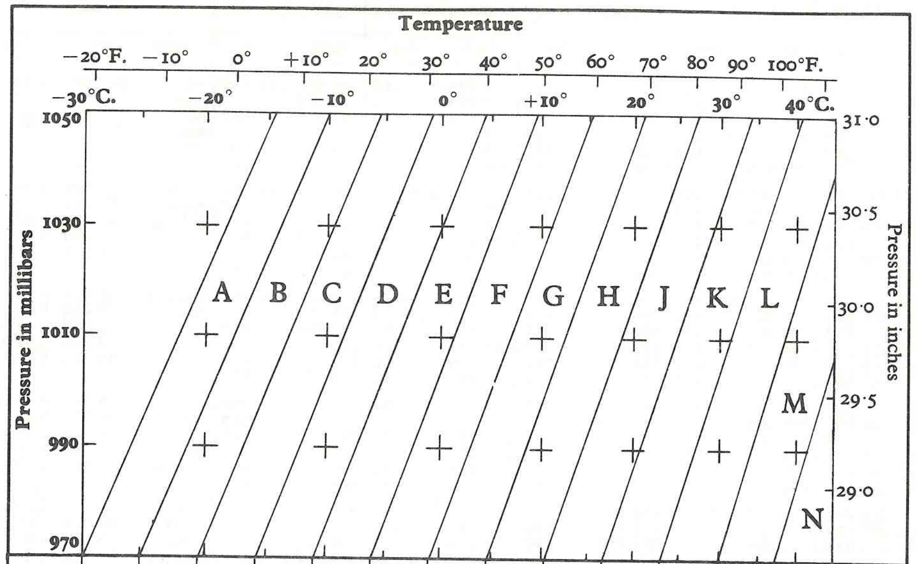
App. Alt.	OCT.–MAR. SUN Lower Limb	OCT.–MAR. SUN Upper Limb	APR.–SEPT. SUN Lower Limb	APR.–SEPT. SUN Upper Limb	STARS PLANETS
° ′	′	′	′	′	′
0 00	−18·2	−50·5	−18·4	−50·2	−34·5
03	17·5	49·8	17·8	49·6	33·8
06	16·9	49·2	17·1	48·9	33·2
09	16·3	48·6	16·5	48·3	32·6
12	15·7	48·0	15·9	47·7	32·0
15	15·1	47·4	15·3	47·1	31·4
0 18	−14·5	−46·8	−14·8	−46·6	−30·8
21	14·0	46·3	14·2	46·0	30·3
24	13·5	45·8	13·7	45·5	29·8
27	12·9	45·2	13·2	45·0	29·2
30	12·4	44·7	12·7	44·5	28·7
33	11·9	44·2	12·2	44·0	28·2
0 36	−11·5	−43·8	−11·7	−43·5	−27·8
39	11·0	43·3	11·2	43·0	27·3
42	10·5	42·8	10·8	42·6	26·8
45	10·1	42·4	10·3	42·1	26·4
48	9·6	41·9	9·9	41·7	25·9
51	9·2	41·5	9·5	41·3	25·5
0 54	− 8·8	−41·1	− 9·1	−40·9	−25·1
0 57	8·4	40·7	8·7	40·5	24·7
1 00	8·0	40·3	8·3	40·1	24·3
03	7·7	40·0	7·9	39·7	24·0
06	7·3	39·6	7·5	39·3	23·6
09	6·9	39·2	7·2	39·0	23·2
1 12	− 6·6	−38·9	− 6·8	−38·6	−22·9
15	6·2	38·5	6·5	38·3	22·5
18	5·9	38·2	6·2	38·0	22·2
21	5·6	37·9	5·8	37·6	21·9
24	5·3	37·6	5·5	37·3	21·6
27	4·9	37·2	5·2	37·0	21·2
1 30	− 4·6	−36·9	− 4·9	−36·7	−20·9
35	4·2	36·5	4·4	36·2	20·5
40	3·7	36·0	4·0	35·8	20·0
45	3·2	35·5	3·5	35·3	19·5
50	2·8	35·1	3·1	34·9	19·1
1 55	2·4	34·7	2·6	34·4	18·7
2 00	− 2·0	−34·3	− 2·2	−34·0	−18·3
05	1·6	33·9	1·8	33·6	17·9
10	1·2	33·5	1·5	33·3	17·5
15	0·9	33·2	1·1	32·9	17·2
20	0·5	32·8	0·8	32·6	16·8
25	− 0·2	32·5	0·4	32·2	16·5
2 30	+ 0·2	−32·1	− 0·1	−31·9	−16·1
35	0·5	31·8	+ 0·2	31·6	15·8
40	0·8	31·5	0·5	31·3	15·5
45	1·1	31·2	0·8	31·0	15·2
50	1·4	30·9	1·1	30·7	14·9
2 55	1·6	30·7	1·4	30·4	14·7
3 00	+ 1·9	−30·4	+ 1·7	−30·1	−14·4
05	2·2	30·1	1·9	29·9	14·1
10	2·4	29·9	2·1	29·7	13·9
15	2·6	29·7	2·4	29·4	13·7
20	2·9	29·4	2·6	29·2	13·4
25	3·1	29·2	2·9	28·9	13·2
3 30	+ 3·3	−29·0	+ 3·1	−28·7	−13·0

App. Alt.	OCT.–MAR. SUN Lower Limb	OCT.–MAR. SUN Upper Limb	APR.–SEPT. SUN Lower Limb	APR.–SEPT. SUN Upper Limb	STARS PLANETS
° ′	′	′	′	′	′
3 30	+ 3·3	−29·0	+ 3·1	−28·7	−13·0
35	3·6	28·7	3·3	28·5	12·7
40	3·8	28·5	3·5	28·3	12·5
45	4·0	28·3	3·7	28·1	12·3
50	4·2	28·1	3·9	27·9	12·1
3 55	4·4	27·9	4·1	27·7	11·9
4 00	+ 4·5	−27·8	+ 4·3	−27·5	−11·8
05	4·7	27·6	4·5	27·3	11·6
10	4·9	27·4	4·6	27·2	11·4
15	5·1	27·2	4·8	27·0	11·2
20	5·2	27·1	5·0	26·8	11·1
25	5·4	26·9	5·1	26·7	10·9
4 30	+ 5·6	−26·7	+ 5·3	−26·5	−10·7
35	5·7	26·6	5·5	26·3	10·6
40	5·9	26·4	5·6	26·2	10·4
45	6·0	26·3	5·8	26·0	10·3
50	6·2	26·1	5·9	25·9	10·1
4 55	6·3	26·0	6·0	25·8	10·0
5 00	+ 6·4	−25·9	+ 6·2	−25·6	− 9·9
05	6·6	25·7	6·3	25·5	9·7
10	6·7	25·6	6·4	25·4	9·6
15	6·8	25·5	6·6	25·2	9·5
20	6·9	25·4	6·7	25·1	9·4
25	7·1	25·2	6·8	25·0	9·2
5 30	+ 7·2	−25·1	+ 6·9	−24·9	− 9·1
35	7·3	25·0	7·0	24·8	9·0
40	7·4	24·9	7·2	24·6	8·9
45	7·5	24·8	7·3	24·5	8·8
50	7·6	24·7	7·4	24·4	8·7
5 55	7·7	24·6	7·5	24·3	8·6
6 00	+ 7·8	−24·5	+ 7·6	−24·2	− 8·5
10	8·0	24·3	7·8	24·0	8·3
20	8·2	24·1	8·0	23·8	8·1
30	8·4	23·9	8·1	23·7	7·9
40	8·6	23·7	8·3	23·5	7·7
6 50	8·7	23·6	8·5	23 3	7·6
7 00	+ 8·9	−23·4	+ 8·6	−23·2	− 7·4
10	9·1	23·2	8·8	23·0	7·2
20	9·2	23·1	9·0	22·8	7·1
30	9·3	23·0	9·1	22·7	7·0
40	9·5	22·8	9·2	22·6	6·8
7 50	9·6	22·7	9·4	22·4	6·7
8 00	+ 9·7	−22·6	+ 9·5	−22·3	− 6·6
10	9·9	22·4	9·6	22·2	6·4
20	10·0	22·3	9·7	22·1	6·3
30	10·1	22·2	9·8	22·0	6·2
40	10·2	22·1	10·0	21·8	6·1
8 50	10·3	22·0	10·1	21·7	6·0
9 00	+10·4	−21·9	+10·2	−21·6	− 5·9
10	10·5	21·8	10·3	21·5	5·8
20	10·6	21·7	10·4	21·4	5·7
30	10·7	21·6	10·5	21·3	5·6
40	10·8	21·5	10·6	21·2	5·5
9 50	10·9	21·4	10·6	21·2	5·4
10 00	+11·0	−21·3	+10·7	−21·1	− 5·3

Additional corrections for temperature and pressure are given on the following page.
For bubble sextant observations ignore dip and use the star corrections for Sun, planets, and stars.

A4 ALTITUDE CORRECTION TABLES—ADDITIONAL CORRECTIONS

ADDITIONAL REFRACTION CORRECTIONS FOR NON-STANDARD CONDITIONS



App. Alt.	A	B	C	D	E	F	G	H	J	K	L	M	N	App. Alt.
° ′	′	′	′	′	′	′	′	′	′	′	′	′	′	° ′
0 00	−6·9	−5·7	−4·6	−3·4	−2·3	−1·1	0·0	+1·1	+2·3	+3·4	+4·6	+5·7	+6·9	0 00
0 30	5·2	4·4	3·5	2·6	1·7	0·9	0·0	0·9	1·7	2·6	3·5	4·4	5·2	0 30
1 00	4·3	3·5	2·8	2·1	1·4	0·7	0·0	0·7	1·4	2·1	2·8	3·5	4·3	1 00
1 30	3·5	2·9	2·4	1·8	1·2	0·6	0·0	0·6	1·2	1·8	2·4	2·9	3·5	1 30
2 00	3·0	2·5	2·0	1·5	1·0	0·5	0·0	0·5	1·0	1·5	2·0	2·5	3·0	2 00
2 30	−2·5	−2·1	−1·6	−1·2	−0·8	−0·4	0·0	+0·4	+0·8	+1·2	+1·6	+2·1	+2·5	2 30
3 00	2·2	1·8	1·5	1·1	0·7	0·4	0·0	0·4	0·7	1·1	1·5	1·8	2·2	3 00
3 30	2·0	1·6	1·3	1·0	0·7	0·3	0·0	0·3	0·7	1·0	1·3	1·6	2·0	3 30
4 00	1·8	1·5	1·2	0·9	0·6	0·3	0·0	0·3	0·6	0·9	1·2	1·5	1·8	4 00
4 30	1·6	1·4	1·1	0·8	0·5	0·3	0·0	0·3	0·5	0·8	1·1	1·4	1·6	4 30
5 00	−1·5	−1·3	−1·0	−0·8	−0·5	−0·2	0·0	+0·2	+0·5	+0·8	+1·0	+1·3	+1·5	5 00
6	1·3	1·1	0·9	0·6	0·4	0·2	0·0	0·2	0·4	0·6	0·9	1·1	1·3	6
7	1·1	0·9	0·7	0·6	0·4	0·2	0·0	0·2	0·4	0·6	0·7	0·9	1·1	7
8	1·0	0·8	0·7	0·5	0·3	0·2	0·0	0·2	0·3	0·5	0·7	0·8	1·0	8
9	0·9	0·7	0·6	0·4	0·3	0·1	0·0	0·1	0·3	0·4	0·6	0·7	0·9	9
10 00	−0·8	−0·7	−0·5	−0·4	−0·3	−0·1	0·0	+0·1	+0·3	+0·4	+0·5	+0·7	+0·8	10 00
12	0·7	0·6	0·5	0·3	0·2	0·1	0·0	0·1	0·2	0·3	0·5	0·6	0·7	12
14	0·6	0·5	0·4	0·3	0·2	0·1	0·0	0·1	0·2	0·3	0·4	0·5	0·6	14
16	0·5	0·4	0·3	0·3	0·2	0·1	0·0	0·1	0·2	0·3	0·3	0·4	0·5	16
18	0·4	0·4	0·3	0·2	0·2	0·1	0·0	0·1	0·2	0·2	0·3	0·4	0·4	18
20 00	−0·4	−0·3	−0·3	−0·2	−0·1	−0·1	0·0	+0·1	+0·1	+0·2	+0·3	+0·3	+0·4	20 00
25	0·3	0·3	0·2	0·2	0·1	−0·1	0·0	+0·1	0·1	0·2	0·2	0·3	0·3	25
30	0·3	0·2	0·2	0·1	0·1	0·0	0·0	0·0	0·1	0·1	0·2	0·2	0·3	30
35	0·2	0·2	0·1	0·1	0·1	0·0	0·0	0·0	0·1	0·1	0·1	0·2	0·2	35
40	0·2	0·1	0·1	0·1	−0·1	0·0	0·0	0·0	+0·1	0·1	0·1	0·1	0·2	40
50 00	−0·1	−0·1	−0·1	−0·1	0·0	0·0	0·0	0·0	0·0	+0·1	+0·1	+0·1	+0·1	50 00

The graph is entered with arguments temperature and pressure to find a zone letter; using as arguments this zone letter and apparent altitude (sextant altitude corrected for dip), a correction is taken from the table. This correction is to be applied to the sextant altitude in addition to the corrections for standard conditions (for the Sun, stars and planets from page A2 and for the Moon from pages xxxiv and xxxv).

JUNE 12, 13, 14 (THURS., FRI., SAT.)

G.M.T. d h	ARIES G.H.A. ° ′	VENUS −3·5 G.H.A. ° ′	VENUS Dec. ° ′	MARS +0·5 G.H.A. ° ′	MARS Dec. ° ′	JUPITER −1·8 G.H.A. ° ′	JUPITER Dec. ° ′	SATURN +0·2 G.H.A. ° ′	SATURN Dec. ° ′	STARS Name	S.H.A. ° ′	Dec. ° ′
12 00	259 48·8	219 05·8	N13 35·1	255 51·9	S 0 37·5	59 07·1	S 7 14·2	357 49·3	S21 48·8	Acamar	315 50·1	S 40 28·2
01	274 51·3	234 05·4	35·9	270 52·7	36·8	74 09·6	14·1	12 52·0	48·7	Achernar	335 57·9	S 57 26·6
02	289 53·8	249 05·0	36·8	285 53·5	36·1	89 12·1	14·1	27 54·6	48·7	Acrux	173 55·3	S 62 52·5
03	304 56·2	264 04·7	· · 37·7	300 54·4	· · 35·4	104 14·6	· · 14·1	42 57·3	· · 48·7	Adhara	255 45·4	S 28 55·1
04	319 58·7	279 04·3	38·6	315 55·2	34·8	119 17·1	14·1	58 00·0	48·7	Aldebaran	291 37·2	N 16 25·5
05	335 01·2	294 03·9	39·5	330 56·0	34·1	134 19·7	14·1	73 02·6	48·7			
06	350 03·6	309 03·5	N13 40·4	345 56·8	S 0 33·4	149 22·2	S 7 14·1	88 05·3	S21 48·7	Alioth	166 56·8	N 56 11·3
07	5 06·1	324 03·2	41·3	0 57·7	32·7	164 24·7	14·1	103 08·0	48·7	Alkaid	153 31·2	N 49 31·4
T 08	20 08·6	339 02·8	42·2	15 58·5	32·0	179 27·2	14·1	118 10·6	48·7	Al Na'ir	28 35·5	S 47 09·5
H 09	35 11·0	354 02·4	· · 43·1	30 59·3	· · 31·4	194 29·7	· · 14·1	133 13·3	· · 48·7	Alnilam	276 28·7	S 1 13·8
U 10	50 13·5	9 02·0	44·0	46 00·1	30·7	209 32·2	14·1	148 15·9	48·7	Alphard	218 36·9	S 8 28·9
R 11	65 15·9	24 01·6	44·9	61 01·0	30·0	224 34·8	14·1	163 18·6	48·7			
S 12	80 18·4	39 01·3	N13 45·8	76 01·8	S 0 29·3	239 37·3	S 7 14·0	178 21·3	S21 48·7	Alphecca	126 45·7	N 26 51·4
D 13	95 20·9	54 00·9	46·7	91 02·6	28·6	254 39·8	14·0	193 23·9	48·7	Alpheratz	358 26·3	N 28 51·5
A 14	110 23·3	69 00·5	47·6	106 03·4	27·9	269 42·3	14·0	208 26·6	48·7	Altair	62 48·3	N 8 45·5
Y 15	125 25·8	84 00·1	· · 48·5	121 04·3	· · 27·3	284 44·8	· · 14·0	223 29·3	· · 48·6	Ankaa	353 56·6	S 42 31·6
16	140 28·3	98 59·7	49·4	136 05·1	26·6	299 47·3	14·0	238 31·9	48·6	Antares	113 16·6	S 26 20·4
17	155 30·7	113 59·4	50·2	151 05·9	25·9	314 49·8	14·0	253 34·6	48·6			
18	170 33·2	128 59·0	N13 51·1	166 06·8	S 0 25·2	329 52·4	S 7 14·0	268 37·2	S21 48·6	Arcturus	146 33·2	N 19 24·0
19	185 35·7	143 58·6	52·0	181 07·6	24·5	344 54·9	14·0	283 39·9	48·6	Atria	108 55·0	S 68 57·2
20	200 38·1	158 58·2	52·9	196 08·4	23·9	359 57·4	14·0	298 42·6	48·6	Avior	234 35·4	S 59 22·9
21	215 40·6	173 57·8	· · 53·8	211 09·2	· · 23·2	14 59·9	· · 14·0	313 45·2	· · 48·6	Bellatrix	279 16·7	N 6 18·7
22	230 43·1	188 57·4	54·7	226 10·1	22·5	30 02·4	14·0	328 47·9	48·6	Betelgeuse	271 46·4	N 7 23·9
23	245 45·5	203 57·1	55·6	241 10·9	21·8	45 04·9	14·0	343 50·6	48·6			
13 00	260 48·0	218 56·7	N13 56·5	256 11·7	S 0 21·2	60 07·4	S 7 14·0	358 53·2	S21 48·6	Canopus	264 15·0	S 52 40·6
01	275 50·4	233 56·3	57·4	271 12·5	20·5	75 09·9	14·0	13 55·9	48·6	Capella	281 36·0	N 45 57·3
02	290 52·9	248 55·9	58·2	286 13·4	19·8	90 12·4	13·9	28 58·5	48·6	Deneb	49 59·3	N 45 07·8
03	305 55·4	263 55·5	13 59·1	301 14·2	· · 19·1	105 15·0	· · 13·9	44 01·2	· · 48·6	Denebola	183 15·8	N 14 48·3
04	320 57·8	278 55·1	14 00·0	316 15·0	18·4	120 17·5	13·9	59 03·9	48·6	Diphda	349 37·5	S 18 12·8
05	336 00·3	293 54·7	00·9	331 15·9	17·8	135 20·0	13·9	74 06·5	48·5			
06	351 02·8	308 54·3	N14 01·8	346 16·7	S 0 17·1	150 22·5	S 7 13·9	89 09·2	S21 48·5	Dubhe	194 42·5	N 61 58·7
07	6 05·2	323 54·0	02·7	1 17·5	16·4	165 25·0	13·9	104 11·9	48·5	Elnath	279 05·3	N 28 34·3
08	21 07·7	338 53·6	03·5	16 18·3	15·7	180 27·5	13·9	119 14·5	48·5	Eltanin	91 04·8	N 51 29·7
F 09	36 10·2	353 53·2	· · 04·4	31 19·2	· · 15·0	195 30·0	· · 13·9	134 17·2	· · 48·5	Enif	34 27·5	N 9 41·1
R 10	51 12·6	8 52·8	05·3	46 20·0	14·4	210 32·5	13·9	149 19·8	48·5	Fomalhaut	16 09·5	S 29 50·3
I 11	66 15·1	23 52·4	06·2	61 20·8	13·7	225 35·0	13·9	164 22·5	48·5			
D 12	81 17·6	38 52·0	N14 07·1	76 21·7	S 0 13·0	240 37·5	S 7 13·9	179 25·2	S21 48·5	Gacrux	172 46·8	S 56 53·2
A 13	96 20·0	53 51·6	08·0	91 22·5	12·3	255 40·0	13·9	194 27·8	48·5	Gienah	176 34·8	S 17 18·8
Y 14	111 22·5	68 51·2	08·8	106 23·3	11·6	270 42·5	13·9	209 30·5	48·5	Hadar	149 46·1	S 60 10·6
15	126 24·9	83 50·8	· · 09·7	121 24·1	· · 11·0	285 45·1	· · 13·9	224 33·2	· · 48·5	Hamal	328 47·7	N 23 15·9
16	141 27·4	98 50·4	10·6	136 25·0	10·3	300 47·6	13·9	239 35·8	48·5	Kaus Aust.	84 38·2	S 34 24·2
17	156 29·9	113 50·0	11·5	151 25·8	09·6	315 50·1	13·9	254 38·5	48·5			
18	171 32·3	128 49·6	N14 12·4	166 26·6	S 0 08·9	330 52·6	S 7 13·9	269 41·1	S21 48·5	Kochab	137 17·4	N 74 19·8
19	186 34·8	143 49·3	13·2	181 27·5	08·3	345 55·1	13·9	284 43·8	48·4	Markab	14 19·5	N 14 58·9
20	201 37·3	158 48·9	14·1	196 28·3	07·6	0 57·6	13·8	299 46·5	48·4	Menkar	314 58·6	N 3 55·6
21	216 39·7	173 48·5	· · 15·0	211 29·1	· · 06·9	16 00·1	· · 13·8	314 49·1	· · 48·4	Menkent	148 56·1	S 36 10·1
22	231 42·2	188 48·1	15·9	226 30·0	06·2	31 02·6	13·8	329 51·8	48·4	Miaplacidus	221 48·9	S 69 33·2
23	246 44·7	203 47·7	16·8	241 30·8	05·6	46 05·1	13·8	344 54·5	48·4			
14 00	261 47·1	218 47·3	N14 17·6	256 31·6	S 0 04·9	61 07·6	S 7 13·8	359 57·1	S21 48·4	Mirfak	309 40·0	N 49 42·7
01	276 49·6	233 46·9	18·5	271 32·4	04·2	76 10·1	13·8	14 59·8	48·4	Nunki	76 49·2	S 26 20·8
02	291 52·0	248 46·5	19·4	286 33·3	03·5	91 12·6	13·8	30 02·4	48·4	Peacock	54 23·9	S 56 51·9
03	306 54·5	263 46·1	· · 20·3	301 34·1	· · 02·8	106 15·1	· · 13·8	45 05·1	· · 48·4	Pollux	244 18·6	N 28 07·6
04	321 57·0	278 45·7	21·1	316 34·9	02·2	121 17·6	13·8	60 07·8	48·4	Procyon	245 43·3	N 5 19·8
05	336 59·4	293 45·3	22·0	331 35·8	01·5	136 20·1	13·8	75 10·4	48·4			
06	352 01·9	308 44·9	N14 22·9	346 36·6	S 0 00·8	151 22·6	S 7 13·8	90 13·1	S21 48·4	Rasalhague	96 44·5	N 12 35·4
07	7 04·4	323 44·5	23·7	1 37·4	S 0 00·1	166 25·1	13·8	105 15·8	48·4	Regulus	208 27·6	N 12 10·2
S 08	22 06·8	338 44·1	24·6	16 38·3	N 0 00·5	181 27·6	13·8	120 18·4	48·3	Rigel	281 52·1	S 8 15·0
A 09	37 09·3	353 43·7	· · 25·5	31 39·1	· · 01·2	196 30·1	· · 13·8	135 21·1	· · 48·3	Rigil Kent.	140 47·7	S 60 40·1
T 10	52 11·8	8 43·3	26·4	46 39·9	01·9	211 32·6	13·8	150 23·7	48·3	Sabik	102 59·6	S 15 40·4
U 11	67 14·2	23 42·9	27·2	61 40·8	02·6	226 35·1	13·8	165 26·4	48·3			
R 12	82 16·7	38 42·5	N14 28·1	76 41·6	N 0 03·3	241 37·6	S 7 13·8	180 29·1	S21 48·3	Schedar	350 27·8	N 56 18·3
D 13	97 19·2	53 42·1	29·0	91 42·4	03·9	256 40·1	13·8	195 31·7	48·3	Shaula	97 17·6	S 37 04·4
A 14	112 21·6	68 41·7	29·8	106 43·3	04·6	271 42·6	13·8	210 34·4	48·3	Sirius	259 10·5	S 16 39·7
Y 15	127 24·1	83 41·3	· · 30·7	121 44·1	· · 05·3	286 45·1	· · 13·8	225 37·1	· · 48·3	Spica	159 14·7	S 10 56·8
16	142 26·5	98 40·9	31·6	136 44·9	06·0	301 47·6	13·8	240 39·7	48·3	Suhail	223 23·1	S 43 16·2
17	157 29·0	113 40·5	32·4	151 45·8	06·6	316 50·1	13·8	255 42·4	48·3			
18	172 31·5	128 40·0	N14 33·3	166 46·6	N 0 07·3	331 52·6	S 7 13·8	270 45·1	S21 48·3	Vega	81 06·5	N 38 44·7
19	187 33·9	143 39·6	34·2	181 47·4	08·0	346 55·1	13·8	285 47·7	48·3	Zuben'ubi	137 50·9	S 15 52·2
20	202 36·4	158 39·2	35·0	196 48·3	08·7	1 57·6	13·8	300 50·4	48·3			
21	217 38·9	173 38·8	· · 35·9	211 49·1	· · 09·3	17 00·1	· · 13·8	315 53·0	· · 48·2			
22	232 41·3	188 38·4	36·8	226 49·9	10·0	32 02·6	13·8	330 55·7	48·2			
23	247 43·8	203 38·0	37·6	241 50·7	10·7	47 05·1	13·8	345 58·4	48·2			
Mer. Pass.	h m 6 35·7	v −0·4	d 0·9	v 0·8	d 0·7	v 2·5	d 0·0	v 2·7	d 0·0			

	S.H.A. ° ′	Mer. Pass. h m
Venus	318 08·7	9 25
Mars	355 23·7	6 55
Jupiter	159 19·4	19 56
Saturn	98 05·2	0 04

JUNE 12, 13, 14 (THURS., FRI., SAT.)

G.M.T. d h	SUN G.H.A. ° ′	SUN Dec. ° ′	MOON G.H.A. ° ′	v ′	Dec. ° ′	d ′	H.P. ′
12 00	180 07·1	N23 06·2	241 41·3	14·8	N 8 40·2	8·8	54·3
01	195 06·9	06·4	256 15·1	14·7	8 49·0	8·7	54·3
02	210 06·8	06·6	270 48·8	14·7	8 57·7	8·6	54·3
03	225 06·7	· · 06·8	285 22·5	14·6	9 06·3	8·7	54·3
04	240 06·5	06·9	299 56·1	14·7	9 15·0	8·6	54·3
05	255 06·4	07·1	314 29·8	14·6	9 23·6	8·5	54·3
06	270 06·3	N23 07·3	329 03·4	14·5	N 9 32·1	8·6	54·3
07	285 06·2	07·4	343 36·9	14·6	9 40·7	8·4	54·3
T 08	300 06·0	07·6	358 10·5	14·4	9 49·1	8·5	54·3
H 09	315 05·9	· · 07·8	12 43·9	14·5	9 57·6	8·4	54·3
U 10	330 05·8	07·9	27 17·4	14·4	10 06·0	8·4	54·3
R 11	345 05·6	08·1	41 50·8	14·4	10 14·4	8·3	54·3
S 12	0 05·5	N23 08·2	56 24·2	14·4	N10 22·7	8·3	54·3
D 13	15 05·4	08·4	70 57·6	14·3	10 31·0	8·3	54·4
A 14	30 05·3	08·6	85 30·9	14·2	10 39·3	8·2	54·4
Y 15	45 05·1	· · 08·7	100 04·1	14·3	10 47·5	8·1	54·4
16	60 05·0	08·9	114 37·4	14·2	10 55·6	8·2	54·4
17	75 04·9	09·0	129 10·6	14·1	11 03·8	8·0	54·4
18	90 04·7	N23 09·2	143 43·7	14·1	N11 11·8	8·1	54·4
19	105 04·6	09·4	158 16·8	14·1	11 19·9	8·0	54·4
20	120 04·5	09·5	172 49·9	14·0	11 27·9	7·9	54·4
21	135 04·4	· · 09·7	187 22·9	14·0	11 35·8	7·9	54·4
22	150 04·2	09·8	201 55·9	14·0	11 43·7	7·9	54·4
23	165 04·1	10·0	216 28·9	13·9	11 51·6	7·8	54·5
13 00	180 04·0	N23 10·1	231 01·8	13·9	N11 59·4	7·7	54·5
01	195 03·8	10·3	245 34·7	13·8	12 07·1	7·8	54·5
02	210 03·7	10·4	260 07·5	13·8	12 14·9	7·6	54·5
03	225 03·6	· · 10·6	274 40·3	13·7	12 22·5	7·6	54·5
04	240 03·4	10·7	289 13·0	13·7	12 30·1	7·6	54·5
05	255 03·3	10·9	303 45·7	13·6	12 37·7	7·5	54·5
06	270 03·2	N23 11·0	318 18·3	13·6	N12 45·2	7·5	54·5
07	285 03·1	11·2	332 50·9	13·6	12 52·7	7·4	54·5
08	300 02·9	11·3	347 23·5	13·5	13 00·1	7·3	54·6
F 09	315 02·8	· · 11·5	1 56·0	13·4	13 07·4	7·3	54·6
R 10	330 02·7	11·6	16 28·4	13·4	13 14·7	7·3	54·6
I 11	345 02·5	11·8	31 00·8	13·4	13 22·0	7·2	54·6
D 12	0 02·4	N23 11·9	45 33·2	13·3	N13 29·2	7·1	54·6
A 13	15 02·3	12·1	60 05·5	13·3	13 36·3	7·1	54·6
Y 14	30 02·1	12·2	74 37·8	13·2	13 43·4	7·0	54·6
15	45 02·0	· · 12·3	89 10·0	13·2	13 50·4	7·0	54·7
16	60 01·9	12·5	103 42·2	13·1	13 57·4	6·9	54·7
17	75 01·7	12·6	118 14·3	13·1	14 04·3	6·9	54·7
18	90 01·6	N23 12·8	132 46·4	13·0	N14 11·2	6·8	54·7
19	105 01·5	12·9	147 18·4	13·0	14 18·0	6·7	54·7
20	120 01·4	13·0	161 50·4	12·9	14 24·7	6·7	54·7
21	135 01·2	· · 13·2	176 22·3	12·9	14 31·4	6·6	54·7
22	150 01·1	13·3	190 54·2	12·8	14 38·0	6·5	54·8
23	165 01·0	13·5	205 26·0	12·8	14 44·5	6·5	54·8
14 00	180 00·8	N23 13·6	219 57·8	12·7	N14 51·0	6·4	54·8
01	195 00·7	13·7	234 29·5	12·6	14 57·4	6·4	54·8
02	210 00·6	13·9	249 01·1	12·7	15 03·8	6·3	54·8
03	225 00·4	· · 14·0	263 32·8	12·5	15 10·1	6·2	54·8
04	240 00·3	14·1	278 04·3	12·5	15 16·3	6·2	54·9
05	255 00·2	14·3	292 35·8	12·5	15 22·5	6·1	54·9
06	270 00·0	N23 14·4	307 07·3	12·4	N15 28·6	6·0	54·9
07	284 59·9	14·5	321 38·7	12·4	15 34·6	6·0	54·9
S 08	299 59·8	14·7	336 10·1	12·3	15 40·6	5·8	54·9
A 09	314 59·6	· · 14·8	350 41·4	12·2	15 46·4	5·9	54·9
T 10	329 59·5	14·9	5 12·6	12·2	15 52·3	5·7	55·0
U 11	344 59·4	15·0	19 43·8	12·2	15 58·0	5·7	55·0
R 12	359 59·2	N23 15·2	34 15·0	12·1	N16 03·7	5·6	55·0
D 13	14 59·1	15·3	48 46·1	12·0	16 09·3	5·5	55·0
A 14	29 59·0	15·4	63 17·1	12·0	16 14·8	5·5	55·0
Y 15	44 58·9	· · 15·5	77 48·1	12·0	16 20·3	5·4	55·0
16	59 58·7	15·7	92 19·1	11·9	16 25·7	5·3	55·1
17	74 58·6	15·8	106 50·0	11·8	16 31·0	5·2	55·1
18	89 58·5	N23 15·9	121 20·8	11·8	N16 36·2	5·2	55·1
19	104 58·3	16·0	135 51·6	11·7	16 41·4	5·1	55·1
20	119 58·2	16·2	150 22·3	11·7	16 46·5	5·0	55·1
21	134 58·1	· · 16·3	164 53·0	11·6	16 51·5	4·9	55·2
22	149 57·9	16·4	179 23·6	11·6	16 56·4	4·9	55·2
23	164 57·8	16·5	193 54·2	11·5	17 01·3	4·8	55·2
	S.D. 15·8	d 0·1	S.D. 14·8		14·9		15·0

Lat. °	Twilight Naut. h m	Twilight Civil h m	Sun-rise h m	Moonrise 12 h m	Moonrise 13 h m	Moonrise 14 h m	Moonrise 15 h m
N 72	▭	▭	▭	{00 03 / 23 54}	23 44	23 26	▭
N 70	▭	▭	▭	00 15	00 14	00 15	00 18
68	▭	▭	▭	00 26	00 30	00 38	00 50
66	▭	▭	▭	00 34	00 43	00 56	01 14
64	////	////	01 36	00 42	00 54	01 10	01 33
62	////	////	02 12	00 48	01 03	01 23	01 48
60	////	00 57	02 37	00 53	01 11	01 33	02 01
N 58	////	01 43	02 57	00 58	01 18	01 42	02 12
56	////	02 12	03 14	01 03	01 25	01 51	02 22
54	00 52	02 34	03 28	01 06	01 30	01 58	02 31
52	01 35	02 51	03 40	01 10	01 35	02 04	02 38
50	02 02	03 06	03 50	01 13	01 40	02 10	02 45
45	02 46	03 35	04 13	01 20	01 50	02 23	03 00
N 40	03 16	03 58	04 30	01 26	01 58	02 33	03 13
35	03 39	04 16	04 45	01 31	02 05	02 42	03 23
30	03 58	04 31	04 58	01 36	02 11	02 50	03 32
20	04 26	04 55	05 20	01 44	02 22	03 04	03 48
N 10	04 49	05 16	05 39	01 51	02 32	03 16	04 02
0	05 08	05 34	05 56	01 57	02 41	03 27	04 15
S 10	05 24	05 51	06 13	02 04	02 50	03 39	04 28
20	05 40	06 08	06 32	02 11	03 00	03 51	04 43
30	05 57	06 26	06 53	02 19	03 11	04 05	04 59
35	06 05	06 37	07 05	02 23	03 18	04 13	05 08
40	06 15	06 48	07 19	02 29	03 25	04 22	05 19
45	06 25	07 02	07 36	02 35	03 34	04 33	05 32
S 50	06 36	07 18	07 56	02 42	03 44	04 46	05 47
52	06 41	07 25	08 06	02 46	03 49	04 52	05 55
54	06 47	07 33	08 17	02 50	03 55	04 59	06 03
56	06 53	07 42	08 29	02 54	04 01	05 07	06 12
58	07 00	07 52	08 44	02 59	04 07	05 16	06 22
S 60	07 07	08 04	09 01	03 04	04 15	05 26	06 34

Lat. °	Sun-set h m	Twilight Civil h m	Twilight Naut. h m	Moonset 12 h m	Moonset 13 h m	Moonset 14 h m	Moonset 15 h m
N 72	▭	▭	▭	16 49	18 34	20 29	▭
N 70	▭	▭	▭	16 30	18 04	19 39	21 12
68	▭	▭	▭	16 15	17 41	19 07	20 28
66	▭	▭	▭	16 03	17 24	18 44	19 58
64	22 25	////	////	15 54	17 10	18 25	19 36
62	21 49	////	////	15 45	16 59	18 10	19 18
60	21 23	23 04	////	15 38	16 49	17 58	19 04
N 58	21 03	22 17	////	15 31	16 40	17 47	18 51
56	20 46	21 48	////	15 26	16 32	17 38	18 40
54	20 32	21 26	23 09	15 21	16 26	17 29	18 31
52	20 20	21 09	22 26	15 16	16 20	17 22	18 22
50	20 10	20 54	21 59	15 12	16 14	17 15	18 15
45	19 47	20 25	21 14	15 03	16 02	17 01	17 59
N 40	19 29	20 02	20 44	14 56	15 53	16 49	17 46
35	19 15	19 44	20 21	14 50	15 44	16 39	17 34
30	19 02	19 29	20 02	14 44	15 37	16 31	17 25
20	18 40	19 04	19 33	14 34	15 24	16 15	17 08
N 10	18 21	18 44	19 11	14 26	15 13	16 02	16 53
0	18 03	18 26	18 52	14 18	15 03	15 50	16 39
S 10	17 46	18 09	18 35	14 10	14 53	15 38	16 25
20	17 28	17 52	18 19	14 02	14 42	15 24	16 11
30	17 07	17 33	18 03	13 52	14 29	15 10	15 54
35	16 55	17 23	17 54	13 47	14 22	15 01	15 44
40	16 41	17 11	17 45	13 41	14 14	14 51	15 33
45	16 24	16 58	17 35	13 33	14 04	14 39	15 20
S 50	16 04	16 42	17 23	13 25	13 53	14 25	15 04
52	15 54	16 35	17 18	13 21	13 48	14 19	14 56
54	15 43	16 26	17 12	13 16	13 42	14 12	14 48
56	15 30	16 17	17 06	13 12	13 35	14 04	14 39
58	15 16	16 07	17 00	13 06	13 28	13 55	14 28
S 60	14 59	15 56	16 52	13 00	13 20	13 44	14 16

Day	SUN Eqn. of Time 00ʰ m s	SUN Eqn. of Time 12ʰ m s	SUN Mer. Pass. h m	MOON Mer. Pass. Upper h m	MOON Mer. Pass. Lower h m	Age d	Phase
12	00 28	00 22	12 00	08 07	20 30	25	●
13	00 16	00 10	12 00	08 52	21 15	26	
14	00 04	00 03	12 00	09 38	22 02	27	

JUNE 15, 16, 17 (SUN., MON., TUES.)

G.M.T.	ARIES	VENUS −3·5		MARS +0·4		JUPITER −1·8		SATURN +0·2	
d h	G.H.A. ° ′	G.H.A. ° ′	Dec. ° ′	G.H.A. ° ′	Dec. ° ′	G.H.A. ° ′	Dec. ° ′	G.H.A. ° ′	Dec. ° ′
15 00	262 46·3	218 37·6	N14 38·5	256 51·6	N 0 11·4	62 07·6	S 7 13·8	1 01·0	S21 48·2
01	277 48·7	233 37·2	39·4	271 52·4	12·0	77 10·1	13·8	16 03·7	48·2
02	292 51·2	248 36·8	40·2	286 53·2	12·7	92 12·6	13·8	31 06·4	48·2
03	307 53·7	263 36·4	· · 41·1	301 54·1	· · 13·4	107 15·1	· · 13·8	46 09·0	· · 48·2
04	322 56·1	278 36·0	42·0	316 54·9	14·1	122 17·6	13·8	61 11·7	48·2
05	337 58·6	293 35·6	42·8	331 55·8	14·7	137 20·1	13·8	76 14·3	48·2
06	353 01·0	308 35·1	N14 43·7	346 56·6	N 0 15·4	152 22·6	S 7 13·8	91 17·0	S21 48·2
07	8 03·5	323 34·7	44·5	1 57·4	16·1	167 25·1	13·8	106 19·7	48·2
08	23 06·0	338 34·3	45·4	16 58·3	16·8	182 27·6	13·8	121 22·3	48·2
S 09	38 08·4	353 33·9	· · 46·3	31 59·1	· · 17·4	197 30·1	· · 13·8	136 25·0	· · 48·2
U 10	53 10·9	8 33·5	47·1	46 59·9	18·1	212 32·6	13·8	151 27·7	48·1
N 11	68 13·4	23 33·1	48·0	62 00·8	18·8	227 35·1	13·8	166 30·3	48·1
D 12	83 15·8	38 32·7	N14 48·8	77 01·6	N 0 19·5	242 37·6	S 7 13·8	181 33·0	S21 48·1
A 13	98 18·3	53 32·3	49·7	92 02·4	20·1	257 40·1	13·8	196 35·6	48·1
Y 14	113 20·8	68 31·8	50·5	107 03·3	20·8	272 42·6	13·8	211 38·3	48·1
15	128 23·2	83 31·4	· · 51·4	122 04·1	· · 21·5	287 45·1	· · 13·8	226 41·0	· · 48·1
16	143 25·7	98 31·0	52·3	137 04·9	22·2	302 47·5	13·8	241 43·6	48·1
17	158 28·1	113 30·6	53·1	152 05·8	22·8	317 50·0	13·8	256 46·3	48·1
18	173 30·6	128 30·2	N14 54·0	167 06·6	N 0 23·5	332 52·5	S 7 13·8	271 49·0	S21 48·1
19	188 33·1	143 29·8	54·8	182 07·4	24·2	347 55·0	13·8	286 51·6	48·1
20	203 35·5	158 29·3	55·7	197 08·3	24·9	2 57·5	13·8	301 54·3	48·1
21	218 38·0	173 28·9	· · 56·5	212 09·1	· · 25·5	18 00·0	· · 13·8	316 56·9	· · 48·1
22	233 40·5	188 28·5	57·4	227 09·9	26·2	33 02·5	13·8	331 59·6	48·1
23	248 42·9	203 28·1	58·2	242 10·8	26·9	48 05·0	13·8	347 02·3	48·1
16 00	263 45·4	218 27·7	N14 59·1	257 11·6	N 0 27·6	63 07·5	S 7 13·8	2 04·9	S21 48·0
01	278 47·9	233 27·2	14 59·9	272 12·5	28·2	78 10·0	13·8	17 07·6	48·0
02	293 50·3	248 26·8	15 00·8	287 13·3	28·9	93 12·5	13·8	32 10·3	48·0
03	308 52·8	263 26·4	· · 01·6	302 14·1	· · 29·6	108 14·9	· · 13·8	47 12·9	· · 48·0
04	323 55·3	278 26·0	02·5	317 15·0	30·3	123 17·4	13·8	62 15·6	48·0
05	338 57·7	293 25·6	03·3	332 15·8	30·9	138 19·9	13·8	77 18·2	48·0
06	354 00·2	308 25·1	N15 04·2	347 16·6	N 0 31·6	153 22·4	S 7 13·8	92 20·9	S21 48·0
07	9 02·6	323 24·7	05·0	2 17·5	32·3	168 24·9	13·8	107 23·6	48·0
08	24 05·1	338 24·3	05·9	17 18·3	33·0	183 27·4	13·8	122 26·2	48·0
M 09	39 07·6	353 23·9	· · 06·7	32 19·1	· · 33·6	198 29·9	· · 13·8	137 28·9	· · 48·0
O 10	54 10·0	8 23·4	07·6	47 20·0	34·3	213 32·4	13·8	152 31·6	48·0
N 11	69 12·5	23 23·0	08·4	62 20·8	35·0	228 34·8	13·8	167 34·2	48·0
D 12	84 15·0	38 22·6	N15 09·3	77 21·7	N 0 35·7	243 37·3	S 7 13·8	182 36·9	S21 48·0
A 13	99 17·4	53 22·2	10·1	92 22·5	36·3	258 39·8	13·8	197 39·5	48·0
Y 14	114 19·9	68 21·7	11·0	107 23·3	37·0	273 42·3	13·8	212 42·2	47·9
15	129 22·4	83 21·3	· · 11·8	122 24·2	· · 37·7	288 44·8	· · 13·8	227 44·9	· · 47·9
16	144 24·8	98 20·9	12·6	137 25·0	38·3	303 47·3	13·8	242 47·5	47·9
17	159 27·3	113 20·5	13·5	152 25·8	39·0	318 49·8	13·8	257 50·2	47·9
18	174 29·8	128 20·0	N15 14·3	167 26·7	N 0 39·7	333 52·2	S 7 13·8	272 52·9	S21 47·9
19	189 32·2	143 19·6	15·2	182 27·5	40·4	348 54·7	13·8	287 55·5	47·9
20	204 34·7	158 19·2	16·0	197 28·4	41·0	3 57·2	13·8	302 58·2	47·9
21	219 37·1	173 18·7	· · 16·9	212 29·2	· · 41·7	18 59·7	· · 13·8	318 00·8	· · 47·9
22	234 39·6	188 18·3	17·7	227 30·0	42·4	34 02·2	13·8	333 03·5	47·9
23	249 42·1	203 17·9	18·5	242 30·9	43·1	49 04·7	13·9	348 06·2	47·9
17 00	264 44·5	218 17·5	N15 19·4	257 31·7	N 0 43·7	64 07·2	S 7 13·9	3 08·8	S21 47·9
01	279 47·0	233 17·0	20·2	272 32·6	44·4	79 09·6	13·9	18 11·5	47·9
02	294 49·5	248 16·6	21·1	287 33·4	45·1	94 12·1	13·9	33 14·2	47·9
03	309 51·9	263 16·2	· · 21·9	302 34·2	· · 45·7	109 14·6	· · 13·9	48 16·8	· · 47·9
04	324 54·4	278 15·7	22·7	317 35·1	46·4	124 17·1	13·9	63 19·5	47·8
05	339 56·9	293 15·3	23·6	332 35·9	47·1	139 19·6	13·9	78 22·1	47·8
06	354 59·3	308 14·9	N15 24·4	347 36·8	N 0 47·8	154 22·0	S 7 13·9	93 24·8	S21 47·8
07	10 01·8	323 14·4	25·2	2 37·6	48·4	169 24·5	13·9	108 27·5	47·8
T 08	25 04·3	338 14·0	26·1	17 38·4	49·1	184 27·0	13·9	123 30·1	47·8
U 09	40 06·7	353 13·6	· · 26·9	32 39·3	· · 49·8	199 29·5	· · 13·9	138 32·8	· · 47·8
E 10	55 09·2	8 13·1	27·7	47 40·1	50·4	214 32·0	13·9	153 35·5	47·8
S 11	70 11·6	23 12·7	28·6	62 41·0	51·1	229 34·4	13·9	168 38·1	47·8
D 12	85 14·1	38 12·2	N15 29·4	77 41·8	N 0 51·8	244 36·9	S 7 13·9	183 40·8	S21 47·8
A 13	100 16·6	53 11·8	30·2	92 42·6	52·5	259 39·4	13·9	198 43·4	47·8
Y 14	115 19·0	68 11·4	31·1	107 43·5	53·1	274 41·9	13·9	213 46·1	47·8
15	130 21·5	83 10·9	· · 31·9	122 44·3	· · 53·8	289 44·4	· · 13·9	228 48·8	· · 47·8
16	145 24·0	98 10·5	32·7	137 45·2	54·5	304 46·8	13·9	243 51·4	47·8
17	160 26·4	113 10·1	33·6	152 46·0	55·1	319 49·3	14·0	258 54·1	47·8
18	175 28·9	128 09·6	N15 34·4	167 46·8	N 0 55·8	334 51·8	S 7 14·0	273 56·8	S21 47·7
19	190 31·4	143 09·2	35·2	182 47·7	56·5	349 54·3	14·0	288 59·4	47·7
20	205 33·8	158 08·7	36·0	197 48·5	57·2	4 56·8	14·0	304 02·1	47·7
21	220 36·3	173 08·3	· · 36·9	212 49·4	· · 57·8	19 59·2	· · 14·0	319 04·7	· · 47·7
22	235 38·8	188 07·8	37·7	227 50·2	58·5	35 01·7	14·0	334 07·4	47·7
23	250 41·2	203 07·4	38·5	242 51·0	59·2	50 04·2	14·0	349 10·1	47·7
Mer. Pass.	h m 6 23·9	v −0·4	d 0·8	v 0·8	d 0·7	v 2·5	d 0·0	v 2·7	d 0·0

STARS Name	S.H.A. ° ′	Dec. ° ′
Acamar	315 50·1	S 40 28·1
Achernar	335 57·9	S 57 26·6
Acrux	173 55·3	S 62 52·5
Adhara	255 45·4	S 28 55·1
Aldebaran	291 37·2	N 16 25·5
Alioth	166 56·8	N 56 11·3
Alkaid	153 31·2	N 49 31·4
Al Na'ir	28 35·4	S 47 09·4
Alnilam	276 28·7	S 1 13·8
Alphard	218 36·9	S 8 28·8
Alphecca	126 45·7	N 26 51·4
Alpheratz	358 26·3	N 28 51·5
Altair	62 48·3	N 8 45·6
Ankaa	353 56·6	S 42 31·6
Antares	113 16·6	S 26 20·4
Arcturus	146 33·2	N 19 24·0
Atria	108 55·0	S 68 57·2
Avior	234 35·4	S 59 22·9
Bellatrix	279 16·7	N 6 18·7
Betelgeuse	271 46·4	N 7 23·9
Canopus	264 15·0	S 52 40·5
Capella	281 36·0	N 45 57·3
Deneb	49 59·3	N 45 07·8
Denebola	183 15·8	N 14 48·3
Diphda	349 37·5	S 18 12·7
Dubhe	194 42·5	N 61 58·7
Elnath	279 05·3	N 28 34·3
Eltanin	91 04·8	N 51 29·7
Enif	34 27·5	N 9 41·1
Fomalhaut	16 09·5	S 29 50·3
Gacrux	172 46·8	S 56 53·2
Gienah	176 34·8	S 17 18·8
Hadar	149 46·1	S 60 10·6
Hamal	328 47·7	N 23 15·9
Kaus Aust.	84 38·2	S 34 24·2
Kochab	137 17·4	N 74 19·8
Markab	14 19·5	N 14 58·9
Menkar	314 58·6	N 3 55·6
Menkent	148 56·1	S 36 10·1
Miaplacidus	221 49·0	S 69 33·1
Mirfak	309 40·0	N 49 42·7
Nunki	76 49·2	S 26 20·8
Peacock	54 23·9	S 56 51·9
Pollux	244 18·6	N 28 07·6
Procyon	245 43·3	N 5 19·8
Rasalhague	96 44·5	N 12 35·5
Regulus	208 27·6	N 12 10·2
Rigel	281 52·1	S 8 15·0
Rigil Kent.	140 47·7	S 60 40·1
Sabik	102 59·6	S 15 40·4
Schedar	350 27·8	N 56 18·3
Shaula	97 17·6	S 37 04·4
Sirius	259 10·5	S 16 39·7
Spica	159 14·7	S 10 56·8
Suhail	223 23·1	S 43 16·2
Vega	81 06·5	N 38 44·7
Zuben'ubi	137 50·9	S 15 52·2

	S.H.A. ° ′	Mer. Pass. h m
Venus	314 42·3	9 26
Mars	353 26·2	6 51
Jupiter	159 22·1	19 44
Saturn	98 19·5	23 47

JUNE 15, 16, 17 (SUN., MON., TUES.)

G.M.T.	SUN		MOON				
d h	G.H.A. ° ′	Dec. ° ′	G.H.A. ° ′	v ′	Dec. ° ′	d ′	H.P. ′
15 00	179 57·7	N23 16·7	208 24·7	11·5	N17 06·1	4·7	55·2
01	194 57·5	16·8	222 55·2	11·4	17 10·8	4·6	55·2
02	209 57·4	16·9	237 25·6	11·4	17 15·4	4·5	55·3
03	224 57·3	· · 17·0	251 56·0	11·3	17 19·9	4·5	55·3
04	239 57·1	17·1	266 26·3	11·2	17 24·4	4·4	55·3
05	254 57·0	17·2	280 56·5	11·3	17 28·8	4·3	55·3
06	269 56·9	N23 17·4	295 26·8	11·1	N17 33·1	4·2	55·3
07	284 56·7	17·5	309 56·9	11·1	17 37·3	4·1	55·4
08	299 56·6	17·6	324 27·0	11·1	17 41·4	4·0	55·4
S 09	314 56·5	· · 17·7	338 57·1	11·0	17 45·4	4·0	55·4
U 10	329 56·3	17·8	353 27·1	11·0	17 49·4	3·8	55·4
N 11	344 56·2	17·9	7 57·1	10·9	17 53·2	3·8	55·4
D 12	359 56·1	N23 18·0	22 27·0	10·9	N17 57·0	3·7	55·5
A 13	14 55·9	18·1	36 56·9	10·8	18 00·7	3·6	55·5
Y 14	29 55·8	18·2	51 26·7	10·7	18 04·3	3·5	55·5
15	44 55·7	· · 18·4	65 56·4	10·8	18 07·8	3·5	55·5
16	59 55·5	18·5	80 26·2	10·6	18 11·3	3·3	55·5
17	74 55·4	18·6	94 55·8	10·6	18 14·6	3·2	55·6
18	89 55·3	N23 18·7	109 25·4	10·6	N18 17·8	3·2	55·6
19	104 55·1	18·8	123 55·0	10·6	18 21·0	3·1	55·6
20	119 55·0	18·9	138 24·6	10·4	18 24·1	2·9	55·6
21	134 54·9	· · 19·0	152 54·0	10·5	18 27·0	2·9	55·7
22	149 54·7	19·1	167 23·5	10·4	18 29·9	2·8	55·7
23	164 54·6	19·2	181 52·9	10·3	18 32·7	2·7	55·7
16 00	179 54·5	N23 19·3	196 22·2	10·3	N18 35·4	2·6	55·7
01	194 54·3	19·4	210 51·5	10·3	18 38·0	2·5	55·7
02	209 54·2	19·5	225 20·8	10·2	18 40·5	2·4	55·8
03	224 54·1	· · 19·6	239 50·0	10·2	18 42·9	2·3	55·8
04	239 53·9	19·7	254 19·2	10·1	18 45·2	2·3	55·8
05	254 53·8	19·8	268 48·3	10·1	18 47·5	2·1	55·8
06	269 53·7	N23 19·9	283 17·4	10·0	N18 49·6	2·0	55·9
07	284 53·5	20·0	297 46·4	10·0	18 51·6	2·0	55·9
08	299 53·4	20·1	312 15·4	10·0	18 53·6	1·8	55·9
M 09	314 53·2	· · 20·2	326 44·4	9·9	18 55·4	1·7	55·9
O 10	329 53·1	20·3	341 13·3	9·9	18 57·1	1·7	55·9
N 11	344 53·0	20·4	355 42·2	9·9	18 58·8	1·5	56·0
D 12	359 52·8	N23 20·5	10 11·1	9·8	N19 00·3	1·4	56·0
A 13	14 52·7	20·6	24 39·9	9·7	19 01·7	1·4	56·0
Y 14	29 52·6	20·7	39 08·6	9·8	19 03·1	1·2	56·0
15	44 52·4	· · 20·7	53 37·4	9·7	19 04·3	1·1	56·1
16	59 52·3	20·8	68 06·1	9·7	19 05·4	1·0	56·1
17	74 52·2	20·9	82 34·8	9·6	19 06·4	1·0	56·1
18	89 52·0	N23 21·0	97 03·4	9·6	N19 07·4	0·8	56·1
19	104 51·9	21·1	111 32·0	9·5	19 08·2	0·7	56·1
20	119 51·8	21·2	126 00·5	9·6	19 08·9	0·7	56·2
21	134 51·6	· · 21·3	140 29·1	9·5	19 09·6	0·5	56·2
22	149 51·5	21·4	154 57·6	9·4	19 10·1	0·4	56·2
23	164 51·4	21·4	169 26·0	9·5	19 10·5	0·3	56·2
17 00	179 51·2	N23 21·5	183 54·5	9·4	N19 10·8	0·2	56·3
01	194 51·1	21·6	198 22·9	9·4	19 11·0	0·1	56·3
02	209 51·0	21·7	212 51·3	9·3	19 11·1	0·0	56·3
03	224 50·8	· · 21·8	227 19·6	9·3	19 11·1	0·1	56·3
04	239 50·7	21·9	241 47·9	9·3	19 11·0	0·2	56·3
05	254 50·5	21·9	256 16·2	9·3	19 10·8	0·3	56·4
06	269 50·4	N23 22·0	270 44·5	9·3	N19 10·5	0·4	56·4
07	284 50·3	22·1	285 12·8	9·2	19 10·1	0·6	56·4
T 08	299 50·1	22·2	299 41·0	9·2	19 09·5	0·6	56·4
U 09	314 50·0	· · 22·3	314 09·2	9·2	19 08·9	0·7	56·5
E 10	329 49·9	22·3	328 37·4	9·1	19 08·2	0·9	56·5
S 11	344 49·7	22·4	343 05·5	9·2	19 07·3	0·9	56·5
D 12	359 49·6	N23 22·5	357 33·7	9·1	N19 06·4	1·1	56·5
A 13	14 49·5	22·6	12 01·8	9·1	19 05·3	1·1	56·6
Y 14	29 49·3	22·6	26 29·9	9·0	19 04·2	1·3	56·6
15	44 49·2	· · 22·7	40 57·9	9·1	19 02·9	1·4	56·6
16	59 49·1	22·8	55 26·0	9·0	19 01·5	1·5	56·6
17	74 48·9	22·9	69 54·0	9·1	19 00·0	1·6	56·6
18	89 48·8	N23 22·9	84 22·1	9·0	N18 58·4	1·7	56·7
19	104 48·6	23·0	98 50·1	9·0	18 56·7	1·8	56·7
20	119 48·5	23·1	113 18·1	8·9	18 54·9	1·9	56·7
21	134 48·4	· · 23·2	127 46·0	9·0	18 53·0	2·0	56·7
22	149 48·2	23·2	142 14·0	9·0	18 51·0	2·1	56·8
23	164 48·1	23·3	156 42·0	8·9	18 48·9	2·3	56·8
	S.D. 15·8	d 0·1	S.D. 15·1		15·3		15·4

Lat.	Twilight Naut.	Twilight Civil	Sunrise	Moonrise 15	16	17	18
°	h m	h m	h m	h m	h m	h m	h m
N 72	▭	▭	▭	▭	▭	▭	▭
N 70	▭	▭	▭	00 18	00 28	00 59	02 11
68	▭	▭	▭	00 50	01 13	01 53	02 57
66	▭	▭	▭	01 14	01 42	02 26	03 28
64	////	////	01 33	01 33	02 05	02 50	03 50
62	////	////	02 10	01 48	02 23	03 09	04 09
60	////	00 52	02 36	02 01	02 37	03 25	04 23
N 58	////	01 41	02 56	02 12	02 50	03 38	04 36
56	////	02 11	03 13	02 22	03 01	03 49	04 47
54	00 48	02 33	03 27	02 31	03 11	03 59	04 57
52	01 33	02 51	03 39	02 38	03 19	04 08	05 05
50	02 00	03 06	03 50	02 45	03 27	04 16	05 13
45	02 46	03 35	04 13	03 00	03 43	04 33	05 29
N 40	03 16	03 58	04 30	03 13	03 57	04 47	05 43
35	03 39	04 16	04 45	03 23	04 09	04 59	05 54
30	03 58	04 31	04 58	03 32	04 19	05 10	06 04
20	04 27	04 56	05 20	03 48	04 36	05 27	06 22
N 10	04 49	05 16	05 39	04 02	04 51	05 43	06 36
0	05 08	05 34	05 57	04 15	05 05	05 58	06 51
S 10	05 25	05 51	06 14	04 28	05 20	06 12	07 05
20	05 41	06 09	06 33	04 43	05 35	06 28	07 19
30	05 58	06 27	06 54	04 59	05 53	06 46	07 37
35	06 06	06 38	07 06	05 08	06 03	06 56	07 47
40	06 16	06 50	07 20	05 19	06 14	07 08	07 58
45	06 26	07 03	07 37	05 32	06 28	07 22	08 11
S 50	06 38	07 19	07 58	05 47	06 45	07 39	08 28
52	06 43	07 27	08 08	05 55	06 53	07 47	08 35
54	06 48	07 35	08 19	06 03	07 02	07 56	08 44
56	06 55	07 44	08 31	06 12	07 12	08 07	08 53
58	07 01	07 54	08 46	06 22	07 23	08 18	09 04
S 60	07 09	08 06	09 03	06 34	07 37	08 32	09 17

Lat.	Sunset	Twilight Civil	Twilight Naut.	Moonset 15	16	17	18
°	h m	h m	h m	h m	h m	h m	h m
N 72	▭	▭	▭	▭	▭	▭	▭
N 70	▭	▭	▭	21 12	22 28	23 07	23 20
68	▭	▭	▭	20 28	21 34	22 20	22 46
66	▭	▭	▭	19 58	21 01	21 49	22 21
64	22 29	////	////	19 36	20 37	21 26	22 02
62	21 51	////	////	19 18	20 18	21 08	21 46
60	21 25	23 10	////	19 04	20 03	20 53	21 33
N 58	21 05	22 20	////	18 51	19 49	20 40	21 22
56	20 48	21 51	////	18 40	19 38	20 29	21 12
54	20 34	21 29	23 14	18 31	19 28	20 19	21 03
52	20 22	21 11	22 29	18 22	19 19	20 10	20 55
50	20 11	20 56	22 01	18 15	19 11	20 02	20 48
45	19 49	20 26	21 16	17 59	18 54	19 46	20 33
N 40	19 31	20 03	20 45	17 46	18 40	19 32	20 20
35	19 16	19 45	20 22	17 34	18 28	19 20	20 09
30	19 03	19 30	20 03	17 25	18 18	19 10	20 00
20	18 41	19 05	19 34	17 08	18 00	18 53	19 44
N 10	18 22	18 45	19 12	16 53	17 45	18 37	19 29
0	18 04	18 27	18 53	16 39	17 30	18 23	19 16
S 10	17 47	18 10	18 36	16 25	17 16	18 08	19 03
20	17 28	17 52	18 20	16 11	17 00	17 53	18 48
30	17 07	17 34	18 03	15 54	16 42	17 35	18 32
35	16 55	17 23	17 55	15 44	16 32	17 25	18 22
40	16 41	17 11	17 45	15 33	16 20	17 13	18 11
45	16 24	16 58	17 35	15 20	16 06	16 59	17 58
S 50	16 03	16 42	17 23	15 04	15 49	16 42	17 42
52	15 53	16 34	17 18	14 56	15 41	16 34	17 35
54	15 42	16 26	17 12	14 48	15 32	16 25	17 26
56	15 30	16 17	17 06	14 39	15 22	16 15	17 17
58	15 15	16 07	16 59	14 28	15 10	16 03	17 07
S 60	14 58	15 55	16 52	14 16	14 57	15 50	16 54

Day	SUN Eqn. of Time 00ʰ	12ʰ	Mer. Pass.	MOON Mer. Pass. Upper	Lower	Age	Phase
	m s	m s	h m	h m	h m	d	
15	00 09	00 15	12 00	10 27	22 52	28	●
16	00 22	00 28	12 00	11 18	23 44	29	
17	00 35	00 41	12 01	12 10	24 37	00	

SEPTEMBER 22, 23, 24 (MON., TUES., WED.)

	G.M.T.	ARIES	VENUS −3·4		MARS −1·0		JUPITER −1·3		SATURN +0·7	
	d h	G.H.A. ° ′	G.H.A. ° ′	Dec. ° ′	G.H.A. ° ′	Dec. ° ′	G.H.A. ° ′	Dec. ° ′	G.H.A. ° ′	Dec. ° ′
	22 00	0 21·0	193 12·3	N 7 01·0	301 54·4	N18 19·2	149 25·8	S11 29·9	101 23·1	S21 54·0
	01	15 23·5	208 11·9	6 59·8	316 56·3	19·4	164 27·8	30·1	116 25·4	54·0
	02	30 25·9	223 11·5	58·6	331 58·1	19·5	179 29·8	30·2	131 27·8	54·0
	03	45 28·4	238 11·0	·· 57·4	347 00·0	·· 19·7	194 31·8	·· 30·4	146 30·1	·· 54·0
	04	60 30·8	253 10·6	56·3	2 01·8	19·9	209 33·8	30·6	161 32·5	54·0
	05	75 33·3	268 10·2	55·1	17 03·7	20·1	224 35·8	30·8	176 34·8	54·1
	06	90 35·8	283 09·8	N 6 53·9	32 05·6	N18 20·3	239 37·8	S11 30·9	191 37·1	S21 54·1
	07	105 38·2	298 09·3	52·7	47 07·4	20·4	254 39·8	31·1	206 39·5	54·1
	08	120 40·7	313 08·9	51·6	62 09·3	20·6	269 41·7	31·3	221 41·8	54·1
M	09	135 43·2	328 08·5	·· 50·4	77 11·1	·· 20·8	284 43·7	·· 31·4	236 44·2	·· 54·1
O	10	150 45·6	343 08·1	49·2	92 13·0	21·0	299 45·7	31·6	251 46·5	54·1
N	11	165 48·1	358 07·6	48·0	107 14·9	21·2	314 47·7	31·8	266 48·8	54·2
D	12	180 50·6	13 07·2	N 6 46·8	122 16·7	N18 21·3	329 49·7	S11 32·0	281 51·2	S21 54·2
A	13	195 53·0	28 06·8	45·7	137 18·6	21·5	344 51·7	32·1	296 53·5	54·2
Y	14	210 55·5	43 06·4	44·5	152 20·5	21·7	359 53·7	32·3	311 55·9	54·2
	15	225 57·9	58 05·9	·· 43·3	167 22·3	·· 21·9	14 55·7	·· 32·5	326 58·2	·· 54·2
	16	241 00·4	73 05·5	42·1	182 24·2	22·0	29 57·7	32·7	342 00·5	54·2
	17	256 02·9	88 05·1	40·9	197 26·1	22·2	44 59·7	32·8	357 02·9	54·2
	18	271 05·3	103 04·7	N 6 39·7	212 27·9	N18 22·4	60 01·7	S11 33·0	12 05·2	S21 54·3
	19	286 07·8	118 04·2	38·6	227 29·8	22·6	75 03·6	33·2	27 07·5	54·3
	20	301 10·3	133 03·8	37·4	242 31·7	22·8	90 05·6	33·4	42 09·9	54·3
	21	316 12·7	148 03·4	·· 36·2	257 33·6	·· 22·9	105 07·6	·· 33·5	57 12·2	·· 54·3
	22	331 15·2	163 03·0	35·0	272 35·4	23·1	120 09·6	33·7	72 14·6	54·3
	23	346 17·7	178 02·5	33·8	287 37·3	23·3	135 11·6	33·9	87 16·9	54·3
	23 00	1 20·1	193 02·1	N 6 32·6	302 39·2	N18 23·5	150 13·6	S11 34·1	102 19·2	S21 54·4
	01	16 22·6	208 01·7	31·5	317 41·1	23·6	165 15·6	34·2	117 21·6	54·4
	02	31 25·1	223 01·3	30·3	332 43·0	23·8	180 17·6	34·4	132 23·9	54·4
	03	46 27·5	238 00·8	·· 29·1	347 44·9	·· 24·0	195 19·6	·· 34·6	147 26·3	·· 54·4
	04	61 30·0	253 00·4	27·9	2 46·7	24·2	210 21·6	34·8	162 28·6	54·4
	05	76 32·4	268 00·0	26·7	17 48·6	24·3	225 23·5	34·9	177 30·9	54·4
	06	91 34·9	282 59·6	N 6 25·5	32 50·5	N18 24·5	240 25·5	S11 35·1	192 33·3	S21 54·5
	07	106 37·4	297 59·1	24·3	47 52·4	24·7	255 27·5	35·3	207 35·6	54·5
T	08	121 39·8	312 58·7	23·1	62 54·3	24·9	270 29·5	35·4	222 37·9	54·5
U	09	136 42·3	327 58·3	·· 22·0	77 56·2	·· 25·0	285 31·5	·· 35·6	237 40·3	·· 54·5
E	10	151 44·8	342 57·9	20·8	92 58·1	25·2	300 33·5	35·8	252 42·6	54·5
S	11	166 47·2	357 57·5	19·6	108 00·0	25·4	315 35·5	36·0	267 44·9	54·5
D	12	181 49·7	12 57·0	N 6 18·4	123 01·9	N18 25·5	330 37·5	S11 36·1	282 47·3	S21 54·6
A	13	196 52·2	27 56·6	17·2	138 03·8	25·7	345 39·4	36·3	297 49·6	54·6
Y	14	211 54·6	42 56·2	16·0	153 05·7	25·9	0 41·4	36·5	312 51·9	54·6
	15	226 57·1	57 55·8	·· 14·8	168 07·5	·· 26·1	15 43·4	·· 36·7	327 54·3	·· 54·6
	16	241 59·5	72 55·4	13·6	183 09·4	26·2	30 45·4	36·8	342 56·6	54·6
	17	257 02·0	87 54·9	12·5	198 11·3	26·4	45 47·4	37·0	357 59·0	54·6
	18	272 04·5	102 54·5	N 6 11·3	213 13·3	N18 26·6	60 49·4	S11 37·2	13 01·3	S21 54·7
	19	287 06·9	117 54·1	10·1	228 15·2	26·8	75 51·4	37·4	28 03·6	54·7
	20	302 09·4	132 53·7	08·9	243 17·1	26·9	90 53·4	37·5	43 06·0	54·7
	21	317 11·9	147 53·3	·· 07·7	258 19·0	·· 27·1	105 55·3	·· 37·7	58 08·3	·· 54·7
	22	332 14·3	162 52·8	06·5	273 20·9	27·3	120 57·3	37·9	73 10·6	54·7
	23	347 16·8	177 52·4	05·3	288 22·8	27·4	135 59·3	38·1	88 13·0	54·7
	24 00	2 19·3	192 52·0	N 6 04·1	303 24·7	N18 27·6	151 01·3	S11 38·2	103 15·3	S21 54·8
	01	17 21·7	207 51·6	02·9	318 26·6	27·8	166 03·3	38·4	118 17·6	54·8
	02	32 24·2	222 51·2	01·7	333 28·5	28·0	181 05·3	38·6	133 20·0	54·8
	03	47 26·7	237 50·7	6 00·5	348 30·4	·· 28·1	196 07·3	·· 38·8	148 22·3	·· 54·8
	04	62 29·1	252 50·3	5 59·3	3 32·3	28·3	211 09·3	38·9	163 24·6	54·8
	05	77 31·6	267 49·9	58·1	18 34·3	28·5	226 11·2	39·1	178 27·0	54·8
	06	92 34·0	282 49·5	N 5 57·0	33 36·2	N18 28·6	241 13·2	S11 39·3	193 29·3	S21 54·9
W	07	107 36·5	297 49·1	55·8	48 38·1	28·8	256 15·2	39·5	208 31·6	54·9
E	08	122 39·0	312 48·7	54·6	63 40·0	29·0	271 17·2	39·6	223 34·0	54·9
D	09	137 41·4	327 48·2	·· 53·4	78 41·9	·· 29·1	286 19·2	·· 39·8	238 36·3	·· 54·9
N	10	152 43·9	342 47·8	52·2	93 43·9	29·3	301 21·2	40·0	253 38·6	54·9
E	11	167 46·4	357 47·4	51·0	108 45·8	29·5	316 23·2	40·2	268 41·0	54·9
S	12	182 48·8	12 47·0	N 5 49·8	123 47·7	N18 29·6	331 25·1	S11 40·3	283 43·3	S21 55·0
D	13	197 51·3	27 46·6	48·6	138 49·6	29·8	346 27·1	40·5	298 45·6	55·0
A	14	212 53·8	42 46·2	47·4	153 51·6	30·0	1 29·1	40·7	313 48·0	55·0
Y	15	227 56·2	57 45·7	·· 46·2	168 53·5	·· 30·2	16 31·1	·· 40·9	328 50·3	·· 55·0
	16	242 58·7	72 45·3	45·0	183 55·4	30·3	31 33·1	41·0	343 52·6	55·0
	17	258 01·2	87 44·9	43·8	198 57·3	30·5	46 35·1	41·2	358 54·9	55·0
	18	273 03·6	102 44·5	N 5 42·6	213 59·3	N18 30·7	61 37·1	S11 41·4	13 57·3	S21 55·1
	19	288 06·1	117 44·1	41·4	229 01·2	30·8	76 39·0	41·6	28 59·6	55·1
	20	303 08·5	132 43·7	40·2	244 03·1	31·0	91 41·0	41·7	44 01·9	55·1
	21	318 11·0	147 43·2	·· 39·0	259 05·1	·· 31·2	106 43·0	·· 41·9	59 04·3	·· 55·1
	22	333 13·5	162 42·8	37·8	274 07·0	31·3	121 45·0	42·1	74 06·6	55·1
	23	348 15·9	177 42·4	36·6	289 09·0	31·5	136 47·0	42·3	89 08·9	55·1
	Mer. Pass.	h m 23 50·7	v −0·4	d 1·2	v 1·9	d 0·2	v 2·0	d 0·2	v 2·3	d 0·0

STARS Name	S.H.A. ° ′	Dec. ° ′
Acamar	315 49·3	S 40 28·0
Achernar	335 56·9	S 57 26·6
Acrux	173 55·9	S 62 52·2
Adhara	255 44·9	S 28 54·8
Aldebaran	291 36·5	N 16 25·6
Alioth	166 57·4	N 56 11·1
Alkaid	153 31·8	N 49 31·3
Al Na'ir	28 35·0	S 47 09·6
Alnilam	276 28·1	S 1 13·6
Alphard	218 36·7	S 8 28·7
Alphecca	126 46·1	N 26 51·5
Alpheratz	358 25·7	N 28 51·9
Altair	62 48·2	N 8 45·8
Ankaa	353 55·9	S 42 31·6
Antares	113 16·9	S 26 20·4
Arcturus	146 33·6	N 19 24·0
Atria	108 55·8	S 68 57·4
Avior	234 35·2	S 59 22·4
Bellatrix	279 16·1	N 6 18·8
Betelgeuse	271 45·8	N 7 24·0
Canopus	264 14·5	S 52 40·2
Capella	281 35·2	N 45 57·2
Deneb	49 59·2	N 45 08·3
Denebola	183 15·9	N 14 48·2
Diphda	349 36·8	S 18 12·6
Dubhe	194 42·8	N 61 58·4
Elnath	279 04·6	N 28 34·3
Eltanin	91 05·3	N 51 30·0
Enif	34 27·3	N 9 41·4
Fomalhaut	16 09·0	S 29 50·3
Gacrux	172 47·3	S 56 52·9
Gienah	176 35·0	S 17 18·7
Hadar	149 46·8	S 60 10·5
Hamal	328 46·9	N 23 16·1
Kaus Aust.	84 38·3	S 34 24·3
Kochab	137 19·2	N 74 19·7
Markab	14 19·0	N 14 59·2
Menkar	314 57·9	N 3 55·8
Menkent	148 56·4	S 36 10·0
Miaplacidus	221 49·0	S 69 32·7
Mirfak	309 39·0	N 49 42·8
Nunki	76 49·2	S 26 20·9
Peacock	54 23·7	S 56 52·2
Pollux	244 18·2	N 28 07·5
Procyon	245 42·9	N 5 19·9
Rasalhague	96 44·7	N 12 35·6
Regulus	208 27·6	N 12 10·2
Rigel	281 51·6	S 8 14·8
Rigil Kent.	140 48·4	S 60 40·0
Sabik	102 59·8	S 15 40·4
Schedar	350 26·9	N 56 18·7
Shaula	97 17·9	S 37 04·5
Sirius	259 10·1	S 16 39·4
Spica	159 14·9	S 10 56·7
Suhail	223 23·0	S 43 15·8
Vega	81 06·8	N 38 45·1
Zuben'ubi	137 51·2	S 15 52·2

	S.H.A. ° ′	Mer. Pass. h m
Venus	191 42·0	11 08
Mars	301 19·1	3 49
Jupiter	148 53·5	13 57
Saturn	100 59·1	17 08

SEPTEMBER 22, 23, 24 (MON., TUES., WED.)

	G.M.T.	SUN		MOON				
	d h	G.H.A. ° ′	Dec. ° ′	G.H.A. ° ′	v ′	Dec. ° ′	d ′	H.P. ′
	22 00	181 44·5	N 0 36·2	68 05·7	10·2	S16 33·5	5·0	56·6
	01	196 44·7	35·2	82 34·9	10·2	16 28·5	5·1	56·6
	02	211 44·9	34·2	97 04·1	10·3	16 23·4	5·1	56·6
	03	226 45·2	·· 33·3	111 33·4	10·4	16 18·3	5·3	56·5
	04	241 45·4	32·3	126 02·8	10·4	16 13·0	5·3	56·5
	05	256 45·6	31·3	140 32·2	10·4	16 07·7	5·4	56·5
	06	271 45·8	N 0 30·3	155 01·6	10·6	S16 02·3	5·4	56·5
	07	286 46·0	29·4	169 31·2	10·5	15 56·9	5·6	56·4
	08	301 46·3	28·4	184 00·7	10·7	15 51·3	5·6	56·4
M	09	316 46·5	·· 27·4	198 30·4	10·7	15 45·7	5·7	56·4
O	10	331 46·7	26·4	213 00·1	10·8	15 40·0	5·8	56·4
N	11	346 46·9	25·5	227 29·9	10·8	15 34·2	5·9	56·3
D	12	1 47·1	N 0 24·5	241 59·7	10·9	S15 28·3	5·9	56·3
A	13	16 47·4	23·5	256 29·6	10·9	15 22·4	6·0	56·3
Y	14	31 47·6	22·6	270 59·5	11·0	15 16·4	6·1	56·3
	15	46 47·8	·· 21·6	285 29·5	11·1	15 10·3	6·2	56·2
	16	61 48·0	20·6	299 59·6	11·1	15 04·1	6·2	56·2
	17	76 48·2	19·6	314 29·7	11·1	14 57·9	6·3	56·2
	18	91 48·5	N 0 18·7	328 59·8	11·3	S14 51·6	6·4	56·2
	19	106 48·7	17·7	343 30·1	11·3	14 45·2	6·4	56·1
	20	121 48·9	16·7	358 00·4	11·3	14 38·8	6·5	56·1
	21	136 49·1	·· 15·7	12 30·7	11·4	14 32·3	6·6	56·1
	22	151 49·3	14·8	27 01·1	11·5	14 25·7	6·6	56·1
	23	166 49·6	13·8	41 31·6	11·5	14 19·1	6·7	56·0
	23 00	181 49·8	N 0 12·8	56 02·1	11·6	S14 12·4	6·8	56·0
	01	196 50·0	11·8	70 32·7	11·6	14 05·6	6·8	56·0
	02	211 50·2	10·9	85 03·3	11·7	13 58·8	6·9	56·0
	03	226 50·4	·· 09·9	99 34·0	11·7	13 51·9	7·0	55·9
	04	241 50·7	08·9	114 04·7	11·8	13 44·9	7·0	55·9
	05	256 50·9	08·0	128 35·5	11·9	13 37·9	7·1	55·9
	06	271 51·1	N 0 07·0	143 06·4	11·9	S13 30·8	7·1	55·9
	07	286 51·3	06·0	157 37·3	11·9	13 23·7	7·2	55·8
T	08	301 51·5	05·0	172 08·2	12·1	13 16·5	7·3	55·8
U	09	316 51·7	·· 04·1	186 39·3	12·0	13 09·2	7·3	55·8
E	10	331 52·0	03·1	201 10·3	12·2	13 01·9	7·4	55·8
S	11	346 52·2	02·1	215 41·5	12·1	12 54·5	7·4	55·8
D	12	1 52·4	N 0 01·1	230 12·6	12·3	S12 47·1	7·5	55·7
A	13	16 52·6	N 0 00·2	244 43·9	12·3	12 39·6	7·5	55·7
Y	14	31 52·8	S 0 00·8	259 15·2	12·3	12 32·1	7·6	55·7
	15	46 53·1	·· 01·8	273 46·5	12·4	12 24·5	7·7	55·7
	16	61 53·3	02·8	288 17·9	12·4	12 16·8	7·7	55·6
	17	76 53·5	03·7	302 49·3	12·5	12 09·1	7·7	55·6
	18	91 53·7	S 0 04·7	317 20·8	12·6	S12 01·4	7·8	55·6
	19	106 53·9	05·7	331 52·4	12·5	11 53·6	7·9	55·6
	20	121 54·2	06·6	346 23·9	12·7	11 45·7	7·9	55·6
	21	136 54·4	·· 07·6	0 55·6	12·7	11 37·8	8·0	55·5
	22	151 54·6	08·6	15 27·3	12·7	11 29·8	8·0	55·5
	23	166 54·8	09·6	29 59·0	12·8	11 21·8	8·0	55·5
	24 00	181 55·0	S 0 10·5	44 30·8	12·9	S11 13·8	8·1	55·5
	01	196 55·2	11·5	59 02·7	12·9	11 05·7	8·2	55·5
	02	211 55·5	12·5	73 34·6	12·9	10 57·5	8·2	55·4
	03	226 55·7	·· 13·5	88 06·5	13·0	10 49·3	8·2	55·4
	04	241 55·9	14·4	102 38·5	13·0	10 41·1	8·3	55·4
	05	256 56·1	15·4	117 10·5	13·1	10 32·8	8·3	55·4
	06	271 56·3	S 0 16·4	131 42·6	13·1	S10 24·5	8·4	55·4
W	07	286 56·6	17·4	146 14·7	13·2	10 16·1	8·4	55·3
E	08	301 56·8	18·3	160 46·9	13·2	10 07·7	8·4	55·3
D	09	316 57·0	·· 19·3	175 19·1	13·3	9 59·3	8·5	55·3
N	10	331 57·2	20·3	189 51·4	13·3	9 50·8	8·5	55·3
E	11	346 57·4	21·3	204 23·7	13·3	9 42·3	8·6	55·3
S	12	1 57·6	S 0 22·2	218 56·0	13·4	S 9 33·7	8·6	55·2
D	13	16 57·9	23·2	233 28·4	13·4	9 25·1	8·6	55·2
A	14	31 58·1	24·2	248 00·8	13·5	9 16·5	8·7	55·2
Y	15	46 58·3	·· 25·1	262 33·3	13·5	9 07·8	8·7	55·2
	16	61 58·5	26·1	277 05·8	13·6	8 59·1	8·8	55·2
	17	76 58·7	27·1	291 38·4	13·6	8 50·3	8·8	55·1
	18	91 59·0	S 0 28·1	306 11·0	13·6	S 8 41·5	8·8	55·1
	19	106 59·2	29·0	320 43·6	13·7	8 32·7	8·8	55·1
	20	121 59·4	30·0	335 16·3	13·7	8 23·9	8·9	55·1
	21	136 59·6	·· 31·0	349 49·0	13·8	8 15·0	8·9	55·1
	22	151 59·8	32·0	4 21·8	13·8	8 06·1	9·0	55·1
	23	167 00·0	32·9	18 54·6	13·8	7 57·1	8·9	55·0
		S.D. 16·0	d 1·0	S.D.	15·3	15·2		15·1

Lat. °	Twilight Naut. h m	Twilight Civil h m	Sunrise h m	Moonrise 22 h m	Moonrise 23 h m	Moonrise 24 h m	Moonrise 25 h m
N 72	03 01	04 32	05 41	17 46	17 37	17 29	17 23
N 70	03 21	04 40	05 42	17 13	17 16	17 16	17 16
68	03 36	04 47	05 43	16 49	16 59	17 05	17 10
66	03 48	04 52	05 44	16 31	16 46	16 56	17 05
64	03 58	04 57	05 44	16 16	16 34	16 49	17 01
62	04 07	05 01	05 45	16 03	16 25	16 42	16 57
60	04 14	05 04	05 45	15 52	16 17	16 37	16 54
N 58	04 20	05 07	05 46	15 43	16 09	16 32	16 51
56	04 25	05 09	05 46	15 35	16 03	16 27	16 48
54	04 29	05 11	05 46	15 27	15 57	16 23	16 46
52	04 33	05 13	05 47	15 21	15 52	16 19	16 44
50	04 37	05 15	05 47	15 15	15 47	16 16	16 42
45	04 44	05 18	05 48	15 02	15 37	16 09	16 38
N 40	04 49	05 21	05 48	14 51	15 29	16 03	16 34
35	04 53	05 23	05 48	14 42	15 21	15 58	16 31
30	04 57	05 25	05 48	14 34	15 15	15 53	16 28
20	05 01	05 27	05 49	14 20	15 04	15 45	16 24
N 10	05 04	05 28	05 49	14 08	14 54	15 38	16 20
0	05 05	05 29	05 49	13 56	14 45	15 31	16 16
S 10	05 04	05 28	05 49	13 45	14 36	15 25	16 12
20	05 02	05 27	05 49	13 33	14 26	15 17	16 08
30	04 57	05 25	05 49	13 19	14 15	15 09	16 03
35	04 54	05 24	05 49	13 11	14 08	15 05	16 00
40	04 50	05 22	05 49	13 01	14 01	14 59	15 57
45	04 45	05 19	05 48	12 51	13 52	14 53	15 54
S 50	04 38	05 16	05 48	12 37	13 41	14 46	15 49
52	04 34	05 14	05 48	12 31	13 37	14 42	15 47
54	04 31	05 12	05 47	12 25	13 31	14 39	15 45
56	04 26	05 10	05 47	12 17	13 25	14 34	15 43
58	04 21	05 08	05 47	12 08	13 19	14 30	15 40
S 60	04 15	05 05	05 47	11 59	13 11	14 24	15 37

Lat. °	Sunset h m	Twilight Civil h m	Twilight Naut. h m	Moonset 22 h m	Moonset 23 h m	Moonset 24 h m	Moonset 25 h m
N 72	18 02	19 10	20 39	22 43	24 32	00 32	02 14
N 70	18 01	19 03	20 21	23 15	24 51	00 51	02 25
68	18 01	18 56	20 06	23 38	25 07	01 07	02 35
66	18 00	18 51	19 54	23 55	25 19	01 19	02 42
64	17 59	18 47	19 45	24 10	00 10	01 30	02 49
62	17 59	18 43	19 37	24 22	00 22	01 38	02 54
60	17 58	18 40	19 30	24 32	00 32	01 46	02 59
N 58	17 58	18 38	19 24	24 41	00 41	01 52	03 03
56	17 58	18 35	19 19	24 49	00 49	01 58	03 07
54	17 58	18 33	19 15	24 56	00 56	02 03	03 10
52	17 57	18 31	19 11	25 02	01 02	02 08	03 14
50	17 57	18 29	19 07	00 04	01 08	02 12	03 16
45	17 57	18 26	19 00	00 18	01 20	02 21	03 22
N 40	17 56	18 23	18 55	00 30	01 30	02 29	03 27
35	17 56	18 21	18 51	00 41	01 38	02 35	03 32
30	17 56	18 20	18 48	00 49	01 46	02 41	03 35
20	17 56	18 18	18 43	01 05	01 58	02 51	03 42
N 10	17 56	18 17	18 41	01 18	02 10	02 59	03 47
0	17 56	18 16	18 40	01 30	02 20	03 07	03 53
S 10	17 56	18 17	18 41	01 43	02 30	03 15	03 58
20	17 56	18 18	18 44	01 56	02 41	03 24	04 04
30	17 56	18 20	18 48	02 11	02 54	03 33	04 10
35	17 57	18 22	18 52	02 19	03 01	03 39	04 14
40	17 57	18 24	18 56	02 29	03 09	03 45	04 18
45	17 58	18 27	19 01	02 41	03 19	03 52	04 22
S 50	17 58	18 31	19 08	02 54	03 30	04 01	04 28
52	17 58	18 32	19 12	03 01	03 35	04 05	04 31
54	17 59	18 34	19 16	03 08	03 41	04 09	04 33
56	17 59	18 36	19 21	03 16	03 48	04 14	04 36
58	17 59	18 39	19 26	03 25	03 55	04 19	04 40
S 60	18 00	18 41	19 32	03 35	04 03	04 25	04 44

Day	SUN Eqn. of Time 00h m s	SUN Eqn. of Time 12h m s	SUN Mer. Pass. h m	MOON Mer. Pass. Upper h m	MOON Mer. Pass. Lower h m	MOON Age d	Phase
22	06 58	07 08	11 53	20 08	07 43	09	
23	07 19	07 29	11 53	20 56	08 32	10	◐
24	07 40	07 50	11 52	21 42	09 19	11	

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G.M.T. d	h	ARIES G.H.A.	VENUS −3·4 G.H.A.	VENUS Dec.	MARS −0·6 G.H.A.	MARS Dec.	JUPITER −1·4 G.H.A.	JUPITER Dec.	SATURN +0·7 G.H.A.	SATURN Dec.	STARS Name	S.H.A.	Dec.
		° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′		° ′	° ′
30	00	97 55·7	166 37·2	S23 08·1	53 46·2	N18 43·8	226 27·9	S17 46·0	188 45·4	S22 29·1	Acamar	315 49·2	S 40 28·4
	01	112 58·2	181 36·3	07·7	68 48·4	44·0	241 29·9	46·1	203 47·5	29·1	Achernar	335 57·1	S 57 27·0
	02	128 00·6	196 35·3	07·3	83 50·6	44·1	256 31·9	46·2	218 49·7	29·1	Acrux	173 55·0	S 62 52·1
	03	143 03·1	211 34·4	·· 06·9	98 52·7	·· 44·2	271 33·9	·· 46·4	233 51·8	·· 29·1	Adhara	255 44·3	S 28 55·1
	04	158 05·6	226 33·5	06·5	113 54·9	44·3	286 35·9	46·5	248 54·0	29·1	Aldebaran	291 36·1	N 16 25·6
	05	173 08·0	241 32·6	06·1	128 57·1	44·4	301 37·9	46·6	263 56·1	29·1			
	06	188 10·5	256 31·6	S23 05·8	143 59·2	N18 44·5	316 39·9	S17 46·7	278 58·3	S22 29·1	Alioth	166 56·7	N 56 10·6
	07	203 13·0	271 30·7	05·4	159 01·4	44·7	331 41·9	46·8	294 00·4	29·1	Alkaid	153 31·4	N 49 30·8
T	08	218 15·4	286 29·8	05·0	174 03·6	44·8	346 43·9	46·9	309 02·5	29·1	Al Na'ir	28 35·5	S 47 09·7
U	09	233 17·9	301 28·9	·· 04·6	189 05·7	·· 44·9	1 45·9	·· 47·0	324 04·7	·· 29·1	Alnilam	276 27·6	S 1 13·8
E	10	248 20·4	316 28·0	04·2	204 07·9	45·0	16 47·8	47·1	339 06·8	29·1	Alphard	218 36·1	S 8 28·9
S	11	263 22·8	331 27·0	03·8	219 10·0	45·1	31 49·8	47·3	354 09·0	29·1			
D	12	278 25·3	346 26·1	S23 03·4	234 12·2	N18 45·3	46 51·8	S17 47·4	9 11·1	S22 29·1	Alphecca	126 46·0	N 26 51·1
A	13	293 27·7	1 25·2	03·0	249 14·4	45·4	61 53·8	47·5	24 13·3	29·1	Alpheratz	358 25·9	N 28 52·0
Y	14	308 30·2	16 24·3	02·6	264 16·5	45·5	76 55·8	47·6	39 15·4	29·1	Altair	62 48·5	N 8 45·7
	15	323 32·7	31 23·4	·· 02·2	279 18·7	·· 45·6	91 57·8	·· 47·7	54 17·6	·· 29·1	Ankaa	353 56·2	S 42 31·9
	16	338 35·1	46 22·5	01·8	294 20·8	45·7	106 59·8	47·8	69 19·7	29·1	Antares	113 16·8	S 26 20·4
	17	353 37·6	61 21·5	01·4	309 23·0	45·9	122 01·8	47·9	84 21·9	29·1			
	18	8 40·1	76 20·6	S23 01·0	324 25·1	N18 46·0	137 03·8	S17 48·0	99 24·0	S22 29·1	Arcturus	146 33·3	N 19 23·6
	19	23 42·5	91 19·7	00·6	339 27·3	46·1	152 05·8	48·2	114 26·2	29·1	Atria	108 55·9	S 68 57·0
	20	38 45·0	106 18·8	23 00·2	354 29·4	46·2	167 07·8	48·3	129 28·3	29·1	Avior	234 34·1	S 59 22·6
	21	53 47·5	121 17·9	22 59·7	9 31·6	·· 46·3	182 09·8	·· 48·4	144 30·5	·· 29·1	Bellatrix	279 15·6	N 6 18·7
	22	68 49·9	136 17·0	59·3	24 33·7	46·5	197 11·7	48·5	159 32·6	29·1	Betelgeuse	271 45·3	N 7 23·9
	23	83 52·4	151 16·0	58·9	39 35·9	46·6	212 13·7	48·6	174 34·8	29·1			
31	00	98 54·9	166 15·1	S22 58·5	54 38·0	N18 46·7	227 15·7	S17 48·7	189 36·9	S22 29·2	Canopus	264 13·8	S 52 40·5
	01	113 57·3	181 14·2	58·1	69 40·1	46·8	242 17·7	48·8	204 39·0	29·2	Capella	281 34·5	N 45 57·4
	02	128 59·8	196 13·3	57·7	84 42·3	46·9	257 19·7	48·9	219 41·2	29·2	Deneb	49 59·8	N 45 08·2
	03	144 02·2	211 12·4	·· 57·3	99 44·4	·· 47·1	272 21·7	·· 49·1	234 43·3	·· 29·2	Denebola	183 15·3	N 14 47·9
	04	159 04·7	226 11·5	56·8	114 46·6	47·2	287 23·7	49·2	249 45·5	29·2	Diphda	349 37·0	S 18 12·8
	05	174 07·2	241 10·6	56·4	129 48·7	47·3	302 25·7	49·3	264 47·6	29·2			
	06	189 09·6	256 09·6	S22 56·0	144 50·8	N18 47·4	317 27·7	S17 49·4	279 49·8	S22 29·2	Dubhe	194 41·7	N 61 58·0
W	07	204 12·1	271 08·7	55·6	159 53·0	47·6	332 29·7	49·5	294 51·9	29·2	Elnath	279 04·0	N 28 34·4
E	08	219 14·6	286 07·8	55·2	174 55·1	47·7	347 31·7	49·6	309 54·1	29·2	Eltanin	91 05·7	N 51 29·7
D	09	234 17·0	301 06·9	·· 54·7	189 57·2	·· 47·8	2 33·7	·· 49·7	324 56·2	·· 29·2	Enif	34 27·6	N 9 41·3
N	10	249 19·5	316 06·0	54·3	204 59·4	47·9	17 35·7	49·8	339 58·4	29·2	Fomalhaut	16 09·3	S 29 50·5
E	11	264 22·0	331 05·1	53·9	220 01·5	48·1	32 37·7	50·0	355 00·5	29·2			
S	12	279 24·4	346 04·2	S22 53·5	235 03·6	N18 48·2	47 39·7	S17 50·1	10 02·7	S22 29·2	Gacrux	172 46·5	S 56 52·8
D	13	294 26·9	1 03·3	53·0	250 05·8	48·3	62 41·6	50·2	25 04·8	29·2	Gienah	176 34·4	S 17 18·8
A	14	309 29·4	16 02·3	52·6	265 07·9	48·4	77 43·6	50·3	40 07·0	29·2	Hadar	149 46·2	S 60 10·3
Y	15	324 31·8	31 01·4	·· 52·2	280 10·0	·· 48·6	92 45·6	·· 50·4	55 09·1	·· 29·2	Hamal	328 46·9	N 23 16·2
	16	339 34·3	46 00·5	51·7	295 12·1	48·7	107 47·6	50·5	70 11·3	29·2	Kaus Aust.	84 38·5	S 34 24·2
	17	354 36·7	60 59·6	51·3	310 14·3	48·8	122 49·6	50·6	85 13·4	29·2			
	18	9 39·2	75 58·7	S22 50·9	325 16·4	N18 48·9	137 51·6	S17 50·7	100 15·6	S22 29·2	Kochab	137 19·1	N 74 19·2
	19	24 41·7	90 57·8	50·4	340 18·5	49·1	152 53·6	50·8	115 17·7	29·2	Markab	14 19·3	N 14 59·2
	20	39 44·1	105 56·9	50·0	355 20·6	49·2	167 55·6	51·0	130 19·8	29·2	Menkar	314 57·7	N 3 55·7
	21	54 46·6	120 56·0	·· 49·6	10 22·7	·· 49·3	182 57·6	·· 51·1	145 22·0	·· 29·2	Menkent	148 56·0	S 36 09·9
	22	69 49·1	135 55·1	49·1	25 24·9	49·5	197 59·6	51·2	160 24·1	29·2	Miaplacidus	221 47·5	S 69 32·8
	23	84 51·5	150 54·2	48·7	40 27·0	49·6	213 01·6	51·3	175 26·3	29·2			
1	00	99 54·0	165 53·3	S22 48·2	55 29·1	N18 49·7	228 03·6	S17 51·4	190 28·4	S22 29·2	Mirfak	309 38·6	N 49 43·1
	01	114 56·5	180 52·4	47·8	70 31·2	49·8	243 05·6	51·5	205 30·6	29·2	Nunki	76 49·4	S 26 20·8
	02	129 58·9	195 51·4	47·3	85 33·3	50·0	258 07·6	51·6	220 32·7	29·2	Peacock	54 24·3	S 56 52·1
	03	145 01·4	210 50·5	·· 46·9	100 35·4	·· 50·1	273 09·6	·· 51·7	235 34·9	·· 29·2	Pollux	244 17·4	N 28 07·4
	04	160 03·9	225 49·6	46·5	115 37·5	50·2	288 11·6	51·8	250 37·0	29·2	Procyon	245 42·3	N 5 19·7
	05	175 06·3	240 48·7	46·0	130 39·7	50·4	303 13·6	51·9	265 39·2	29·2			
	06	190 08·8	255 47·8	S22 45·6	145 41·8	N18 50·5	318 15·6	S17 52·1	280 41·3	S22 29·3	Rasalhague	96 44·8	N 12 35·4
	07	205 11·2	270 46·9	45·1	160 43·9	50·6	333 17·6	52·2	295 43·5	29·3	Regulus	208 26·9	N 12 09·9
T	08	220 13·7	285 46·0	44·7	175 46·0	50·7	348 19·6	52·3	310 45·6	29·3	Rigel	281 51·1	S 8 15·0
H	09	235 16·2	300 45·1	·· 44·2	190 48·1	·· 50·9	3 21·6	·· 52·4	325 47·8	·· 29·3	Rigil Kent.	140 48·0	S 60 39·7
U	10	250 18·6	315 44·2	43·7	205 50·2	51·0	18 23·6	52·5	340 49·9	29·3	Sabik	102 59·9	S 15 40·4
R	11	265 21·1	330 43·3	43·3	220 52·3	51·1	33 25·6	52·6	355 52·1	29·3			
S	12	280 23·6	345 42·4	S22 42·8	235 54·4	N18 51·3	48 27·6	S17 52·7	10 54·2	S22 29·3	Schedar	350 27·2	N 56 19·0
D	13	295 26·0	0 41·5	42·4	250 56·5	51·4	63 29·6	52·8	25 56·4	29·3	Shaula	97 18·0	S 37 04·4
A	14	310 28·5	15 40·6	41·9	265 58·6	51·5	78 31·5	52·9	40 58·5	29·3	Sirius	259 09·5	S 16 39·7
Y	15	325 31·0	30 39·7	·· 41·5	281 00·7	·· 51·7	93 33·5	·· 53·0	56 00·7	·· 29·3	Spica	159 14·5	S 10 56·8
	16	340 33·4	45 38·8	41·0	296 02·8	51·8	108 35·5	53·2	71 02·8	29·3	Suhail	223 22·2	S 43 16·0
	17	355 35·9	60 37·9	40·5	311 04·9	51·9	123 37·5	53·3	86 05·0	29·3			
	18	10 38·3	75 37·0	S22 40·1	326 07·0	N18 52·1	138 39·5	S17 53·4	101 07·1	S22 29·3	Vega	81 07·1	N 38 44·8
	19	25 40·8	90 36·1	39·6	341 09·1	52·2	153 41·5	53·5	116 09·3	29·3	Zuben'ubi	137 51·0	S 15 52·2
	20	40 43·3	105 35·2	39·1	356 11·2	52·3	168 43·5	53·6	131 11·4	29·3			
	21	55 45·7	120 34·3	·· 38·7	11 13·2	·· 52·5	183 45·5	·· 53·7	146 13·6	·· 29·3			
	22	70 48·2	135 33·4	38·2	26 15·3	52·6	198 47·5	53·8	161 15·7	29·3			
	23	85 50·7	150 32·5	37·7	41 17·4	52·7	213 49·5	53·9	176 17·9	29·3			
Mer. Pass.		17 21·5 (h m)	v −0·9	d 0·4	v 2·1	d 0·1	v 2·0	d 0·1	v 2·1	d 0·0			

	S.H.A. ° ′	Mer. Pass. h m
Venus	67 20·2	12 56
Mars	315 43·1	20 19
Jupiter	128 20·8	8 50
Saturn	90 42·0	11 20

DEC. 30, 31, 1959 JAN. 1 (TUES., WED., THURS.)

G.M.T. d	h	SUN G.H.A.	SUN Dec.	MOON G.H.A.	v	Dec.	d	H.P.
		° ′	° ′	° ′	′	° ′	′	′
30	00	179 27·3	S23 12·8	312 58·5	10·8	N 9 39·0	9·3	57·6
	01	194 27·0	12·7	327 28·3	10·8	9 29·7	9·4	57·6
	02	209 26·7	12·5	341 58·1	10·9	9 20·3	9·5	57·6
	03	224 26·4	·· 12·4	356 28·0	10·8	9 10·8	9·5	57·6
	04	239 26·1	12·2	10 57·8	10·9	9 01·3	9·5	57·6
	05	254 25·8	12·1	25 27·7	10·8	8 51·8	9·6	57·7
	06	269 25·5	S23 11·9	39 57·5	10·9	N 8 42·2	9·7	57·7
	07	284 25·2	11·7	54 27·4	10·9	8 32·5	9·7	57·7
T	08	299 24·9	11·6	68 57·3	10·9	8 22·8	9·8	57·7
U	09	314 24·6	·· 11·4	83 27·2	10·9	8 13·0	9·8	57·7
E	10	329 24·3	11·3	97 57·1	10·9	8 03·2	9·9	57·7
S	11	344 24·0	11·1	112 27·0	10·9	7 53·3	9·9	57·8
D	12	359 23·7	S23 11·0	126 56·9	10·9	N 7 43·4	9·9	57·8
A	13	14 23·4	10·8	141 26·8	11·0	7 33·5	10·0	57·8
Y	14	29 23·1	10·6	155 56·8	10·9	7 23·5	10·1	57·8
	15	44 22·8	·· 10·5	170 26·7	10·9	7 13·4	10·1	57·8
	16	59 22·5	10·3	184 56·6	11·0	7 03·3	10·1	57·9
	17	74 22·2	10·2	199 26·6	10·9	6 53·2	10·2	57·9
	18	89 21·9	S23 10·0	213 56·5	11·0	N 6 43·0	10·2	57·9
	19	104 21·6	09·8	228 26·5	10·9	6 32·8	10·2	57·9
	20	119 21·3	09·7	242 56·4	11·0	6 22·6	10·3	57·9
	21	134 21·0	·· 09·5	257 26·4	11·0	6 12·3	10·4	58·0
	22	149 20·7	09·3	271 56·4	10·9	6 01·9	10·3	58·0
	23	164 20·4	09·2	286 26·3	11·0	5 51·6	10·4	58·0
31	00	179 20·1	S23 09·0	300 56·3	11·0	N 5 41·2	10·5	58·0
	01	194 19·8	08·8	315 26·3	10·9	5 30·7	10·5	58·0
	02	209 19·5	08·7	329 56·2	11·0	5 20·2	10·5	58·0
	03	224 19·2	·· 08·5	344 26·2	11·0	5 09·7	10·5	58·1
	04	239 18·9	08·3	358 56·2	11·0	4 59·2	10·6	58·1
	05	254 18·6	08·1	13 26·2	10·9	4 48·6	10·6	58·1
	06	269 18·3	S23 08·0	27 56·1	11·0	N 4 38·0	10·7	58·1
W	07	284 18·0	07·8	42 26·1	10·9	4 27·3	10·6	58·1
E	08	299 17·7	07·6	56 56·0	11·0	4 16·7	10·7	58·2
D	09	314 17·4	·· 07·5	71 26·0	11·0	4 06·0	10·8	58·2
N	10	329 17·1	07·3	85 56·0	10·9	3 55·2	10·7	58·2
E	11	344 16·8	07·1	100 25·9	11·0	3 44·5	10·8	58·2
S	12	359 16·5	S23 06·9	114 55·9	10·9	N 3 33·7	10·8	58·2
D	13	14 16·2	06·7	129 25·8	10·9	3 22·9	10·8	58·2
A	14	29 15·9	06·6	143 55·7	11·0	3 12·1	10·9	58·3
Y	15	44 15·6	·· 06·4	158 25·7	10·9	3 01·2	10·8	58·3
	16	59 15·3	06·2	172 55·6	10·9	2 50·4	10·9	58·3
	17	74 15·0	06·0	187 25·5	10·9	2 39·5	11·0	58·3
	18	89 14·7	S23 05·8	201 55·4	10·9	N 2 28·5	10·9	58·3
	19	104 14·4	05·7	216 25·3	10·9	2 17·6	10·9	58·4
	20	119 14·1	05·5	230 55·2	10·9	2 06·7	11·0	58·4
	21	134 13·8	·· 05·3	245 25·1	10·8	1 55·7	11·0	58·4
	22	149 13·5	05·1	259 54·9	10·9	1 44·7	11·0	58·4
	23	164 13·2	04·9	274 24·8	10·8	1 33·7	11·0	58·4
1	00	179 12·9	S23 04·7	288 54·6	10·9	N 1 22·7	11·0	58·4
	01	194 12·6	04·5	303 24·5	10·8	1 11·7	11·1	58·5
	02	209 12·3	04·3	317 54·3	10·8	1 00·6	11·0	58·5
	03	224 12·0	·· 04·2	332 24·1	10·8	0 49·6	11·1	58·5
	04	239 11·7	04·0	346 53·9	10·7	0 38·5	11·1	58·5
	05	254 11·4	03·8	1 23·6	10·8	0 27·4	11·1	58·5
	06	269 11·1	S23 03·6	15 53·4	10·7	N 0 16·3	11·0	58·5
	07	284 10·8	03·4	30 23·1	10·7	N 0 05·3	11·1	58·6
T	08	299 10·5	03·2	44 52·8	10·7	S 0 05·8	11·1	58·6
H	09	314 10·2	·· 03·0	59 22·5	10·7	0 16·9	11·1	58·6
U	10	329 09·9	02·8	73 52·2	10·7	0 28·0	11·1	58·6
R	11	344 09·6	02·6	88 21·9	10·6	0 39·1	11·2	58·6
S	12	359 09·3	S23 02·4	102 51·5	10·6	S 0 50·3	11·1	58·6
D	13	14 09·0	02·2	117 21·1	10·6	1 01·4	11·1	58·7
A	14	29 08·7	02·0	131 50·7	10·6	1 12·5	11·1	58·7
Y	15	44 08·4	·· 01·8	146 20·3	10·6	1 23·6	11·1	58·7
	16	59 08·1	01·6	160 49·9	10·5	1 34·7	11·1	58·7
	17	74 07·8	01·4	175 19·4	10·5	1 45·8	11·1	58·7
	18	89 07·5	S23 01·2	189 48·9	10·5	S 1 56·9	11·1	58·7
	19	104 07·3	01·0	204 18·4	10·5	2 08·0	11·1	58·8
	20	119 07·0	00·8	218 47·9	10·4	2 19·1	11·1	58·8
	21	134 06·7	·· 00·6	233 17·3	10·4	2 30·2	11·1	58·8
	22	149 06·4	00·4	247 46·7	10·4	2 41·3	11·0	58·8
	23	164 06·1	00·2	262 16·1	10·3	2 52·3	11·1	58·8
		S.D. 16·3	d 0·2	S.D.	15·7	15·9		16·0

Lat.	Twilight Naut.	Twilight Civil	Sunrise	Moonrise 30	Moonrise 31	Moonrise 1	Moonrise 2
°	h m	h m	h m	h m	h m	h m	h m
N 72	08 25	10 46	▬	20 32	22 22	24 14	00 14
N 70	08 06	09 51	▬	20 41	22 25	24 09	00 09
68	07 50	09 18	▬	20 49	22 26	24 05	00 05
66	07 38	08 54	10 30	20 55	22 28	24 02	00 02
64	07 27	08 34	09 51	21 00	22 29	23 59	25 31
62	07 17	08 19	09 23	21 04	22 30	23 57	25 25
60	07 09	08 06	09 03	21 08	22 31	23 55	25 20
N 58	07 02	07 54	08 46	21 12	22 32	23 53	25 16
56	06 55	07 44	08 32	21 15	22 33	23 52	25 12
54	06 49	07 36	08 19	21 18	22 33	23 50	25 08
52	06 44	07 28	08 08	21 20	22 34	23 49	25 05
50	06 39	07 20	07 59	21 22	22 35	23 48	25 02
45	06 27	07 05	07 38	21 27	22 36	23 45	24 56
N 40	06 17	06 51	07 22	21 31	22 37	23 43	24 51
35	06 08	06 40	07 08	21 35	22 38	23 42	24 46
30	05 59	06 29	06 55	21 38	22 39	23 40	24 42
20	05 43	06 11	06 35	21 43	22 40	23 37	24 36
N 10	05 27	05 54	06 16	21 48	22 41	23 35	24 30
0	05 11	05 37	05 59	21 53	22 42	23 33	24 24
S 10	04 52	05 19	05 42	21 57	22 44	23 31	24 19
20	04 29	04 59	05 23	22 02	22 45	23 28	24 13
30	04 01	04 34	05 01	22 07	22 46	23 26	24 07
35	03 42	04 19	04 48	22 10	22 47	23 24	24 03
40	03 19	04 01	04 34	22 14	22 48	23 23	23 59
45	02 49	03 39	04 16	22 18	22 49	23 21	23 54
S 50	02 05	03 10	03 54	22 22	22 51	23 19	23 48
52	01 38	02 55	03 43	22 25	22 51	23 17	23 45
54	00 56	02 37	03 31	22 27	22 52	23 16	23 42
56	////	02 16	03 17	22 30	22 53	23 15	23 39
58	////	01 47	03 01	22 33	22 53	23 14	23 36
S 60	////	01 01	02 41	22 36	22 54	23 12	23 32

Lat.	Sunset	Twilight Civil	Twilight Naut.	Moonset 30	Moonset 31	Moonset 1	Moonset 2
°	h m	h m	h m	h m	h m	h m	h m
N 72	▬	13 19	15 41	11 09	11 03	10 57	10 50
N 70	▬	14 14	16 00	10 58	10 58	10 58	10 57
68	▬	14 47	16 15	10 49	10 54	10 59	11 03
66	13 35	15 12	16 28	10 41	10 51	10 59	11 08
64	14 15	15 31	16 39	10 35	10 48	11 00	11 12
62	14 42	15 46	16 48	10 29	10 45	11 00	11 16
60	15 03	16 00	16 56	10 24	10 43	11 01	11 19
N 58	15 20	16 11	17 03	10 20	10 41	11 01	11 22
56	15 34	16 21	17 10	10 16	10 39	11 02	11 25
54	15 47	16 30	17 16	10 12	10 38	11 02	11 27
52	15 58	16 39	17 22	10 09	10 36	11 02	11 29
50	16 07	16 46	17 27	10 06	10 35	11 03	11 31
45	16 28	17 01	17 39	10 00	10 32	11 03	11 35
N 40	16 44	17 15	17 49	09 55	10 29	11 04	11 39
35	16 58	17 26	17 58	09 50	10 27	11 04	11 42
30	17 10	17 37	18 06	09 46	10 25	11 05	11 44
20	17 31	17 55	18 23	09 39	10 22	11 05	11 49
N 10	17 50	18 12	18 39	09 33	10 19	11 06	11 53
0	18 07	18 29	18 55	09 27	10 16	11 06	11 57
S 10	18 24	18 47	19 14	09 21	10 13	11 07	12 01
20	18 43	19 07	19 36	09 14	10 10	11 07	12 05
30	19 04	19 32	20 05	09 07	10 07	11 08	12 09
35	19 17	19 47	20 23	09 02	10 05	11 08	12 12
40	19 32	20 05	20 46	08 58	10 02	11 08	12 15
45	19 50	20 27	21 16	08 52	10 00	11 09	12 19
S 50	20 12	20 56	22 00	08 45	09 56	11 09	12 23
52	20 22	21 11	22 26	08 42	09 55	11 10	12 25
54	20 34	21 28	23 07	08 38	09 53	11 10	12 27
56	20 48	21 49	////	08 34	09 51	11 10	12 29
58	21 05	22 18	////	08 30	09 49	11 10	12 32
S 60	21 24	23 03	////	08 25	09 47	11 11	12 35

Day	SUN Eqn. of Time 00h	SUN Eqn. of Time 12h	SUN Mer. Pass.	MOON Mer. Pass. Upper	MOON Mer. Pass. Lower	Age	Phase
	m s	m s	h m	h m	h m	d	
30	02 10	02 25	12 02	03 15	15 39	20	◑
31	02 39	02 53	12 03	04 04	16 29	21	
1	03 08	03 22	12 03	04 54	17 19	22	

36ᵐ INCREMENTS AND CORRECTIONS 37ᵐ

36ᵐ	SUN PLANETS	ARIES	MOON	v or d	Corrⁿ	v or d	Corrⁿ	v or d	Corrⁿ
s	° ′	° ′	° ′	′	′	′	′	′	′
00	9 00·0	9 01·5	8 35·4	0·0	0·0	6·0	3·7	12·0	7·3
01	9 00·3	9 01·7	8 35·6	0·1	0·1	6·1	3·7	12·1	7·4
02	9 00·5	9 02·0	8 35·9	0·2	0·1	6·2	3·8	12·2	7·4
03	9 00·8	9 02·2	8 36·1	0·3	0·2	6·3	3·8	12·3	7·5
04	9 01·0	9 02·5	8 36·4	0·4	0·2	6·4	3·9	12·4	7·5
05	9 01·3	9 02·7	8 36·6	0·5	0·3	6·5	4·0	12·5	7·6
06	9 01·5	9 03·0	8 36·8	0·6	0·4	6·6	4·0	12·6	7·7
07	9 01·8	9 03·2	8 37·1	0·7	0·4	6·7	4·1	12·7	7·7
08	9 02·0	9 03·5	8 37·3	0·8	0·5	6·8	4·1	12·8	7·8
09	9 02·3	9 03·7	8 37·5	0·9	0·5	6·9	4·2	12·9	7·8
10	9 02·5	9 04·0	8 37·8	1·0	0·6	7·0	4·3	13·0	7·9
11	9 02·8	9 04·2	8 38·0	1·1	0·7	7·1	4·3	13·1	8·0
12	9 03·0	9 04·5	8 38·3	1·2	0·7	7·2	4·4	13·2	8·0
13	9 03·3	9 04·7	8 38·5	1·3	0·8	7·3	4·4	13·3	8·1
14	9 03·5	9 05·0	8 38·7	1·4	0·9	7·4	4·5	13·4	8·2
15	9 03·8	9 05·2	8 39·0	1·5	0·9	7·5	4·6	13·5	8·2
16	9 04·0	9 05·5	8 39·2	1·6	1·0	7·6	4·6	13·6	8·3
17	9 04·3	9 05·7	8 39·5	1·7	1·0	7·7	4·7	13·7	8·3
18	9 04·5	9 06·0	8 39·7	1·8	1·1	7·8	4·7	13·8	8·4
19	9 04·8	9 06·2	8 39·9	1·9	1·2	7·9	4·8	13·9	8·5
20	9 05·0	9 06·5	8 40·2	2·0	1·2	8·0	4·9	14·0	8·5
21	9 05·3	9 06·7	8 40·4	2·1	1·3	8·1	4·9	14·1	8·6
22	9 05·5	9 07·0	8 40·6	2·2	1·3	8·2	5·0	14·2	8·6
23	9 05·8	9 07·2	8 40·9	2·3	1·4	8·3	5·0	14·3	8·7
24	9 06·0	9 07·5	8 41·1	2·4	1·5	8·4	5·1	14·4	8·8
25	9 06·3	9 07·7	8 41·4	2·5	1·5	8·5	5·2	14·5	8·8
26	9 06·5	9 08·0	8 41·6	2·6	1·6	8·6	5·2	14·6	8·9
27	9 06·8	9 08·2	8 41·8	2·7	1·6	8·7	5·3	14·7	8·9
28	9 07·0	9 08·5	8 42·1	2·8	1·7	8·8	5·4	14·8	9·0
29	9 07·3	9 08·7	8 42·3	2·9	1·8	8·9	5·4	14·9	9·1
30	9 07·5	9 09·0	8 42·6	3·0	1·8	9·0	5·5	15·0	9·1
31	9 07·8	9 09·2	8 42·8	3·1	1·9	9·1	5·5	15·1	9·2
32	9 08·0	9 09·5	8 43·0	3·2	1·9	9·2	5·6	15·2	9·2
33	9 08·3	9 09·8	8 43·3	3·3	2·0	9·3	5·7	15·3	9·3
34	9 08·5	9 10·0	8 43·5	3·4	2·1	9·4	5·7	15·4	9·4
35	9 08·8	9 10·3	8 43·8	3·5	2·1	9·5	5·8	15·5	9·4
36	9 09·0	9 10·5	8 44·0	3·6	2·2	9·6	5·8	15·6	9·5
37	9 09·3	9 10·8	8 44·2	3·7	2·3	9·7	5·9	15·7	9·6
38	9 09·5	9 11·0	8 44·5	3·8	2·3	9·8	6·0	15·8	9·6
39	9 09·8	9 11·3	8 44·7	3·9	2·4	9·9	6·0	15·9	9·7
40	9 10·0	9 11·5	8 44·9	4·0	2·4	10·0	6·1	16·0	9·7
41	9 10·3	9 11·8	8 45·2	4·1	2·5	10·1	6·1	16·1	9·8
42	9 10·5	9 12·0	8 45·4	4·2	2·6	10·2	6·2	16·2	9·9
43	9 10·8	9 12·3	8 45·7	4·3	2·6	10·3	6·3	16·3	9·9
44	9 11·0	9 12·5	8 45·9	4·4	2·7	10·4	6·3	16·4	10·0
45	9 11·3	9 12·8	8 46·1	4·5	2·7	10·5	6·4	16·5	10·0
46	9 11·5	9 13·0	8 46·4	4·6	2·8	10·6	6·4	16·6	10·1
47	9 11·8	9 13·3	8 46·6	4·7	2·9	10·7	6·5	16·7	10·2
48	9 12·0	9 13·5	8 46·9	4·8	2·9	10·8	6·6	16·8	10·2
49	9 12·3	9 13·8	8 47·1	4·9	3·0	10·9	6·6	16·9	10·3
50	9 12·5	9 14·0	8 47·3	5·0	3·0	11·0	6·7	17·0	10·3
51	9 12·8	9 14·3	8 47·6	5·1	3·1	11·1	6·8	17·1	10·4
52	9 13·0	9 14·5	8 47·8	5·2	3·2	11·2	6·8	17·2	10·5
53	9 13·3	9 14·8	8 48·0	5·3	3·2	11·3	6·9	17·3	10·5
54	9 13·5	9 15·0	8 48·3	5·4	3·3	11·4	6·9	17·4	10·6
55	9 13·8	9 15·3	8 48·5	5·5	3·3	11·5	7·0	17·5	10·6
56	9 14·0	9 15·5	8 48·8	5·6	3·4	11·6	7·1	17·6	10·7
57	9 14·3	9 15·8	8 49·0	5·7	3·5	11·7	7·1	17·7	10·8
58	9 14·5	9 16·0	8 49·2	5·8	3·5	11·8	7·2	17·8	10·8
59	9 14·8	9 16·3	8 49·5	5·9	3·6	11·9	7·2	17·9	10·9
60	9 15·0	9 16·5	8 49·7	6·0	3·7	12·0	7·3	18·0	11·0

37ᵐ	SUN PLANETS	ARIES	MOON	v or d	Corrⁿ	v or d	Corrⁿ	v or d	Corrⁿ
s	° ′	° ′	° ′	′	′	′	′	′	′
00	9 15·0	9 16·5	8 49·7	0·0	0·0	6·0	3·8	12·0	7·5
01	9 15·3	9 16·8	8 50·0	0·1	0·1	6·1	3·8	12·1	7·6
02	9 15·5	9 17·0	8 50·2	0·2	0·1	6·2	3·9	12·2	7·6
03	9 15·8	9 17·3	8 50·4	0·3	0·2	6·3	3·9	12·3	7·7
04	9 16·0	9 17·5	8 50·7	0·4	0·3	6·4	4·0	12·4	7·8
05	9 16·3	9 17·8	8 50·9	0·5	0·3	6·5	4·1	12·5	7·8
06	9 16·5	9 18·0	8 51·1	0·6	0·4	6·6	4·1	12·6	7·9
07	9 16·8	9 18·3	8 51·4	0·7	0·4	6·7	4·2	12·7	7·9
08	9 17·0	9 18·5	8 51·6	0·8	0·5	6·8	4·3	12·8	8·0
09	9 17·3	9 18·8	8 51·9	0·9	0·6	6·9	4·3	12·9	8·1
10	9 17·5	9 19·0	8 52·1	1·0	0·6	7·0	4·4	13·0	8·1
11	9 17·8	9 19·3	8 52·3	1·1	0·7	7·1	4·4	13·1	8·2
12	9 18·0	9 19·5	8 52·6	1·2	0·8	7·2	4·5	13·2	8·3
13	9 18·3	9 19·8	8 52·8	1·3	0·8	7·3	4·6	13·3	8·3
14	9 18·5	9 20·0	8 53·1	1·4	0·9	7·4	4·6	13·4	8·4
15	9 18·8	9 20·3	8 53·3	1·5	0·9	7·5	4·7	13·5	8·4
16	9 19·0	9 20·5	8 53·5	1·6	1·0	7·6	4·8	13·6	8·5
17	9 19·3	9 20·8	8 53·8	1·7	1·1	7·7	4·8	13·7	8·6
18	9 19·5	9 21·0	8 54·0	1·8	1·1	7·8	4·9	13·8	8·6
19	9 19·8	9 21·3	8 54·3	1·9	1·2	7·9	4·9	13·9	8·7
20	9 20·0	9 21·5	8 54·5	2·0	1·3	8·0	5·0	14·0	8·8
21	9 20·3	9 21·8	8 54·7	2·1	1·3	8·1	5·1	14·1	8·8
22	9 20·5	9 22·0	8 55·0	2·2	1·4	8·2	5·1	14·2	8·9
23	9 20·8	9 22·3	8 55·2	2·3	1·4	8·3	5·2	14·3	8·9
24	9 21·0	9 22·5	8 55·4	2·4	1·5	8·4	5·3	14·4	9·0
25	9 21·3	9 22·8	8 55·7	2·5	1·6	8·5	5·3	14·5	9·1
26	9 21·5	9 23·0	8 55·9	2·6	1·6	8·6	5·4	14·6	9·1
27	9 21·8	9 23·3	8 56·2	2·7	1·7	8·7	5·4	14·7	9·2
28	9 22·0	9 23·5	8 56·4	2·8	1·8	8·8	5·5	14·8	9·3
29	9 22·3	9 23·8	8 56·6	2·9	1·8	8·9	5·6	14·9	9·3
30	9 22·5	9 24·0	8 56·9	3·0	1·9	9·0	5·6	15·0	9·4
31	9 22·8	9 24·3	8 57·1	3·1	1·9	9·1	5·7	15·1	9·4
32	9 23·0	9 24·5	8 57·4	3·2	2·0	9·2	5·8	15·2	9·5
33	9 23·3	9 24·8	8 57·6	3·3	2·1	9·3	5·8	15·3	9·6
34	9 23·5	9 25·0	8 57·8	3·4	2·1	9·4	5·9	15·4	9·6
35	9 23·8	9 25·3	8 58·1	3·5	2·2	9·5	5·9	15·5	9·7
36	9 24·0	9 25·5	8 58·3	3·6	2·3	9·6	6·0	15·6	9·8
37	9 24·3	9 25·8	8 58·5	3·7	2·3	9·7	6·1	15·7	9·8
38	9 24·5	9 26·0	8 58·8	3·8	2·4	9·8	6·1	15·8	9·9
39	9 24·8	9 26·3	8 59·0	3·9	2·4	9·9	6·2	15·9	9·9
40	9 25·0	9 26·5	8 59·3	4·0	2·5	10·0	6·3	16·0	10·0
41	9 25·3	9 26·8	8 59·5	4·1	2·6	10·1	6·3	16·1	10·1
42	9 25·5	9 27·0	8 59·7	4·2	2·6	10·2	6·4	16·2	10·1
43	9 25·8	9 27·3	9 00·0	4·3	2·7	10·3	6·4	16·3	10·2
44	9 26·0	9 27·5	9 00·2	4·4	2·8	10·4	6·5	16·4	10·3
45	9 26·3	9 27·8	9 00·5	4·5	2·8	10·5	6·6	16·5	10·3
46	9 26·5	9 28·1	9 00·7	4·6	2·9	10·6	6·6	16·6	10·4
47	9 26·8	9 28·3	9 00·9	4·7	2·9	10·7	6·7	16·7	10·4
48	9 27·0	9 28·6	9 01·2	4·8	3·0	10·8	6·8	16·8	10·5
49	9 27·3	9 28·8	9 01·4	4·9	3·1	10·9	6·8	16·9	10·6
50	9 27·5	9 29·1	9 01·6	5·0	3·1	11·0	6·9	17·0	10·6
51	9 27·8	9 29·3	9 01·9	5·1	3·2	11·1	6·9	17·1	10·7
52	9 28·0	9 29·6	9 02·1	5·2	3·3	11·2	7·0	17·2	10·8
53	9 28·3	9 29·8	9 02·4	5·3	3·3	11·3	7·1	17·3	10·8
54	9 28·5	9 30·1	9 02·6	5·4	3·4	11·4	7·1	17·4	10·9
55	9 28·8	9 30·3	9 02·8	5·5	3·4	11·5	7·2	17·5	10·9
56	9 29·0	9 30·6	9 03·1	5·6	3·5	11·6	7·3	17·6	11·0
57	9 29·3	9 30·8	9 03·3	5·7	3·6	11·7	7·3	17·7	11·1
58	9 29·5	9 31·1	9 03·6	5·8	3·6	11·8	7·4	17·8	11·1
59	9 29·8	9 31·3	9 03·8	5·9	3·7	11·9	7·4	17·9	11·2
60	9 30·0	9 31·6	9 04·0	6·0	3·8	12·0	7·5	18·0	11·3

38ᵐ INCREMENTS AND CORRECTIONS 39ᵐ

38ᵐ	SUN PLANETS	ARIES	MOON	v or d	Corrⁿ	v or d	Corrⁿ	v or d	Corrⁿ
s	° ′	° ′	° ′	′	′	′	′	′	′
00	9 30·0	9 31·6	9 04·0	0·0	0·0	6·0	3·9	12·0	7·7
01	9 30·3	9 31·8	9 04·3	0·1	0·1	6·1	3·9	12·1	7·8
02	9 30·5	9 32·1	9 04·5	0·2	0·1	6·2	4·0	12·2	7·8
03	9 30·8	9 32·3	9 04·7	0·3	0·2	6·3	4·0	12·3	7·9
04	9 31·0	9 32·6	9 05·0	0·4	0·3	6·4	4·1	12·4	8·0
05	9 31·3	9 32·8	9 05·2	0·5	0·3	6·5	4·2	12·5	8·0
06	9 31·5	9 33·1	9 05·5	0·6	0·4	6·6	4·2	12·6	8·1
07	9 31·8	9 33·3	9 05·7	0·7	0·4	6·7	4·3	12·7	8·1
08	9 32·0	9 33·6	9 05·9	0·8	0·5	6·8	4·4	12·8	8·2
09	9 32·3	9 33·8	9 06·2	0·9	0·6	6·9	4·4	12·9	8·3
10	9 32·5	9 34·1	9 06·4	1·0	0·6	7·0	4·5	13·0	8·3
11	9 32·8	9 34·3	9 06·7	1·1	0·7	7·1	4·6	13·1	8·4
12	9 33·0	9 34·6	9 06·9	1·2	0·8	7·2	4·6	13·2	8·5
13	9 33·3	9 34·8	9 07·1	1·3	0·8	7·3	4·7	13·3	8·5
14	9 33·5	9 35·1	9 07·4	1·4	0·9	7·4	4·7	13·4	8·6
15	9 33·8	9 35·3	9 07·6	1·5	1·0	7·5	4·8	13·5	8·7
16	9 34·0	9 35·6	9 07·9	1·6	1·0	7·6	4·9	13·6	8·7
17	9 34·3	9 35·8	9 08·1	1·7	1·1	7·7	4·9	13·7	8·8
18	9 34·5	9 36·1	9 08·3	1·8	1·2	7·8	5·0	13·8	8·9
19	9 34·8	9 36·3	9 08·6	1·9	1·2	7·9	5·1	13·9	8·9
20	9 35·0	9 36·6	9 08·8	2·0	1·3	8·0	5·1	14·0	9·0
21	9 35·3	9 36·8	9 09·0	2·1	1·3	8·1	5·2	14·1	9·0
22	9 35·5	9 37·1	9 09·3	2·2	1·4	8·2	5·3	14·2	9·1
23	9 35·8	9 37·3	9 09·5	2·3	1·5	8·3	5·3	14·3	9·2
24	9 36·0	9 37·6	9 09·8	2·4	1·5	8·4	5·4	14·4	9·2
25	9 36·3	9 37·8	9 10·0	2·5	1·6	8·5	5·5	14·5	9·3
26	9 36·5	9 38·1	9 10·2	2·6	1·7	8·6	5·5	14·6	9·4
27	9 36·8	9 38·3	9 10·5	2·7	1·7	8·7	5·6	14·7	9·4
28	9 37·0	9 38·6	9 10·7	2·8	1·8	8·8	5·6	14·8	9·5
29	9 37·3	9 38·8	9 11·0	2·9	1·9	8·9	5·7	14·9	9·6
30	9 37·5	9 39·1	9 11·2	3·0	1·9	9·0	5·8	15·0	9·6
31	9 37·8	9 39·3	9 11·4	3·1	2·0	9·1	5·8	15·1	9·7
32	9 38·0	9 39·6	9 11·7	3·2	2·1	9·2	5·9	15·2	9·8
33	9 38·3	9 39·8	9 11·9	3·3	2·1	9·3	6·0	15·3	9·8
34	9 38·5	9 40·1	9 12·1	3·4	2·2	9·4	6·0	15·4	9·9
35	9 38·8	9 40·3	9 12·4	3·5	2·2	9·5	6·1	15·5	9·9
36	9 39·0	9 40·6	9 12·6	3·6	2·3	9·6	6·2	15·6	10·0
37	9 39·3	9 40·8	9 12·9	3·7	2·4	9·7	6·2	15·7	10·1
38	9 39·5	9 41·1	9 13·1	3·8	2·4	9·8	6·3	15·8	10·1
39	9 39·8	9 41·3	9 13·3	3·9	2·5	9·9	6·4	15·9	10·2
40	9 40·0	9 41·6	9 13·6	4·0	2·6	10·0	6·4	16·0	10·3
41	9 40·3	9 41·8	9 13·8	4·1	2·6	10·1	6·5	16·1	10·3
42	9 40·5	9 42·1	9 14·1	4·2	2·7	10·2	6·5	16·2	10·4
43	9 40·8	9 42·3	9 14·3	4·3	2·8	10·3	6·6	16·3	10·5
44	9 41·0	9 42·6	9 14·5	4·4	2·8	10·4	6·7	16·4	10·5
45	9 41·3	9 42·8	9 14·8	4·5	2·9	10·5	6·7	16·5	10·6
46	9 41·5	9 43·1	9 15·0	4·6	3·0	10·6	6·8	16·6	10·7
47	9 41·8	9 43·3	9 15·2	4·7	3·0	10·7	6·9	16·7	10·7
48	9 42·0	9 43·6	9 15·5	4·8	3·1	10·8	6·9	16·8	10·8
49	9 42·3	9 43·8	9 15·7	4·9	3·1	10·9	7·0	16·9	10·8
50	9 42·5	9 44·1	9 16·0	5·0	3·2	11·0	7·1	17·0	10·9
51	9 42·8	9 44·3	9 16·2	5·1	3·3	11·1	7·1	17·1	11·0
52	9 43·0	9 44·6	9 16·4	5·2	3·3	11·2	7·2	17·2	11·0
53	9 43·3	9 44·8	9 16·7	5·3	3·4	11·3	7·3	17·3	11·1
54	9 43·5	9 45·1	9 16·9	5·4	3·5	11·4	7·3	17·4	11·2
55	9 43·8	9 45·3	9 17·2	5·5	3·5	11·5	7·4	17·5	11·2
56	9 44·0	9 45·6	9 17·4	5·6	3·6	11·6	7·4	17·6	11·3
57	9 44·3	9 45·8	9 17·6	5·7	3·7	11·7	7·5	17·7	11·4
58	9 44·5	9 46·1	9 17·9	5·8	3·7	11·8	7·6	17·8	11·4
59	9 44·8	9 46·4	9 18·1	5·9	3·8	11·9	7·6	17·9	11·5
60	9 45·0	9 46·6	9 18·4	6·0	3·9	12·0	7·7	18·0	11·6

39ᵐ	SUN PLANETS	ARIES	MOON	v or d	Corrⁿ	v or d	Corrⁿ	v or d	Corrⁿ
s	° ′	° ′	° ′	′	′	′	′	′	′
00	9 45·0	9 46·6	9 18·4	0·0	0·0	6·0	4·0	12·0	7·9
01	9 45·3	9 46·9	9 18·6	0·1	0·1	6·1	4·0	12·1	8·0
02	9 45·5	9 47·1	9 18·8	0·2	0·1	6·2	4·1	12·2	8·0
03	9 45·8	9 47·4	9 19·1	0·3	0·2	6·3	4·1	12·3	8·1
04	9 46·0	9 47·6	9 19·3	0·4	0·3	6·4	4·2	12·4	8·2
05	9 46·3	9 47·9	9 19·5	0·5	0·3	6·5	4·3	12·5	8·2
06	9 46·5	9 48·1	9 19·8	0·6	0·4	6·6	4·3	12·6	8·3
07	9 46·8	9 48·4	9 20·0	0·7	0·5	6·7	4·4	12·7	8·4
08	9 47·0	9 48·6	9 20·3	0·8	0·5	6·8	4·5	12·8	8·4
09	9 47·3	9 48·9	9 20·5	0·9	0·6	6·9	4·5	12·9	8·5
10	9 47·5	9 49·1	9 20·7	1·0	0·7	7·0	4·6	13·0	8·6
11	9 47·8	9 49·4	9 21·0	1·1	0·7	7·1	4·7	13·1	8·6
12	9 48·0	9 49·6	9 21·2	1·2	0·8	7·2	4·7	13·2	8·7
13	9 48·3	9 49·9	9 21·5	1·3	0·9	7·3	4·8	13·3	8·8
14	9 48·5	9 50·1	9 21·7	1·4	0·9	7·4	4·9	13·4	8·8
15	9 48·8	9 50·4	9 21·9	1·5	1·0	7·5	4·9	13·5	8·9
16	9 49·0	9 50·6	9 22·2	1·6	1·1	7·6	5·0	13·6	9·0
17	9 49·3	9 50·9	9 22·4	1·7	1·1	7·7	5·1	13·7	9·0
18	9 49·5	9 51·1	9 22·6	1·8	1·2	7·8	5·1	13·8	9·1
19	9 49·8	9 51·4	9 22·9	1·9	1·3	7·9	5·2	13·9	9·2
20	9 50·0	9 51·6	9 23·1	2·0	1·3	8·0	5·3	14·0	9·2
21	9 50·3	9 51·9	9 23·4	2·1	1·4	8·1	5·3	14·1	9·3
22	9 50·5	9 52·1	9 23·6	2·2	1·4	8·2	5·4	14·2	9·3
23	9 50·8	9 52·4	9 23·8	2·3	1·5	8·3	5·5	14·3	9·4
24	9 51·0	9 52·6	9 24·1	2·4	1·6	8·4	5·5	14·4	9·5
25	9 51·3	9 52·9	9 24·3	2·5	1·6	8·5	5·6	14·5	9·5
26	9 51·5	9 53·1	9 24·6	2·6	1·7	8·6	5·7	14·6	9·6
27	9 51·8	9 53·4	9 24·8	2·7	1·8	8·7	5·7	14·7	9·7
28	9 52·0	9 53·6	9 25·0	2·8	1·8	8·8	5·8	14·8	9·7
29	9 52·3	9 53·9	9 25·3	2·9	1·9	8·9	5·9	14·9	9·8
30	9 52·5	9 54·1	9 25·5	3·0	2·0	9·0	5·9	15·0	9·9
31	9 52·8	9 54·4	9 25·7	3·1	2·0	9·1	6·0	15·1	9·9
32	9 53·0	9 54·6	9 26·0	3·2	2·1	9·2	6·1	15·2	10·0
33	9 53·3	9 54·9	9 26·2	3·3	2·2	9·3	6·1	15·3	10·1
34	9 53·5	9 55·1	9 26·5	3·4	2·2	9·4	6·2	15·4	10·1
35	9 53·8	9 55·4	9 26·7	3·5	2·3	9·5	6·3	15·5	10·2
36	9 54·0	9 55·6	9 26·9	3·6	2·4	9·6	6·3	15·6	10·3
37	9 54·3	9 55·9	9 27·2	3·7	2·4	9·7	6·4	15·7	10·3
38	9 54·5	9 56·1	9 27·4	3·8	2·5	9·8	6·5	15·8	10·4
39	9 54·8	9 56·4	9 27·7	3·9	2·6	9·9	6·5	15·9	10·5
40	9 55·0	9 56·6	9 27·9	4·0	2·6	10·0	6·6	16·0	10·5
41	9 55·3	9 56·9	9 28·1	4·1	2·7	10·1	6·6	16·1	10·6
42	9 55·5	9 57·1	9 28·4	4·2	2·8	10·2	6·7	16·2	10·7
43	9 55·8	9 57·4	9 28·6	4·3	2·8	10·3	6·8	16·3	10·7
44	9 56·0	9 57·6	9 28·8	4·4	2·9	10·4	6·8	16·4	10·8
45	9 56·3	9 57·9	9 29·1	4·5	3·0	10·5	6·9	16·5	10·9
46	9 56·5	9 58·1	9 29·3	4·6	3·0	10·6	7·0	16·6	10·9
47	9 56·8	9 58·4	9 29·6	4·7	3·1	10·7	7·0	16·7	11·0
48	9 57·0	9 58·6	9 29·8	4·8	3·2	10·8	7·1	16·8	11·1
49	9 57·3	9 58·9	9 30·0	4·9	3·2	10·9	7·2	16·9	11·1
50	9 57·5	9 59·1	9 30·3	5·0	3·3	11·0	7·2	17·0	11·2
51	9 57·8	9 59·4	9 30·5	5·1	3·4	11·1	7·3	17·1	11·3
52	9 58·0	9 59·6	9 30·8	5·2	3·4	11·2	7·4	17·2	11·3
53	9 58·3	9 59·9	9 31·0	5·3	3·5	11·3	7·4	17·3	11·4
54	9 58·5	10 00·1	9 31·2	5·4	3·6	11·4	7·5	17·4	11·5
55	9 58·8	10 00·4	9 31·5	5·5	3·6	11·5	7·6	17·5	11·5
56	9 59·0	10 00·6	9 31·7	5·6	3·7	11·6	7·6	17·6	11·6
57	9 59·3	10 00·9	9 32·0	5·7	3·8	11·7	7·7	17·7	11·7
58	9 59·5	10 01·1	9 32·2	5·8	3·8	11·8	7·8	17·8	11·7
59	9 59·8	10 01·4	9 32·4	5·9	3·9	11·9	7·8	17·9	11·8
60	10 00·0	10 01·6	9 32·7	6·0	4·0	12·0	7·9	18·0	11·9

40m INCREMENTS AND CORRECTIONS 41m

40m	SUN PLANETS	ARIES	MOON	v or d	Corrn	v or d	Corrn	v or d	Corrn
s	° ′	° ′	° ′	′	′	′	′	′	′
00	10 00·0	10 01·6	9 32·7	0·0	0·0	6·0	4·1	12·0	8·1
01	10 00·3	10 01·9	9 32·9	0·1	0·1	6·1	4·1	12·1	8·2
02	10 00·5	10 02·1	9 33·1	0·2	0·1	6·2	4·2	12·2	8·2
03	10 00·8	10 02·4	9 33·4	0·3	0·2	6·3	4·3	12·3	8·3
04	10 01·0	10 02·6	9 33·6	0·4	0·3	6·4	4·3	12·4	8·4
05	10 01·3	10 02·9	9 33·9	0·5	0·3	6·5	4·4	12·5	8·4
06	10 01·5	10 03·1	9 34·1	0·6	0·4	6·6	4·5	12·6	8·5
07	10 01·8	10 03·4	9 34·3	0·7	0·5	6·7	4·5	12·7	8·6
08	10 02·0	10 03·6	9 34·6	0·8	0·5	6·8	4·6	12·8	8·6
09	10 02·3	10 03·9	9 34·8	0·9	0·6	6·9	4·7	12·9	8·7
10	10 02·5	10 04·1	9 35·1	1·0	0·7	7·0	4·7	13·0	8·8
11	10 02·8	10 04·4	9 35·3	1·1	0·7	7·1	4·8	13·1	8·8
12	10 03·0	10 04·7	9 35·5	1·2	0·8	7·2	4·9	13·2	8·9
13	10 03·3	10 04·9	9 35·8	1·3	0·9	7·3	4·9	13·3	9·0
14	10 03·5	10 05·2	9 36·0	1·4	0·9	7·4	5·0	13·4	9·0
15	10 03·8	10 05·4	9 36·2	1·5	1·0	7·5	5·1	13·5	9·1
16	10 04·0	10 05·7	9 36·5	1·6	1·1	7·6	5·1	13·6	9·2
17	10 04·3	10 05·9	9 36·7	1·7	1·1	7·7	5·2	13·7	9·2
18	10 04·5	10 06·2	9 37·0	1·8	1·2	7·8	5·3	13·8	9·3
19	10 04·8	10 06·4	9 37·2	1·9	1·3	7·9	5·3	13·9	9·4
20	10 05·0	10 06·7	9 37·4	2·0	1·4	8·0	5·4	14·0	9·5
21	10 05·3	10 06·9	9 37·7	2·1	1·4	8·1	5·5	14·1	9·5
22	10 05·5	10 07·2	9 37·9	2·2	1·5	8·2	5·5	14·2	9·6
23	10 05·8	10 07·4	9 38·2	2·3	1·6	8·3	5·6	14·3	9·7
24	10 06·0	10 07·7	9 38·4	2·4	1·6	8·4	5·7	14·4	9·7
25	10 06·3	10 07·9	9 38·6	2·5	1·7	8·5	5·7	14·5	9·8
26	10 06·5	10 08·2	9 38·9	2·6	1·8	8·6	5·8	14·6	9·9
27	10 06·8	10 08·4	9 39·1	2·7	1·8	8·7	5·9	14·7	9·9
28	10 07·0	10 08·7	9 39·3	2·8	1·9	8·8	5·9	14·8	10·0
29	10 07·3	10 08·9	9 39·6	2·9	2·0	8·9	6·0	14·9	10·1
30	10 07·5	10 09·2	9 39·8	3·0	2·0	9·0	6·1	15·0	10·1
31	10 07·8	10 09·4	9 40·1	3·1	2·1	9·1	6·1	15·1	10·2
32	10 08·0	10 09·7	9 40·3	3·2	2·2	9·2	6·2	15·2	10·3
33	10 08·3	10 09·9	9 40·5	3·3	2·2	9·3	6·3	15·3	10·3
34	10 08·5	10 10·2	9 40·8	3·4	2·3	9·4	6·3	15·4	10·4
35	10 08·8	10 10·4	9 41·0	3·5	2·4	9·5	6·4	15·5	10·5
36	10 09·0	10 10·7	9 41·3	3·6	2·4	9·6	6·5	15·6	10·5
37	10 09·3	10 10·9	9 41·5	3·7	2·5	9·7	6·5	15·7	10·6
38	10 09·5	10 11·2	9 41·7	3·8	2·6	9·8	6·6	15·8	10·7
39	10 09·8	10 11·4	9 42·0	3·9	2·6	9·9	6·7	15·9	10·7
40	10 10·0	10 11·7	9 42·2	4·0	2·7	10·0	6·8	16·0	10·8
41	10 10·3	10 11·9	9 42·4	4·1	2·8	10·1	6·8	16·1	10·9
42	10 10·5	10 12·2	9 42·7	4·2	2·8	10·2	6·9	16·2	10·9
43	10 10·8	10 12·4	9 42·9	4·3	2·9	10·3	7·0	16·3	11·0
44	10 11·0	10 12·7	9 43·2	4·4	3·0	10·4	7·0	16·4	11·1
45	10 11·3	10 12·9	9 43·4	4·5	3·0	10·5	7·1	16·5	11·1
46	10 11·5	10 13·2	9 43·6	4·6	3·1	10·6	7·2	16·6	11·2
47	10 11·8	10 13·4	9 43·9	4·7	3·2	10·7	7·2	16·7	11·3
48	10 12·0	10 13·7	9 44·1	4·8	3·2	10·8	7·3	16·8	11·3
49	10 12·3	10 13·9	9 44·4	4·9	3·3	10·9	7·4	16·9	11·4
50	10 12·5	10 14·2	9 44·6	5·0	3·4	11·0	7·4	17·0	11·5
51	10 12·8	10 14·4	9 44·8	5·1	3·4	11·1	7·5	17·1	11·5
52	10 13·0	10 14·7	9 45·1	5·2	3·5	11·2	7·6	17·2	11·6
53	10 13·3	10 14·9	9 45·3	5·3	3·6	11·3	7·6	17·3	11·7
54	10 13·5	10 15·2	9 45·6	5·4	3·6	11·4	7·7	17·4	11·7
55	10 13·8	10 15·4	9 45·8	5·5	3·7	11·5	7·8	17·5	11·8
56	10 14·0	10 15·7	9 46·0	5·6	3·8	11·6	7·8	17·6	11·9
57	10 14·3	10 15·9	9 46·3	5·7	3·8	11·7	7·9	17·7	11·9
58	10 14·5	10 16·2	9 46·5	5·8	3·9	11·8	8·0	17·8	12·0
59	10 14·8	10 16·4	9 46·7	5·9	4·0	11·9	8·0	17·9	12·1
60	10 15·0	10 16·7	9 47·0	6·0	4·1	12·0	8·1	18·0	12·2

41m	SUN PLANETS	ARIES	MOON	v or d	Corrn	v or d	Corrn	v or d	Corrn
s	° ′	° ′	° ′	′	′	′	′	′	′
00	10 15·0	10 16·7	9 47·0	0·0	0·0	6·0	4·2	12·0	8·3
01	10 15·3	10 16·9	9 47·2	0·1	0·1	6·1	4·2	12·1	8·4
02	10 15·5	10 17·2	9 47·5	0·2	0·1	6·2	4·3	12·2	8·4
03	10 15·8	10 17·4	9 47·7	0·3	0·2	6·3	4·4	12·3	8·5
04	10 16·0	10 17·7	9 47·9	0·4	0·3	6·4	4·4	12·4	8·6
05	10 16·3	10 17·9	9 48·2	0·5	0·3	6·5	4·5	12·5	8·6
06	10 16·5	10 18·2	9 48·4	0·6	0·4	6·6	4·6	12·6	8·7
07	10 16·8	10 18·4	9 48·7	0·7	0·5	6·7	4·6	12·7	8·8
08	10 17·0	10 18·7	9 48·9	0·8	0·6	6·8	4·7	12·8	8·9
09	10 17·3	10 18·9	9 49·1	0·9	0·6	6·9	4·8	12·9	8·9
10	10 17·5	10 19·2	9 49·4	1·0	0·7	7·0	4·8	13·0	9·0
11	10 17·8	10 19·4	9 49·6	1·1	0·8	7·1	4·9	13·1	9·1
12	10 18·0	10 19·7	9 49·8	1·2	0·8	7·2	5·0	13·2	9·1
13	10 18·3	10 19·9	9 50·1	1·3	0·9	7·3	5·0	13·3	9·2
14	10 18·5	10 20·2	9 50·3	1·4	1·0	7·4	5·1	13·4	9·3
15	10 18·8	10 20·4	9 50·6	1·5	1·0	7·5	5·2	13·5	9·3
16	10 19·0	10 20·7	9 50·8	1·6	1·1	7·6	5·3	13·6	9·4
17	10 19·3	10 20·9	9 51·0	1·7	1·2	7·7	5·3	13·7	9·5
18	10 19·5	10 21·2	9 51·3	1·8	1·2	7·8	5·4	13·8	9·5
19	10 19·8	10 21·4	9 51·5	1·9	1·3	7·9	5·5	13·9	9·6
20	10 20·0	10 21·7	9 51·8	2·0	1·4	8·0	5·5	14·0	9·7
21	10 20·3	10 21·9	9 52·0	2·1	1·5	8·1	5·6	14·1	9·8
22	10 20·5	10 22·2	9 52·2	2·2	1·5	8·2	5·7	14·2	9·8
23	10 20·8	10 22·4	9 52·5	2·3	1·6	8·3	5·7	14·3	9·9
24	10 21·0	10 22·7	9 52·7	2·4	1·7	8·4	5·8	14·4	10·0
25	10 21·3	10 23·0	9 52·9	2·5	1·7	8·5	5·9	14·5	10·0
26	10 21·5	10 23·2	9 53·2	2·6	1·8	8·6	5·9	14·6	10·1
27	10 21·8	10 23·5	9 53·4	2·7	1·9	8·7	6·0	14·7	10·2
28	10 22·0	10 23·7	9 53·7	2·8	1·9	8·8	6·1	14·8	10·2
29	10 22·3	10 24·0	9 53·9	2·9	2·0	8·9	6·2	14·9	10·3
30	10 22·5	10 24·2	9 54·1	3·0	2·1	9·0	6·2	15·0	10·4
31	10 22·8	10 24·5	9 54·4	3·1	2·1	9·1	6·3	15·1	10·4
32	10 23·0	10 24·7	9 54·6	3·2	2·2	9·2	6·4	15·2	10·5
33	10 23·3	10 25·0	9 54·9	3·3	2·3	9·3	6·4	15·3	10·6
34	10 23·5	10 25·2	9 55·1	3·4	2·4	9·4	6·5	15·4	10·7
35	10 23·8	10 25·5	9 55·3	3·5	2·4	9·5	6·6	15·5	10·7
36	10 24·0	10 25·7	9 55·6	3·6	2·5	9·6	6·6	15·6	10·8
37	10 24·3	10 26·0	9 55·8	3·7	2·6	9·7	6·7	15·7	10·9
38	10 24·5	10 26·2	9 56·1	3·8	2·6	9·8	6·8	15·8	10·9
39	10 24·8	10 26·5	9 56·3	3·9	2·7	9·9	6·8	15·9	11·0
40	10 25·0	10 26·7	9 56·5	4·0	2·8	10·0	6·9	16·0	11·1
41	10 25·3	10 27·0	9 56·8	4·1	2·8	10·1	7·0	16·1	11·1
42	10 25·5	10 27·2	9 57·0	4·2	2·9	10·2	7·1	16·2	11·2
43	10 25·8	10 27·5	9 57·2	4·3	3·0	10·3	7·1	16·3	11·3
44	10 26·0	10 27·7	9 57·5	4·4	3·0	10·4	7·2	16·4	11·3
45	10 26·3	10 28·0	9 57·7	4·5	3·1	10·5	7·3	16·5	11·4
46	10 26·5	10 28·2	9 58·0	4·6	3·2	10·6	7·3	16·6	11·5
47	10 26·8	10 28·5	9 58·2	4·7	3·3	10·7	7·4	16·7	11·6
48	10 27·0	10 28·7	9 58·4	4·8	3·3	10·8	7·5	16·8	11·6
49	10 27·3	10 29·0	9 58·7	4·9	3·4	10·9	7·5	16·9	11·7
50	10 27·5	10 29·2	9 58·9	5·0	3·5	11·0	7·6	17·0	11·8
51	10 27·8	10 29·5	9 59·2	5·1	3·5	11·1	7·7	17·1	11·8
52	10 28·0	10 29·7	9 59·4	5·2	3·6	11·2	7·7	17·2	11·9
53	10 28·3	10 30·0	9 59·6	5·3	3·7	11·3	7·8	17·3	12·0
54	10 28·5	10 30·2	9 59·9	5·4	3·7	11·4	7·9	17·4	12·0
55	10 28·8	10 30·5	10 00·1	5·5	3·8	11·5	8·0	17·5	12·1
56	10 29·0	10 30·7	10 00·3	5·6	3·9	11·6	8·0	17·6	12·2
57	10 29·3	10 31·0	10 00·6	5·7	3·9	11·7	8·1	17·7	12·2
58	10 29·5	10 31·2	10 00·8	5·8	4·0	11·8	8·2	17·8	12·3
59	10 29·8	10 31·5	10 01·1	5·9	4·1	11·9	8·2	17·9	12·4
60	10 30·0	10 31·7	10 01·3	6·0	4·2	12·0	8·3	18·0	12·5

42m INCREMENTS AND CORRECTIONS 43m

42m	SUN PLANETS	ARIES	MOON	v or d	Corrn	v or d	Corrn	v or d	Corrn
s	° ′	° ′	° ′	′	′	′	′	′	′
00	10 30·0	10 31·7	10 01·3	0·0	0·0	6·0	4·3	12·0	8·5
01	10 30·3	10 32·0	10 01·5	0·1	0·1	6·1	4·3	12·1	8·6
02	10 30·5	10 32·2	10 01·8	0·2	0·1	6·2	4·4	12·2	8·6
03	10 30·8	10 32·5	10 02·0	0·3	0·2	6·3	4·5	12·3	8·7
04	10 31·0	10 32·7	10 02·3	0·4	0·3	6·4	4·5	12·4	8·8
05	10 31·3	10 33·0	10 02·5	0·5	0·4	6·5	4·6	12·5	8·9
06	10 31·5	10 33·2	10 02·7	0·6	0·4	6·6	4·7	12·6	8·9
07	10 31·8	10 33·5	10 03·0	0·7	0·5	6·7	4·7	12·7	9·0
08	10 32·0	10 33·7	10 03·2	0·8	0·6	6·8	4·8	12·8	9·1
09	10 32·3	10 34·0	10 03·4	0·9	0·6	6·9	4·9	12·9	9·1
10	10 32·5	10 34·2	10 03·7	1·0	0·7	7·0	5·0	13·0	9·2
11	10 32·8	10 34·5	10 03·9	1·1	0·8	7·1	5·0	13·1	9·3
12	10 33·0	10 34·7	10 04·2	1·2	0·9	7·2	5·1	13·2	9·4
13	10 33·3	10 35·0	10 04·4	1·3	0·9	7·3	5·2	13·3	9·4
14	10 33·5	10 35·2	10 04·6	1·4	1·0	7·4	5·2	13·4	9·5
15	10 33·8	10 35·5	10 04·9	1·5	1·1	7·5	5·3	13·5	9·6
16	10 34·0	10 35·7	10 05·1	1·6	1·1	7·6	5·4	13·6	9·6
17	10 34·3	10 36·0	10 05·4	1·7	1·2	7·7	5·5	13·7	9·7
18	10 34·5	10 36·2	10 05·6	1·8	1·3	7·8	5·5	13·8	9·8
19	10 34·8	10 36·5	10 05·8	1·9	1·3	7·9	5·6	13·9	9·8
20	10 35·0	10 36·7	10 06·1	2·0	1·4	8·0	5·7	14·0	9·9
21	10 35·3	10 37·0	10 06·3	2·1	1·5	8·1	5·7	14·1	10·0
22	10 35·5	10 37·2	10 06·5	2·2	1·6	8·2	5·8	14·2	10·1
23	10 35·8	10 37·5	10 06·8	2·3	1·6	8·3	5·9	14·3	10·1
24	10 36·0	10 37·7	10 07·0	2·4	1·7	8·4	6·0	14·4	10·2
25	10 36·3	10 38·0	10 07·3	2·5	1·8	8·5	6·0	14·5	10·3
26	10 36·5	10 38·2	10 07·5	2·6	1·8	8·6	6·1	14·6	10·3
27	10 36·8	10 38·5	10 07·7	2·7	1·9	8·7	6·2	14·7	10·4
28	10 37·0	10 38·7	10 08·0	2·8	2·0	8·8	6·2	14·8	10·5
29	10 37·3	10 39·0	10 08·2	2·9	2·1	8·9	6·3	14·9	10·6
30	10 37·5	10 39·2	10 08·5	3·0	2·1	9·0	6·4	15·0	10·6
31	10 37·8	10 39·5	10 08·7	3·1	2·2	9·1	6·4	15·1	10·7
32	10 38·0	10 39·7	10 08·9	3·2	2·3	9·2	6·5	15·2	10·8
33	10 38·3	10 40·0	10 09·2	3·3	2·3	9·3	6·6	15·3	10·8
34	10 38·5	10 40·2	10 09·4	3·4	2·4	9·4	6·7	15·4	10·9
35	10 38·8	10 40·5	10 09·7	3·5	2·5	9·5	6·7	15·5	11·0
36	10 39·0	10 40·7	10 09·9	3·6	2·6	9·6	6·8	15·6	11·1
37	10 39·3	10 41·0	10 10·1	3·7	2·6	9·7	6·9	15·7	11·1
38	10 39·5	10 41·3	10 10·4	3·8	2·7	9·8	6·9	15·8	11·2
39	10 39·8	10 41·5	10 10·6	3·9	2·8	9·9	7·0	15·9	11·3
40	10 40·0	10 41·8	10 10·8	4·0	2·8	10·0	7·1	16·0	11·3
41	10 40·3	10 42·0	10 11·1	4·1	2·9	10·1	7·2	16·1	11·4
42	10 40·5	10 42·3	10 11·3	4·2	3·0	10·2	7·2	16·2	11·5
43	10 40·8	10 42·5	10 11·6	4·3	3·0	10·3	7·3	16·3	11·5
44	10 41·0	10 42·8	10 11·8	4·4	3·1	10·4	7·4	16·4	11·6
45	10 41·3	10 43·0	10 12·0	4·5	3·2	10·5	7·4	16·5	11·7
46	10 41·5	10 43·3	10 12·3	4·6	3·3	10·6	7·5	16·6	11·8
47	10 41·8	10 43·5	10 12·5	4·7	3·3	10·7	7·6	16·7	11·8
48	10 42·0	10 43·8	10 12·8	4·8	3·4	10·8	7·7	16·8	11·9
49	10 42·3	10 44·0	10 13·0	4·9	3·5	10·9	7·7	16·9	12·0
50	10 42·5	10 44·3	10 13·2	5·0	3·5	11·0	7·8	17·0	12·0
51	10 42·8	10 44·5	10 13·5	5·1	3·6	11·1	7·9	17·1	12·1
52	10 43·0	10 44·8	10 13·7	5·2	3·7	11·2	7·9	17·2	12·2
53	10 43·3	10 45·0	10 13·9	5·3	3·8	11·3	8·0	17·3	12·3
54	10 43·5	10 45·3	10 14·2	5·4	3·8	11·4	8·1	17·4	12·3
55	10 43·8	10 45·5	10 14·4	5·5	3·9	11·5	8·1	17·5	12·4
56	10 44·0	10 45·8	10 14·7	5·6	4·0	11·6	8·2	17·6	12·5
57	10 44·3	10 46·0	10 14·9	5·7	4·0	11·7	8·3	17·7	12·5
58	10 44·5	10 46·3	10 15·1	5·8	4·1	11·8	8·4	17·8	12·6
59	10 44·8	10 46·5	10 15·4	5·9	4·2	11·9	8·4	17·9	12·7
60	10 45·0	10 46·8	10 15·6	6·0	4·3	12·0	8·5	18·0	12·8

43m	SUN PLANETS	ARIES	MOON	v or d	Corrn	v or d	Corrn	v or d	Corrn
s	° ′	° ′	° ′	′	′	′	′	′	′
00	10 45·0	10 46·8	10 15·6	0·0	0·0	6·0	4·4	12·0	8·7
01	10 45·3	10 47·0	10 15·9	0·1	0·1	6·1	4·4	12·1	8·8
02	10 45·5	10 47·3	10 16·1	0·2	0·1	6·2	4·5	12·2	8·8
03	10 45·8	10 47·5	10 16·3	0·3	0·2	6·3	4·6	12·3	8·9
04	10 46·0	10 47·8	10 16·6	0·4	0·3	6·4	4·6	12·4	9·0
05	10 46·3	10 48·0	10 16·8	0·5	0·4	6·5	4·7	12·5	9·1
06	10 46·5	10 48·3	10 17·0	0·6	0·4	6·6	4·8	12·6	9·1
07	10 46·8	10 48·5	10 17·3	0·7	0·5	6·7	4·9	12·7	9·2
08	10 47·0	10 48·8	10 17·5	0·8	0·6	6·8	4·9	12·8	9·3
09	10 47·3	10 49·0	10 17·8	0·9	0·7	6·9	5·0	12·9	9·4
10	10 47·5	10 49·3	10 18·0	1·0	0·7	7·0	5·1	13·0	9·4
11	10 47·8	10 49·5	10 18·2	1·1	0·8	7·1	5·1	13·1	9·5
12	10 48·0	10 49·8	10 18·5	1·2	0·9	7·2	5·2	13·2	9·6
13	10 48·3	10 50·0	10 18·7	1·3	0·9	7·3	5·3	13·3	9·6
14	10 48·5	10 50·3	10 19·0	1·4	1·0	7·4	5·4	13·4	9·7
15	10 48·8	10 50·5	10 19·2	1·5	1·1	7·5	5·4	13·5	9·8
16	10 49·0	10 50·8	10 19·4	1·6	1·2	7·6	5·5	13·6	9·9
17	10 49·3	10 51·0	10 19·7	1·7	1·2	7·7	5·6	13·7	9·9
18	10 49·5	10 51·3	10 19·9	1·8	1·3	7·8	5·7	13·8	10·0
19	10 49·8	10 51·5	10 20·2	1·9	1·4	7·9	5·7	13·9	10·1
20	10 50·0	10 51·8	10 20·4	2·0	1·5	8·0	5·8	14·0	10·2
21	10 50·3	10 52·0	10 20·6	2·1	1·5	8·1	5·9	14·1	10·2
22	10 50·5	10 52·3	10 20·9	2·2	1·6	8·2	5·9	14·2	10·3
23	10 50·8	10 52·5	10 21·1	2·3	1·7	8·3	6·0	14·3	10·4
24	10 51·0	10 52·8	10 21·3	2·4	1·7	8·4	6·1	14·4	10·4
25	10 51·3	10 53·0	10 21·6	2·5	1·8	8·5	6·2	14·5	10·5
26	10 51·5	10 53·3	10 21·8	2·6	1·9	8·6	6·2	14·6	10·6
27	10 51·8	10 53·5	10 22·1	2·7	2·0	8·7	6·3	14·7	10·7
28	10 52·0	10 53·8	10 22·3	2·8	2·0	8·8	6·4	14·8	10·7
29	10 52·3	10 54·0	10 22·5	2·9	2·1	8·9	6·5	14·9	10·8
30	10 52·5	10 54·3	10 22·8	3·0	2·2	9·0	6·5	15·0	10·9
31	10 52·8	10 54·5	10 23·0	3·1	2·2	9·1	6·6	15·1	10·9
32	10 53·0	10 54·8	10 23·3	3·2	2·3	9·2	6·7	15·2	11·0
33	10 53·3	10 55·0	10 23·5	3·3	2·4	9·3	6·7	15·3	11·1
34	10 53·5	10 55·3	10 23·7	3·4	2·5	9·4	6·8	15·4	11·2
35	10 53·8	10 55·5	10 24·0	3·5	2·5	9·5	6·9	15·5	11·2
36	10 54·0	10 55·8	10 24·2	3·6	2·6	9·6	7·0	15·6	11·3
37	10 54·3	10 56·0	10 24·4	3·7	2·7	9·7	7·0	15·7	11·4
38	10 54·5	10 56·3	10 24·7	3·8	2·8	9·8	7·1	15·8	11·5
39	10 54·8	10 56·5	10 24·9	3·9	2·8	9·9	7·2	15·9	11·5
40	10 55·0	10 56·8	10 25·2	4·0	2·9	10·0	7·3	16·0	11·6
41	10 55·3	10 57·0	10 25·4	4·1	3·0	10·1	7·3	16·1	11·7
42	10 55·5	10 57·3	10 25·6	4·2	3·0	10·2	7·4	16·2	11·7
43	10 55·8	10 57·5	10 25·9	4·3	3·1	10·3	7·5	16·3	11·8
44	10 56·0	10 57·8	10 26·1	4·4	3·2	10·4	7·5	16·4	11·9
45	10 56·3	10 58·0	10 26·4	4·5	3·3	10·5	7·6	16·5	12·0
46	10 56·5	10 58·3	10 26·6	4·6	3·3	10·6	7·7	16·6	12·0
47	10 56·8	10 58·5	10 26·8	4·7	3·4	10·7	7·8	16·7	12·1
48	10 57·0	10 58·8	10 27·1	4·8	3·5	10·8	7·8	16·8	12·2
49	10 57·3	10 59·0	10 27·3	4·9	3·6	10·9	7·9	16·9	12·3
50	10 57·5	10 59·3	10 27·5	5·0	3·6	11·0	8·0	17·0	12·3
51	10 57·8	10 59·6	10 27·8	5·1	3·7	11·1	8·0	17·1	12·4
52	10 58·0	10 59·8	10 28·0	5·2	3·8	11·2	8·1	17·2	12·5
53	10 58·3	11 00·1	10 28·3	5·3	3·8	11·3	8·2	17·3	12·5
54	10 58·5	11 00·3	10 28·5	5·4	3·9	11·4	8·3	17·4	12·6
55	10 58·8	11 00·6	10 28·7	5·5	4·0	11·5	8·3	17·5	12·7
56	10 59·0	11 00·8	10 29·0	5·6	4·1	11·6	8·4	17·6	12·8
57	10 59·3	11 01·1	10 29·2	5·7	4·1	11·7	8·5	17·7	12·8
58	10 59·5	11 01·3	10 29·5	5·8	4·2	11·8	8·6	17·8	12·9
59	10 59·8	11 01·6	10 29·7	5·9	4·3	11·9	8·6	17·9	13·0
60	11 00·0	11 01·8	10 29·9	6·0	4·4	12·0	8·7	18·0	13·1

STARS, JANUARY—JUNE

Mag.	Name and No.		S.H.A.	JAN.	FEB.	MAR.	APR.	MAY	JUNE	Declination	JAN.	FEB.	MAR.	APR.	MAY	JUNE
			°	′	′	′	′	′	′	°	′	′	′	′	′	′
1·6	α Geminorum		247	00·6	00·6	00·7	00·8	00·9	01·0	N. 31	58·8	58·8	58·8	58·9	58·8	58·8
3·3	σ Puppis*		248	00·9	01·0	01·1	01·3	01·5	01·6	S. 43	13·1	13·3	13·4	13·4	13·3	13·2
3·1	β Canis Minoris		248	46·4	46·4	46·5	46·6	46·7	46·7	N. 8	22·3	22·3	22·3	22·3	22·3	22·3
2·4	η Canis Majoris		249	23·0	23·0	23·1	23·3	23·4	23·5	S. 29	13·4	13·5	13·6	13·6	13·6	13·5
2·7	π Puppis*		251	04·6	04·6	04·7	04·9	05·1	05·1	S. 37	01·5	01·6	01·7	01·7	01·7	01·6
2·0	δ Canis Majoris		253	19·2	19·3	19·4	19·5	19·7	19·7	S. 26	19·7	19·9	19·9	19·9	19·9	19·8
3·1	ο Canis Majoris		254	40·4	40·5	40·6	40·7	40·8	40·9	S. 23	46·5	46·6	46·6	46·6	46·6	46·5
1·6	ε Canis Majoris	19	255	44·9	44·9	45·0	45·2	45·3	45·4	S. 28	55·1	55·2	55·2	55·2	55·2	55·1
2·8	τ Puppis*		257	46·0	46·1	46·3	46·5	46·8	46·8	S. 50	34·1	34·2	34·3	34·3	34·2	34·1
−1·6	α Canis Majoris	18	259	10·1	10·1	10·2	10·4	10·5	10·5	S. 16	39·7	39·7	39·8	39·8	39·7	39·7
1·9	γ Geminorum		261	10·2	10·2	10·3	10·4	10·5	10·5	N. 16	26·0	26·0	26·0	26·0	26·0	26·0
−0·9	α Carinæ*	17	264	14·1	14·3	14·5	14·7	15·0	15·0	S. 52	40·6	40·7	40·8	40·8	40·7	40·5
2·0	β Canis Majoris		264	46·8	46·8	47·0	47·1	47·2	47·2	S. 17	56·3	56·3	56·4	56·4	56·3	56·2
2·7	θ Aurigæ		270	46·5	46·6	46·7	46·9	47·0	47·0	N. 37	12·6	12·6	12·7	12·6	12·6	12·6
2·1	β Aurigæ		270	52·6	52·7	52·8	53·0	53·1	53·1	N. 44	56·7	56·7	56·7	56·7	56·7	56·6
Var.‡	α Orionis	16	271	46·0	46·1	46·2	46·3	46·4	46·4	N. 7	23·9	23·9	23·9	23·9	23·9	23·9
2·2	κ Orionis		273	33·1	33·1	33·3	33·4	33·5	33·5	S. 9	41·2	41·3	41·3	41·3	41·2	41·1
1·9	ζ Orionis		275	20·0	20·0	20·1	20·3	20·3	20·3	S. 1	58·0	58·0	58·0	58·0	58·0	57·9
2·8	α Columbæ		275	27·6	27·7	27·9	28·1	28·2	28·2	S. 34	06·0	06·1	06·1	06·1	06·0	05·9
3·0	ζ Tauri		276	12·5	12·5	12·7	12·8	12·9	12·8	N. 21	07·0	07·0	07·0	07·0	07·0	07·0
1·8	ε Orionis	15	276	28·3	28·4	28·5	28·6	28·7	28·7	S. 1	13·8	13·9	13·9	13·9	13·8	13·8
2·9	ι Orionis		276	38·9	39·0	39·1	39·2	39·3	39·2	S. 5	56·3	56·4	56·4	56·4	56·4	56·3
2·7	α Leporis		277	16·4	16·5	16·6	16·7	16·8	16·8	S. 17	51·3	51·4	51·4	51·4	51·3	51·2
2·5	δ Orionis		277	31·6	31·7	31·8	32·0	32·0	32·0	S. 0	19·9	19·9	20·0	19·9	19·9	19·8
3·0	β Leporis		278	22·9	23·0	23·1	23·2	23·3	23·3	S. 20	47·7	47·8	47·8	47·8	47·7	47·6
1·8	β Tauri	14	279	04·9	05·0	05·1	05·3	05·3	05·3	N. 28	34·4	34·4	34·4	34·3	34·3	34·3
1·7	γ Orionis	13	279	16·4	16·4	16·6	16·7	16·7	16·7	N. 6	18·6	18·6	18·6	18·6	18·6	18·7
0·2	α Aurigæ	12	281	35·6	35·7	35·9	36·0	36·1	36·0	N. 45	57·4	57·5	57·5	57·5	57·4	57·3
0·3	β Orionis	11	281	51·8	51·9	52·0	52·1	52·2	52·1	S. 8	15·1	15·2	15·2	15·2	15·1	15·0
2·9	β Eridani		283	32·8	32·9	33·0	33·1	33·2	33·1	S. 5	08·5	08·6	08·6	08·6	08·5	08·5
2·9	ι Aurigæ †		286	25·6	25·7	25·8	26·0	26·0	25·9	N. 33	06·1	06·1	06·1	06·0	06·0	06·0
1·1	α Tauri	10	291	36·9	37·0	37·1	37·2	37·3	37·2	N. 16	25·5	25·5	25·5	25·5	25·5	25·5
3·2	γ Eridani		300	58·6	58·7	58·9	59·0	59·0	58·9	S. 13	37·8	37·8	37·8	37·8	37·7	37·6
3·0	ε Persei		301	14·0	14·2	14·3	14·4	14·4	14·3	N. 39	53·5	53·5	53·5	53·4	53·4	53·3
2·9	ζ Persei		302	07·2	07·3	07·5	07·6	07·6	07·4	N. 31	45·7	45·7	45·6	45·6	45·6	45·5
3·0	η Tauri		303	44·8	44·9	45·0	45·1	45·1	45·0	N. 23	58·6	58·6	58·6	58·5	58·5	58·5
1·9	α Persei	9	309	39·7	39·9	40·1	40·2	40·1	40·0	N. 49	43·0	42·9	42·9	42·8	42·7	42·7
Var.§	β Persei †		313	38·1	38·2	38·4	38·5	38·4	38·3	N. 40	47·9	47·8	47·8	47·7	47·6	47·6
2·8	α Ceti	8	314	58·5	58·6	58·7	58·7	58·7	58·6	N. 3	55·5	55·5	55·5	55·5	55·5	55·6
3·1	θ Eridani	7	315	49·8	49·9	50·1	50·2	50·2	50·1	S. 40	28·6	28·6	28·6	28·5	28·3	28·2
3·1	β Trianguli		328	14·1	14·2	14·3	14·4	14·3	14·1	N. 34	47·5	47·5	47·4	47·4	47·3	47·3
2·2	α Arietis	6	328	47·7	47·8	47·9	47·9	47·8	47·6	N. 23	16·0	15·9	15·9	15·9	15·8	15·9
2·2	γ Andromedæ		329	39·9	40·1	40·2	40·2	40·1	39·9	N. 42	07·9	07·9	07·8	07·7	07·7	07·7
3·0	α Hydri		330	38·3	38·6	38·8	38·9	38·9	38·6	S. 61	46·7	46·7	46·6	46·4	46·2	46·1
2·7	β Arietis		331	55·0	55·1	55·2	55·2	55·1	54·9	N. 20	36·3	36·3	36·2	36·2	36·2	36·2
2·1	α Ursæ Minoris †		331	14·7	23·9	30·8	33·7	30·5	23·1	N. 89	04·4	04·4	04·3	04·2	04·0	03·9
0·6	α Eridani	5	335	57·8	58·0	58·2	58·2	58·1	57·9	S. 57	27·3	27·2	27·1	26·9	26·8	26·6
2·8	δ Cassiopeiæ		339	13·8	14·1	14·2	14·2	14·1	13·7	N. 60	01·4	01·3	01·2	01·1	01·0	00·9
2·4	β Andromedæ		343	09·2	09·3	09·3	09·3	09·2	09·0	N. 35	24·1	24·0	24·0	23·9	23·9	23·9
Var. ‖	γ Cassiopeiæ		346	27·3	27·6	27·7	27·7	27·4	27·1	N. 60	29·7	29·6	29·5	29·4	29·3	29·3
2·2	β Ceti	4	349	37·7	37·8	37·8	37·8	37·7	37·5	S. 18	13·1	13·1	13·0	13·0	12·9	12·7
2·5	α Cassiopeiæ	3	350	28·1	28·3	28·4	28·3	28·1	27·8	N. 56	18·7	18·7	18·6	18·4	18·4	18·4
2·4	α Phœnicis	2	353	56·9	57·0	57·0	57·0	56·8	56·6	S. 42	32·2	32·2	32·1	31·9	31·8	31·6
2·9	β Hydri †		354	07·2	07·7	08·0	07·9	07·5	06·8	S. 77	29·7	29·6	29·4	29·2	29·0	28·9
2·9	γ Pegasi		357	13·8	13·9	13·9	13·9	13·7	13·5	N. 14	57·1	57·1	57·0	57·0	57·0	57·1
2·4	β Cassiopeiæ		358	16·0	16·2	16·2	16·1	15·9	15·5	N. 58	55·4	55·3	55·2	55·1	55·0	55·0
2·2	α Andromedæ	1	358	26·7	26·8	26·8	26·7	26·6	26·3	N. 28	51·7	51·6	51·6	51·5	51·5	51·6

* Formerly Argus ‡ 0·1—1·2 § 2·3—3·5 ‖ Irregular variable; 1955 mag. 2·8
† Not suitable for use with H.O. 214 (H.D. 486)

STARS JULY—DECEMBER

Mag.	Name and No.		S.H.A.	JULY	AUG.	SEPT.	OCT.	NOV.	DEC.	Declination	JULY	AUG.	SEPT.	OCT.	NOV.	DEC.
			°	′	′	′	′	′	′	°	′	′	′	′	′	′
1·6	*Castor*		246	60·9	60·8	60·6	60·3	60·1	59·9	N. 31	58·8	58·7	58·7	58·7	58·6	58·6
3·3	σ Puppis*		248	01·6	01·5	01·3	01·1	00·8	00·6	S. 43	13·1	13·0	12·9	12·9	12·9	13·1
3·1	β Canis Minoris		248	46·7	46·6	46·4	46·2	45·9	45·8	N. 8	22·4	22·4	22·4	22·4	22·3	22·3
2·4	η Canis Majoris		249	23·4	23·3	23·2	22·9	22·7	22·5	S. 29	13·4	13·3	13·2	13·2	13·3	13·4
2·7	π Puppis*		251	05·1	05·0	04·8	04·6	04·3	04·2	S. 37	01·4	01·3	01·2	01·2	01·3	01·4
2·0	*Wezen*		253	19·7	19·6	19·4	19·1	18·9	18·8	S. 26	19·7	19·6	19·5	19·5	19·6	19·7
3·1	ο Canis Majoris		254	40·9	40·7	40·5	40·3	40·1	39·9	S. 23	46·4	46·3	46·2	46·2	46·3	46·4
1·6	*Adhara*	19	255	45·3	45·2	45·0	44·8	44·5	44·4	S. 28	55·0	54·8	54·8	54·8	54·9	55·0
2·8	τ Puppis*		257	46·8	46·7	46·4	46·1	45·9	45·7	S. 50	34·0	33·8	33·7	33·7	33·8	34·0
−1·6	*Sirius*	18	259	10·4	10·3	10·1	09·9	09·7	09·5	S. 16	39·6	39·5	39·4	39·4	39·5	39·6
1·9	*Alhena*		261	10·4	10·3	10·1	09·9	09·6	09·5	N. 16	26·1	26·1	26·1	26·1	26·0	26·0
−0·9	*Canopus*	17	264	15·0	14·8	14·5	14·3	14·0	13·8	S. 52	40·4	40·2	40·2	40·2	40·3	40·5
2·0	*Mirzam*		264	47·1	47·0	46·8	46·6	46·4	46·2	S. 17	56·1	56·0	56·0	56·0	56·1	56·2
2·7	θ Aurigæ		270	46·8	46·6	46·3	46·1	45·8	45·7	N. 37	12·5	12·5	12·5	12·5	12·5	12·5
2·1	*Menkalinan*		270	52·9	52·7	52·4	52·1	51·9	51·7	N. 44	56·5	56·5	56·5	56·5	56·5	56·6
Var.‡	*Betelgeuse*	16	271	46·3	46·1	45·9	45·7	45·5	45·4	N. 7	24·0	24·0	24·0	24·0	24·0	23·9
2·2	κ Orionis		273	33·4	33·2	33·0	32·8	32·6	32·5	S. 9	41·0	41·0	40·9	41·0	41·0	41·1
1·9	*Alnitak*		275	20·2	20·0	19·8	19·6	19·4	19·3	S. 1	57·9	57·8	57·8	57·8	57·8	57·9
2·8	*Phact*		275	28·1	27·9	27·7	27·4	27·2	27·1	S. 34	05·7	05·6	05·6	05·6	05·7	05·8
3·0	ζ Tauri		276	12·7	12·5	12·3	12·0	11·8	11·7	N. 21	07·0	07·0	07·0	07·0	07·0	07·0
1·8	*Alnilam*	15	276	28·5	28·4	28·2	27·9	27·8	27·6	S. 1	13·7	13·6	13·6	13·6	13·7	13·7
2·9	ι Orionis		276	39·1	38·9	38·7	38·5	38·4	38·2	S. 5	56·2	56·1	56·1	56·1	56·2	56·3
2·7	α Leporis		277	16·7	16·5	16·3	16·1	15·9	15·8	S. 17	51·1	51·0	50·9	51·0	51·1	51·2
2·5	δ Orionis		277	31·9	31·7	31·5	31·3	31·1	31·0	S. 0	19·8	19·7	19·7	19·7	19·8	19·8
3·0	β Leporis		278	23·2	23·0	22·8	22·6	22·4	22·3	S. 20	47·5	47·4	47·4	47·4	47·5	47·6
1·8	*Elnath*	14	279	05·1	04·9	04·7	04·4	04·2	04·1	N. 28	34·3	34·3	34·3	34·3	34·3	34·4
1·7	*Bellatrix*	13	279	16·6	16·4	16·2	16·0	15·8	15·7	N. 6	18·7	18·8	18·8	18·8	18·8	18·7
0·2	*Capella*	12	281	35·8	35·6	35·3	35·0	34·7	34·6	N. 45	57·3	57·3	57·3	57·3	57·3	57·4
0·3	*Rigel*	11	281	52·0	51·8	51·6	51·4	51·2	51·1	S. 8	14·9	14·9	14·8	14·8	14·9	15·0
2·9	β Eridani		283	33·0	32·8	32·6	32·4	32·3	32·2	S. 5	08·4	08·3	08·3	08·3	08·3	08·4
2·9	ι Aurigæ †		286	25·8	25·5	25·3	25·0	24·9	24·7	N. 33	06·0	06·0	06·0	06·0	06·0	06·1
1·1	*Aldebaran*	10	291	37·0	36·8	36·6	36·4	36·2	36·1	N. 16	25·5	25·6	25·6	25·6	25·6	25·6
3·2	γ Eridani		300	58·7	58·5	58·3	58·1	58·0	57·9	S. 13	37·5	37·4	37·4	37·4	37·5	37·6
3·0	ε Persei		301	14·1	13·8	13·5	13·3	13·1	13·1	N. 39	53·3	53·3	53·4	53·5	53·5	53·6
2·9	ζ Persei		302	07·2	07·0	06·7	06·5	06·4	06·3	N. 31	45·5	45·6	45·6	45·7	45·7	45·8
3·0	*Alcyone*		303	44·8	44·6	44·3	44·1	44·0	44·0	N. 23	58·6	58·6	58·7	58·7	58·8	58·8
1·9	*Mirfak*	9	309	39·7	39·4	39·1	38·8	38·7	38·6	N. 49	42·7	42·7	42·8	42·9	43·0	43·1
Var.§	*Algol* †		313	38·0	37·7	37·5	37·3	37·2	37·1	N. 40	47·6	47·7	47·8	47·8	47·9	48·0
2·8	*Menkar*	8	314	58·4	58·1	57·9	57·8	57·7	57·7	N. 3	55·7	55·7	55·8	55·8	55·8	55·7
3·1	*Acamar*	7	315	49·8	49·6	49·3	49·2	49·1	49·1	S. 40	28·0	28·0	28·0	28·1	28·2	28·3
3·1	β Trianguli		328	13·8	13·6	13·3	13·2	13·2	13·2	N. 34	47·4	47·5	47·5	47·6	47·7	47·8
2·2	*Hamal*	6	328	47·4	47·2	47·0	46·9	46·8	46·8	N. 23	15·9	16·0	16·1	16·2	16·2	16·2
2·2	*Almak*		329	39·6	39·4	39·1	39·0	38·9	39·0	N. 42	07·7	07·8	07·9	08·0	08·1	08·2
3·0	α Hydri		330	38·3	37·9	37·6	37·5	37·5	37·7	S. 61	46·0	46·0	46·0	46·2	46·3	46·4
2·7	*Sheratan*		331	54·7	54·5	54·3	54·2	54·1	54·2	N. 20	36·3	36·4	36·5	36·5	36·6	36·6
2·1	*Polaris* †		330	73·3	63·1	54·9	50·1	50·0	54·9	N. 89	03·9	04·0	04·1	04·3	04·5	04·6
0·6	*Achernar*	5	335	57·5	57·2	57·0	56·8	56·9	57·0	S. 57	26·5	26·5	26·6	26·7	26·8	26·9
2·8	*Ruchbah*		339	13·3	13·0	12·7	12·6	12·6	12·7	N. 60	01·0	01·1	01·2	01·4	01·5	01·6
2·4	*Mirach*		343	08·7	08·5	08·3	08·2	08·2	08·3	N. 35	23·9	24·1	24·2	24·3	24·4	24·4
Var. ‖	γ Cassiopeiæ		346	26·7	26·4	26·1	26·0	26·1	26·2	N. 60	29·3	29·5	29·6	29·8	29·9	30·0
2·2	*Diphda*	4	349	37·2	37·0	36·9	36·8	36·9	36·9	S. 18	12·7	12·6	12·6	12·7	12·7	12·8
2·5	*Schedar*	3	350	27·4	27·1	26·9	26·9	26·9	27·1	N. 56	18·4	18·5	18·7	18·8	19·0	19·0
2·4	*Ankaa*	2	353	56·3	56·1	55·9	55·9	56·0	56·1	S. 42	31·6	31·6	31·6	31·7	31·8	31·9
2·9	β Hydri †		354	06·0	05·3	04·9	04·9	05·3	05·9	S. 77	28·9	28·9	29·1	29·2	29·3	29·4
2·9	*Algenib*		357	13·3	13·1	13·0	12·9	13·0	13·0	N. 14	57·2	57·3	57·4	57·5	57·5	57·5
2·4	*Caph*		358	15·2	14·9	14·7	14·7	14·8	15·0	N. 58	55·1	55·2	55·4	55·6	55·7	55·7
2·2	*Alpheratz*	1	358	26·1	25·9	25·8	25·7	25·8	25·9	N. 28	51·7	51·8	51·9	52·0	52·0	52·0

* Formerly Argus ‡ 0·1—1·2 § 2·3—3·5 ‖ Irregular variable; 1955 mag. 2·8
† Not suitable for use with H.O. 214 (H.D. 486)

POLARIS (POLE STAR) TABLES

FOR DETERMINING LATITUDE FROM SEXTANT ALTITUDE AND FOR AZIMUTH

L.H.A. ARIES	0°– 9°	10°– 19°	20°– 29°	30°– 39°	40°– 49°	50°– 59°	60°– 69°	70°– 79°	80°– 89°	90°– 99°	100°– 109°	110°– 119°
	a_0	a_0	a_0	a_0	a_0	a_0	a_0	a_0	a_0	a_0	a_0	a_0
°	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′
0	0 10·1	0 06·1	0 03·8	0 03·1	0 04·2	0 06·9	0 11·3	0 17·1	0 24·2	0 32·4	0 41·3	0 50·8
1	09·7	05·8	03·6	03·1	04·4	07·3	11·8	17·8	25·0	33·2	42·2	51·7
2	09·2	05·5	03·5	03·2	04·6	07·7	12·3	18·4	25·8	34·1	43·2	52·7
3	08·8	05·3	03·4	03·2	04·8	08·1	12·9	19·1	26·6	35·0	44·1	53·7
4	08·3	05·0	03·3	03·3	05·1	08·5	13·4	19·8	27·4	35·9	45·0	54·6
5	0 07·9	0 04·7	0 03·2	0 03·4	0 05·3	0 08·9	0 14·0	0 20·5	0 28·2	0 36·7	0 46·0	0 55·6
6	07·5	04·5	03·2	03·5	05·6	09·4	14·6	21·2	29·0	37·6	46·9	56·6
7	07·2	04·3	03·1	03·7	05·9	09·8	15·2	22·0	29·8	38·5	47·9	57·6
8	06·8	04·1	03·1	03·8	06·2	10·3	15·8	22·7	30·7	39·5	48·9	58·5
9	06·5	03·9	03·1	04·0	06·6	10·8	16·5	23·5	31·5	40·4	49·8	0 59·5
10	0 06·1	0 03·8	0 03·1	0 04·2	0 06·9	0 11·3	0 17·1	0 24·2	0 32·4	0 41·3	0 50·8	1 00·5
Lat.	a_1	a_1	a_1	a_1	a_1	a_1	a_1	a_1	a_1	a_1	a_1	a_1
°	′	′	′	′	′	′	′	′	′	′	′	′
0	0·5	0·6	0·6	0·6	0·6	0·5	0·4	0·3	0·2	0·2	0·1	0·1
10	·5	·6	·6	·6	·6	·5	·4	·4	·3	·2	·2	·1
20	·5	·6	·6	·6	·6	·5	·5	·4	·3	·3	·3	·2
30	·6	·6	·6	·6	·6	·5	·5	·5	·4	·4	·3	·3
40	0·6	0·6	0·6	0·6	0·6	0·6	0·5	0·5	0·5	0·5	0·5	0·4
45	·6	·6	·6	·6	·6	·6	·6	·6	·5	·5	·5	·5
50	·6	·6	·6	·6	·6	·6	·6	·6	·6	·6	·6	·6
55	·6	·6	·6	·6	·6	·6	·6	·7	·7	·7	·7	·7
60	·6	·6	·6	·6	·6	·6	·7	·7	·8	·8	·8	·8
62	0·7	0·6	0·6	0·6	0·6	0·7	0·7	0·8	0·8	0·9	0·9	0·9
64	·7	·6	·6	·6	·6	·7	·7	·8	·9	0·9	1·0	1·0
66	·7	·6	·6	·6	·6	·7	·8	·9	0·9	1·0	1·0	1·1
68	0·7	0·6	0·6	0·6	0·7	0·7	0·8	0·9	1·0	1·1	1·1	1·2
Month	a_2	a_2	a_2	a_2	a_2	a_2	a_2	a_2	a_2	a_2	a_2	a_2
	′	′	′	′	′	′	′	′	′	′	′	′
Jan.	0·7	0·7	0·7	0·7	0·7	0·7	0·7	0·7	0·7	0·7	0·7	0·7
Feb.	·6	·7	·7	·7	·8	·8	·8	·8	·8	·8	·8	·8
Mar.	·5	·5	·6	·6	·7	·8	·8	·8	·9	·9	·9	0·9
Apr.	0·3	0·4	0·4	0·5	0·6	0·6	0·7	0·8	0·8	0·9	0·9	1·0
May	·2	·3	·3	·4	·4	·5	·6	·6	·7	·8	·8	0·9
June	·2	·2	·2	·3	·3	·4	·4	·5	·5	·6	·7	·8
July	0·2	0·2	0·2	0·2	0·2	0·3	0·3	0·4	0·4	0·5	0·5	0·6
Aug.	·4	·3	·3	·3	·2	·2	·3	·3	·3	·4	·4	·4
Sept.	·5	·5	·4	·4	·3	·3	·3	·3	·3	·3	·3	·3
Oct.	0·7	0·6	0·6	0·5	0·5	0·4	0·4	0·3	0·3	0·3	0·3	0·3
Nov.	0·9	·8	·8	·7	·7	·6	·5	·5	·4	·4	·3	·3
Dec.	1·0	0·9	0·9	0·9	0·8	0·8	0·7	0·6	0·6	0·5	0·4	0·4
Lat.	AZIMUTH											
°	°	°	°	°	°	°	°	°	°	°	°	°
0	0·4	0·2	0·1	359·9	359·7	359·6	359·5	359·3	359·2	359·2	359·1	359·1
20	0·4	0·2	0·1	359·9	359·7	359·6	359·4	359·3	359·2	359·1	359·1	359·0
40	0·5	0·3	0·1	359·9	359·7	359·5	359·3	359·1	359·0	358·9	358·8	358·8
50	0·6	0·4	0·1	359·8	359·6	359·4	359·1	359·0	358·8	358·7	358·6	358·6
55	0·7	0·4	0·1	359·8	359·6	359·3	359·0	358·8	358·6	358·5	358·4	358·4
60	0·8	0·5	0·1	359·8	359·5	359·2	358·9	358·6	358·4	358·3	358·2	358·1
65	0·9	0·6	0·2	359·8	359·4	359·1	358·7	358·4	358·1	358·0	357·9	357·8

Latitude = corrected sextant altitude $- 1° + a_0 + a_1 + a_2$

The table is entered with L.H.A. Aries to determine the column to be used; each column refers to a range of 10°. a_0 is taken, with mental interpolation, from the upper table with the units of L.H.A. Aries in degrees as argument; a_1, a_2 are taken, without interpolation, from the second and third tables with arguments latitude and month respectively. a_0, a_1, a_2 are always positive. The final table gives the azimuth of *Polaris*.

POLARIS (POLE STAR) TABLES

FOR DETERMINING LATITUDE FROM SEXTANT ALTITUDE AND FOR AZIMUTH

L.H.A. ARIES	120°– 129°	130°– 139°	140°– 149°	150°– 159°	160°– 169°	170°– 179°	180°– 189°	190°– 199°	200°– 209°	210°– 219°	220°– 229°	230°– 239°
	a_0	a_0	a_0	a_0	a_0	a_0	a_0	a_0	a_0	a_0	a_0	a_0
°	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′
0	1 00·5	1 10·1	1 19·4	1 28·0	1 35·8	1 42·4	1 47·7	1 51·6	1 53·9	1 54·5	1 53·5	1 50·8
1	01·5	11·1	20·3	28·9	36·5	43·0	48·2	51·9	54·0	54·5	53·3	50·5
2	02·4	12·0	21·2	29·7	37·2	43·6	48·6	52·2	54·1	54·4	53·1	50·1
3	03·4	13·0	22·1	30·5	37·9	44·1	49·0	52·4	54·2	54·4	52·9	49·7
4	04·4	13·9	22·9	31·3	38·6	44·7	49·4	52·7	54·3	54·3	52·6	49·3
5	1 05·3	1 14·8	1 23·8	1 32·0	1 39·2	1 45·2	1 49·8	1 52·9	1 54·4	1 54·2	1 52·4	1 48·9
6	06·3	15·7	24·7	32·8	39·9	45·8	50·2	53·1	54·4	54·1	52·1	48·5
7	07·3	16·7	25·5	33·6	40·5	46·3	50·6	53·3	54·5	54·0	51·8	48·0
8	08·2	17·6	26·4	34·3	41·2	46·8	50·9	53·5	54·5	53·8	51·5	47·6
9	09·2	18·5	27·2	35·1	41·8	47·3	51·3	53·7	54·5	53·7	51·2	47·1
10	1 10·1	1 19·4	1 28·0	1 35·8	1 42·4	1 47·7	1 51·6	1 53·9	1 54·5	1 53·5	1 50·8	1 46·6
Lat.	a_1	a_1	a_1	a_1	a_1	a_1	a_1	a_1	a_1	a_1	a_1	a_1
°	′	′	′	′	′	′	′	′	′	′	′	′
0	0·1	0·1	0·2	0·3	0·3	0·4	0·5	0·6	0·6	0·6	0·6	0·5
10	·1	·2	·2	·3	·4	·5	·5	·6	·6	·6	·6	·5
20	·2	·3	·3	·4	·4	·5	·5	·6	·6	·6	·6	·5
30	·3	·3	·4	·4	·5	·5	·6	·6	·6	·6	·6	·5
40	0·4	0·5	0·5	0·5	0·5	0·5	0·6	0·6	0·6	0·6	0·6	0·6
45	·5	·5	·5	·5	·6	·6	·6	·6	·6	·6	·6	·6
50	·6	·6	·6	·6	·6	·6	·6	·6	·6	·6	·6	·6
55	·7	·7	·7	·7	·7	·6	·6	·6	·6	·6	·6	·6
60	·8	·8	·8	·8	·7	·7	·6	·6	·6	·6	·6	·6
62	0·9	0·9	0·8	0·8	0·8	0·7	0·7	0·6	0·6	0·6	0·6	0·7
64	1·0	1·0	0·9	·9	·8	·7	·7	·6	·6	·6	·6	·7
66	1·1	1·0	1·0	0·9	·8	·8	·7	·6	·6	·6	·6	·7
68	1·2	1·1	1·1	1·0	0·9	0·8	0·7	0·6	0·6	0·6	0·7	0·7
Month	a_2	a_2	a_2	a_2	a_2	a_2	a_2	a_2	a_2	a_2	a_2	a_2
	′	′	′	′	′	′	′	′	′	′	′	′
Jan.	0·6	0·6	0·6	0·6	0·6	0·5	0·5	0·5	0·5	0·5	0·5	0·5
Feb.	·8	·8	·7	·7	·7	·6	·6	·5	·5	·5	·4	·4
Mar.	0·9	0·9	0·9	0·8	0·8	·8	·7	·7	·6	·6	·5	·4
Apr.	1·0	1·0	1·0	1·0	1·0	0·9	0·9	0·8	0·8	0·7	0·6	0·6
May	0·9	1·0	1·0	1·0	1·0	1·0	1·0	0·9	0·9	·8	·8	·7
June	·8	0·9	0·9	1·0	1·0	1·0	1·0	1·0	1·0	0·9	0·9	·8
July	0·7	0·7	0·8	0·9	0·9	0·9	1·0	1·0	1·0	1·0	1·0	0·9
Aug.	·5	·6	·6	·7	·7	·8	0·8	0·9	0·9	0·9	1·0	1·0
Sept.	·4	·4	·5	·5	·6	·6	·7	·7	·8	·8	0·9	0·9
Oct.	0·3	0·3	0·3	0·4	0·4	0·4	0·5	0·6	0·6	0·7	0·7	0·8
Nov.	·2	·2	·2	·2	·3	·3	·3	·4	·4	·5	·5	·6
Dec.	0·3	0·3	0·2	0·2	0·2	0·2	0·2	0·3	0·3	0·3	0·4	0·4
Lat.	AZIMUTH											
°	°	°	°	°	°	°	°	°	°	°	°	°
0	359·1	359·1	359·2	359·3	359·4	359·5	359·6	359·8	359·9	0·1	0·3	0·4
20	359·0	359·1	359·1	359·2	359·3	359·5	359·6	359·8	359·9	0·1	0·3	0·4
40	358·8	358·8	358·9	359·0	359·2	359·3	359·5	359·7	359·9	0·1	0·3	0·5
50	358·6	358·6	358·7	358·8	359·0	359·2	359·4	359·7	359·9	0·2	0·4	0·6
55	358·4	358·5	358·6	358·7	358·9	359·1	359·4	359·6	359·9	0·2	0·4	0·7
60	358·2	358·2	358·3	358·5	358·7	359·0	359·3	359·6	359·9	0·2	0·5	0·8
65	357·8	357·9	358·1	358·3	358·5	358·8	359·1	359·5	359·9	0·2	0·6	0·9

ILLUSTRATION

January 10 at G.M.T. $22^h\ 17^m\ 50^s$ in longitude W. 27° 34′ the corrected sextant altitude of *Polaris* was 49° 31′·6.

From the daily pages	° ′
G.H.A. Aries (22^h)	79 54·9
Increment ($17^m\ 50^s$)	4 28·2
Longitude (west)	−27 34
L.H.A. Aries	56 49

Corr. Sext. Alt.	49°31′·6
a_0 (argument 56° 49′)	0 09·7
a_1 (lat. 50° approx.)	0·6
a_2 (January)	0·7
Sum − 1° = Lat. =	48 42·6

POLARIS (POLE STAR) TABLES

FOR DETERMINING LATITUDE FROM SEXTANT ALTITUDE AND FOR AZIMUTH

L.H.A. ARIES	240°–249°	250°–259°	260°–269°	270°–279°	280°–289°	290°–299°	300°–309°	310°–319°	320°–329°	330°–339°	340°–349°	350°–359°
	a_0	a_0	a_0	a_0	a_0	a_0	a_0	a_0	a_0	a_0	a_0	a_0
°	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′
0	1 46·6	1 41·0	1 34·0	1 26·1	1 17·3	1 07·9	0 58·2	0 48·5	0 39·1	0 30·4	0 22·4	0 15·6
1	46·1	40·3	33·3	25·2	16·3	06·9	57·2	47·6	38·2	29·5	21·7	15·0
2	45·6	39·7	32·5	24·4	15·4	06·0	56·3	46·6	37·3	28·7	21·0	14·4
3	45·1	39·0	31·7	23·5	14·5	05·0	55·3	45·7	36·4	27·9	20·2	13·8
4	44·5	38·3	31·0	22·6	13·6	04·0	54·3	44·7	35·5	27·1	19·5	13·2
5	1 43·9	1 37·6	1 30·2	1 21·7	1 12·6	1 03·1	0 53·3	0 43·8	0 34·7	0 26·3	0 18·9	0 12·7
6	43·4	36·9	29·4	20·9	11·7	02·1	52·4	42·8	33·8	25 5	18·2	12·1
7	42·8	36·2	28·6	20·0	10·7	01·1	51·4	41·9	32·9	24·7	17·5	11·6
8	42·2	35·5	27·7	19·1	09·8	1 00·1	50·4	41·0	32·1	23·9	16·9	11·1
9	41·6	34·8	26·9	18·2	08·8	0 59·2	49·5	40·1	31·2	23·2	16·2	10·6
10	1 41·0	1 34·0	1 26·1	1 17·3	1 07·9	0 58·2	0 48·5	0 39·1	0 30·4	0 22·4	0 15·6	0 10·1
Lat.	a_1	a_1	a_1	a_1	a_1	a_1	a_1	a_1	a_1	a_1	a_1	a_1
°	′	′	′	′	′	′	′	′	′	′	′	′
0	0·4	0·3	0·2	0·2	0·1	0·1	0·1	0·1	0·2	0·3	0·3	0·4
10	·4	·4	·3	·2	·2	·1	·1	·2	·2	·3	·4	·5
20	·5	·4	·3	·3	·3	·2	·2	·3	·3	·4	·4	·5
30	·5	·5	·4	·4	·3	·3	·3	·3	·4	·4	·5	·5
40	0·5	0·5	0·5	0·5	0·5	0·4	0·4	0·5	0·5	0·5	0·5	0·5
45	·6	·6	·5	·5	·5	·5	·5	·5	·5	·5	·6	·6
50	·6	·6	·6	·6	·6	·6	·6	·6	·6	·6	·6	·6
55	·6	·7	·7	·7	·7	·7	·7	·7	·7	·7	·7	·6
60	·7	·7	·8	·8	·8	·8	·8	·8	·8	·8	·7	·7
62	0·7	0·8	0·8	0·9	0·9	0·9	0·9	0·9	0·8	0·8	0·8	0·7
64	·7	·8	·9	0·9	1·0	1·0	1·0	1·0	0·9	·9	·8	·7
66	·8	·9	0·9	1·0	1·0	1·1	1·1	1·0	1·0	0·9	·8	·8
68	0·8	0·9	1·0	1·1	1·1	1·2	1·2	1·1	1·1	1·0	0·9	0·8
Month	a_2	a_2	a_2	a_2	a_2	a_2	a_2	a_2	a_2	a_2	a_2	a_2
	′	′	′	′	′	′	′	′	′	′	′	′
Jan.	0·5	0·5	0·5	0·5	0·5	0·5	0·6	0·6	0·6	0·6	0·6	0·7
Feb.	·4	·4	·4	·4	·4	·4	·4	·4	·5	·5	·5	·6
Mar.	·4	·4	·3	·3	·3	·3	·3	·3	·3	·4	·4	·4
Apr.	0·5	0·4	0·4	0·3	0·3	0·2	0·2	0·2	0·2	0·2	0·2	0·3
May	·6	·6	·5	·4	·4	·3	·3	·2	·2	·2	·2	·2
June	·8	·7	·7	·6	·5	·4	·4	·3	·3	·2	·2	·2
July	0·9	0·8	0·8	0·7	0·7	0·6	0·5	0·5	0·4	0·3	0·3	0·3
Aug.	·9	·9	·9	·8	·8	·8	·7	·6	·6	·5	·5	·4
Sept.	·9	·9	·9	·9	·9	·9	·8	·8	·7	·7	·6	·6
Oct.	0·8	0·9	0·9	0·9	0·9	0·9	0·9	0·9	0·9	0·8	0·8	0·8
Nov.	·7	·7	·8	·8	·9	·9	1·0	1·0	1·0	1·0	0·9	0·9
Dec.	0·5	0·6	0·6	0·7	0·8	0·8	0·9	0·9	1·0	1·0	1·0	1·0
Lat.	AZIMUTH											
°	°	°	°	°	°	°	°	°	°	°	°	°
0	0·5	0·7	0·8	0·8	0·9	0·9	0·9	0·9	0·8	0·7	0·6	0·5
20	0·6	0·7	0·8	0·9	0·9	1·0	1·0	0·9	0·9	0·8	0·7	0·5
40	0·7	0·9	1·0	1·1	1·2	1·2	1·2	1·2	1·1	1·0	0·8	0·7
50	0·8	1·0	1·2	1·3	1·4	1·4	1·4	1·4	1·3	1·2	1·0	0·8
55	0·9	1·1	1·3	1·5	1·6	1·6	1·6	1·6	1·5	1·3	1·1	0·9
60	1·1	1·3	1·5	1·7	1·8	1·8	1·8	1·8	1·7	1·5	1·3	1·0
65	1·3	1·5	1·8	1·9	2·1	2·2	2·2	2·1	2·0	1·8	1·6	1·3

Latitude = corrected sextant altitude $-1° + a_0 + a_1 + a_2$

The table is entered with L.H.A. Aries to determine the column to be used; each column refers to a range of 10°. a_0 is taken, with mental interpolation, from the upper table with the units of L.H.A. Aries in degrees as argument; a_1, a_2 are taken, without interpolation, from the second and third tables with arguments latitude and month respectively. a_0, a_1, a_2 are always positive. The final table gives the azimuth of *Polaris*.

Answers

to

Exercises

ANSWERS TO EXERCISES:

Exercises to Chapter 1

1. (i) 0·488; (ii) 0·469; (iii) 1·494.
2. (i) 55°; (ii) 44°; (iii) 37°.
3. $\theta = 24°$.
   $\cos\theta = 0·9$; $\sec\theta = 1·1$; $\tan\theta = 0·4$;
   $\cot\theta = 2·2$; $\operatorname{cosec}\theta = 2·4$.

10. 101½°.

15. 55° 09′.

Exercises to Chapter 2

1. (i) 2·393; (ii) 1·045; (iii) 0·011.
2. (i) 2·133; (ii) 0·518; (iii) 3·69.
3. (i) 46·97 cm.; (ii) 273·1 yds.

6. 3,438.
7. 6·72 ml.
8. 67½°; $3\pi/8$ or $1·18^c$.
9. 6,876 ft.

11. (i) sin 5° = 0·872620; (ii) cosec 2° = 28·731669; (iii) tan 30′ = 0°0087262; (iv) cot 89° = 0·0174524.
12. (i) 0·01745400; (ii) 85·940185.

Exercises to Chapter 3

2. (i) 0·766; (ii) 2·28; (iii) 0·726.
3. 29° (D. Lat.: Dep.: :180 : 100).
4. 25°
5. 8°·9.
6. 18° 421 lire; 161·9 Fr.
7. 13′·65.
8. 94·87 sq ml.
9. 25 sq. cms.
10. 035°.
11. Corr. = 48′·41; Lat. = 41° 21′·6.
12. 48′·5.
13. (i) 5° E.; (ii) 8°·7 W.
14. (i) 5° E.; (ii) 3°·4 W.
15. 6°·4.
16. 3°·25.
17. 5·8 ml.
18. 86·6 cms.

Exercises to Chapter 3 (*continued*)

19. 4·195 metres.
20. 070° 32′.
21. 4·216 ml.
22. 21° 48′ N. or S.
23. 43·01 sq. cms.
24. 26° 27′.

Exercises to Chapter 4

5. cos 15° = 0·966; sin 75° = 0·966.
6. tan 15° = 0·268; tan 75° = 3·732.

Exercises to Chapter 5

1. 2·873 ml.
2. 4·607 ml.
3. A = 58° 24¾′; B = 96° 22¾′; C = 25° 13½′.
4. 10·57 ml.
5. 2·21 ml.
6. 007° 43′.
7. 292½°; 3 hr. 11 m.
8. 16·44 ml.
9. 20·1 km.
10. AD = 165·4 m.; BD = 123·2 m.; CD = 234·5 m.
11. AC = 800 yds.
12. B = 27° 04′ or 78° 16′.

Exercises to Chapter 8

1. 51° 19′ or 128° 41′.
2. 54° 01¾′.
3. 168° 41½′.
4. 98° 30½′.
5. 32° 51′.
6. 49° 36′.
7. 154° 16¾′.
8. Z = 112° 28′; ZX = 57° 07¾′.

Exercises to Chapter 9

1. a = 32° 18¼′; b = 56° 14½′; c = 48° 53½′.
2. e = 27° 57′; f = 77° 00¼′; G = 82° 58′.
3. X = 42° 04′; y = 108° 51′; z = 104° 13¼′.
4. Q = 72° 31¾′; p = 116° 55′; q = 70° 33½′.
5. Y = 110° 29½′; x = 144° 52′; z = 63° 33′.
6. A = 60° 47½′; a = 74° 23′; b = 146° 31′.
7. X = 52° 23′; Y = 65° 19′; Z = 104° 46′.

Exercises to Chapter 9 (*continued*)

8. Z = 61° 29½′ or 118° 30½′.
9. { C = 58° 41½′ or 121° 18½′; B = 123° 26′ or 22° 27′.
10. 49° 40′.

Exercises to Chapter 10

12. Lat. 20° N., Long. 165° E.
13. Lat. 46° S., Long. 00°.
14. Lat. 00° 00′, Long. 15° 18′ W.
15. (*a*) 04° 09′; 24° 01′.
    (*b*) 11° 26′; 176° 34′.
19. 59° 50′ N.
20. 1850 metres.

Exercises to Chapter 11

9. 263′·0 E.
10. 335½ ml.
11. 58° 37′.
12. 17,693·5 ml.
13. 450 kn.
14. 090° × 0·88 kn.
15. 40° 30′ N.; 16° 33½′ W.
16. 178° 04′ W.
17. 090° × 11·1 ml.
18. 25° 17′ N.; 583 ml.
19. Dep. = 378·4 ml.; Dist. = 462 ml.
20. Dep. = 91·2 ml.; Change in Lat. = 04° 58′·4 S.
21. Dist. = 354 ml.; Change in Lat. = 04° 39′ S.
22. Dist. = 100 ml.; Course 144°.
23. Course = 212°; Dist. = 324 ml.; Lat = 46° 39′ N.
24. Course = 176°; Dist. = 270 ml.; Lat. = 19° 19′·6 S.
25. Course = 328½° Dist. = 625 ml.; Lat. = 25°29′ S.
26. 44·59 ml.
27. Dist. = 3,686·2 ml.

Exercises to Chapter 12

9. 33·6.
10. 2·388 cms. to 1°.
11. 3·40 cms.
12. 1·943 cms. to 1′ of Long.
13. 2·87 cms. to 1′ of Lat.

Exercises to Chapter 13

5. 337·3.
6. Co. = 244° 32½′; Dist. = 667·1 ml.
7. Lat = 46° 50′ N.; Long. = 23° 44′·3 W.
8. Co. = 248° 20′; Dist. = 443·6 ml.
9. Co. = 086° 13½′; Dist. = 1,820·1 ml.
10. Dist. = 973·1 ml.; Long. = 79° 42′·8 W.
11. Lat. = 04° 09′ S.; Dist. = 1,711·4 ml.
12. Co. = 027° 27′; Dist. = 928·4 ml.
13. Bearing = 158°; Dist. = 18·3 ml.
14. Lat. = 38° 16′ N.; Long. = 175° 20′ W.
15. Lat. = 51° 10′·1 N.; Long. = 08° 32′·5 W.
16. Set = 147° Drift = 14·9 ml.
17. Lat. = 32° 21′·6 S.; Long. = 51°36′·0 E.
18. Set = 166°; Rate = 4·6 ml. in 24 hr.

Exercises to Chapter 14

7. 7·694 in.
12. Lat. = 36° 25′ S.; Long. = 53° 42′ W.
14. Within.
18. Lat. = 29° 00′ N.; Long. 50° 00′ E. or 130° 00′ W.
19. Lat. = 30° 00′ S.; Long. = 80° 00′ E.
20. Long. = 70° 00′ E. or 110° 00′ W.
21. Co. = 270° 58′; Dist. = 2,769¾ ml.
Points: (i) 49° 24½′ N., 16° 31′ W.
(ii) 48° 04½′ N., 26° 31′ W.
(iii) 45° 44½′ N., 36° 31′ W.
(iv) 42° 10′ N., 46° 31′ W.
(v) 37° 18′ N., 56° 31′ W.
22. Dist. = 1,677·5 ml.; Co. = 224° 50′.
23. Co. = 270° 10′; Dist. = 3,396 ml.
Points: (i) 35° 58½′ N., 10° 00′ W.
(ii) 35° 13′ N., 20° 00′ W.
(iii) 33° 38′ N., 30° 00′ W.
(iv) 31° 09′ N., 40° 00′ W.
(v) 27° 42½′ N., 50° 00′ W.
(vi) 23° 16′ N., 60° 00′ W.
24. Dist. = 6,188 ml.; Co. = 092° 39′.
25. Dist. = 5,958 ml.; Co. = 166° 06′.
26. Co. = 228° 48′; Dist = 4,264½ ml.
27. Co. = 285° 35′; Dist. = 1,651½ ml.
Points: (i) 52° 36′ N., 20° 00′ W.
(ii) 53° 00′ N., 30° 00′ W.
(iii) 53° 00′ N., 40° 00′ W.
(iv) 52° 53′ N., 50° 00′ W.
28. 1,052 ml.

Exercises to Chapter 15

5. (*a*) 1 : 729600; (*b*) 1 : 1216.
6. 3·704 cm.

Exercises to Chapter 16

9. (i) Error = 3° W.; True Co. = 315°.
(ii) Dev. = 0°; Error = 5° E.
(iii) Dev = 24° W.; True Co. = 171°.
(iv) Comp. Co. = 088°; Dev. = 2° W.
(v) Var. = 14° E.; Comp. Co. = 247°.
(vi) Error = 9° W.; True Co. = 351°.
(vii) Var. = 1° W.; True Co. = 266°.
(viii) Var. = 14° E.; True Co. = 124°.
10. 4° E.
11. Nil.
12. 055°.
13. 313°.
14. (*a*) 117°.
(*b*) 225°.
(*c*) 193¾°.
(*d*) 290°.
(*e*) 283°.
15. 330°; 255°; 183°.

Exercises to Chapter 18

6. 13¾ ml.
7. 9½ ml.; 18° 27′ N.; 63° 25½′ W.
8. 16 ml.; 22 ml.
9. 2·5 ml.
10. 14·63 ml.
11. 13·99.
12. 9·01 ml.
13. 3·5 ml.
14. 9·01 ml.
15. 13·05 ml.

Exercises to Chapter 19

6. 0·50 ml.
8. 23·97 ml.
10. 1° 04′·24.

Exercises to Chapter 20

8. 5·0 ml.; Error = 6° W.
9. 3·1 ml.

Exercises to Chapter 20 (*continued*)

10. 220° × 2·3 ml.
11. 1·6 ml. or 4·7 ml.
12. 6·3 ml.

Exercises to Chapter 21

2. 070°
3. 332°
4. 1·1 ml.
5. Dist. = 6·2 ml.; Set = 060°; Rate = 1·1 kn.
6. Co. = 350°; Drift = 4·5 ml.

Exercises to Chapter 22

3. 184°.
4. 122°; A/C 23° to starboard.
5. 158°; 13¼ kn.
6. W. Magnetic.
7. Co. = 255°; Current: 288° × 13 ml.
8. Co. = 159°; Time = 1 hr. 45 m.; Dist. = 17·4 ml.
9. 280°.

Exercises to Chapter 23

6. 090¾°.

Exercises to Chapter 24

8. 7 : 3 approx.
13. 2·7 m. above cd.
14. 22·2 m.
15. 1·3 m. U.K.C.
16. 1005 hours.
17. 0604 hours.

Exercises to Chapter 25

5. $(2\frac{1}{2})^5$.
6. $(2\frac{1}{2})^{27}$.

Exercises to Chapter 27

5. 11° 31½′ N.; 54° 21½′ E.
6. (*a*) 00° 00′; 90° 00′ E.
   (*b*) 23½° N.; 270°.
7. 46° 16′ N.; 323° 43′·8.

Exercises to Chapter 31

24. 34° 57′·1.
25. 36° 39′·8.
26. 43° 37′·1.
27. 49° 57′·1.
28. 52° 35′·2.
29. 32° 28·9.
30. 24° 54′·3.
31. 54° 29′·5.
32. 36° 49′·3.
33. 56° 46′·2.
34. 34° 22′·9.
35. 42° 54′·6.
36. 36° 19′·6.
37. 47° 40′·6.
38. 62° 16′·4.
39. 79° 26′·2.
40. 64° 09′·2.
41. 41° 21′·4.
42. 48° 14′·6.

Exercises to Chapter 32

3. (i) 00° 20′·3 S.; 28° 02′·8 E.
   (ii) 00° 23′·5 N.; 16° 47′·4 W.
   (iii) 00° 10′·5 S.; 178° 05′·0 E.
4. (i) 00° 10′·0 N.; 175° 40′·4 W.
   (ii) 23° 13′·9 N.; 139° 00′·5 E.
5. 23° 20′·5 N.; 00° 07′·2 E.
6. 23° 15′·3 N.; 25° 34′·1 E.
7. 56° 11′·3 N.; 39° 26′·8 E.
8. 23° 15′·9 N.; 175° 10′·5 E.
9. 56° 18′·3 N.; 39° 31′·3 W.
10. 15° 58′·6 S.; 165° 04′·7 W.
11. 14° 34′·8 S.; 6° 53′·7 W.
12. 23° 16′·0 N.; 174° 32′·0 E.
13. 23° 20′·0 S.; 45° 20′·5 W.

Exercises to Chapter 33

6. (i) 10 h. 00 m.; (ii) 22 h. 01 m.; (iii) 00 h. 33 m.
7. (i) 13 h. 22 m.; (ii) 10 h. 16 m.; (iii) 22 h. 44 m.
8. (i) 15 h. 58 m.; (ii) 00 h. 51 m.
9. (i) 09 h. 44 m.; (ii) 03 h. 59 m.; (iii) 23 h. 31 m.
10. 63° 06′ N.
11. 48° 34′ S.
12. 090°–270° through Lat. 33° 45′·0 S. Long. 78° 45′ E.
13. 090°–270° through Lat. 44° 51′·8 N. Long. 29° 45′ W.
14. 090°–270° through Lat. 50° 00′·3 N. Long. 160° 45′ W.

Exercises to Chapter 33 (*continued*)

15. 090°–270° through Lat. 32° 34′·0 S. Long. 156° 00′ E.
16. 090°–270° through Lat. 56° 53′·9 N. Long. 53° 15′ W.
17. 090°–270° through Lat. 50° 32′·9 N. Long. 70° 00′ W.
18. 090°–270° through Lat. 29° 39′·1 S. Long. 75° 00′ E.
19. 090°–270° through Lat. 4° 42′·0 N. Long. 152° 00′ E.
20. 090°–270° through Lat. 68° 25′·0 N. Long. 120° 00′ W.
21. 090°–270° through Lat. 00° 12′·2 S. Long. 23° 00′ W.

Exercises to Chapter 34

14. P.L. 000°–180°. Intercept 10·3 ml. *Towards.* Azimuth 090°.
15. P.L. 144½°–324½°. Intercept 8·7 ml. *Towards.* Azimuth 234½°.
16. P.L. 053°–233°. Intercept 4·4 ml. *Away.* Azimuth 324°.
17. P.L. 179°–359°. Intercept 4·6 ml. *Towards.* Azimuth 089°.
18. P.L. 159½°–339½°. Intercept 5·8 ml. *Away.* Azimuth 069½°.
19. P.L. 073½°–253½°. Intercept 6·8 ml. *Towards.* Azimuth 343½°.
20. P.L. 138½°–318½°. Intercept 32·3 ml. *Away.* Azimuth 228½°.
21. P.L. 029°–209°. Intercept 2·5 ml. *Away.* Azimuth 299°.
22. P.L. 061½°–241½°. Intercept 6·3 ml. *Towards.* Azimuth 151½°.
23. P.L. 006½°–186½°. Intercept 7·2 ml. *Away.* Azimuth 096½°.
24. P.L. 042°–222°. Intercept 3·5 ml. *Towards.* Azimuth 132°.
25. P.L. 136½°–316½°. Intercept 23·4 ml. *Away.* Azimuth 226½°.

Exercises to Chapter 35

10. 40° 10′ N., 46° 13′ W.
11. 25° 10′ S., 120° 25¼′ W.
12. 44° 20′ N., 155° 26′ W.
13. 60° 29½′ S., 30° 21′ W.
14. 51° 00′ N., 43° 55′ W.
15. 40° 30′ S., 123° 44½′ W.
16. 39° 52′ N., 00° 58′ W.
17. 36° 25′ N., 09° 18′ W.

Exercises to Chapter 36

2. Alioth (32) will cross North.
   Gacrux (31) will cross South with a small altitude.
   Gienah (29) will cross South.
3. Nunki (50) has crossed North.
4. Canopus (17) will cross South with a low altitude.
   Betelgeuse (16) will cross South.
   Alnilam (15) has just crossed South.
   Elnath (14) has crossed South.
   Bellatrix (13) has crossed South.
   Capella (12) has crossed North.
   Rigel (11) has crossed North.
5. Antares (42) will cross South.
   Atria (43) will cross South.
6. Eltanin (47) will cross North.

Exercises to Chapter 37

4. 45° 26′·1 N.
5. 42° 21′·2 N.
6. P.L. 089½–269½° through Lat. 38° 27′·2 N. Long. 33° 00′ W.
7. P.L. 089–269° through Lat. 27° 49′·5 N. Long. 75° 00′ W.
8. 7°·5 W.
9. 2°·8 W.
10. 2°·0 W.

Exercises to Chapter 40

3. 942′·5 per hour.
12. 2 m. 36 s.
13. 1 m. 26 s.

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